PERTURBATIONS ALONG HEADCUT AND THEIR EFFECTS ON GULLY FORMATION

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Formation process of gully-headcut has been simulated through numerical methods. 2-D depth-averaged flow model with non-orthogonal boundary fitted grid system is employed to describe discharge distribution around gully-headcut. The heterogeneity of unit discharge along headcut promotes gully development process. Simulated result shows initial shape of the gully-headcut is very important for its further development and advancement. Headcut convex shape from up-stream side is a necessary condition to develop gully, while the concave shape is disadvantageous in gully development process. Higher convexity, especially around gully-head, makes the development process faster.

\textit{Key Words:} overland flow, gully, headcut migration, depth-averaged flow simulation, perturbation

1. INTRODUCTION

Gully erosion is a process in which the flow concentrates in a narrow channel and removes soil to bring spatial variety of depth within short period. The erosion process gradually migrates upstream with time. Headcut is a sudden change in bed elevation or a knickpoint at the leading edge (head) of a gully. Gully erosion is usually accompanied with headcuts. A gully-headcut is one of important fluvial processes not only on slopes but also in rivers. For example, Gay et al.\textsuperscript{3} observed that overland flow causes cut-off in meandering channels with gully-headcuts. Figure 1 shows the typical gully erosion process caused by the overland flow. Formation and movement of a gully-headcut is a complex phenomenon, which is controlled by the hydraulic characteristics of flow and by the erosion resistant properties of earth material at the flow boundary.

Several experimental studies on a gully-headcut have been done by many researchers\textsuperscript{2,3,4,5,6}. Numerical simulation models on the headcut migration in vertical 2-D framework were also investigated\textsuperscript{7,8}. On the contrary, the number of experimental and analytical studies on the heterogeneous gully-headcut is relatively small. Izumi conducted an analytical study on the channelization process. He employed a linear stability analysis on the development of the small disturbance along laterally uniform headcut. His analysis clarified that when there is small disturbance along headcut, the unit discharge through the brink is also disturbed, which makes the erosion rate change, and then the gully can be developed. He proposed a model including erosion upstream of headcut and provided an explanation for the finite wavelength selection. According to that theory, the dominant wavelength was found to be 1000 times of the critical depth\textsuperscript{9,10}. However, his analysis failed not explain which wave number is dominant for the development of gully in case of no upstream erosion\textsuperscript{11}. In the present paper the effects of lateral disturbances along headcut on gully development process when there is no upstream erosion have been analyzed. Dey et al.\textsuperscript{12} conducted fixed bed flume experiments with oblique headcuts to observe the flow pattern around gully-headcut. They found that most of the flow concentrates to the head (upstream part) of the headcut. This variation of unit discharge along headcut plays the most important role in gully development process.

In this study, the non-linear equations to govern the flow field around headcut have been solved, and then the development process of gully for different hydraulic and geometric conditions has been investigated. 2-D depth-averaged flow model in coupling with headcut migration model is used to
investigate the effect of initial perturbation shapes along headcut on gully development process.

2. MODEL FOR FLOW SIMULATION

Overland flow characteristics around gully-headcut have been simulated through 2-D depth-averaged flow model, as the discharge distribution along headcut line is a key factor for gully development. Along the headcut line, almost critical condition of the flow appears. The variation of the water depth around the headcut controls the discharge distribution. On the other hand, the headcut shape in a plane is not usually so simple. It is difficult to express natural plane geometry of the headcut accurately with rectangular grid system. In the present study, a non-orthogonal boundary fitted grid system is employed to express the variable shapes of the headcut.

(1) Governing equations

The governing equations for a depth-averaged 2-D flow model can be written as follows:

\[
div(\phi \mathbf{V}) = S_\phi
\]

Continuity equation:

\[
\phi = 1 ; \quad S_\phi = 0
\]

x-momentum equation:

\[
S_\phi = -gh \frac{\partial \phi}{\partial x} - C_j u \sqrt{u^2 + v^2} + \frac{\partial}{\partial x} (h \tau_{xx}) + \frac{\partial}{\partial y} (h \tau_{xy})
\]

y-momentum equation:

\[
S_\phi = -gh \frac{\partial \phi}{\partial y} - C_j v \sqrt{u^2 + v^2} + \frac{\partial}{\partial x} (h \tau_{yx}) + \frac{\partial}{\partial y} (h \tau_{yy})
\]

where \( \phi \) = transported physical quantity; \( h \) = water depth; \( \mathbf{V} \) = depth-averaged velocity vector; \( S_\phi \) = source term; \( x, y \) = Cartesian coordinates; \( u, v \) = depth-averaged flow velocity, respectively; \( g \) = gravitational acceleration; \( \xi \) = water surface elevation; \( C_j \) = coefficient for bed-shear stress; and \( \tau \) = turbulent shear stress (in plane), which is assumed to follow the Boussinesq approximation with eddy viscosity. The following 0-equation turbulence model is used to express the eddy viscosity:

\[
\nu_\tau = \alpha u_s h
\]

where \( \alpha \) = empirical constant; and \( u_s \) = shear velocity. \( \alpha = 0.10 \) is used for all cases. In general, turbulence stresses are treated as a diffusion term. However, it is more stable for conducting numerical simulation with the collocate grid to make the diffusion term zero virtually and to take into account the stress term in the source term. The turbulence stress is thereby taken into account in the source term instead of the diffusion term in the present study.

(2) Simulation techniques

The governing equations are discretized with volume integration in each cell with the collocate grid. The linear interpolation with the gradient of the physical quantities at the cell center is basically employed to express the cell face values. However, in order to discretize the convection term the cell face values are basically obtained with the upwind scheme, and then the correction term to reduce the numerical diffusion is taken into account in the source term as shown in Fig. 2 and Eq. (6).

\[
\phi_e = \begin{cases} 
\phi_p + (\text{grad} \phi)_p \cdot \mathbf{e}_p & (\mathbf{n} \cdot \mathbf{V}_e > 0) \\
\phi_E + (\text{grad} \phi)_E \cdot \mathbf{e}_E & (\mathbf{n} \cdot \mathbf{V}_e < 0)
\end{cases}
\]

where \( \mathbf{e} \) = vector from the cell center to the cell face; \( \mathbf{n} \) = unit normal vector to the cell face; and the subscript \( e \) represents the cell face value, the subscripts \( p \) and \( E \) represent the cell center values. The SIMPLE algorithm is employed for the water surface-velocity coupling. The interpolation technique by Rhie and Chow is adopted to prevent the numerical oscillation.
3. MODEL FOR GULLY DEVELOPMENT

The main driving mechanism of headcut migration is the energy at the overfall. Many of previous researches postulate that the headcut migration speed is function of the hydraulic attack parameters and the material-dependent coefficient. The hydraulic attack parameters are the flow discharge and the headcut height, while the material-dependent coefficient reflects the effects of soil properties such as density, strength, composition etc. The heterogeneity of the unit discharge along headcut produces the heterogeneous driving energy for headcut migration, which causes the differences in migration speed and thus promotes the gully advancement. In the present model, it is assumed that the headcut migrates perpendicular to the headcut face, which was also assumed in the linear analysis by Izumi. Following relationship is used to predict the headcut migration speed in gully development process, which was proposed by Temple. He did laboratory experiments to get this model, which was then verified with the field data:

\[ c = Aq_y^mH^n \]  

where \( c \) = headcut migration speed; \( q_y \) = unit discharge at brink perpendicular to headcut face; \( H \) = elevation change in energy grade line through the headcut, which can be represented by the headcut height; \( A \) = material-dependent advance rate coefficient; and \( m, n \) = exponents, the experimentally determined constants which are nearly equal to 1/3 and 1/2, respectively. Finally, the changes in gully shape are simulated by coupling the flow model with Eq.(7).

4. MODEL VERIFICATION

The model to simulate the flow around headcut is verified with the result obtained from a fixed-bed laboratory experiment and is compared with the analytical solution of Izumi. The details of the verifications and the comparisons are described in following sub-headings:

(1) Verification through experimental investigations with oblique headcut

The oblique shape of headcut is chosen to get a heterogeneous distribution of unit discharge as we mentioned in section-2 that the lateral variation of unit discharge is a key factor of gully formation. However, the following articles describe the experimental procedure and its verification with the results of numerical simulation.

a) Set-up and procedure

All experiments were conducted in a laboratory flume of 20m long and 0.50m wide. The glass-made side-walls of the flume provided us an opportunity to observe the flow pattern along the headcut. Flow discharge was controlled by an adjustable inlet valve and monitored through a digital display-meter. Water was initially discharged into an inlet tank, which acted as a reservoir for damping the turbulence in the test section. The flume bed in the test section was made of wood in which a 0.10m vertical-drop was located at the end of it. A movable point-gauge was used to measure the water surface and bed elevation. The velocities along the flow direction at different locations were measured by a miniature propeller current-meter, which was connected with a digital data recorder. To get the accurate and stable value, the average velocity of a 30sec time-span was taken. The bed slope was fixed as 1:1000 for all experiments. Figure 3 shows the schematic diagram of the experimental set-up.
b) Results and discussions

In experiments, three different cases were conducted by changing the discharge and headcut pattern. Table 1 summarizes all of the experimental conditions. The straight oblique headcut is considered in experiments because of simplicity. Within the total test section, the zone of interest was only a length of 4m upstream from the point B’ of Fig.3, because the water depth at upstream of AA’ line could be regarded as the normal depth of the flow. So, the point A was set as the origin of all measurements.

Figure 4 shows change of the water depth (h) along x-direction for different transverse locations. At far upstream of headcut h remains at its uniform value and decreases gradually with a uniform rate along x-distance as it approaches to headcut location. At the vicinity of headcut location h suddenly decreases with a high rate and reaches its minimum value at brink. Further, around the brink h gradually increases with the y-distance for the same x-location. Similar phenomena observed in all cases clearly indicate that the flow concentrates around the point B, the head of the headcut BB’.

Figure 5 shows the variation of h_b (water depth at brink) and q_y (discharge through unit length of headcut) along yy-direction. Both of h_b and q_y are observed higher at the point B and decrease gradually with the distance along headcut. The decreasing rates in h_b and q_y highly depend on magnitude of a_0 (Fig.3), which clearly indicates that a_0 has a significant effect on gully development. (10)

(2) Comparison with previous linear analysis

Izumi (11) conducted an analytical study on channelization process and formation of drainage basin. Firstly he divided the total simulation domain into two zones: (i) around vicinity of headcut (zone-1) and (ii) far upstream from headcut (zone-2). The non-linear flow equations were made linear by omitting the non-linear terms and then the linear equations were applied for both zones separately. The different sets of equations were combined finally by introducing some interface boundary conditions to get the linear solutions.

However, his linear analysis clarified that when there is a small disturbance along headcut, the unit discharge through brink is also disturbed. In his analysis the sinusoidal disturbance along headcut (Fig.6) was defined by the following equation:

\[ x' = a_0 [1 - \cos(ky)] \]

(8)

where \( x' \) = distance along x-direction form gully-head; \( a_0 \) = amplitude of the disturbance; \( k \) = wave number \( (2\pi/L) \); \( L \) = wavelength of the disturbance;
Table 2 Numerical conditions used in flow verification

<table>
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<tr>
<th>case no.</th>
<th>$a_d$ (m)</th>
<th>$L$ (m)</th>
<th>$S$</th>
<th>$q_0$ (m$^3$/s)</th>
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and $y$-distance along $y$-direction. For the given disturbance as Eq.(8), he found the following relation to express the discharge per unit width ($q$) at brink

$$q = q_0 \left( 1 + \frac{3a_d S}{h_m} \frac{F^{2/3}}{C \cos(ky)} \right) \tag{9}$$

where $q_0$=discharge per unit width at far-upstream; $S$=longitudinal bed slope; $h_m$=water depth at far-upstream; $F$=Froude number; and $C$=constant which controls the fluctuation of $q$. The numerical conditions of flow model used in the comparison are tabulated in Table 2.

For a given value of $a_d$, higher wave number indicates the higher disturbances along headcut. Figure 7 shows the variation of $q$ along $y$-direction. Higher $q$ is observed around head of the initial gully and gradually decreases in $y$-direction. Figure 8 shows the values of $C$ against the dimensionless wave number ($kh_m/S$). $C$ is small for the small wave number and gradually increases as the wave number increases. Close agreements among the experimental, analytical and numerical results ensure that the numerical model is good enough to simulate the flow field around gully-headcut. From Figs. 7 and 8, it is found that the higher disturbance along headcut causes the larger fluctuation in $q$, which promotes the gully development process.

5. SIMULATION FOR GULLY DEVELOPMENT

Gully development process has been investigated through numerical simulation. A sinusoidal disturbance is applied along the headcut initially (Fig.6). The change in elevation of energy grade line ($H$) is assumed to be equal to the vertical-drop of bed elevation through headcut. The migration speed of gully-headcut is determined according to Eq.(7), which has been modified as $c = Bq_{yy}^m$ where $B = AH^n$, which is kept constant as 0.001 arbitrarily in all simulations because the main intention of the present paper is to investigate the relative change in plane geometry of the gully shape with time only. The effect of $B$ changes the migration speed while the relative movement of gully shape remains unaffected. The exponents $m=1/3$ is used in all simulations. The conditions for simulation are shown in Table 3. The horizontal distance between gully-head and -tail is referred as gully-length, which is twice of the disturbance amplitude ($a_d$).
Table 3 Numerical conditions used in gully simulation for sinusoidal disturbance

<table>
<thead>
<tr>
<th>Case no.</th>
<th>( a_0 ) (m)</th>
<th>( L ) (m)</th>
<th>( S )</th>
<th>( q_w ) (m/s)</th>
<th>( F )</th>
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<td>G-4</td>
<td></td>
<td>5</td>
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\[
D'' = \frac{d^2}{dy^2} \left( \frac{x'}{a} \right)
\]

![Fig.9] Growth ratio \( a(a_0) \) of gully with time

![Fig.10] Changes in gully shapes

Figures 9 and 10 show the effect of sinusoidal disturbance on gully development process. Parameter \( a_0 \) and \( a \) used in figures represent the initial gully-length and the gully-length at any particular time, respectively. These figures show that the gully develops initially but it fails to continue for long time. Initially when the disturbance is very small, the variation of \( q \) along headcut can be governed by Eq.(9). The sinusoidal shape of disturbance changes with time as gully develops. The variation of \( q \) along headcut, which is a key factor in gully development process, also changes accordingly. To find the reason why the gully development process failed to continue, the details of shape changes have been investigated carefully. In order to investigate the nature of gully shape with time, the second derivative \( (D'') \) is chosen as the parameter, which determines the convexity or concavity of the shape. Initial sinusoidal wave is concave around gully-head and convex around gully-tail. The convexity of the shape gradually disappears as the gully develops. The development process continues as long as the convexity is dominant over the concavity of the shape. Figure 11 shows the variation of \( D'' \) along \( y \)-direction. Initially \( D'' \) has a positive value for \( y \) equal to 0-5m, while \( D'' \) is negative for \( y \) equal to 5-10m. Positive and negative values of \( D'' \) indicate the concavity and convexity of the shape, respectively. The value of \( D'' \) changes with time. The significant changes are observed at higher values of \( y \) (gully-tail). The negative values of \( D'' \) increase with time and remain up to a certain period. Finally \( D'' \) around gully-tail becomes positive. These phenomena clarify that the initial convex shape of gully-tail gradually changes to the concave shape with the gully development process. Figure 12 shows the average \( D'' \) through the channel width against the growth rate \( \{d(a/a_0)/dt\} \). This figure concludes the positive growth rate in gully development is only expectable as long as the average \( D'' \) remains negative or zero.

It is clear from the above simulation that 2-D plane shape of initial gully is very important on its further development. To investigate more about the effect of shape on gully development, simulation for fully concave and fully convex shape have been done in the present study. Two types of exponential
functions are employed as the initial gully shapes to investigate the effect of each shape on its further development (Fig.13). Table 4 shows the different hydraulic and geometric conditions used to investigate the shape effect in gully formation. Case-1 and Case-2 represent the concave and convex shapes, respectively.

The changes in the gully shape strongly depend on the initial shape of gully. Figure 14 indicates the effect of initial shape on gully development process. Gully-length gradually increases with time in Case-2B (convex shape) but decreases in Case-1 (concave shape). The changes in longitudinal distance between gully-head and tail (a) with time are shown in Fig.15. Growth ratio \(\frac{a}{a_0}\) increases with time in Case-2B while decreases in Case-1. The changing
Fig. 15 Changes in growth ratio ($a/a_o$) with time 
($p=10$, $a_o=0.5m$ and $q=0.01$ m$^2$/s)

Fig. 16 Effect of $p$ on gully development 
($a_o=0.5m$ and $q=0.01$ m$^2$/s)

Fig. 17 Changes in growth ratio ($a/a_o$) for different $p$ 
($a_o=0.5m$ and $q=0.01$ m$^2$/s)

Fig. 18 Changes in growth ratio ($a/a_o$) for different $a_o$ 
($p=25$ and $q=0.01$ m$^2$/s)

Fig. 19 Changes in growth ratio ($a/a_o$) for discharge 
($p=25$ and $a_o=0.5m$)

rate in growth ratio for Case-2B is almost constant with time, while it is higher initially and then gradually decreases for Case-1. Further, the convexity of the initial shape, which is defined by the parameter $p$ in present study, controls the speed of gully development process. Figure 16 shows the changes in gully shape for different values of $p$. The initial shape with higher value of $p$, which indicates the higher convexity of shape, develops gully faster. The growth ratio ($a/a_o$) of gully development is much higher in Case-2C than that of Case-2A and Case-2B (Fig.17). Almost same value of $a/a_o$ is observed initially for all conditions of Case-2C (Fig.18), which indicates that the initial gully-length ($a_o$) does not have any significant effect on growth ratio. The unit discharge ($q$) also has a significant effect on gully development process. Higher values of $q$ enhance the gully development speed (Fig.19).
The above results show that the headcut shape with a sudden variation in stream-wise direction around the gully-head (convex shape) is required for gully to develop. Previous study based on linear analysis showed that the any small disturbance around headcut could grow gullies. However, our results with non-linear simulation have clarified that the disturbances with a mild variation in stream-wise direction around the gully-head (concave shape) are not able to develop gully.

6. CONCLUSION

2-D depth-averaged flow model in coupling with headcut migration model is used to investigate the effect of initial disturbance on gully development process. The variation of unit discharge (q) along the brink depends on gully-length (a) and -shape. The heterogeneity of unit discharge along headcut promotes the gully advancement. Concave shapes are disadvantageous in gully development process. All convex exponential shapes are able to develop gully. The most important factor for the gully to develop is the parameter p, which represents the convexity of shape. High value of p promotes the gully development process.

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ヘッドカットにおける平面的擾乱がガリーの発達過程に及ぼす効果

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ヘッドカットにおける平面的擾乱の発達過程が数値解析モデルによって検討された。ヘッドカットの平面的擾乱は流量集中部の空間的変動をもたらして擾乱を発達させ、ついにはガリー形成に至るものと考えられる。水深平均流れモデルを任意の平面形状を有するヘッドカットに適合する境界適合格子上で解くことによりヘッドカットにおける流量分布状況が再現され、さらに計算された流量によるヘッドカットの進行速度からヘッドカットの平面形状変化が得られた。非線形効果を考慮できる本数値解析結果から、ヘッドカットの平面形状の遷移が擾乱の発達に大きな影響を与えることが明らかとなった。