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STUDY ON BEHAVIOR OF EXTERNALLY PRESTRESSED CONCRETE BEAMS USING THE DEFORMATION COMPATIBILITY OF CABLE

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In this study, the frictional resistance at the deviators is incorporated in the calculation of cable strain. The equation of cable strain, which is based on the deformation compatibility of cable, is expressed in the general form for the analysis of externally prestressed concrete beams with normally external cable as well as externally prestressed concrete beams with large eccentricities. Application of the developed equation in the numerical analysis is carried out to verify its accuracy. It is found that the structural behavior of externally prestressed concrete beams can satisfactorily predict throughout the entire loading range up to failure. A good agreement with experimental data is found.

Key Words: *external cable, deviator, unbonded, beam with large eccentricities.*

1. INTRODUCTION

External prestressing is defined as prestress by the high strength cable, which is placed outside of the cross section and attached to the beam at some deviator points along the beam. In an external prestressing system, depending on the location of deviators, there are two kinds of beam, namely, prestressed concrete beam with normally external cables (below referred as typical beam) and prestressed concrete beam with large eccentricities (beam with large eccentricities). In the former, the deviators are located within the depth of cross section. In the latter, the deviators can be located outside of the depth of cross section, above the top surface or under the bottom surface. In the analysis of externally prestressed concrete beams, questions are always raised in the calculation of cable strain. Because, there is no bonding between the concrete and the cable, thus the cable strain cannot be maintained at the critical section as in the conventional prestressed concrete beams. The strain variation in the external cable is considered to be a function of the overall deformation of the beams. Meaning that the strain change in the cable is member-dependent and is influenced by the initial cable profile, span to depth ratio, deflected shape of the beam, friction at the deviator, etc [1].

When the structural behavior of externally prestressed concrete beams is investigated, many researchers have calculated the cable stress either by using the equations, which are provided in the codes or by adopted assumption that the total elongation of the cable element is equal to the total elongation of the concrete element at the cable level. This assumption is considered to be a effective tool for the evaluation of cable strain in the analysis of internally unbonded prestressed concrete beams as well as the typical beams, and good agreement with experimental data has been reported [2)~9)].

For the analysis of the typical beams, an analytical methodology, which is based on the deformation compatibility between concrete and a cable, has been developed [4]. In principle, the analysis of the beam with large eccentricities can use the analytical methodology for the typical beams. However, the only additional point is to be considered in the case of the beam with large eccentricities, namely eccentricity of the cable due to the location of the external cable. The difference in the analysis of both kinds of the beams with external cables is that the overall deformation of the beam in terms of the concrete strain, which is usually used in the calculation of cable strain for the typical beams, cannot be used in the analysis of the beam with large eccentricities. Because, in the case of the beam with large eccentricities, almost the cable portions are located outside of the depth of cross section, thus the concrete does not exist at the cable level.

Therefore, there is a need to have a computation method for the cable strain that should take account of the cable eccentricity, friction at the deviators and continuity of the structure, and the method can be used in the analysis of both kinds of the beams with external cables. To satisfy these conditions, the elongation of the cable must be in consistent with its deflection and to that effect, geometrical deformation of the cable must be correctly evaluated regardless of the deformed configuration of the beam. One of the possibilities is that the cable strain depends only on the deformation of the points, to which the cable is attached. It turns out that the cable strain depends on the total length variation of the cable between the extreme ends. Therefore, before an analysis model is carried out, a proposed formulation, which is to be used in the analysis of externally prestressed concrete beams, should be expressed in the general form for the evaluation of cable strain of the beams as above mentioned.

2. RESEARCH SIGNIFICANCE

A perusal of the relatively limited number of analytical studies available on the behavior of externally prestressed concrete beams reveals that several investigators attempted to calculate the cable stress by their formulations with some parameters involved for the certain cases of the beams with external cables. However, there were extremely few formulations or developed method, which were common use for the analysis of both kinds of the beams as above mentioned. It will be useful contribution towards to develop a method of analysis of externally prestressed concrete beams that will be included the eccentricity of cable and the friction at the deviators. Also, the strain variation in the cable can be correctly evaluated by the same method for the analysis of the typical beams and for the beam with large eccentricities.

3. REVIEW OF COMPUTING METHOD FOR CABLE STRAIN

As mentioned earlier, when the externally prestressed concrete beam is subjected to bending, the deflection of the external cable does not follow the beam deflection except at the deviator points. As a result, the cable strain cannot be determined from the local strain compatibility between the concrete and the cable. When the structural behavior of the typical beams was investigated, while calculating the cable strain, two extreme cases are usually considered, namely, free slip (no friction) and perfectly fixed (no movement) at the deviators. In the first case, the cable moves freely throughout the deviators without any restraint and the cable is treated as an internally unbonded cable. The cable strain variation is constant over its whole length and it can be expressed as:

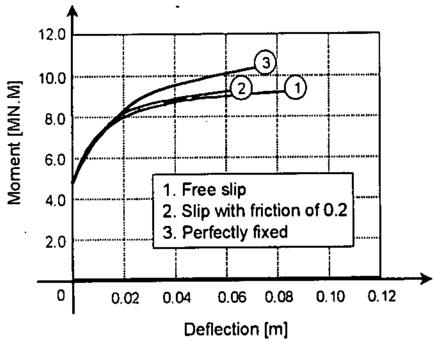

Fig.1 Effect of friction at the deviator

$$\Delta\varepsilon_s = \frac{1}{l}\int_0^l \Delta\varepsilon_{cs}\,dx \tag{1}$$

where $\Delta\varepsilon_s, \Delta\varepsilon_{cs}$ is the increments of cable strain and concrete strain at the cable level, respectively; l is the total length of cable between the extreme ends.

For the second case, the cable is considered perfectly fixed at the deviators. This means that the cable strain variation of each segment is independent of the others. The increment of cable strain depends only on the deformations of the two successive deviators or anchorages, to which the cable is attached. The strain variation in a cable can be expressed as:

$$\Delta\varepsilon_{si} = \frac{\Delta l_i}{l_i} \tag{2}$$

where $\Delta l_i, l_i$ are the incremental and original lengths of a cable segment under consideration, respectively.

For the former, if the frictional resistance at the deviators is neglected, deflection and cracking may be overestimated at the service loading range. For the latter, if the cable is assumed to be a perfectly fixed, the ultimate loading capacity may be overestimated [1]. This phenomenon can be seen in **Fig.1** by showing the effect of bond condition of the cable at the deviators in the analysis of three cases (free slip, slip with friction of 0.2 and perfectly fixed), which has been reported in Virlogeux, M.[10].

In many studies [4)~9)], when the structural behavior of the typical beams was investigated, most analytical approaches are usually based on the assumption in that the total elongation of the cable element must be equal to the total elongation of the concrete element at the cable level between the extreme ends. This can be expressed in the following equation:

$$\sum_{i=1}^{n} l_i \Delta \varepsilon_{si} = \int_0^l \Delta \varepsilon_{cs} dx \tag{3}$$

where $\Delta\varepsilon_{si}, \Delta\varepsilon_{cs}$ are the increments of cable strain and concrete strain at the cable level, respectively; l_i is the length of a cable element under consideration; l is the total length of cable between the extreme ends.

From the analytical results, it points out that the strain variation in the cable mainly depends on the overall deformation of the beams because of the lack of bond between the concrete and the cable.

Some researchers [2), 3), 6), 7)] extended this approach for the analysis of externally prestressed concrete beams with large eccentricities by the additional assumption of an imaginary concrete strain at the cable portion, at which the concrete does not exist. However, this extension seems to be limited because of the difficulties in defining value of the imaginary concrete strain at the cable level.

Virlogeux, M.[10)] proposed another approach based on a geometrical compatibility of external cable. Due to the rectilinear shape of external cable between the points, at which the cable attaches to the concrete beam, the strain variation of cable can be defined on the basis of deformations of the contacted points. Therefore, the cable strain can be evaluated regardless of the deformed shape of the concrete beam and it depends only on the deformations of the deviator points. By using this concept, Eakarat, W. et al.[6)] have been developed a computing program for the analysis of simply supported beam with large eccentricities and good agreement with experimental data had been reported. However, the effect of cable friction at the deviator did not consider in the Eakarat's calculation.

Normally, there is a frictional resistance between the cable and the deviator, and the cable strain depends on the coefficient of friction. In previous study [4)], when the friction at the deviators is considered, the increments of cable strains at the both side of the deviator are different. The difference in the strain increment can be expressed in terms of the friction coefficient, k_{Di} as:

$$\Delta\varepsilon_{si+1} - \Delta\varepsilon_{si} = \frac{k_{Di}}{l_{i+1} + l_i} \int_0^{l_{i+1}+l_i} \Delta\varepsilon_{cs} dx \tag{4}$$

where $\Delta\varepsilon_{si}, \Delta\varepsilon_{si+1}$ are the strain increments of (i) and $(i+1)$ cable elements, respectively; $\Delta\varepsilon_{cs}$ is the increment of concrete strain at the cable level; l_i, l_{i+1} are the length of (i) and $(i+1)$ cable elements, respectively.

In Eq.(4), the friction coefficient k_{Di} is not known at present and is assumed to be a function of the inclination angle of cable, and have a value between 0 and 1.0. The value of the friction coefficient indicates the extent of fixity of the cables at the deviator. When the beam is subjected to bending, according to the deflected shape of the beam, the inclination angle of cable will change. As a result, the friction coefficients at the deviators will change as well. Therefore, the value of friction coefficients is not constant during the loading step and should be changed depending on the loading condition. For a beam with having many deviators or multiple span continuous beams, the value and sign of this coefficient are often arisen in the calculation and the computing process should be repeated until a desirable result is obtained. Moreover, this friction coefficient is not familiar in the engineering design for the prestressed structures, and it only has the mathematical meaning. To overcome these difficulties, a formulation of the cable strain based on the force equilibrium condition at the deviator, will be presented hereinafter.

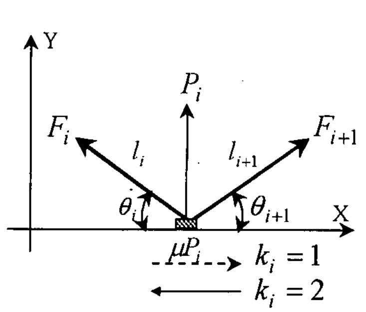

Fig.2 Force equilibrium at deviator

4. FORCE EQUILIBRIUM CONDITION AT DEVIATORS

Fig.2 shows that F_i, F_{i+1} are tensile forces in the cable segments (i) and $(i+1)$ at the deviator, correspondingly, θ_i, θ_{i+1} are cable angles, respectively. Thus, the force equilibrium condition at a deviator on the X direction can be expressed as:

$$F_i \cos\theta_i + (-1)^{k_i} \mu(F_i \sin\theta_i + F_{i+1} \sin\theta_{i+1}) = F_{i+1} \cos\theta_{i+1} \tag{5}$$

where

$$k_i = \begin{cases} 1 & if \quad F_i \cos\theta_i > F_{i+1} \cos\theta_{i+1} \\ 2 & if \quad F_i \cos\theta_i < F_{i+1} \cos\theta_{i+1} \end{cases}$$

and coefficient, k_i depends on the slipping direction; μ is a friction coefficient at the deviator and is assumed to be known at each deviator.

Dividing both sides of Eq.(5) by $E_{ps}A_{ps}$, the force equilibrium condition can be expressed in terms of the increments of cable strain as:

$$\Delta\varepsilon_{si}\cos\theta_i + (-1)^{k_i}\mu(\Delta\varepsilon_{si}\sin\theta_i + \Delta\varepsilon_{si+1}\sin\theta_{i+1}) = \Delta\varepsilon_{si+1}\cos\theta_{i+1}$$

$$\left[\cos\theta_i + (-1)^{k_i}\mu\sin\theta_i\right]\Delta\varepsilon_{si} + \left[-\cos\theta_{i+1} + (-1)^{k_i}\mu\sin\theta_{i+1}\right]\Delta\varepsilon_{si+1} = 0 \tag{6}$$

where E_{ps} and A_{ps} are the elastic modulus and area of the prestressing cable; $\Delta\varepsilon_{si}, \Delta\varepsilon_{si+1}$ are the increments of cable strains at both sides of the deviator.

5. FORMULATION OF CABLE STRAIN

Unlikely the previous assumptions for the calculation of cable strain in the analysis of the typical beams, for the beam with large eccentricities, instead of using the overall deformation of the concrete beam, the only deformations at the extreme top of the deviators are considered in formulation of the cable strain. In the case of cable having perfectly fixed at the deviators, the cable strain of each segment is independent of the others and the strain variation can be defined as in Eq.(2). On the other hand, the cable can be allowed to slip at the deviators, the total elongation of cable must be equal to the total cable length variation of each segment and expressed as:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \sum_{i=1}^{n} \Delta l_i \tag{7}$$

where $l_i, \Delta\varepsilon_{si}$ are the length and strain increase of the cable element under consideration, respectively; Δl_i is the cable length variation of the cable element.

The cable length variation Δl_i can directly derive from the deformations of the deviators, for example the deviators 1 and 2 are shown in **Fig.3**. Before deformation, the coordinates of the deviators 1 and 2 at the extreme top are x_1, y_1 and x_2, y_2, respectively. After deformation, the deviators 1 and 2 shift to new positions and their coordinates are $x_1 + \Delta x_1, y_1 + \Delta y_1$ and $x_2 + \Delta x_2, y_2 + \Delta y_2$. Thus, the cable length variation can be calculated as:

$$\Delta l_i = \left[(x_2 + \Delta x_2 - x_1 - \Delta x_1)^2 + (y_2 + \Delta y_2 - y_1 - \Delta y_1)^2\right]^{1/2} - l_i \tag{8}$$

where l_i is the original length of the cable segment under consideration.

After some manipulations with neglecting the high order terms, the cable length variation Δl_i is

$$\Delta l_i = \frac{(x_2 - x_1)}{l_i}(\Delta x_2 - \Delta x_1) + \frac{(y_2 - y_1)}{l_i}(\Delta y_2 - \Delta y_1)$$

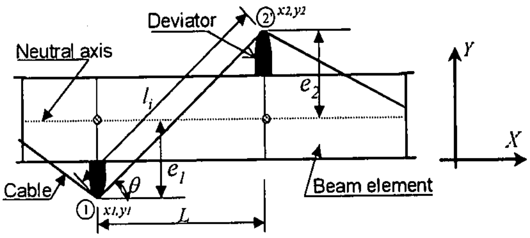

Fig.3 Beam with large eccentricities

or

$$\Delta l_i = \cos\theta(\Delta x_2 - \Delta x_1) + \sin\theta(\Delta y_2 - \Delta y_1) \tag{9}$$

where θ is the angle of the external cable to the horizontal line; $\Delta x_1, \Delta x_2$ and $\Delta y_1, \Delta y_2$ are called as increments of the horizontal and the vertical displacements at the deviators 1 and 2, respectively.

Based on the displacement functions for the beam element [4], these incremental displacements can be defined. It should be noted that the nodal displacement vector $\{d^*\}$ at the extreme top of the deviators should be used instead of the nodal displacement vector $\{d\}$ for the beam element. Therefore, the cable length variation between the deviators 1 and 2 can be rewritten in Eq.(10) as:

$$\begin{aligned}\Delta l_i = {} & \cos\theta\left(\left[N_u^2\right]\{d_2^*\} - \left[N_u^1\right]\{d_1^*\}\right) + \\ & + \sin\theta\left(\left[N_{vb}^2\right]\{d_2^*\} - \left[N_{vb}^1\right]\{d_1^*\}\right)\end{aligned} \tag{10}$$

where $\{d^*\}^T = \{d_1^* \quad d_2^*\} = \{u_1^* \quad v_1^* \quad \theta_1^* \quad u_2^* \quad v_2^* \quad \theta_2^*\}$ is the nodal displacement vector at the extreme top of the deviators; $[N_u] = [N_u^1 \quad N_u^2]$ and $[N_{vb}] = \{N_{vb}^1 \quad N_{vb}^2\}$ are the displacement functions for the beam element in the horizontal and the vertical directions, respectively and are defined in the following:

$$[N_u] = \left[1 - \frac{x}{L} \quad 0 \quad 0 \quad \frac{x}{L} \quad 0 \quad 0\right]$$

$$\begin{aligned}[N_{vb}] = \frac{1}{Ts}\Bigg[& 0 \;;\; \left(1 + \frac{6K}{L^2} - \frac{3}{L^2}x^2 + \frac{2}{L^3}x^3\right) \;; \\ & -\left(\frac{4K}{L} + \frac{12K^2}{L^3}\right) + \left(1 + \frac{12K}{L^3}\right)x - \left(\frac{2}{L} + \frac{6K}{L^3}\right)x^2 + \frac{1}{L^2}x^3 \;;\; 0 \;; \\ & \left(\frac{6K}{L^2} + \frac{3}{L^2}x^2 - \frac{2}{L^3}x^3\right) \;;\; \left(\frac{12K^2}{L^3} - \frac{2K}{L}\right) + \left(\frac{6K}{L^3} - \frac{1}{L}\right)x^2 + \frac{1}{L^2}x^3\Bigg]\end{aligned}$$

where $Ts = 1 + \dfrac{12K}{L^2}$; $K = \dfrac{EI}{GA}$ is a stiffness ratio and L is the length of a beam element.

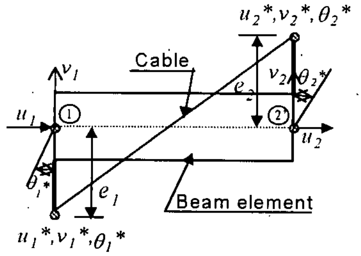

Fig.4 Arrangement of the external cable

Substituting $[N_u]$ and $[N_{vb}]$ into Eq.(10) and the cable length variation between the deviators 1 and 2 can be expressed as:

$$\Delta l_i = [A]\{d^*\} \tag{11}$$

where

$$[A] = \Bigg[-\cos\theta\left(1-\frac{x}{L}\right) \;;\; -\frac{\sin\theta}{Ts}\left(1+\frac{6K}{L^2}-\frac{3}{L^2}x^2+\frac{2}{L^3}x^3\right) \;;$$
$$-\frac{\sin\theta}{Ts}\left(-\left(\frac{4K}{L}+\frac{12K^2}{L^3}\right)+\left(1+\frac{12K}{L^3}\right)x-\left(\frac{2}{L}+\frac{6K}{L^3}\right)x^2+\frac{1}{L^2}x^3\right) \;;$$
$$\cos\theta\left(\frac{x}{L}\right) \;;\; \frac{\sin\theta}{Ts}\left(\frac{6K}{L^2}+\frac{3}{L^2}x^2-\frac{2}{L^3}x^3\right) \;;$$
$$\frac{\sin\theta}{Ts}\left(\left(\frac{12K^2}{L^3}-\frac{2K}{L}\right)+\left(\frac{6K}{L^3}-\frac{1}{L}\right)x^2+\frac{1}{L^2}x^3\right)\Bigg]$$

Fig.4 shows the arrangement of the external cable in the beam with large eccentricities. The nodal displacement vector at the extreme top of the deviators can be expressed in terms of the nodal displacement vector for the beam element as follows:

$$\begin{aligned} &u_1^* = u_1 + e_1\theta_1; \quad v_1^* = v_1; \quad \theta_1^* = \theta_1; \\ &u_2^* = u_2 + e_2\theta_2; \quad v_2^* = v_2; \quad \theta_2^* = \theta_2; \end{aligned} \tag{12}$$

where e_1 and e_2 are the eccentricities of cable at the deviators 1 and 2, respectively.

This nodal displacement vector can be rewritten in a matrix form as:

$$\begin{Bmatrix} u_1^* \\ v_1^* \\ \theta_1^* \\ u_2^* \\ v_2^* \\ \theta_2^* \end{Bmatrix} = \begin{bmatrix} 1 & 0 & e_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & e_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

$$\{d^*\} = [B]\{d\} \tag{13}$$

Substituting Eq.(13) into Eq.(11) to obtain the cable length variation between the deviators 1 and 2, which is directly related to the nodal displacement vector for the beam element and expressed as:

$$\Delta l_i = [A]\{d^*\} = [A][B]\{d\} \tag{14}$$

where matrices $[A]$ and $[B]$ are defined in Eq.(11) and Eq.(13), respectively.

Substituting Eq.(14) into Eq.(7), the total elongation of the cable can be expressed as:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \sum_{i=1}^{n} [A][B]\{d\} \tag{15}$$

Combining Eq.(15) with the force equilibrium condition at the deviators, which is expressed in Eq.(6), the increment of cable strain can be incorporated in a matrix form as:

$$\begin{bmatrix} l_1 & l_2 & l_3 & \cdots & l_{n-1} & l_n \\ c_1+(-1)^{k_1}\mu s_1 & -c_2+(-1)^{k_1}\mu s_2 & 0 & \cdots & 0 & 0 \\ 0 & c_2+(-1)^{k_2}\mu s_2 & -c_3+(-1)^{k_2}\mu s_3 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -c_{n-1}+(-1)^{k_{n-2}}\mu s_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & c_{n-1}+(-1)^{k_{n-1}}\mu s_{n-1} & -c_n+(-1)^{k_{n-1}}\mu s_n \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{s1} \\ \Delta\varepsilon_{s2} \\ \Delta\varepsilon_{s3} \\ \vdots \\ \vdots \\ \Delta\varepsilon_{sn-1} \\ \Delta\varepsilon_{sn} \end{Bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}[A][B]\{d\} \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

or

$$[M]\{\Delta\varepsilon_s\} = [N]\{d\} \tag{16}$$

where c_i and s_i are denoted as cosine and sine of cable angle; the subscripts under these letters indicate the cable angle number; l_i is the length of cable segment. $\{d\}^T = \{u_1 \;\; v_1 \;\; \theta_1 \;\; u_2 \;\; v_2 \;\; \theta_2\}$ is the nodal displacement vector for the beam element.

Therefore, the increment of cable strain is defined as:

$$\{\Delta\varepsilon_s\} = [M]^{-1}[N]\{d\} \tag{17}$$

By using Eq.(17), a numerical analysis of externally prestressed concrete beams is carried out. The analytical results are compared with the

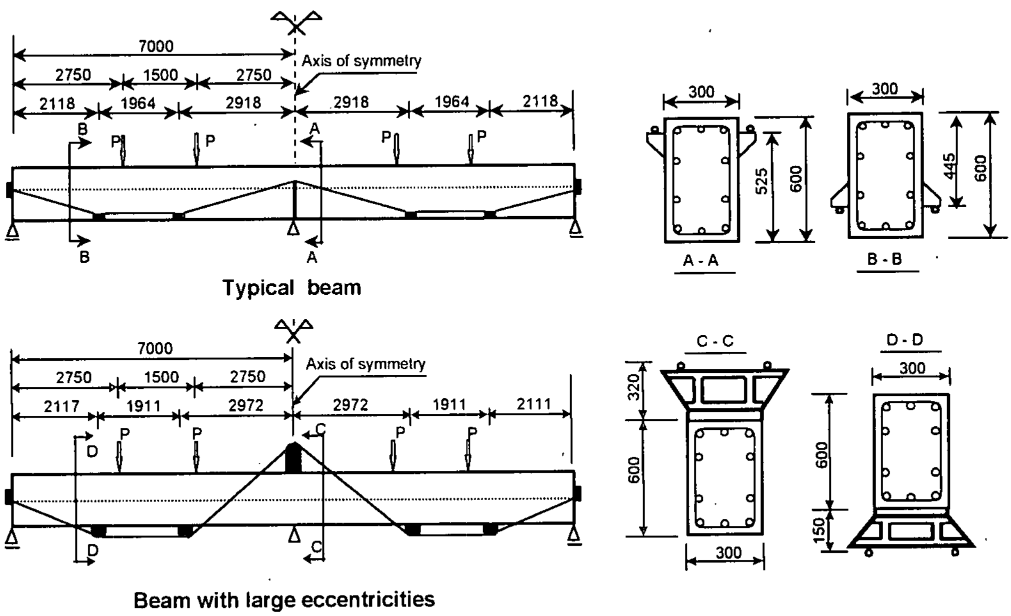

Fig.6 Layout scheme of two span continuous beams
(All dimensions in mm)

experimental data to verify the accuracy of the proposed method.

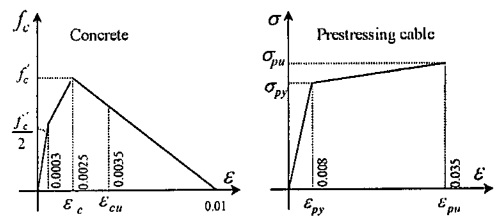

Fig.5 Stress-strain relationships

6. NUMERICAL EXAMPLES

A computer program, which was developed for the analysis of the typical beam [4] is to be applied in this study. The Eq.(17) is implemented in the program for the evaluation of cable strain.

To demonstrate the applicability of the developed formulation, a numerical analysis of two examples is carried out. One of them is a typical beam and the other is the beam with large eccentricities. An effect of cable eccentricity on the structural behavior is also investigated. Hereinafter, the analytical results in comparison with experimental data and the analytical results obtained by other investigators will be presented.

Table 1 Material properties (Mpa)

Name of the beams		Typical beam	Beam with large eccentricities
Concrete	f_c'	42.4	42.4
	E_c	$2.58\text{x}10^4$	$2.58\text{x}10^4$
Reinforcing steel, bar	σ_{sy}	364	364
	E_s	$2.1\text{x}10^5$	$2.1\text{x}10^5$
Prestressing cable	σ_{py}	1600	1500
	σ_{pu}	1900	1750
	E_{ps}	$1.98\text{x}10^5$	$1.97\text{x}10^5$

(1) Introduction of analytical models

The numerical examples are carried out following the experimental work [8), 9)], which was conducted by Sumitomo Co. In the analysis, the following assumptions are adopted:

1. Plane sections remain plane after bending.
2. Shear deformations are considered.
3. Compatibility of cable deformation is considered as that the total elongation of cable elements must be equal to the total length variation of each cable element between the extreme ends.
4. The ultimate limit state is defined when either the concrete strain at the extreme compression fiber reaches 0.0035 or the tensile stress of reinforcement or prestressing cable exceeds the nominal tensile strength.
5. In the analysis, the stress-strain curve for the concrete is assumed to be tri-linear, whereas it is assumed to be a bilinear curve for the prestressing cable as shown in **Fig.5**.

The layout of the analytical scheme for the typical beam and the beam with large eccentricities is presented in **Fig.6**.

The beams have a rectangular section, using two cables type of 1T17.8 SWPR19 (2.084cm^2/cable) for the typical beam, and two cables type 1T12.4 SWPR7A (0.929cm^2/cable) for the beam with large eccentricities. Two beams were designed to achieve the same ultimate strength, but with different prestressing force. At the initial stage, the cables were stressed approximately 50% of the ultimate strength of the cable. Material properties are shown in **Table 1**. Two loading points of the applied load are provided on each span with the symmetrical loading condition. During the test, the maximum values of the applied load were about 308.0 kN and 315.0 kN, corresponding to the maximum displacements of about 48.0mm and 69.0 mm for the typical beam and for the beam with large eccentricities, respectively. The crushing of concrete was found in the compression zone in the both cases. At the final stage of loading, the yielding of the external cables was found in the case of the beam with large eccentricities, whereas the cables did not yield in the case of the typical beam. At the ultimate state, the increase of cable stress was equal to 370.0 N/mm^2 and 780.0 N/mm^2 for the typical beam and for the beam with large eccentricities, respectively.

It is assumed that at the center-support, the cables cannot slip, meaning that they have a perfectly fixed because of the symmetrical loading condition. While, the cable can be allowed to slip at the deviators with the friction coefficients $\mu = 0.12$ and $\mu = 0.15$ for the typical beam and for the beam with large eccentricities, respectively. For the no-slip points of cable, the friction coefficients are referred to the previous study [11], which were assumed to be equal to 2.0.

(2) Discussion of analytical results

a) Load-displacement responses

Fig.7 and **Fig.8** show the load-displacement relationships at the midspan section for the typical beams and for the beam with large eccentricities, respectively. From these figures, it can be seen that the predicted load-displacement responses are in full agreement with the experimental data. The maximum values of the applied load are 310.8 kN and 310.2 kN, corresponding to the maximum displacements of about 50.0 mm and 69.8 mm for the typical beams and for the beam with large eccentricities, respectively. After reaching to the peak point, the applied load gradually reduces, and simultaneously the crushing of concrete occurs in the compression zone on the right span of the typical beam and on the left span of the beam with large eccentricities. This phenomenon could be considered as a local failure of the structure.

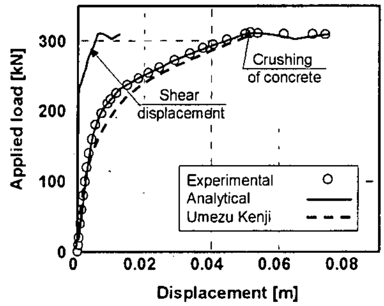

Fig.7 Load-displacement relation (Typical beam)

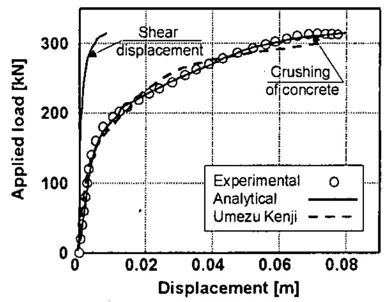

Fig.8 Load-displacement relation (Beam with large eccentricities)

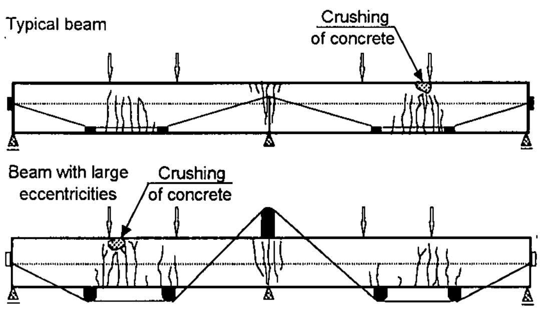

Fig.9 The ultimate state of the test beams

The crushing of concrete was also found in the experimental observation, which is shown in **Fig.9**. In comparison with results calculated by Umezu, K. et al. [8), 9)], which show by the dash-line, it can be seen that the predicted results from the proposed analytical model are somewhat closer to the experimental observation. The difference between two analytical models can be explained that the proposed analytical model takes both the cable friction at the deviator and the shear deformation into account, whereas Umezu's model did not. As a result, the Umezu's model slightly overestimated the displacement after the decompression.

The effect of shear deformation in the comparison with the total deformation of the beams is also presented in **Fig.7** and **Fig.8**. It can be seen that the affect of shear deformation is extremely small in the non-cracked elastic zone. However, in the cracked non-elastic zone, the shear deformation gradually develops with further increasing the applied load. At the ultimate state, the shear deformation is about 12 % and 11% of the total beam deflection for the typical beam and for the beam with large eccentricities, respectively.

b) Stress increase in the external cable

Fig.10 and **Fig.11** show the increase of cable stress against the applied load for the typical beams and for the beam with large eccentricities, respectively. From these figures, it shows that before the appearance of crack, the increase of cable stress is very small. However, it gradually increases with further increasing of the applied load after the occurrence of cracks. When the crushing of concrete happens in the compression zone, the cable stress reduces a little; simultaneously the applied load reduces as well. For the typical beam, the stress in the external cable increases only slowly so that when the crushing strain has been reached in the concrete, the stress in the cable is far below its ultimate strength. The prestressing cable undergoes small stress and remains in the elastic range. While, for the beam with large eccentricities, the stress increases very fast and almost proportionally increases with further increasing the applied load after the decompression. Finally, the cable stress increases approximately 366.0 N/mm^2 at the ultimate state and it is about 80% of the yielding strength of cable including the initial stress at the prestressing stage for the typical beam. For the beam with large eccentricities, the cable has yielded when the applied load reaches about 285.0 kN and the increase of cable stress is about 807.3 N/mm^2 at the ultimate state. The predicted responses have a good accuracy with the experimental data. In comparison with the results calculated by Umezu, K. et al. [8), 9)] showing by the dash-lines in **Fig.10** and **Fig.11**, it can be seen that for the Umezu's model, at the certain loading stage, especially after cracking, the predicted value of the cable stress is bigger than that in the experimental observation. This can be understood that a big displacement induces a big increase of cable stress by means of Eq.(1), which had been used in the author's calculation. From these figures, it can be seen that the analytical model gives somewhat better results in comparison with the Umezu's calculations.

(3) Effect of cable eccentricity on the structural behavior of the beam

To show the effect of cable eccentricity, the predicted results and also the experimental data of both kinds of the beams will be compared to each other and discussed in this section.

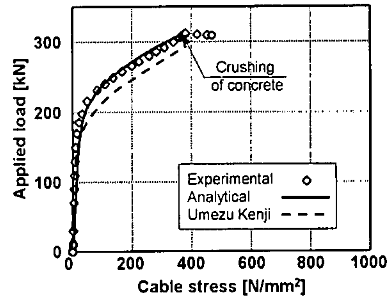

Fig.10 Increase of cable stress (Typical beam)

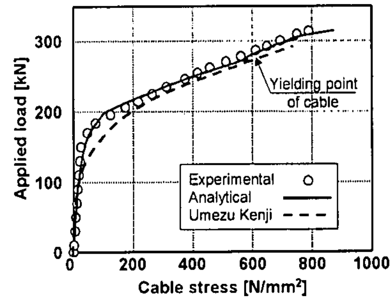

Fig.11 Increase of cable stress (Beam with large eccentricities)

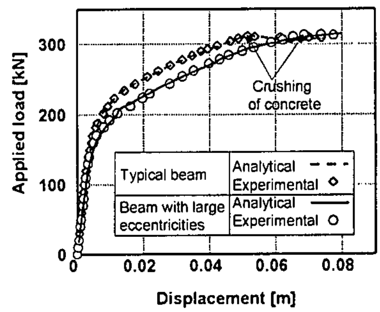

Fig.12 Load-displacement relationship

Fig.12 shows the load-displacement responses of the beams against the applied load. It can be seen that in the elastic zone, both kinds of the beams behave the same and indicating no influence of the arrangement of the external cables. However, in the non-elastic cracked zone, the displacement responses deviate from each other. For a given the applied load, the larger deflection is found in the beam with large eccentricities than that in the typical beam. It is clearly shown that in the non-elastic cracked zone, the eccentricity of cable has a significant effect on the displacement response of the beams. Even though, the ultimate load capacity of the beams is nearly the same, but the maximum displacements are significantly different at the ultimate state and they are about 50.0 mm and 69.8 mm for the typical beam and the beam

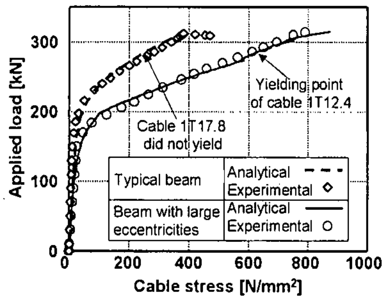

Fig.13 increase of cable stress

with large eccentricities, respectively. The difference in displacement could be mainly attributed to the lower prestressing force applied in the beam with large eccentricities at the prestressing stage than that in the typical beam. Moreover, the external cable in the beam with large eccentricities reaches the yielding strength before the ultimate load capacity of the beam.

Fig.13 shows the increase of cable stress of the beams against the applied load. It can be clearly seen that at a given the applied load, the stress increase in the cable of the beam with large eccentricities is greater than that in the typical beam. This is because the stress increase in the cable is a function of the total elongation of cable and essentially depends on the deformation of the beam. Therefore, the large deflection in the beam with large eccentricities would contribute to the great cable stress than that obtained in the typical beam. Moreover, at the prestressing stage, the beam with large eccentricities was prestressed with lesser prestressing force, which was approximately 45% of the prestressing force in the typical beam. For the typical beam, the prestressing cable remains in the elastic range even if at the final loading stage. While, for the beam with large eccentricities, the prestressing cable yields before the applied load reaches the ultimate load capacity of the beam.

Therefore, it clearly shown that for the beam with large eccentricities, the ultimate load capacity is almost the same as in the typical beam, while the required amount of the prestressing cable reduces significantly. The external cable in the beam with large eccentricities can reach to the yielding point before the ultimate state, i.e. utilized the cable material leaded to economical structures.

From analytical results, it can be concluded that by using the developed method, the structural behavior of the typical beam and the beam with large eccentricities can satisfactorily predict from the zero loading stage up to the ultimate loading stage. Good results are found in comparison with the experimental data. The structural responses are predicted very well in both cases.

7. CONCLUSIONS

A nonlinear analysis of prestressed concrete beams with external cables is carried out by using a finite element algorithm including coupled effects of the shear deformation and the friction at the deviators. The following conclusions can be made in this study.

1. A new equation of cable strain is formulated and it can be satisfactorily predicted the structural behavior of externally prestressed concrete beams from the zero loading stage up to the ultimate loading stage. The predicted responses for the displacement and the cable stress are in very good agreement with experimental data.
2. The formulated equation for cable strain is in the general form, and it can be used in the analysis of the typical beam as well as the beam with large eccentricities.
3. A big eccentricity of cable induces a great strain variation in the cable. Therefore, in the same conditions, the increase of cable stress in the beam with large eccentricities is usually greater than that in the typical beam at the certain loading stage after the decompression.

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ケーブル変形の適合性を考慮した外ケーブル方式PC桁の挙動に関する研究

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これまでの研究では，典型的な外ケーブル方式PC桁のケーブルのひずみは，コンクリート桁全体の変形に基づいて示されている．しかし，大偏心外ケーブルを持つPC桁の断面では，ケーブルの位置にコンクリートが存在しないために，桁全体の変形に基づいて解析を行うことはできない．そこで，ケーブルのひずみとディビエーターの変形に基づいて，ディビエーターにおける力の釣り合いをもとにケーブルのひずみの定式化を新たに行った．この手法により，大偏心外ケーブルを持つPC桁の解析を一般の外ケーブル方式PC桁の解析と同じように取り扱うことができるようにした．