

# COST-TIME SCHEDULING OF CONSTRUCTION WORKS EXECUTION

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Cost and time are essential parts of construction works accomplishment. According to the existing circumstances, construction works are analyzed under the deterministic or stochastic conditions. The deterministic conditions are assumed when random factors can be sufficiently eliminated. In opposite case the stochastic conditions must be considered. The requirements of construction works execution are described by using models of construction technology and resources. Scheduling of the works execution are realized by solving mixed linear programming problems under the requirement. In this paper, these are applied to cost-time scheduling of the small bridge erection as an example under stochastic conditions.

**KeyWords:** construction works execution, scheduling, determinism, risk, uncertainty, resources.

## 1. INTRODUCTION.

The term of construction works is used here for defining a set of connected operations of any construction project which must be realized in order to achieve the customers' goals. The construction works execution, however, signifies a process of all operations carrying out. The customer commissions the project execution and demands the given results. In most practical situations there are a lot of possibilities for the construction works execution. The problem is how and when to carry out the operations in existing circumstances by using the resources being at the contractor's disposal, to meet the customer's goals in the best way. The basic practical criteria for the selection of the accomplishment possibilities are minimum cost and the minimum time of the construction works execution. The execution schedule fixed by using these criteria separately, are two different optimum cases. The situation is most complicated when neither of the single criteria solutions satisfies the client. Then, one must find a compromise solution which is satisfactory. In an interactive heuristic process the compromise solution may be found when both criteria are fulfilled to some extent simultaneously. The approach makes possible to apply relatively independent models of construction works execution technology, which can be worked out strictly from the project accomplishment point of view, as well as models of the operations execution resources defining only their accomplishment abilities. That means that the models of the construction works execution technology describe

technical, technological, organizational and economic conditions only from the project realization point of view, such as the range, relationship, extent, accuracy and quality of construction works. The models of the construction works execution resources, however, describe their whole structure, capability and suitability for operations, efficiency or productivity, consumption of energy or fuel or other resources and unit performing cost. Depending on circumstances, individual characteristics of the construction works and the set of executors, the situation can be considered as deterministic or stochastic <sup>1) , 2)</sup>.

The deterministic situation is assumed when:

- The technical, technological and organizational conditions of construction works execution are certain.
- The exact measurement of the size of all operations is possible.
- The needed resources ensure established efficiency and quality and they are at the executor's disposal at that moment and for the whole time of the operations execution.
- The state of a facility after each specified operations execution is entirely determined and completely attainable.
- There are no essential disruptions of construction works execution.

The stochastic models must be used when there are some or all following conditions:

- Random technical, technological, organizational and economic conditions cannot be sufficiently eliminated or reduced.

- Exact measurement of operations is unfeasible or inadvisable.
- Execution efficiency of operations by the executors, being at contractor's disposal, is only probable or even uncertain.
- Natural or intentional disturbances of operations execution exist.
- Time limit of access or amount of operations execution resources are random.
- Quality and usefulness of passive resources such as materials or semi-finished product have a random character and are detected just during execution of operations.
- The state of a facility after all specified operations execution is well described but some or all of the operations are able to be accomplished with random productivity.

Under the stochastic circumstances two kinds of cases are distinguished – the risk situation and the uncertain situation. In the first one the probability of all specified events can be determined. In the latter, the probability of all or some specified events cannot be determined. Those conditions are important for choosing the method of modeling and scheduling of the construction works execution of the given project.

## 2. CONSTRUCTION WORKS EXECUTION TECHNOLOGY

The operations are separated and described depending on necessary connections and sequence of events from the project completion viewpoint only <sup>1)</sup> 2). The structure of construction works and each operation are separately defined and modelled solely for the sake of the project demands and conditions. The resources limitations are not taken into consideration here. All distinguished and modelled operations must be done in technical and technological order, so that the project can be accomplished. The project, from the execution viewpoint can be divided in some different consecutive or parallel stages and sets of operations. Each division constitute a single variant of construction works. Each variant can be analyzed separately but all of its operations must be done in a determined order – the variant has the deterministic structure but the operations can have deterministic or stochastic characterization. Then, the construction works execution technology under the deterministic and the stochastic circumstances can be modelled by using the following network  $S^T = \langle G, B, L \rangle$ :

- $G = \langle Y, U, P \rangle$  – a unigraph, coherent, a-cyclic, with a single initial node and single final node which describing the structure of the whole construction works and the required sequence of the operations execution;
  - $Y = \{y_1, y_2, \dots, y_j, \dots, y_k, \dots, y_W\}$  – a set of vertices of the graph  $G$ , representing the events of starting and ending of each operation  $u_j \in U$ ;
  - $U = \{u_1, u_2, \dots, u_j, \dots, u_k, \dots, u_N\}$  – a set of arcs of the graph  $G$ , representing relatively independent operations making up of the construction works;
  - $P \subset Y \times U \times Y$  – a three-term relation assigning the initial and final vertex of each arc of the graph  $G$ , representing technical and technological dependence of the operations  $u_j \in U$ ;
- $B: (Y \times W) \rightarrow R^+$  – function determined on the set of vertices  $y_i \in Y$  of the graph  $G$ , characterizing the state of execution  $w_k \in W_i$ , after all operations  $u_j \in U_i \subset U$  preceding vertex  $y_i \in Y$  have been completed ( $W_i$  – the set of parameters and characteristics of the desired state of a facility at vertex  $y_i \in Y$ , after completing all operations preceding it  $u_j \in U_i$ ,  $W = \{w_1, w_2, \dots, w_k, \dots, w_K\}$ ,  $W_i = \{w_k\}$ ,  $W = \bigcup_{i=1}^{i=W} W_i$ ;  $U_i$  – a set of operations preceding vertex  $y_i$  and decisive to the state  $W_i$  at this vertex;  $R^+$  – a set of positive real numbers).
- $L: U \rightarrow R^+$  – function defined on the set of arcs  $u_j \in U$  of the graph  $G$ , describing the extent of each operation. When deterministic circumstances can be supposed, the constant extent of operations are assumed, that is  $l_j = \text{const}$ . If one must suppose stochastic circumstances, the extent of operations is assumed as a random quantity  $L_j$  which receives the given values  $l_j$  depending on current random interactions.

The mentioned approach indicates that the deterministic models and the stochastic models of construction works execution technology differ only with the characteristic of operations  $u_j \in U$ , that is the form of the function  $L: U \rightarrow R^+$ . The risk and uncertainty concern only the execution characteristic of operations  $u_j \in U$ . It means that the desired state  $w_k \in W_i$  of the facility are achieved after correct execution of all operations  $u_j \in U_i$  preceding the vertex  $y_i \in Y$ . If those conditions are not met, the additional risk or uncertainty of timetable or cost execution must be considered. Such cases are not analyzed here. In the

cases of risk, when the density function or distribution of probability of random variables  $L_j$  are known, the expected values  $E[L_j]$  of those random variables can be used to compute the schedule of the construction works execution. This approach is easier and more effective than the exact method. In the case of uncertainty, when the exact density function or distribution of the random variables  $L_j$  are not known, the expected values  $E[L_j]$  can be used if the subjective probability can be determined by experts. The models formulated in the above way describe only technical and technological demands of construction works execution technology. At the same time it means that the resource limitations are not taken in consideration.

### 3. CONSTRUCTION WORKS EXECUTION RESOURCES

Modeling of the construction works execution resources is realized independently from modeling of the construction works execution technology (1, 2). In this process the elementary resources (laborers or specialists, equipment) are combined into the elementary executors, which are useful in performing the operations. For such combination are described the structure, capability and suitability for carrying out the operations, efficiency or productivity, consumption of energy or fuel and other resources, and unit performing cost of operations of each executor. The elementary executors are characterized under detailed analysis of their skills, and usefulness and needs of the project and the construction works execution. Obviously the models can be worked out in advance for any specialization of construction works execution and used every time they are needed. Depending on existing circumstances the deterministic and stochastic models of the resources are distinguished. The executors are described by using the parameters or characteristics of executors constant under deterministic circumstances and random under stochastic circumstances. Under the risk circumstances, the density function or distribution of probability all parameters can be determined. Under the uncertainty circumstances, those quantities cannot be determined. In this case the models of the resources can be used only if the experts are able to quantify subjective probability. The random characteristics of the operation executors are specific for each type of the staff, equipment and materials. They are relatively independent from random factors of construction works modelled by using the network  $S^r$ . The basic sources

of random characteristics of executors are random phenomena and random technical, technological, organizational and economic events caused by circumstances of the action and individual properties of the executors.

There are two kinds of elementary resources – active elementary resources  $H = \{1, \dots, h, \dots, H\}$  and passive elementary resources  $Q = \{1, \dots, q, \dots, Q\}$ . If the situation is deterministic the quantities  $z_h$  and  $z_q$  of the elementary resources  $h \in H$  and  $q \in Q$  are identified and described with a given accuracy. Their amounts and changeability are surely known. If the situation is stochastic the quantities  $Z_h$  and  $Z_q$  of the resources  $h \in H$  and  $q \in Q$  are random variables. They are identified and described suitably to the random characterization of the resources. It is strongly suggested to use the expected values  $E[Z_h]$  and  $E[Z_q]$  of those variables.

The active elementary resources are combined into the permanent or temporary (for the execution of some operations only) elementary executors  $H^m = \{h\}$ ,  $H^m \subset H$ ,  $H = \bigcup_{m \in M} H^m$ ,  $m \in M$ ,  $M = \{1, \dots, m, \dots, n, \dots, M\}$ . Each of the elementary executor consist of subset of human resources  $H_R^m$  (individuals or groups) and subset of production means  $H_S^m$  (equipment, machines, appliances or tools). The relations between the subsets are described by the relationships  $H^m = H_R^m \cup H_S^m$  and  $H_R^m \cap H_S^m = \emptyset$ . The number of the elementary executors of type  $m \in M$  is the certain quantity  $z_m$  under the deterministic circumstances, and the random quantity  $Z_m$  under the stochastic circumstances. It is suggested to use the expected values  $E[Z_m]$  of the variables  $Z_m$  as well. Generally, the elementary executor  $H^m \subset H$  can perform the set of operations  $J^m = J_e^m \cup J_b^m \cup J_l^m \cup J_g^m \cup J_a^m$  with given productivity. In this set the following operations can be distinguished: maintenance supplying –  $j \in J_e^m$ , material supplying –  $j \in J_b^m$ , externally useful works –  $j \in J_l^m$ , activity waste utilization –  $j \in J_g^m$  and servicing or selling the final product –  $j \in J_a^m$ . At the given moment  $t \in T$  the elementary executor can perform only one operation, that is  $|J^m(t)| \in \{0, 1\}$  for  $t \in T$  and  $J^m(t) \subset J^m$ . The execution of each operation  $j \in J^m$  means transformation of the passive resources into the final product or into the indirect passive resources, or

performing servicing works. During the operations execution the maintenance resources  $q \in Q_e^m$  are used up. The materials  $q \in Q_b^m$  and production „in progress“  $q \in Q_l^m$  are processed into final products  $q \in Q_a^m$  or into another indirect products  $q \in Q_l^m$ . The final products  $q \in Q_a^m$  are sold or stored. The activity wastes  $q \in Q_g^m$  are expelled. The subsets of the resources make up the set of all passive resources  $Q^m = Q_e^m \cup Q_b^m \cup Q_l^m \cup Q_g^m \cup Q_a^m$ . In the certain period  $T = [t', t'']$ , the efficiency of the operations execution is assumed as a fixed quantity  $\lambda_j^m$  under the deterministic circumstances, and as a random quantity  $\lambda_j^m$  under the stochastic circumstances. Then, if the execution of any operation starts at the moment  $t_0$  and finishes at the moment  $t_k$ ,  $[t_0, t_k] \subset T$ , the fixed extent  $l_j^m(t_k)$  and expected extent  $E[L_j^m(t_k)]$  of the operations execution, can be calculated by using formulas 1, 2, 3 or 4:

- Continuous execution:

$$l_j^m(t_k) = \lambda_j^m(t_k - t_0) + l_j^m(t_0) \quad (1)$$

$$E[L_j^m(t_k)] = E[\lambda_j^m(t_k - t_0) + l_j^m(t_0)] \quad (2)$$

- Discrete execution:

$$l_j^m(t_k) = \sum_{t=t_0}^{t=t_k} \lambda_j^m(t) + l_j^m(t_0) \quad (3)$$

$$E[L_j^m(t_k)] = \sum_{t=t_0}^{t=t_k} E[\lambda_j^m(t)] + E[l_j^m(t_0)] \quad (4)$$

Under the deterministic circumstances, in the period  $[t', t''] \in T$ , the execution of operations  $j \in J^m$  with the efficiency  $\lambda_j^m = \text{const}$  by the elementary executor  $H^m \in H$  are characterized by the unit expenditure  $p_{h,j}^m = \text{const}$  of the active elementary resources  $h \in H^m$ , and the unit production or consumption  $r_{q,j}^m = \text{const}$  of the passive elementary resources  $q \in Q^m$ . Under the stochastic circumstances the unit expenditure  $P_{h,j}^m$  of the active resources  $h \in H^m$  and the unit production or consumption  $R_{q,j}^m$  of the passive resources  $q \in Q^m$  are random quantities. The expected values  $E[P_{h,j}^m]$  and  $E[R_{q,j}^m]$  are suggested to be used as well. Then, the quantities  $l_q^m(t_k)$  and  $E[L_q^m(t_k)]$  describing the processing

scale of the passive resources  $q \in Q^m$  after the period  $[t_0, t_k] \in T$ , during the execution of the operations  $j \in J^m$ , can be calculated by using the formulas 5, 6, 7 or 8:

- Continuous execution:

$$\begin{aligned} \langle l_q^m(t_k) \rangle_{Q^m \times 1} &= [r_{q,j}^m]_{Q^m \times J^m} \langle \lambda_j^m \rangle_{J^m \times 1} (t_k - t_0) + \\ &+ \langle l_q^m(t_0) \rangle_{Q^m \times 1} \end{aligned} \quad (5)$$

$$\begin{aligned} \langle E[L_q^m(t_k)] \rangle_{Q^m \times 1} &= [E[R_{q,j}^m]]_{Q^m \times J^m} \langle E[\lambda_j^m] \rangle_{J^m \times 1} (t_k - t_0) + \\ &+ \langle E[L_q^m(t_0)] \rangle_{Q^m \times 1} \end{aligned} \quad (6)$$

- Discrete execution:

$$\begin{aligned} \langle l_q^m(t_k) \rangle_{Q^m \times 1} &= \sum_{t=t_0}^{t=t_k} \left( [r_{q,j}^m]_{Q^m \times J^m} \langle \lambda_j^m(t) \rangle_{J^m \times 1} \right) + \\ &+ \langle l_q^m(t_0) \rangle_{Q^m \times 1} \end{aligned} \quad (7)$$

$$\begin{aligned} \langle E[L_q^m(t_k)] \rangle_{Q^m \times 1} &= \sum_{t=t_0}^{t=t_k} \left( [E[R_{q,j}^m]]_{Q^m \times J^m} \langle E[\lambda_j^m(t)] \rangle_{J^m \times 1} \right) + \\ &+ \langle E[L_q^m(t_0)] \rangle_{Q^m \times 1} \end{aligned} \quad (8)$$

If the elementary executor  $H^m \in H$  can accumulate and store the passive resources  $q \in Q^m$ , the quantities  $z_q^m$  and  $Z_q^m$  must range from the fixed minimum value  $\underline{z}_q^m$  to the fixed maximum value  $\bar{z}_q^m$ , that is  $\underline{z}_q^m \leq z_q^m \leq \bar{z}_q^m$  under the deterministic circumstances or  $\underline{z}_q^m \leq Z_q^m \leq \bar{z}_q^m$  under the stochastic circumstances. In the opposite case the resources must be provided from and generated to the external cooperating systems at the moment of consumption or production. The "just in time" supplying, distribution and waste expelling system is forced. The states  $z_q^m(t_k)$  or  $E[Z_q^m(t_k)]$  of the passive resources  $q \in Q^m$  at the moment  $t_k$ , can be calculated by using the formulas 9 or 10:

$$\langle z_q^m(t_k) \rangle_{Q^m \times 1} = \langle z_q^m(t_0) \rangle_{Q^m \times 1} \pm \langle l_q^m(t_k) \rangle_{Q^m \times 1} \quad (9)$$

$$\langle E[Z_q^m(t_k)] \rangle_{Q^m \times 1} = \langle E[Z_q^m(t_0)] \rangle_{Q^m \times 1} \pm \langle E[L_q^m(t_k)] \rangle_{Q^m \times 1} \quad (10)$$

The direct cost  $k_f^m(t_k)$  and the expected direct cost  $E[K_f^m(t_k)]$  of the execution of the operations  $j \in J^m$  by the elementary executor  $H^m \in H$  after the

period  $[t_0, t_k] \in T$  can be calculated in the currency  $f \in F^m$  by using the formulas 11 or 12:

$$\langle k_f^m(t_k) \rangle_{F^m \times 1} = [\kappa_{f,j}^m]_{F^m \times J^m} \langle l_j^m(t_k) \rangle_{J^m \times 1} \quad (11)$$

$$\langle E[K_f^m(t_k)] \rangle_{F^m \times 1} = [E[\kappa_{f,j}^m]]_{F^m \times J^m} \langle E[L_j^m(t_k)] \rangle_{J^m \times 1} \quad (12)$$

- $K_f^m(t_k)$  – random variable describing the random direct costs expressed in the currency  $f \in F^m$  of the operations  $j \in J^m$  carried out by the executor  $H^m \in H$  to the moment  $t_k$ ;
- $\kappa_{f,j}^m$  – fixed unit cost expressed in the currency  $f \in F^m$  of the operations  $j \in J^m$  carried out by the elementary executor  $H^m \in H$ ;
- $K_{f,j}^m$  – random unit cost expressed in the currency  $f \in F^m$  of the operations  $j \in J^m$  carried out by the elementary executor  $H^m \in H$ ;
- $E[K_{f,j}^m]$  – expected value of the variable  $K_{f,j}^m$ ;
- $F^m = |F^m|$ ,  $J^m = |J^m|$ .

Then, the fixed income  $d_f^m(t_k)$  or the expected income  $E[D_f^m(t_k)]$  of the elementary executor  $H^m \in H$ , in currency  $f \in F^m$  in time  $[t_0, t_k] \in T$ , can be calculated by using formulas 13 or 14:

$$\langle d_f^m(t_k) \rangle_{F^m \times 1} = [\delta_{f,q}^m]_{F^m \times Q_a^m} \langle l_q^m(t_k) \rangle_{Q_a^m \times 1} \quad (13)$$

$$\langle E[D_f^m(t_k)] \rangle_{F^m \times 1} = [E[\delta_{f,q}^m]]_{F^m \times Q_a^m} \langle E[L_q^m(t_k)] \rangle_{Q_a^m \times 1} \quad (14)$$

- $\delta_{f,q}^m$  – fixed unit price of the product  $q \in Q_a^m$  in currency  $f \in F^m$ , obtained by the elementary executor  $H^m \in H$  in the period  $[t_0, t_k] \in T$ ;
- $\Delta_{f,q}^m$  – random unit price of the product  $q \in Q_a^m$  in the currency  $f \in F^m$ , obtained by the elementary executor  $H^m \in H$ , in the period  $[t_0, t_k] \in T$ ;
- $D_f^m(t_k)$  – random income obtained in currency  $f \in F^m$ , by the elementary executor  $H^m \in H$ , in the period  $[t_0, t_k] \in T$ ;
- $E[\Delta_{f,q}^m]$  – expected value of the random variable  $\Delta_{f,q}^m$ .

The fixed direct balance  $u_f^m(t_k)$  and the expected direct balance  $E[U_f^m(t_k)]$  of the costs and income, expressed in the currency  $f \in F^m$ , for the elementary

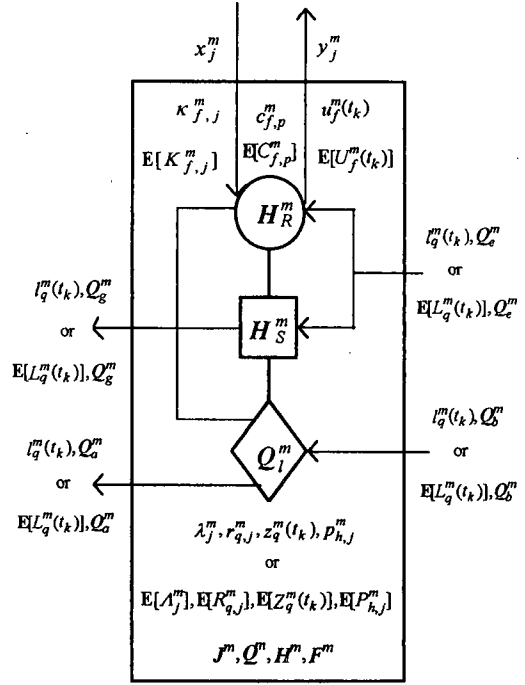


Fig. 1 Integrated model of the elementary executor in graphical form.

executor  $H^m \in H$  after a lapse of time  $[t_0, t_k] \in T$  can be calculated by using the formulas 15, 16, 17 or 18:

- In individual currency units  $f \in F^m$ :  

$$\langle u_f^m(t_k) \rangle_{F^m \times 1} = \langle u_f^m(t_0) \rangle_{F^m \times 1} + \langle d_f^m(t_k) \rangle_{F^m \times 1} - \langle k_f^m(t_k) \rangle_{F^m \times 1} \quad (15)$$

$$\langle E[U_f^m(t_k)] \rangle_{F^m \times 1} = \langle E[U_f^m(t_0)] \rangle_{F^m \times 1} + \langle E[D_f^m(t_k)] \rangle_{F^m \times 1} - \langle E[K_f^m(t_k)] \rangle_{F^m \times 1} \quad (16)$$

-  $U_f^m(t_k)$  – random direct balance expressed in the currency  $f \in F^m$  after a lapse of time  $[t_0, t_k] \in T$  – the operations  $j \in J^m$  carried out by the executor  $H^m \in H$ .

- In the currency  $h \in F^m$  and the fixed exchange rate  $c_{h,f}^m$  or the expected exchange rate  $E[C_{h,f}^m]$ :

$$u_h^m(t_k) = \langle c_{h,f}^m \rangle_{1 \times F^m} \langle u_f^m(t_k) \rangle_{F^m \times 1} \quad (17)$$

$$E[U_h^m(t_k)] = \langle E[C_{h,f}^m] \rangle_{1 \times F^m} \langle E[U_f^m(t_k)] \rangle_{F^m \times 1} \quad (18)$$

The integrated model of the elementary executor in graphical form is shown in Figure 1.

#### 4. SCHEDULING OF THE CONSTRUCTION WORKS EXECUTION.

If the technology of the construction works of any project is modelled by using the network  $S^T = \langle G, B, L \rangle$ , the execution schedule can be calculated by determining the following functions in the graph  $G$  (1, 2):

- $V: Y \rightarrow R^+$  function defined on the set  $Y$  of the vertices of the graph  $G$ , characterizing for each operation  $u_j \in U$ ,  $\langle y_i, u_j, y_k \rangle \in P$ : the fixed starting moment  $v_i$  and the fixed finishing moment  $v_k$  under the deterministic circumstances or the expected starting moment  $E[V_i]$  and the expected finishing moment  $E[V_k]$  under the risk or uncertainty circumstances;
- $X: (H \times U) \rightarrow \{0, 1\}$ ,  $T: (H \times U) \rightarrow R^+$  and  $K: (H \times U) \rightarrow R^+$  – functions defined on the set of arcs  $u_j \in U$  of the graph  $G$ , characterizing:
  - function  $X$  – the deterministic allocation of the executors  $H^m \in H$  to the operations  $u_j \in U$  ( $x_{j,m} = 1$  or  $x_{j,m} = 0$ ),
  - functions  $T$  and  $K$  – defining for the given set  $H = \bigcup_{m \in M} H^m$  of the executors, which perform the operations  $u_j \in U$ , correspondingly: the fixed duration  $t_{j,m}$  and the fixed cost  $k_{f,j}^m$  under the deterministic circumstances or the expected duration  $E[T_{j,m}]$  and the expected cost  $E[K_{f,j}^m]$  under the stochastic circumstances.

Finally, the technology and organization of the construction works execution are described by the network  $S = \langle G, \{B, V\}, \{X, L, T, K\} \rangle$ . The functions  $T$  and  $K$  are determined on the basis of the analysis of suitability of the elementary executors for carrying out the individual operations. The functions  $V$  and  $X$  for the independent executors ( $H^m \cap H^n = \emptyset$ ) can be calculated by solving the following mixed linear scheduling problem:

1. Under the deterministic circumstances:

- to determine the vector  $\langle v_1, \dots, v_i, \dots, v_k, \dots, v_W, x_{1,1}, \dots, x_{j,m}, \dots, x_{N,R}, x_{i,e}, \dots \rangle$ , so that to minimize the fixed time and fixed cost of the construction works execution, that is:

$$v = \min \sum_{i=1}^{i=W} v_i \quad (19)$$

and

$$k = \min \sum_{j=1}^N \sum_{m \in R_j} k_{f,j}^m x_{j,m} \quad (20)$$

under the following conditions of the execution:

$$\left\{ \begin{array}{l} v_k - v_i \geq \sum_{m \in R_j} t_{j,m} x_{j,m} \text{ for } u_j \in U, \langle y_i, u_j, y_k \rangle \in P, \\ \left. \begin{array}{l} v_i - v_e \geq t_{l,m} x_{l,m} + \\ - B(1 - x_{i,e}) - D(1 - x_{j,m}) \end{array} \right\} \text{ for } H^m \in H, \\ \left. \begin{array}{l} v_e - v_i \geq t_{j,m} x_{j,m} + \\ - Bx_{i,e} - D(1 - x_{l,m}) \end{array} \right\} \begin{array}{l} m = 1, 2, \dots, R, \\ \langle y_i, u_j, y_k \rangle \in P, \\ \langle y_e, u_l, y_s \rangle \in P, \end{array} \\ \sum_{m \in R_j} x_{j,m} = 1 \quad u_j \in U, j = 1, 2, \dots, N. \end{array} \right. \quad (21)$$

- $x_{j,m} = \begin{cases} 1, & \text{if the operation } u_j \in U \text{ is carried} \\ & \text{out by the executor } H^m \in H, \\ 0, & \text{in the opposite case.} \end{cases}$
- $x_{l,m} = \begin{cases} 1, & \text{if the operation } u_l \in U \text{ is carried} \\ & \text{out by the executor } H^m \in H, \\ 0, & \text{in the opposite case.} \end{cases}$
- $x_{i,e} = \begin{cases} 1, & \text{if for } y_i \text{ and } y_e, v_i \geq v_e, \langle y_i, u_j, y_k \rangle \in P, \\ 0, & \text{if for } y_i \text{ and } y_e, v_i < v_e, \langle y_e, u_l, y_s \rangle \in P. \end{cases}$
- $v_i, v_e, v_k \geq 0$ ,
- $B, D$  – sufficiently large numbers.

2. Under the stochastic circumstances:

- to determine the vector  $\langle E[V_1], \dots, E[V_i], \dots, E[V_k], \dots, E[V_W], x_{1,1}, \dots, x_{j,m}, \dots, x_{N,R}, x_{i,e}, \dots \rangle$ , so as to minimize the expected time and expected costs of the construction works execution, that is:

$$E[V] = \min \sum_{i=1}^{i=W} E[V_i] \quad (22)$$

and

$$E[K] = \min \sum_{j=1}^N \sum_{m \in R_j} E[K_{f,j}^m] x_{j,m} \quad (23)$$

under the following conditions of the execution:

$$\left\{ \begin{array}{l} E[V_k] - E[V_i] \geq \sum_{m \in R_j} E[T_{j,m}] x_{j,m} \\ \text{for } u_j \in U, \langle y_i, u_j, y_k \rangle \in P, \\ \left. \begin{array}{l} E[V_i] - E[V_e] \geq E[T_{l,m}] x_{l,m} + \\ - B(1 - x_{i,e}) - D(1 - x_{j,m}) \end{array} \right\} \text{ for } H^m \in H, \\ \left. \begin{array}{l} E[V_e] - E[V_i] \geq E[T_{j,m}] x_{j,m} + \\ - Bx_{i,e} - D(1 - x_{l,m}) \end{array} \right\} \begin{array}{l} m = 1, \dots, R, \\ \langle y_i, u_j, y_k \rangle \in P, \\ \langle y_e, u_l, y_s \rangle \in P. \end{array} \\ \sum_{m \in R_j} x_{j,m} = 1 \quad \text{for } u_j \in U, j = 1, \dots, N, \end{array} \right. \quad (24)$$

- $x_{i,e} = \begin{cases} 1, & \text{if for } y_i, y_e, E[V_i] \geq E[V_e], \langle y_i, u_j, y_k \rangle \in P, \\ 0, & \text{if for } y_i, y_e, E[V_i] < E[V_e], \langle y_e, u_l, y_s \rangle \in P. \end{cases}$
- $E[V_i], E[V_e], E[V_k] \geq 0$ ,

- $R_j$  – the set of executors  $H^m \in H$  suitable for carrying out the operations  $u_j \in U$ ,
- $B, D, x_{j,m}$  and  $x_{l,m}$  as above.

In both cases the mixed linear scheduling problem can be solved as unicriterial solutions when the time criterion or the cost criterion are taken into account individually, and as bi-criterial solution when the criteria are considered simultaneously. The solutions determine the allocation of the executors for carrying out the operations and the fixed earliest moment of starting the operations under the deterministic circumstances or the expected earliest moment of starting the operations under the stochastic circumstances. The generated solutions can be accomplished if the needed resources are at the executors' disposal at the starting moment and during the time of the operations execution. It is not always fulfilled.

## 5. IMPROVING OF THE SCHEDULE

The executors  $H^m \in H, m \in M$ , can carry out the operations  $u_j \in U$  according to the schedule  $S$  if the active elementary resources  $h \in H^m$ , and the passive elementary resources  $q \in Q^m$  are accessible in a sufficient amount. In the random circumstances those conditions are not always met. If so, the calculated expected starting moment  $E[V_i]$  can be delayed. It is assumed that the activities are rational. Under such circumstances, the indispensable elementary resources  $h \in H^m$  and  $q \in Q^m$  are available at the expected moment  $E[V_i^z] = E[V_i]$ , and that the expected delays  $E[\Delta V_i]$  of the events  $y_i \in Y$  are equal the expected resource delays  $E[\Delta V_i^z]$ . Hence, for the independent random variables  $T_{j,m}$  and  $\Delta V_i$ , the computational expected duration can be calculated by using the formula  $E[T_{j,m}^z] = E[T_{j,m}] + E[\Delta V_i]$  with the variance  $Var[T_{j,m}^z] = Var[T_{j,m}] + Var[\Delta V_i]$ . The influence of the resource delays  $E[\Delta V_i^z]$  can be determined by solving twice the formulated above scheduling problem: first using the expected duration  $E[T_{j,m}]$ , and the second using the computational expected duration  $E[T_{j,m}^z]$ . In this way the allocation of the executors  $H^m \in H$  and the earliest starting moment  $E[V_i]$  and  $E[V_i^z]$  for all operations are calculated. Next, the network  $S = \langle G, \{B, V\}, \{X, L, T, K\} \rangle$  can be analy-

zed for both situations by using the approach representative of the well known PERT method. But in this case, when there are the resource delays  $E[\Delta V_i^z] > 0$ , the starting moment  $E[V_i^z]$  of the event  $y_i \in Y$  is uncertain. The delay of this event depends on the resource delays only, that is  $E[\Delta V_i] = E[\Delta V_i^z]$ . The disruption and in the aftermath the delays  $E[\Delta V_i]$  of the next expected earliest starting moments  $E[V_i^z]$  of the events  $y_i \in Y$  are generated by the random technique, technology, organization, natural or system conditions of the construction works execution as well as the random accessibility of the needed resources.

## 6. EXPLANATORY EXAMPLE

The small bridge erection is considered as a construction project. The construction works execution technology of the bridge erection is modelled by using the network  $S^T = \langle G, B, L \rangle$ . The contractor, under the existing circumstances, determines the elementary executors  $H^m \in H$  available and suitable for carrying out the individual operations  $u_j \in U$  of the network  $S^T$ . Then, the fixed duration  $t_{j,m}$  or the expected duration  $E[T_{j,m}]$  and the fixed cost  $k_{f,j}^m$  or the expected cost  $E[K_{f,j}^m]$  are correspondingly estimated. One must note that the contractor may have more than one executor for any of the operations. The desirable allocation of the executors  $H^m \subset H$  to the operations  $u_j \in U$  and the fixed earliest starting moments  $v_i$  or the expected earliest starting moments  $E[V_i]$  can be fixed by solving the above described mixed linear scheduling problem. In this way, the network  $S = \langle G, \{B, V\}, \{X, L, T, K\} \rangle$  is fully defined, that is the schedule of the construction works execution is determined. If the resource disruptions exist, the schedule must be improved for that reason. With this end in view the expected delays  $E[\Delta V_i^z] = E[\Delta V_i]$  and the computational expected durations  $E[T_{j,m}^z] = E[T_{j,m}] + E[\Delta V_i^z]$  are defined for each operation  $u_j \in U$  and the suitable executor  $H^m \subset H$ .

**Table 1** List of the technological operations of construction works, the expected durations and the expected costs of the operations execution by the available and suitable elementary executors.

Operations	Nodes	Names of operations	Executors	Execution when the resources are fully available		Execution under the resource disruptions		Notes
				Expected duration	Expected cost	Expected duration	Expected cost	
1	2	3	4	5	6	7	8	9
1	1-2	Development works	1	25	15	28	25	
2	2-5	Earth works at abutment No. 1	2	22	28	24	28	
			3	16	22	18	32	
			4	12	45	14	45	
5	5-7	Shuttering of abutment No. 1	7	48	55	50	55	
			8	40	62	42	62	
			9	34	70	36	70	
7	7-9	Reinforcement of abutment No. 1	10	80	90	83	90	
			11	72	100	75	100	
			12	65	130	68	138	
9	9-11	Concreting abutment No. 1	13	3	20	4	30	
4	4-6	Earth works at abutment No. 2	2	20	28	26	28	
			3	12	35	18	25	
			4	10	36	16	36	
6	6-8	Shuttering of abutment No. 2	7	45	50	521	50	
			8	36	56	42	56	
			9	30	65	36	65	
8	8-10	Reinforcement of abutment No. 2	10	76	80	86	80	
			11	68	92	78	92	
			12	60	125	70	125	
10	10-11	Concreting abutment No. 2	13	2	20	3	20	
3	3-11	Preparing the auxiliary intermediate support	5	15	28	17	28	
			6	20	20	22	20	
11	11-14	Shuttering of the bridge span	7	52	48	35	60	
			8	42	70	51	70	
			9	40	80	42	80	
14	14-17	Reinforcement of the bridge span	10	90	110	95	110	
			11	80	120	84	120	
			12	70	140	73	140	
17	17-20	Concreting the bridge span	13	3	40	4	40	
12	12-15	Shaping the embankment at the abutment No. 1	2	35	30	38	30	
			3	30	34	33	34	
			4	24	40	26	40	
15	15-18	Preparation of the roadway bed at abutment No. 1	14	11	12	12	12	
			15	8	16	10	16	
18	18-20	Execution of the base course for the approach to abutment No. 1	14	12	10	14	16	
			15	9	18	10	18	
13	13-16	Shaping the embankment at abutment No. 2	2	20	20	22	20	
			3	18	24	20	24	
			4	16	30	18	24	
16	16-19	Preparation of the roadway bed at abutment No. 2	14	7	8	8	8	
			15	5	10	6	16	
19	19-20	Execution of the base course for the approach to abutment No. 2	14	12	10	12	10	
			15	9	18	7	12	
20	20-21	Laying the bridge roadway	16	7	20	8	20	
21	21-22	Finishing works	1	30	20	33	20	



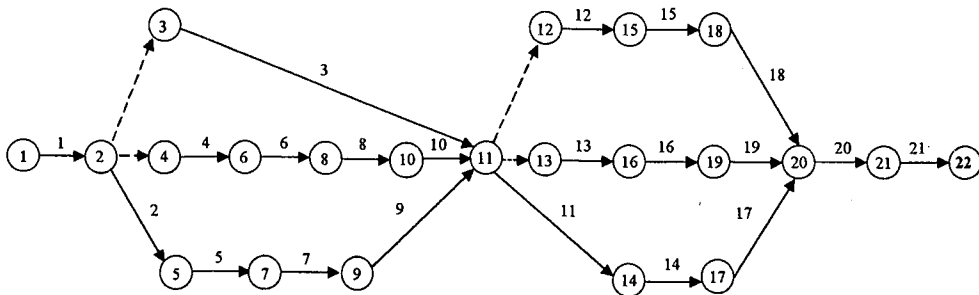


Fig. 2. Network model of the construction works execution technology

Table 2. The schedule of the construction works execution – the compromise solutions.

Operations	Nodes	Names of operations	Execution when the resources are fully available			Execution under the resource disruptions		
			Executors	Expected starting moments	Expected finishing moments	Executors	Expected starting moments	Expected finishing moments
1	2	3	4	5	6	7	8	9
1	1-2	Development works	1	0	25	1	0	28
2	2-5	Earth works at abutment No. 1	4	25	37	4	28	42
5	5-7	Shuttering of abutment No. 1	9	37	71	8	42	84
7	7-9	Reinforcement of abutment No. 1	11	71	143	10	84	167
9	9-11	Concreting abutment No. 1	13	143	151	13	167	171
4	4-6	Earth works at abutment No. 2	3	25	37	3	28	46
6	6-8	Shuttering of abutment No. 2	8	37	73	9	46	82
8	8-10	Reinforcement of abutment No. 2	10	73	149	11	82	160
10	10-11	Concreting abutment No. 2	13	149	151	13	160	171
3	3-11	Preparing the auxiliary intermediate support	6	25	151	6	28	171
11	11-14	Shuttering of the bridge span	7	151	203	7	171	206
14	14-17	Reinforcement of the bridge span	11	203	283	11	206	290
7	17-20	Concreting the bridge span	13	283	286	13	290	294
12	12-15	Shaping the embankment at the abutment No. 1	2	151	186	2	171	211
15	15-18	Preparation of the roadway bed at abutment No. 1	14	186	197	14	211	223
18	18-20	Execution of the base course for the approach to abutment No. 1	14	197	286	14	223	194
13	13-16	Shaping the embankment bed at abutment No. 2	3	151	169	2	171	191
16	16-19	Preparation of the roadway bed at abutment No. 2	14	169	176	14	191	199
19	19-20	Execution of the base course for the approach to abutment No. 2	14	176	286	14	199	294
20	20-21	Laying the bridge roadway	16	286	293	16	294	302
21	21-22	Finishing works	1	293	323	1	302	335

Solving the scheduling problem for the corrected data, the new allocation of the executors and the earliest expected starting moment are determined. The main data of the construction works execution technology are included in Table 1. The graphical model  $\mathcal{S}^T$  connected with the data is presented in Figure 2. The calculated results when the needed resources are fully available and when the resource disturbances exist are shown in Table 2.

## 7. FINAL CONCLUSIONS.

The method of cost-time scheduling presented above applies to the modeling and scheduling of construction works execution under the deterministic and stochastic circumstances. But in a practice it is possible only by using the computer scheduling system. Such system has been worked out. The data describing the construction works execution technology, the available and suitable resources, that is the network  $\mathcal{S}^T = \langle G, B, \mathcal{S}^T \rangle$

$L$ ), the functions  $T$  and  $K$  must be prepared by specialists who know the technical, technological, organizational and economic problems of the construction works execution of the given project. Entering the data into the computer system is easy and can be done almost by anyone. The data and the set of the acceptable solutions are automatically analyzed. The unicriterial and bi-criteria solutions are generated. The method can be easily expanded into multi-criteria method by using also other criteria simultaneously with the two used above. The submitted example of the small bridge erection illustrates the case of scheduling under risk circumstances. It indicates the differences between the solutions when the needed resources are fully available at the expected starting moments and when the resource disruptions exist. The differences concern the executors' allocation, the expected starting and finishing moments and the probability of the final deadline of the construction works execution. This information enable one to evaluate the process of the execution and, if necessary, to make corrections. The method is

important for analyzing the cost and time of the construction works execution from the following viewpoint: the minimum time, the minimum cost and the compromise way (the minimum cost and the minimum time simultaneously) of the execution. In the explanatory example – respectively: 289 days and 954 thousand zlotych (Polish currency), 618 days and 744 thousand zlotych, 323 days and 809 thousand zlotych. In the case of uncertainty the method can be used when the subjective probability for all mentioned above quantities can be determined. The described above problems are currently researched.

## REFERENCES

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