J. Materials, Conc. Struct., Pavements, JSCE, No.648／V-47, 243-252, 2000 May

INFLUENCE OF CORRELATION AMONG DEFLECTION DATA ON THE BACKCALCULATION RESULTS

James MAINA[1], Hiroshi YOKOTA[2], Kunihito MATSUI[3] and Isao KUROBAYASHI[4]

[1]Student Member of JSCE, Postgraduate Student, Dept. of Civil and Environmental Eng., Miyazaki University (1-1 Gakuen Kibanadai Nishi, Miyazaki 889-2192, Japan)
[2]Member of JSCE, Professor, Dept. of Civil and Environmental Eng., Miyazaki University (1-1 Gakuen Kibanadai Nishi, Miyazaki 889-2192, Japan)
[3]Fellow of JSCE & Member of ASCE, Professor, Dept. of Civil Eng., Tokyo Denki University (Hatoyama, Hiki, Saitama 350-0394, Japan)
[4]Student Member of JSCE, Graduate Student, Dept. of Civil Eng., Tokyo Denki University (Hatoyama, Hiki, Saitama 350-0394, Japan)

A study on how correlation among FWD deflection data influences accuracy of the backcalculated pavement layer moduli is presented. FWD data were obtained from the Ministry of Construction, Road Test Section 609 and the data with stronger correlation were used in this study. Deflections without correlation and with similar correlation to the FWD data were generated using Monte Carlo simulation in order to investigate the effect of correlation on the backcalculation results. Furthermore, backcalculation analysis using theoretical deflection data with the same standard deviation but different correlations showed smaller variation in the backcalculation results for the deflection data with stronger correlation.

Key Words: *falling weight deflectometer (FWD), deflection, correlation, backcalculation accuracy, pavement layer moduli, Monte Carlo simulation.*

1. INTRODUCTION

Presently, there are several nondestructive testing (NDT) devices that are being used by various organizations to perform pavement deflection tests. Pavement deflections, although provide valuable information for structural pavement evaluation, are highly dependent on loading mode and magnitude as well as the precision of the device[1),2),3)].

In Japan, pavement deflection tests are mostly carried out using falling weight deflectometer (FWD) devices. When different FWD devices are used to measure pavement deflections at one section, in some cases, their deflection values as well as coefficients of variation (COV) turn out to be different. The differences could be attributed to the fact that even though FWD devices operate on similar principles, they have three significant differences;

1. The force generating unit - One-mass or Two-mass system
2. The method of load distribution on the pavement - Un-segmented plate with thin hard rubber pad or segmented plate with thick, soft rubber pad
3. The method of deflection measurement – Velocity transducers, geophones or seismometers.

Furthermore, a number of researchers have also studied the correlation between different NDT devices, and results have showed higher as well as lower coefficients of correlation[1),2),3)].

This study attempts to move a step further and investigate the effects that correlation among the deflection data from the same FWD device on one pavement section would have on the backcalculation results. It is hoped that pavement engineers will use findings from this study to evaluate reliability of the deflection data and, consequently, the accuracy of their backcalculation results.

FWD data from Ministry of Construction, Road Test Section 609 were used in this study. Backcalculated layer moduli obtained by using these data were compared with the results from the data that were obtained from Monte Carlo simulation[4)], which randomly generated deflections without correlation and with similar correlation to that of FWD data. Another investigation on the influence of correlation on the backcalculated results was performed by using theoretical correlation with theoretical deflection data. By using strong theoretical correlation matrix of deflections, the effects of errors in the asphalt concrete layer thickness were also studied and are reported in this paper.

Table 1.1 FWD deflections using 49 kN (FWD1)

	D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}
Mean deflection (μm)	1033	815	495	378	243	136
Standard Deviation	14.4	12.2	3.4	3.7	3.5	2.6
Coefficient of Variation	0.014	0.015	0.007	0.010	0.014	0.019

Table 1.2 FWD deflections using 49 kN (FWD2)

	D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}
Mean deflection (μm)	998	827	533	413	269	146
Standard Deviation	6.7	6.0	5.7	2.9	3.3	1.8
Coefficient of Variation	0.007	0.007	0.011	0.007	0.012	0.013

Table 2.1 Correlation matrix of FWD test results (FWD1)

	D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}
D_0	1	0.921	0.862	0.672	0.378	0.093
D_{20}		1	0.851	0.678	0.295	0.073
D_{45}			1	0.685	0.346	0.264
D_{60}				1	0.369	0.157
D_{90}					1	0.327
D_{150}						1

Table 2.2 Correlation matrix of FWD test results (FWD2)

	D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}
D_0	1	0.485	0.578	0.097	0.437	0.103
D_{20}		1	0.440	0.205	0.356	0.012
D_{45}			1	-0.112	0.446	-0.019
D_{60}				1	0.420	0.156
D_{90}					1	0.260
D_{150}						1

E_{O1}, $\nu_1 = 0.35$, $h_1 = 9$ cm

E_{O2}, $\nu_2 = 0.35$, $h_2 = 14$ cm

E_{O3}, $\nu_3 = 0.35$, $h_3 = 22$cm

E_{O4}, $\nu_4 = 0.35$

(E_{Oi}, ν_i, h_i = Young's modulus, Poisson's ratio, and thickness of layer i)

Fig.1 Pavement structure for Road Test Section 609

2. DESCRIPTION OF FWD RESULTS

FWD measurement tests were conducted in 1991 on the Ministry of Construction, Road Test Section 609. Pavement structure of the Road Test Section 609 is shown in **Fig.1**. Since in-situ Poisson's ratios are not well defined and a small variation does not have any significant effect on the backcalculation results[5], practically reasonable values were assumed. Two deflection tests represented as FWD1 and FWD2 were carried out at the same point, 50 times each. Deflections were measured at 0 cm, 20 cm, 45 cm, 60 cm, 90 cm, and 150 cm points relative to the center point of loading and normalized with respect to 49 kN in order to remove the influence of test load variation on deflections. Diameter of the loading plate was 30 cm. Temperature correction was not considered because, if necessary, it should be performed on the backcalculated layer moduli. Mean, standard deviation as well as coefficient of variation of the measured deflection data for FWD1 and FWD2 were computed and are shown in **Table 1.1** and **Table 1.2**, respectively. Mean deflection data were well compared. Since the tests were performed at the same point, the differences in standard deviations as well as coefficients of variation could be attributed to the fundamental differences between the FWD devices. Correlation matrices of the measured deflection data for FWD1 and FWD2 were also computed and are presented in **Table 2.1** and **Table 2.2**, respectively. Their differences were also due to the differences between the FWD devices.

3. BACKCALCULATION OF LAYER MODULI

(1) General

Backcalculation of pavement layer moduli was performed using a computer program called MUJI-F, which was developed by one of the authors[6]. This program uses BISAR[7] as one of its subroutine. 50 sets of deflection data that were obtained from actual FWD deflection test for FWD1 were used in the analysis because they had relatively stronger correlation (see **Table 2.1**) and their backcalculation results are presented in this paper.

The procedure followed here was to calculate mean deflection values for every 5 sets of FWD deflection data. This resulted into 10 sets of new deflection data. Mean deflection values were computed and later used in the backcalculation analysis in order to minimize the effect of repeatability error on the measured deflections[8), 9)]. Repeatability errors are random. The mean deflection value would, therefore, give an improved estimate of the true reading. According to statistical theory, for N repeated loading drops, the standard error of the mean would be reduced from that of a single observation by the reciprocal of the square root of N [9)].

In the system identification process, the aim is to compute for pavement layer parameters that would give deflection basin, which closely resembles that of the measured one. As explained earlier, the only pavement layer parameters that are still considered unknown are the layer moduli values E_j (j = 1,2,..,m), where m is the number of pavement layers. In this research, m = 4 (see **Fig.1**). Symbolically, the computed response would then be written as $z_i(E_1,E_2,E_3,E_4) = z_i(E_j)$ while, the measured response would be written as u_i where i (= 1,2,...,n) means a deflection measurement point. In this research, the number of deflection measurement points, n, was equal to 6.

(2) Formulation of the system identification

A modified Gauss-Newton[10)] method was used in the process of estimating pavement layer moduli values, E_j, which would give the computed deflection basin that is well compared to the measured one. The process of estimating pavement layer moduli is an iterative one, where computations are repeated while modifying layer moduli until the rate of change of layer moduli is less or equal to a tolerance value. In this study, a tolerance value for all computations was set to 0.001. A very small tolerance value assures that the results obtained would be independent of the initial value. This would be achieved by minimizing the sum of squares of the differences between measured and computed pavement surface deflections. The objective function, **J**, in the minimization could then be written as

$$\mathbf{J} = \sum_{i=1}^{n}(u_i - z_i(E_j))^T(u_i - z_i(E_j)) \qquad (1)$$

However, since this is an iterative procedure, an increment dE_j^k should be found and added to E_j^k, at the k-th iteration and j-th layer, in order to estimate E_j^{k+1}, at the (k+1)-th iteration and j-th layer, thus making the calculated deflection equal to $z_i(E_1,\ldots,E_j+dE_j,\ldots,E_4)$. On generalizing and employing Taylor series expansion, yields

$$\mathbf{z}(\mathbf{E}+d\mathbf{E}) = \mathbf{z}(\mathbf{E}) + \mathbf{A}d\mathbf{E} \qquad (2)$$

where

$\mathbf{A} = \partial\mathbf{z}/\partial\mathbf{E}$ is an $n \times m$ matrix

The approximation of this differential equation could be obtained by using the modified Gauss-Newton method. Replacing **z(E)** with **z(E**+d**E)** and substituting Eq. (2) into Eq. (1), yields

$$\mathbf{J} = \{[\mathbf{u} - \mathbf{z}(\mathbf{E})] - \mathbf{A}d\mathbf{E}\}^T\{[\mathbf{u} - \mathbf{z}(\mathbf{E})] - \mathbf{A}d\mathbf{E}\} \qquad (3)$$

On minimizing, by partially differentiating **J** with respect to d**E**, yields

$$\frac{\partial\mathbf{J}}{\partial(d\mathbf{E})} = 0 = -2\mathbf{A}^T(\mathbf{u} - \mathbf{z}(\mathbf{E})) + 2\mathbf{A}^T\mathbf{A}d\mathbf{E} \qquad (4)$$

On simplification, Eq.(4) becomes

$$\mathbf{A}^T\mathbf{A}d\mathbf{E} = \mathbf{A}^T(\mathbf{u} - \mathbf{z}(\mathbf{E})) \qquad (5)$$

By applying singular value decomposition (SVD)[11)] to **A** gives

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{T}} \qquad (6)$$

where

$\mathbf{U} = n \times m$ matrix,
$\mathbf{D} = m \times m$ diagonal matrix,
$\mathbf{V}^{\mathrm{T}} = m \times m$ matrix and is a transpose of **V**, and
$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{V}\mathbf{V}^{\mathrm{T}} = 1$ (since **V** is a square matrix)

Substituting Eq.(6) into Eq.(5) and expand while considering the above conditions for **A** yields

$$\mathbf{V}\mathbf{D}^2\mathbf{V}^T d\mathbf{E} = \mathbf{V}\mathbf{D}\mathbf{U}^T(\mathbf{u} - \mathbf{z}(\mathbf{E})) \qquad (7)$$

Stepwise pre-multiplication of both sides of Eq.(7) by $\mathbf{V}^T$ followed by $\mathbf{D}^{-2}$ and **V** yields

$$d\mathbf{E} = \mathbf{V}\mathbf{D}^{-1}\mathbf{U}^T(\mathbf{u} - \mathbf{z}(\mathbf{E})) \qquad (8)$$

Using Eq.(8), a back substitution routine could be used to obtain a solution vector d**E** given that SVD of matrix **A** has already been obtained[11)].

Modified layer moduli would then become

$$E_j^{k+1} = E_j^k + dE_j^k \qquad (9)$$

Table 3 Mean backcalculation results using FWD deflection data

[1 kgf/cm² = 98 kPa]

	E_1	E_2	E_3	E_4
Mean Layer Moduli (kgf/cm²)	33179.4	1670.5	570.5	658.8
Standard Deviation	1859.0	229.8	21.0	5.5
Coefficient of Variation	5.60	13.76	3.68	0.84
Confidence Interval (95%)	[32922, 33437]	[1639, 1702]	[568, 573]	[658, 660]

As explained earlier, iteration process would continue until rate of change of the layer moduli is less or equal to the tolerance value. Backcalculated results obtained are shown in **Table 3**. Mean layer moduli shown in **Table 3** are averages of the 10 backcalculated layer moduli obtained by using 10 sets of deflection data.

4. GENERATING DEFLECTION BASINS

(1) General

Having performed backcalculation of the measured FWD deflection data, the next step was to investigate the effect of correlation among the deflection data on the backcalculation results. This objective would only be achieved through the use of two types of theoretically generated deflection data. These two types of deflection data would be generated by (1) not considering correlation among actual FWD deflection data and (2) considering correlation among actual FWD deflection data.

In order to obtain the first type of deflection data mentioned above, Monte Carlo simulation technique, was used. This technique was selected because it can make use of randomly generated numbers to obtain sensor deflections that would be independent of each other. The random number generator that was used can generate a very large number of data, up to 10 million, with no periodicity[11]. The technique of deflection simulation involves addition or subtraction of a random portion of the standard deviation to the measured FWD deflection at each sensor point. The deflection data generated using this technique may, therefore, be considered as true representatives of the FWD data.

In this research, Monte Carlo simulation technique was used to generate 1000 sets of normally distributed random deflection basins with same mean deflection values and standard deviations as the FWD data. Since only limited FWD tests are performed at one measurement point and considering the influence of random errors on the FWD data, it was important to generate a larger number of deflection sets in order to minimize the repeatability errors caused by deviations in sensor measurements.

Since deflection data that are randomly generated using Monte Carlo simulation are independent, there would be no correlation among the generated deflection data (see **Table 4**). These deflection data would be of the first type explained earlier.

(2) Introduction of correlation matrix into Monte Carlo simulation

For the purpose of generating the second type of deflection data mentioned earlier, correlation matrix was introduced into the Monte Carlo simulation procedure. The resulting 1000 sets of randomly generated deflection data would have similar correlation to the FWD deflection data. The method of introducing correlation matrix into the random generation of deflection data is as follows:

1. Consider the objective correlation matrix, which is a square matrix, as S. Applying singular value decomposition to S, gives

$$\mathbf{S} = \mathbf{U}\mathbf{D}\mathbf{V}^T \tag{10}$$

Since S is a square matrix and symmetric, then

$$\mathbf{U} = \mathbf{V} \tag{11}$$

thus making

$$\mathbf{S} = \mathbf{V}\mathbf{D}\mathbf{V}^T \tag{12}$$

On decomposing the correlation matrix while taking into consideration that $\boldsymbol{D}$ is a diagonal matrix, yields

$$\begin{aligned} \mathbf{S} &= (\mathbf{V}\mathbf{D}^{1/2})\mathbf{D}^{1/2}\mathbf{V}^T \\ \mathbf{S} &= \mathbf{V}\mathbf{D}^{1/2}\cdot\mathbf{D}^{1/2}\mathbf{V}^T \\ \Rightarrow \mathbf{V}\mathbf{D}^{1/2} &= \overline{\mathbf{S}},\quad \mathbf{D}^{1/2}\mathbf{V}^T = \overline{\mathbf{S}}^T \\ \therefore \mathbf{S} &= \overline{\mathbf{S}}\cdot\overline{\mathbf{S}}^T \end{aligned} \tag{13}$$

2. Consider $\varepsilon = \{\varepsilon_i\}$ as a vector of uniform random numbers with average value of 0 and standard deviation of 1.

3. Consider the average value, $\overline{u_i}$, of the FWD test

Table 4 Correlation matrix of generated deflection data without correlation and with similar correlation to the FWD data

	D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}		D_0	D_{20}	D_{45}	D_{60}	D_{90}	D_{150}
D_0	1	-0.028	-0.007	0.007	-0.003	0.0004		1	0.923	0.868	0.665	0.355	0.074
D_{20}		1	0.019	0.030	0.006	0.021			1	0.846	0.670	0.269	0.037
D_{45}			1	-0.012	-0.008	0.014				1	0.681	0.328	0.231
D_{60}				1	-0.040	-0.034					1	0.364	0.142
D_{90}					1	-0.005						1	0.340
D_{150}						1							1

Table 5 Mean backcalculation results using generated deflection data without correlation

[1 kgf/cm² = 98 kPa]

	E_1	E_2	E_3	E_4
Mean Layer Moduli (kgf/cm²)	34038	1617	611	658
Standard Deviation	6351.6	496.3	165.2	6.4
Coefficient of Variation	18.66	30.70	27.03	0.97
Confidence Interval (95%)	[33158, 34919]	[1548, 1685]	[588, 634]	[658, 659]

Table 6 Mean backcalculation results using generated deflection data with similar correlation to the FWD data

[1 kgf/cm² = 98 kPa]

	E_1	E_2	E_3	E_4
Mean Layer Moduli (kgf/cm²)	33400	1652	571	659
Standard Deviation	1897.3	186.8	28.6	5.4
Coefficient of Variation	5.68	11.31	5.00	0.82
Confidence Interval (95%)	[33138, 33663]	[1626, 1678]	[567, 575]	[658, 660]

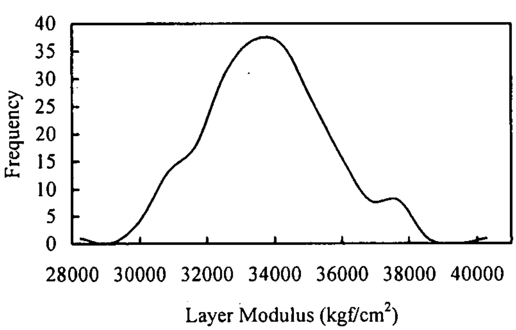

Fig.2 Normal distribution of backcalculated layer modulus, E_1

results and standard deviation as σ_i. Then

$$U_i = \frac{u_i - \overline{u_i}}{\sigma_i} \tag{14}$$

where

u_i : randomly generated normally distributed number

U_i : normalized u_i values. This value is related to the uniform random number in the following manner;

$$\mathbf{U} = \overline{\mathbf{S}} \cdot \varepsilon \tag{15}$$

where

$$\mathbf{U} = \{ U_i \} \tag{16}$$

Considering $\overline{\mathbf{u}} = \{ \overline{u_i} \}$ and a diagonal matrix $\mathbf{Q} = \{ \sigma_i \}$ then, transformation of Eq. (14), gives

$$\mathbf{u} = \mathbf{QU} + \overline{\mathbf{u}} \tag{17}$$

Substituting Eq. (15) into Eq. (17), yields

$$\mathbf{u} = \mathbf{Q}\overline{\mathbf{S}}\varepsilon + \overline{\mathbf{u}} \tag{18}$$

Using Eq. (18), deflection data with similar correlation to that of the FWD data would be randomly generated (see **Table 4**).

(3) Backcalculation analysis using randomly generated deflection data.

Backcalculation analysis of the generated deflection data was carried out in a similar manner to the

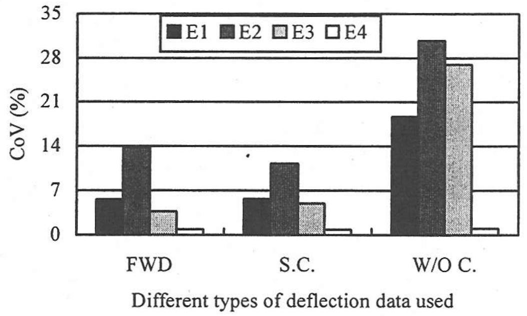

Fig.3 Coeffiecient of variation for layer moduli

E_{O1} = 29342 kgf/cm^2, ν_1 = 0.35, h_1 = 9 cm

E_{O2} = 1770 kgf/cm^2, ν_2 = 0.35, h_2 = 14 cm

E_{O3} = 665 kgf/cm^2, ν_3 = 0.35, h_3 = 22cm

E_{O4} = 639 kgf/cm^2, ν_4 = 0.35

(E_{Oi} , ν_i , h_i = Young's modulus, Poisson ratio, and height of layer i)
[1 kgf/cm^2 = 98 kPa]

Fig.4 Hypothetical pavement model

case of the measured FWD deflection data. Mean deflection values for every 5 sets of the generated deflections were used. Since the generated sets of deflection data were 1000, the resulting backcalculated pavement layer moduli were 200. Then, the mean values of the 200 sets of backcalculation results were taken as final backcalculation results of the pavement layers.

Overall, mean deflection values and standard deviations for the generated deflection data without correlation and with similar correlation to that of the FWD data were well compared. Backcalculation results for the two cases are presented in **Table 5** and **Table 6**.

As an example, distribution pattern of the backcalculated layer modulus, E_1, of the first layer of the pavement, is presented in **Fig.2**. The results presented in the Figure were obtained by using deflection data with similar correlation to that of the FWD data. This figure depicts a very well defined normally distribution pattern of the backcalculation results. The normal distribution pattern of the backcalculation results might be due to relatively smaller coefficient of variation of the deflection data used, which were randomly generated by Monte Carlo simulation technique.

Backcalculation results presented in **Table 6** are very close to the results obtained by using the originally measured FWD deflection data (see **Table 3**). Standard deviations as well as coefficients of correlation in **Table 6** and **Table 3** are well compared. The opposite case could be found for the results presented in **Table 5**. Backcalculation results are much bigger than the ones shown in **Table 3** and also standard deviation and coefficient of variation values do not agree well with the ones shown in **Table 3**. These findings quite obviously indicate the importance of randomly generating deflection data that have similar correlation to the original data in order to reduce variation in the backcalculation results as well as to obtain backcalculation results that quite closely represent the results of the original deflection data.

For a clear understanding of the backcalculation results, the results of coefficients of correlation obtained by using measured FWD deflections, deflection data without correlation and deflection data with similar correlation to the measured FWD deflections are presented in **Fig.3**. In this figure, FWD means results obtained by using the measured FWD deflection data, S.C. means results obtained by using deflection data with similar correlation to the measure FWD deflection data, and W/O C. means results obtained by using deflection data without correlation among the data. Coefficient of variation is presented in order to show the effect of correlation among deflection data on the accuracy of the backcalculation results.

5. BACKCALCULATION ANALYSIS OF THEORETICAL DEFLECTION DATA

A four-layer hypothetical pavement system shown in **Fig.4** was used for the theoretical backcalculation analysis. The pavement cross section is similar to that of the Test Road Section 609, but the layer properties are only assumed values. In order to obtain theoretical pavement deflections, a theoretical static loading with the magnitude of 49 kN and a loading plate of 30 cm in diameter, were assumed.

A static elastic multilayer program, BISAR was used for the computation of static deflection basins of the pavement. Theoretical deflections were computed for the following six sensor positions relative to the point of loading; 0 cm, 20 cm, 45 cm, 60 cm, 90 cm, and 150 cm.

Table 7 Mean backcalculation results of generated deflection data using theoretical correlation matrix

	E_1	E_2	E_3	E_4
Theoretical Layer Moduli	29342	1770	665	639
ρ_{30} = **0.5**	29587	1741	672	638
ρ_{30} = **0.3**	29570	1742	673	638
ρ_{30} = **0.1**	29553	1743	674	638
ρ_{30} = **0.001** (a is negative)	29643	1738	679	638
ρ_{30} = **0.001** (a is positive)	29614	1742	678	638
Without Correlation	30143	1704	689	638

(unit: kgf/cm^2 : [1 kgf/cm^2 = 98 kPa])

Table 8 Coefficient of variation for the backcalculation results of generated deflection data using theoretical correlation matrix

	E_1	E_2	E_3	E_4
ρ_{30} = **0.5**	3.59	5.30	2.75	0.48
ρ_{30} = **0.3**	4.65	6.92	3.47	0.48
ρ_{30} = **0.1**	6.25	9.34	4.46	0.49
ρ_{30} = **0.001** (a is negative)	10.10	15.01	6.86	0.50
ρ_{30} = **0.001** (a is positive)	10.40	16.19	7.14	0.65
Without Correlation	13.33	19.76	9.14	0.64

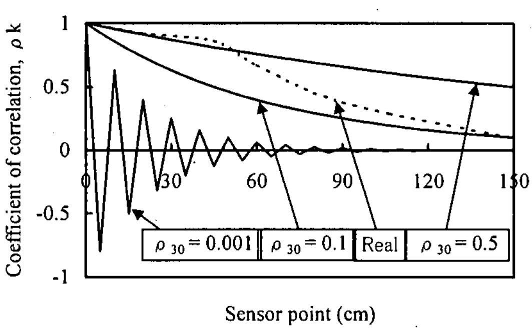

Fig.5 Theoretical correlation coefficient

(1) Generation of theoretical deflection basins

Since the objective of this study was to investigate the effect of correlation among deflection data on the backcalculation results, theoretical study was aimed at establishing the effect of strong, weak and no correlation by the use/no use of theoretical correlation matrices.

Theoretical correlation matrices were developed using[12)]

$$\rho_k = (-a)^k \qquad (19)$$

where

k : number of 5 cm spans covering the whole length to the sensor point, e.g. sensor at 150 cm → k = 30 (according to this study)

ρ_k : coefficient of correlation at the *k-th* number

$\rho_0 = 1$

$\rho_{i,j} = \rho_{j,i} = \rho_{|i-j|}$

a : constant

Setting ρ_{30} = 0.5, 0.3, 0.1, 0.001, and 0.001 with corresponding values of a = -0.977, -0.961, -0.926, -0.794 and 0.794, it was possible to compute the required theoretical correlation matrix for each case. The above values would determine how strong the correlation matrix would be. Bigger values of ρ_{30} would mean relatively stronger correlation matrix. An example of the plots of coefficients of correlation for ρ_{30} = 0.5, 0.1, 0.001 (a is positive), and real correlation for FWD data is shown in **Fig.5**.

In order to reduce error of deflection computations for the hypothetical pavement model, 1000 sets of normally distributed deflection data were randomly generated. The procedure followed was similar to the one already explained in the preceding chapters. However, through theoretical computations, only one set of deflection data was obtained. In this regard, the theoretical deflections at each measured sensor point were considered as mean deflections. A standard deviation of deflection values was assumed to be 1 % of the deflection value at each sensor point. By using theoretical mean deflection data, assumed standard deviation, and each of

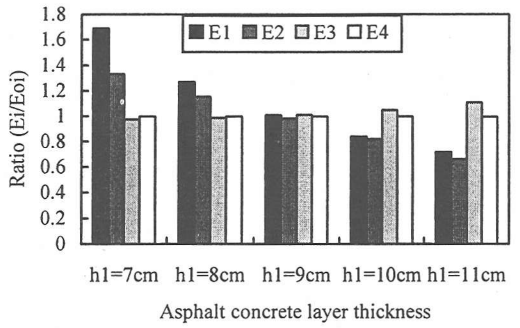

Fig.6 Ratio of backcalculated to theoretical layer moduli

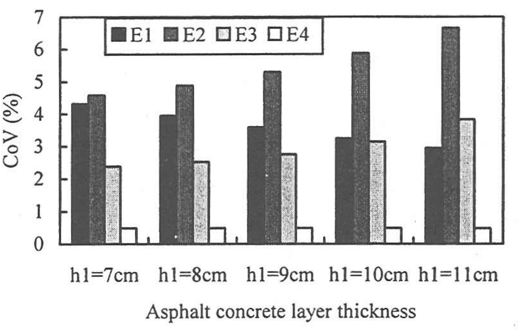

Fig.7 Coefficient of variation for the backcalculation results

the 5 theoretical correlation matrices or no correlation, 1000 sets of normally distributed deflection data (for each case) were randomly generated by the Monte Carlo simulation technique.

Backcalculation results obtained from the theoretical analysis are shown in **Table 7** and **Table 8**. The backcalculated layer moduli, when compared to the theoretical values assumed in the hypothetical pavement system, show that when correlation matrix is used, the results would not differ very much from the actual values. Furthermore, coefficients of variation obtained show that stronger correlation would give backcalculation results with higher accuracy. However weaker the correlation matrix used in the generation of deflection data might be, the overall backcalculation results would have lower coefficient of variation than for the case where deflection data without correlation were used (see **Table 8**). The differences in the results are clearer in the top layers.

6. ERRORS IN LAYER THICKNESS

After carrying out a number of computational investigation as stipulated in the preceding chapters, there was a need to investigate the influence of strong correlation on the backcalculation results in case of errors in the first pavement layer (asphalt concrete layer) thickness, h_1. Pavement first layer was selected for the analysis because previous studies have indicated that backcalculation results are mostly affected when there are errors in the pavement first layer thickness[13]. Strong correlation in this case was taken to mean, $\rho_{30} = 0.5$. Strong correlation was selected because it was found, in the preceding chapter, that higher accuracy of the backcalculation results was obtained when deflection data were generated using strong correlation matrix. Backcalculation analysis was performed for the cases where h_1, for the hypothetical pavement model (**Fig.5**), was varied by ± 1 cm and ± 2 cm while base course and subbase course thicknesses were kept constant. Pavement layer thickness errors were introduced during backcalculation analysis. This means original deflection basin was computed using true values of the pavement cross section. Backcalculation procedure followed was similar to all the preceding backcalculation cases.

Fig.6 shows ratios of mean backcalculated layer moduli, E_i (i =1 ~ 4) to theoretical layer moduli, E_{Oi} (i =1 ~ 4), as shown in **Fig.4**, for each case of asphalt concrete layer thickness, h_1. Coefficients of variation of layer moduli are shown in **Fig.7**. These results show that however strong the correlation matrix might be, if there is an error in h_1, backcalculation results would be affected. The effect was higher in the case of pavement first and second layer moduli values.

7. CONFIDENCE REGION FOR STRAINS

Strains play a key role in pavement design and management, especially, in the computations of the number of axle load applications that cause rutting failure or fatigue cracking during the determination of pavement remaining life. Radial strain ε_x at the bottom of the top layer, which is normally correlated to fatigue cracking and vertical strain ε_z at the top of the bottom layer, which has been correlated to rutting failure are computed for that purpose.

These strains, which can be used as input to the distress models (fatigue and rutting), would be affected by the accuracy of the deflection measurements as well as the backcalculated layer moduli. The distress models are very sensitive to the input values and a small variation in the strains is expected to cause a substantial difference in the number of axle load applications for both models.

If ε_x and ε_z are normally distributed, then their confidence region could be given by

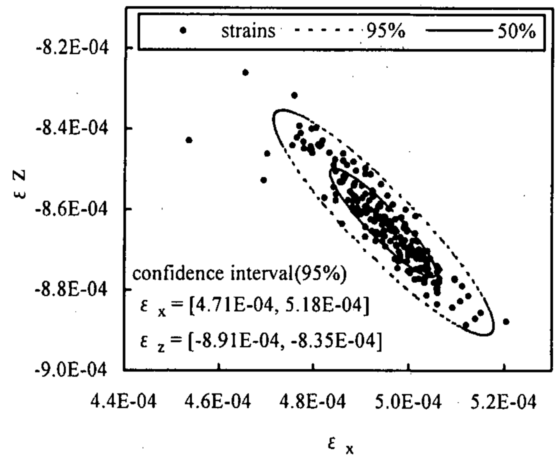

Fig.8 Confidence region for strains (W/O C.)

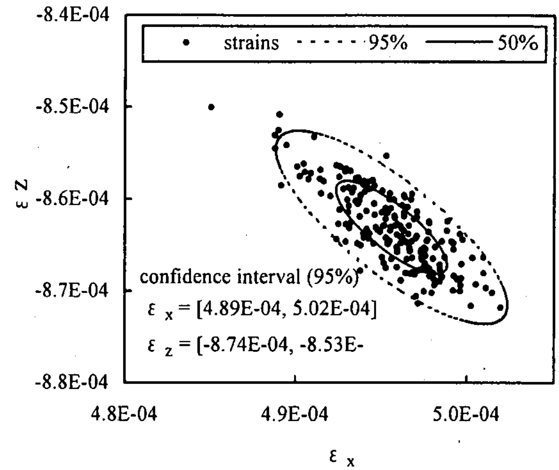

Fig.9 Confidence region for strains (S.C.)

$$(\boldsymbol{\varepsilon}-\bar{\boldsymbol{\varepsilon}})^T \Sigma^{-1}(\boldsymbol{\varepsilon}-\bar{\boldsymbol{\varepsilon}}) \le \chi^2_{\alpha,2} \quad (20)$$

where

$\boldsymbol{\varepsilon}$: a vector composed of strains ε_x and ε_z

$\bar{\boldsymbol{\varepsilon}}$: a vector composed of mean values of strains ε_x and ε_z

Σ^{-1} : inverse of covariance matrix

$\chi^2_{\alpha,2}$: chi-squared distribution for confidence limit of 100(1-α)% and a number of degrees of freedom equal to the number of variables (=2)

Covariance matrix may be written as

$$\Sigma = \begin{bmatrix} \sigma_{x,x} & \sigma_{x,z} \\ \sigma_{x,z} & \sigma_{z,z} \end{bmatrix} \quad (21)$$

where

$\sigma_{x,x}$: Variance of strain ε_x

$\sigma_{z,z}$: Variance of strain ε_z

$\sigma_{x,z} (= \rho_{x,z}\sqrt{\sigma_{x,x}}\sqrt{\sigma_{z,z}})$: Variance of strains ε_x and ε_z

$\rho_{x,z}$: Correlation of strains ε_x and ε_z

Thus, inverse matrix would be

$$\Sigma^{-1} = \frac{1}{\sigma_{x,x}\sigma_{z,z} - \sigma^2_{x,z}} \begin{bmatrix} \sigma_{z,z} & -\sigma_{x,z} \\ -\sigma_{x,z} & \sigma_{x,x} \end{bmatrix} \quad (22)$$

By substituting Eq.(22) into Eq.(20) and expand, yields

$$\therefore \quad \frac{\sigma_{z,z}(\varepsilon_x - \bar{\varepsilon}_x)^2 + \sigma_{x,x}(\varepsilon_z - \bar{\varepsilon}_z)^2}{\sigma_{x,x}\sigma_{z,z}(1-\rho^2_{x,z})} - \frac{2\rho_{x,z}\sqrt{\sigma_{x,x}}\sqrt{\sigma_{z,z}}(\varepsilon_x - \bar{\varepsilon}_x)(\varepsilon_z - \bar{\varepsilon}_z)}{\sigma_{x,x}\sigma_{z,z}(1-\rho^2_{x,z})} \le \chi^2_{\alpha,2} \quad (23)$$

By specifying the value of α equals to 0.05 and 0.5, the values of chi-squared distributions could be determined from the standard chi-squared distribution table. Having determined the chi-squared distributions, the step that followed was to compute the confidence regions that fell within the two dimensional limits provided. By using Eq.(20) with the equality and perform rotation of axes, the confidence region obtained would be in the form of an ellipse with an inclined plane. As examples, results of strains for the case of deflection data without correlation and the case of deflection data generated by using theoretical correlation matrix of $\rho_{30} = 0.5$ are shown in **Fig.8** and **Fig.9**, respectively. The dotted and solid lines represent 95% and 50% confidence regions, respectively. The total number of plotted dots is 200 and about 6 points and 9 points fall out of the 95% confidence regions in **Fig.8** and **Fig.9**, respectively. Confidence intervals (95%) in **Fig.9** in both vertical [0.21E-04] and horizontal [0.13E-04] axes cover about one-third (1/3) of the vertical [0.56E-04] and horizontal [0.47E-04] intervals shown in **Fig.8**. Smaller intervals would mean higher accuracy.

Effects of errors in the estimation of strains will be quantitatively examined in our future research work.

8. CONCLUSIONS

Backcalculation analyses were performed covering a wider range of deflection data like actual FWD deflection data, randomly generated deflection data without correlation and with similar correlation to FWD data, theoretical deflection generated using 5 theoretical correlation matrices, and generated theoretical deflection without correlation. Furthermore, analysis was performed while considering errors in pavement first layer thickness. Findings from this research will help pavement engineers determine the reliability of their FWD deflection data and the accuracy of the backcalculation results obtained.

The overall results obtained have led into the following conclusions;

(1) It is possible to randomly generate deflection data with similar correlation to the original FWD deflection data.

(2) It is possible to derive theoretical correlation matrix using mathematical relations.

(3) Backcalculation results obtained by using FWD data and deflection data with similar correlation to that of FWD data are well compared.

(4) Backcalculation results obtained by using deflection data without correlation among the deflection data have relatively poor accuracy.

(5) Backcalculation results obtained when stronger correlation matrix is used in the generation of deflection data have relatively smaller variation.

(6) Effects of errors in the pavement first layer thickness on the backcalculation results would not be cancelled even when there is a stronger correlation among the deflection data.

(7) There is a small variation of strains when deflections with correlation among the data are used in the backcalculation of layer moduli.

ACKNOWLEDGEMENT: The authors acknowledge the support from all the people, at Miyazaki University and Tokyo Denki University whose hard work contributed to the successful conclusion of this study.

REFERENCES

1) Hoffman, M. S., and Thompson, M. R.: Comparative Study of Selected Nondestructive Testing Devices, *Transportation Research Record*, *852*, pp.32-41, 1982.
2) Thollen, O., Sharma, J., and Terrel, R. L.: Comparison of Falling Weight Deflectometer with Other Deflection Testing Devices, *Transportation Research Record*, *1007*, pp.20-25, 1985.
3) Bentsen, R. A., Nazarian, S., and Harrison, J. A.: Reliability Testing of Seven Nondestructive Pavement Testing Devices, *Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026*, A. J. Bush III and G. Y. Baladi, Eds., American Society of Testing and Materials, Philadelphia, pp.41-58, 1989.
4) Ang, A. H-S., and Tang, W. H.: Probability Concepts in Engineering Planning and Design, Vol. I and Vol. II, *John Wiley and Sons*, New York, 1984.
5) Maestas, J. M., and Mamlouk, M. S.: Comparison of Pavement Deflection Analysis Methods Using Overlay Design, *Transportation Research Record*, *1377*, pp.17-25, 1992.
6) Matsui, K., Inoue, T., and Sanpei, T.: Development of Analytic Method for the Stiffness Estimation of Pavement System, *JSCE*, No. 420/V-13, pp.107-114, 1990. (in Japanese)
7) De Jong, D. L., Peutz, M. G. F., and Korswagen, A. R.: Computer Program Bisar, Layered Systems under normal and tangential loads, *External Report*, Koninkljke/Shell-Laboratorium, Amsterdam, 1979.
8) Matsui, K., Kasahara, A., and Okada, K.: Effect of Errors in Deflection Readings and Pavement Model on Backcalculated Layer Moduli, *JSCE, no. 526/V-29*, pp.55-62, 1995. (in Japanese)
9) Irwin, L. H., Yang, W. S., and Stubstad, R. N.: Deflection Reading Accuracy and Layer Thickness Accuracy in Backcalculation of Pavement Layer Moduli, *Nondestructive Testing of Pavements and Backcalculation of Moduli, ASTM STP 1026*, A.J. Bush III and G.Y. Baladi, Eds., American Society of Testing and Materials, pp.229-244, 1989.
10) Matsui, K., Kurobayashi, I., and Nishiyama, T.: Effort for Improving Accuracy of Pavement Layer Moduli Estimated from FWD Data, , *Journal of Pavement Engineering, JSCE*, Vol.3, pp. 39-47, 1998. (in Japanese)
11) Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P.: *Numerical Recipes in Fortran, The Art of Scientific Computing*, Cambridge University Press, 1992.
12) Tokumaru, H., Soeda, T., Nakamizo, T., and Akizuki, K.: *Computation and Measurement: Random Data Processing, Its Theory and Application*, Baifuukan Co., 1982. (in Japanese)
13) Maina, J. W., Yokota, H. and Matsui, K.: Effect of Errors in Layer Thickness on Backcalculated Layer Moduli, *Journal of Pavement Engineering, JSCE*, Vol.3, pp. 49-56, 1998.

(Received June 18, 1999)

FWD 試験データの相関が逆解析におよぼす影響

マイナ　ジェイムス・横田　漠・松井　邦人・黒林　功

本研究の目的は，FWD たわみ相互の相関が逆解析結果にどのように影響するかを検討することである．まず，建設省土木研究所で行われた FWD 試験の中でたわみ相関の比較的強いデータを用いて，相関のない表面たわみ及び実測値とほぼ同じ相関を持つ表面たわみの二種類をモンテカルロシミュレーションして，たわみの相関が逆解析に与える影響を検討した．さらに，相関の程度が異なる表面たわみを理論的に算出のうえ逆解析を行い，相関の程度が逆解析に与える影響を調べた．その結果，たわみの標準偏差が同じ場合相関が強いほど，逆解析結果のばらつきが小さくなることが明らかになった．