J. Materials, Conc. Struct., Pavements, JSCE, No. 606/V-41, 165-169, 1998 November

BACK-CALCULATION FOR STRUCTURAL PARAMETERS OF PAVEMENT SLAB ON WINKLER AND ELASTIC SOLID SUBGRADES

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Approximate formulas for calculating the pavement slab deflections on both Winkler and elastic solid subgrades are presented on the basis of thin elastic slab theory. The back-calculation process for estimating pavement structural parameters can be implemented by adopting these formulas. The validity of the procedures proposed in this paper was demonstrated by examples. The subgrade idealization influences the result of back-calculations; that is, the elastic modulus of pavement slab back-calculated in the Winkler subgrade idealization case was higher than that in the elastic solid subgrade idealization case for the same deflection basin.

Key Words: *thin elastic slab theory, Winkler subgrade, elastic solid subgrade, back-calculation*

1. INTRODUCTION

Portland cement concrete pavement is modeled as a thin elastic slab supported on a subgrade [1)~3)], as illustrated in **Fig. 1**. The governing equation of a thin elastic slab on subgrade is expressed by Eq. (1).

$$D\nabla^2\nabla^2 w(r) = p(r) - q(r) \qquad (1)$$

where,

D – stiffness of the slab. $D = \dfrac{E_c \cdot h_c^3}{12(1-\mu_c^2)}$, in which E_c, h_c and μ_c are the elastic modulus, thickness and Poisson's ratio of the slab, respectively

∇^2 – Laplace operator in polar coordinates under radial symmetry case. $\nabla^2 = \dfrac{d^2}{dr^2} + \dfrac{1}{r}\dfrac{d}{dr}$

$w(r)$ – deflection of the slab

$p(r)$ – intensity of the distributed load

$q(r)$ – intensity of subgrade reaction

The Winkler subgrade and elastic solid subgrade are two fundamental subgrade idealizations. The Winkler subgrade idealization considers the subgrade as a bed of closely spaced, independent, linear springs. The vertical stress occurring at any point of the subgrade is proportional to the vertical displacement at that point alone, as expressed by Eq. (2), in which k is the coefficient of subgrade reaction. The elastic solid subgrade idealization considers the subgrade as a homogenous, linearly elastic, isotropic, semi-infinite solid. Unlike the behavior of Winkler subgrade, the displacements at any point of the elastic solid subgrade are dependent on not only the stress acting at that particular point alone, but also at surrounding points, as expressed by Eq. (3).

$$w(r) = \frac{q(r)}{k} \qquad (2)$$

$$w(r) = \frac{2(1-\mu_0^2)}{E_0}\int_0^\infty \bar{q}(\xi) J_0(\xi r) d\xi \qquad (3)$$

where,

$\bar{q}(\xi)$ – Bessel transformation of the vertical stress $q(r)$ on subgrade

k – coefficient of subgrade reaction

E_0 – elastic modulus of subgrade

μ_0 – Poisson's ratio of subgrade

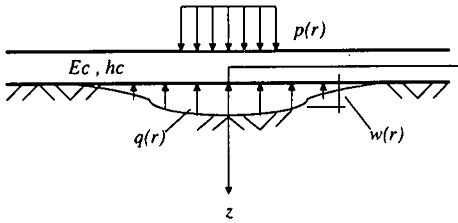

Fig. 1 Slab-on-subgrade model

Significant effort has been emphasized on the calculation and back-calculation of the slab-on-subgrade problem [4)-11)]. However, some studies have very complicated solutions, while the illustration of others is very ambiguous. Therefore, the results are highly subjective and are difficult to use directly. In this paper, based on solutions of the thin elastic slab theory, the approximate formulas for calculating pavement deflection and a back-calculation procedure for estimating the structural parameters of pavement slab on both Winkler and elastic solid subgrade are presented.

2. CALCULATION

Under an interior circular distributed load with radius of a and amplitude of P, the surface deflection of pavement slab on the Winkler subgrade and elastic solid subgrade are expressed by Eq. (4) and Eq. (5), respectively [1)-3)].

$$w_k(r) = \frac{P}{\pi k a^2}\cdot \bar{w}_k(r,l_k) \qquad (4)$$

$$w_e(r) = \frac{P\cdot(1-\mu_0^2)}{E_0\cdot l_e}\cdot \bar{w}_e(r,l_e) \qquad (5)$$

where, $\bar{w}_k(r,l_k)$ and $\bar{w}_e(r,l_e)$ are coefficients of pavement slab deflection on the Winkler and elastic solid subgrade, respectively.

$$\bar{w}_k(r,l_k) = \frac{a}{l_k}\int_0^\infty \frac{J_0(\frac{r}{l_k}\xi)\cdot J_1(\frac{a}{l_k}\xi)}{1+\xi^4}d\xi \qquad (6)$$

$$\bar{w}_e(r,l_e) = \frac{2l_e}{\pi a}\int_0^\infty \frac{J_0(\frac{r}{l_e}\xi)\cdot J_1(\frac{a}{l_e}\xi)}{\xi(1+\xi^3)}d\xi \qquad (7)$$

Both Eq. (6) and Eq. (7) can be calculated by numerical integration, in which l_k and l_e are relative stiffness radius of pavement slab on each subgrade, as expressed by Eq. (8) and Eq. (9), respectively. J_0 and J_1 are expressions of Bessel functions.

$$l_k = \sqrt[4]{\frac{D}{k}} \qquad (8)$$

$$l_e = \sqrt[3]{\frac{2D(1-\mu_0^2)}{E_0}} \qquad (9)$$

With a series of values assigned to the non-dimensional independent variables a/l_k and r/l_k (a/l_e and r/l_e), Eq. (6) and Eq. (7) were calculated numerically. By regressing the calculated results, two polynomial formulas, as expressed by Eq. (10) and Eq. (11), were developed to approximately calculate the $\bar{w}_k(r,l_k)$ and $\bar{w}_e(r,l_e)$ terms, respectively. It was verified that both formulas are valid within the range of $r/l_k \le 4$ or $r/l_e \le 6$ and have an absolute precision of ± 0.002 or a relative precision of more than 97% while compared with solutions of the thin elastic slab theory.

$$\begin{aligned}\bar{w}_k(r,l_k) = {} & 0.3683\cdot(\frac{a}{l_k})^{1.9714} - 0.00058915\cdot(\frac{r}{l_k}) + 0.010789\cdot(\frac{a}{l_k})\cdot(\frac{r}{l_k}) - 0.1209\cdot(\frac{a}{l_k})^2\cdot(\frac{r}{l_k}) \\ & + 0.22984\cdot(\frac{a}{l_k})^3\cdot(\frac{r}{l_k}) - 0.22841\cdot(\frac{a}{l_k})^{2.1026}\cdot(\frac{r}{l_k})^2 + 0.13219\cdot(\frac{a}{l_k})^{2.0822}\cdot(\frac{r}{l_k})^3 \\ & - 0.033879\cdot(\frac{a}{l_k})^{2.0835}\cdot(\frac{r}{l_k})^4 + 0.0042492\cdot(\frac{a}{l_k})^{2.091}\cdot(\frac{r}{l_k})^5 - 0.00020943\cdot(\frac{a}{l_k})^{2.098}\cdot(\frac{r}{l_k})^6\end{aligned} \qquad (10)$$

$$\bar{w}_e(r,l_e) = 0.38624 - 0.012605\cdot(\frac{a}{l_e}) - 0.059544\cdot(\frac{a}{l_e})^2 - 0.042044\cdot(\frac{r}{l_e}) + 0.077774\cdot(\frac{a}{l_e})\cdot(\frac{r}{l_e})$$
$$- 0.023741\cdot(\frac{a}{l_e})^2\cdot(\frac{r}{l_e}) - 0.12652\cdot(\frac{a}{l_e})^{0.094472}\cdot(\frac{r}{l_e})^2 + 0.065273\cdot(\frac{a}{l_e})^{0.071838}\cdot(\frac{r}{l_e})^3 \quad (11)$$
$$- 0.014594\cdot(\frac{a}{l_e})^{0.067112}\cdot(\frac{r}{l_e})^4 + 0.0015227\cdot(\frac{a}{l_e})^{0.065626}\cdot(\frac{r}{l_e})^5 - 0.000059238\cdot(\frac{a}{l_e})^{0.064908}\cdot(\frac{r}{l_e})^6$$

3. BACK-CALCULATION

Deflections $w_k(r)$ and $w_e(r)$ are functions of $\bar{w}_k(r,l_k)$ and $\bar{w}_e(r,l_e)$, respectively. That is to say, they are functions of only r and l_k (or l_e). Hence, if the pavement deflection at any two positions, r_i and r_j, are obtained, another unknown variable, l_k (or l_e), can be back-calculated from Eq. (12) or Eq. (13), which are induced from Eq. (4) and Eq. (5), respectively.

$$F(l_k) = \bar{w}_k(r_i,l_k) - \frac{w_k(r_i)}{w_k(r_j)}\cdot\bar{w}_k(r_j,l_k) = 0 \quad (12)$$

$$G(l_e) = \bar{w}_e(r_i,l_e) - \frac{w_e(r_i)}{w_e(r_j)}\cdot\bar{w}_e(r_j,l_e) = 0 \quad (13)$$

Newton-Raphson method was introduced to solve the polynomial equations. Because both equations are not the ill-conditioned polynomials within the practical ranges of loading plate radius a (e.g. 150mm, 225mm or 250mm), the relative stiffness radius l_k or l_e (500 - 2000mm) and the radial distance of deflection measurement r (0 - 1500mm), the solutions of Eq. (12) and Eq. (13) are unique.

The back-calculation procedure for a pavement deflection basin is illustrated in **Fig. 2**, in which m is the number of deflection sensors. The representative values of E_c and k (or E_0) can be obtained statistically by considering the distributions of the back-calculated results.

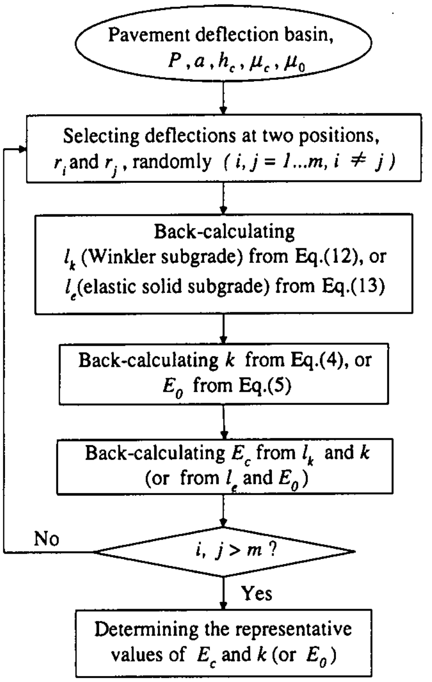

Fig. 2 Flow chart of the back-calculation procedure

4. VERIFICATIONS

(1) Calculation

An example calculation is used to show the validity of the approximate formulas proposed in this paper. A 254 mm thick slab with elastic modulus of 31,000 MPa and Poisson's ratio of 0.15 rests on both the Winkler subgrade and elastic solid subgrade. For the former, k = 57.3 MPa/m. For the latter, E_0 = 191 MPa and μ_0 = 0.45. In both cases, the applied load is 34.7 kN and the radius of loading plate is 150 mm [8].

The theoretical and calculated pavement deflections on the Winkler and elastic solid subgrades are listed in **Table 1**. It is revealed that the deflections calculated by the approximate formulas proposed herein coincide well with the theoretical solutions.

(2) Back-calculation

The theoretical deflections of both subgrade idealizations in **Table 1** were used to back-calculate the pavement structural parameters, i.e., elastic modulus of the slab E_c, coefficient of subgrade

Table 1 Calculated pavement deflection

Subgrade	Solution	Deflection (mm)			
		d_0	d_1	d_2	d_3
Winkler	Theoretical	0.0860	0.0798	0.0684	0.0556
	Approximate	0.0860	0.0795	0.0683	0.0556
Elastic Solid	Theoretical	0.0771	0.0720	0.0626	0.0533
	Approximate	0.0775	0.0719	0.0628	0.0527

Note: d_0, d_1, d_2 and d_3 are deflections at 0, 305, 610 and 914 mm away from the load center, respectively.

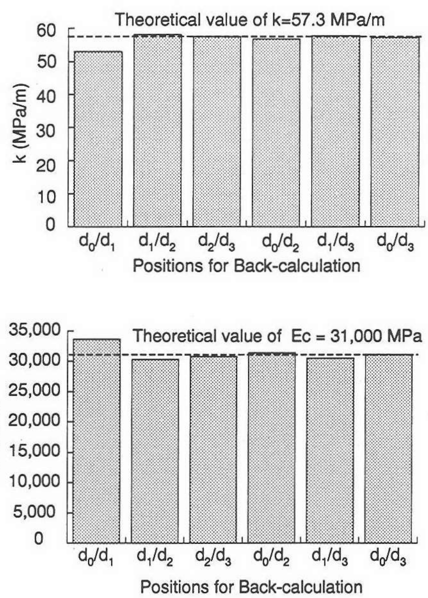

Fig. 3 Back-calculation for the Winkler subgrade case

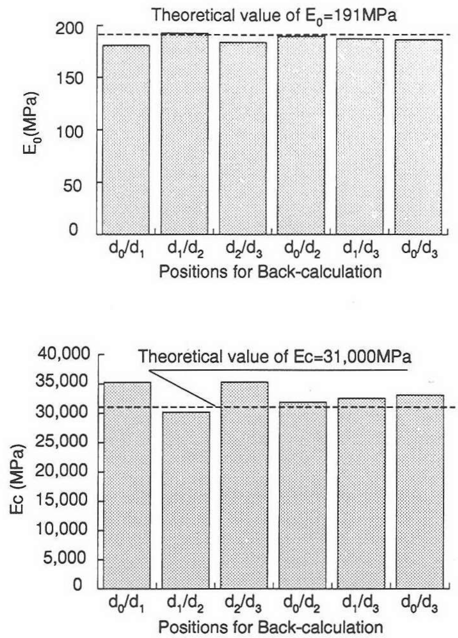

Fig. 4 Back-calculation for the elastic solid subgrade case

Table 2 Influence of subgrade idealization on the back-calculation results

Subgrade idealization	E_c (MPa)	k (MPa/m)	E_0 (MPa)
Winkler	36,100	54.9	-
Elastic solid	27,300	-	187.4

reaction k for the Winkler subgrade idealization, and elastic modulus of the subgrade E_0 for the elastic solid subgrade idealization. **Fig. 3** shows the estimated structural parameters for the Winkler subgrade idealization case, back-calculated by Eq. (12). **Fig. 4** shows those for the elastic solid subgrade idealization case, back-calculated by Eq. (13). It is revealed that the back-calculation procedures proposed in this paper have satisfied accuracy for pavement structural evaluation.

(3) Influence of subgrade idealization on back-calculation

For investigating the influence of different subgrade idealizations on back-calculation, the average value of the theoretical pavement deflections shown in **Table 1** (0.0816, 0.0759, 0.0655 and 0.0545 mm at 0, 305, 610 and 914 mm away from the load center, respectively) were back-calculated. The representative pavement structural parameters that average the obtained values after excluding the maximum and minimum are listed in **Table 2**. It is revealed that the elastic modulus of pavement slab back-calculated in the Winkler subgrade idealization case was higher than that in the elastic solid subgrade idealization case. Therefore, The back-calculated pavement structural parameters not only depend on the obtained deflection basin, but also on the deployed subgrade idealization.

5. SUMMARY AND CONCLUSIONS

1)Approximate formulas for calculating the defections of pavement slab on both Winkler and elastic solid subgrades are proposed, and their validity was verified.

2)Back-calculation procedures using these formulas are proposed for estimating the structural parameters of pavement slab on Winkler and elastic solid subgrades.

3)For the same deflection basin, the elastic modulus of pavement slab back-calculated in the Winkler subgrade idealization case was higher than that in the elastic solid subgrade idealization case.

Successive studies will be:

1)Back-calculating the measured deflections at a variety of pavement structures to find out the rational sensor arrangements for implementing the deflection measurement and back-calculation.

2)Identifying the discrepancies between the solutions of theoretical idealizations and the responses of actual pavements, and investigating the back-calculation results derived from different deflection measurement conditions.

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(Received February 16, 1998)

ウィンクラーならびに弾性路床上の舗装版構造における力学定数の逆解析

殷　建軍・八谷好高

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