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A STUDY ON THE FUNDAMENTAL CHARACTERISTICS OF LATTICE MODEL FOR REINFORCED CONCRETE BEAM ANALYSIS

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A new technique to modify the lattice model is described by the authors. This new method significantly depends on the calculation of the minimum total potential energy of the structure starting from the elastic stage up to the failure stage inside each increment of the calculation. Adoption of the minimum total potential energy for the structure is studied. Angle of inclination of the diagonals and the appropriate discretization method for the truss member are very important parameters affecting the results of the lattice model and are studied in this paper. The applicability of the Modified Lattice Model is examined by proposed shear strength equations and existing experimental data.

Key Words: *shear-resisting mechanism, modified lattice model, arch element, total potential energy, and subdiagonal element.*

1. INTRODUCTION

It is generally agreed that the truss analogy concept for shear resistance is easily applied to reinforced concrete structures. However, there exist several different truss models to analyze the shear resisting mechanism in reinforced concrete beams. But in each model, there are still some problems to be investigated. For example, Lattice Model, which is first proposed by Niwa et al.[14)] and extended later by the authors in three dimensions [6), 7)], has several fundamental points to be clarified and other points to be modified. In this model the arch member is a very important concept, because after the yielding of shear reinforcement, the model can explain the increase in the shear capacity, while the simple truss model cannot, especially in the case of deep beams. Arch element has some important effects in the shear carrying capacity [14)]. The thickness of the arch element is determined by minimizing the total potential energy for the whole structure. But, any physical explanation for the minimization of total potential energy is not given, and once the value of the thickness of the arch element is determined in the elastic stage, this value is unchanged throughout the whole loading history. The thickness of the arch element may be changed during the loading stages, but its change is simply neglected. In this paper, we clarify this point in the first place and show the improved accuracy by performing the minimization at every loading stage.

Also, during the different loading stages, the change of the thickness of the arch element with the corresponding load carrying capacity is studied. In addition, the rational reasoning for the strain incompatibility through the width of a beam by separating the arch member and the truss member within one beam will be explained. Experimentally, it is found that a two-dimensional stress analyses is not adequate for reinforced concrete members[11)]. With this clarification, the fundamental characteristics of the arch element mechanism for shear resistance of reinforced concrete members are discussed; especially the strain values between the arch and diagonal elements in the same cross-section are not equal. The strains may not be uniform in the direction of member width. The third point to be clarified is the direction of most

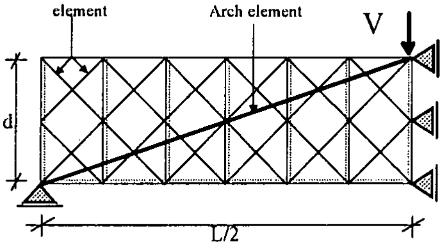


Fig.1 Modified Lattice Model
of concrete beam

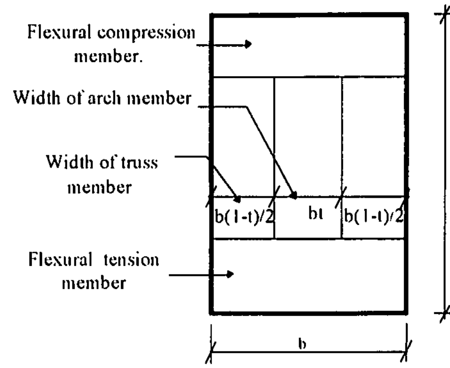


Fig.2 Cross -section of concrete beam
in the Modified Lattice Model

appropriate discretization for subdiagonal members. The appropriate discretization is studied to find out the suitable form of applying he Modified Lattice Model which gives a similar response close to the experimental results depending on the change of spacing of shear reinforcement and the subdiagonal angle. Finally, the application of the "Modified Lattice Model" to simulate the shear failure of reinforced concrete beams is carried out. The change of the stress states in each member inside the beam is investigated.

In this paper, authors try to give rational reasoning or rational explanation for all the previous problems. Furthermore, based on the modified and rational model we give some numerical calculation results, which may be useful in its real application.

2. OUTLINE ON THE MODIFIE LATTICE MODEL

The chosen element discretization and structural geometry of the Modified Lattice Model is illustrated in **Fig.1**. The reason behind this truss discretization will be verified in the following sections. The reinforced concrete beam has been simulated under bending and shear as simple truss components. The compressive stress in the upper part of the beam is resisted by concrete in the form of a horizontal strut with a cross-section area equal to the area of the upper rectangle in **Fig.2**. The tensile stress in the lower part is taken by the bottom steel in the form of horizontal members in addition to the horizontal concrete fibers in the lower part with a cross section area equal to the lower rectangle area in **Fig.2**. To resist the shear forces inside the beam, the truss model has diagonal concrete tension and compression members with the area as shown in **Fig.2**, which can be fixed after the value of "t" is determined as it will be shown in section 3. For the vertical members the effect of concrete is not considered because the resistance of concrete for tension is already incorporated in the diagonal tension member. That is in addition to the vertical steel members, which represent the shear reinforcement in the web. **Fig.2** shows the cross section of a concrete beam modeled as a Modified Lattice Model.

In **Fig.1**, the thick solid line represents the arch element, which is assumed to be a flat and slender one connecting the nodes at both ends of the beam with an area as shown in **Fig.2**. In this analysis, the arch element and the diagonal elements are separated and each one of them has its stress and strain distribution. The reason for this element separation is that the structural action is normally a combination of series and parallel couplings of the cracking zones and the uncracked (elastic) zones. In the Modified Lattice Model, we simulated these zones with continuous pairs of tension and compression members. The arch member is considered as a very important element in this study, because it represents the core of the beam [13]. A design code in AIJ [1] and many other codes [4] assume two dimensional stress fields; but if the member section is wide enough, the stress may not be uniform in the direction of member width. It is also known experimentally by Ichinose [11] that the values of the strains or the stresses of the beam are not uniform along the width in the same cross-section. It means that the stress or the strain diagram is not constant in the direction of the width of the beam [11]. So, in this model we separate the arch element and the diagonal element, and each one of them has its stress and strain distribution. The arch element has the ability to resist a large portion of the applied load [13]. So it is very

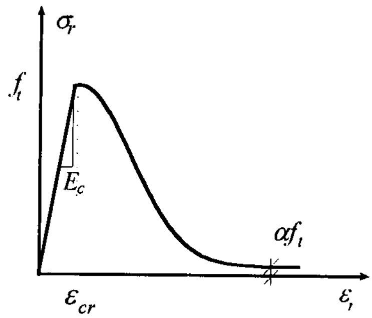


Fig. 3 Tensile stress-strain curve of concrete

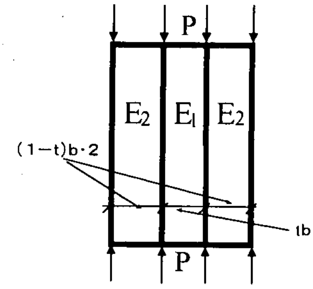


Fig. 4 Cross- section of a spring

important to look for the change of the area of the arch element during the different loading stages, as it will be shown in section 3. In the Modified Lattice Model, the diagonal tension member of concrete resists the principal tensile stress resulting from shear force. The stress-strain relation of tension member of concrete has been taken as expressed in Eq. (1) and Eq. (2) [8), 9)] and as shown in **Fig.3**.

For ascending branch $(\varepsilon_r < \varepsilon_{cr})$

$$\sigma_r = E_c \varepsilon_r \qquad (1)$$

For descending branch $(\varepsilon_r \geq \varepsilon_{cr})$

$$\sigma_r = (1-\alpha) f_t \exp\left[-m^2\left(\frac{\varepsilon_r}{\varepsilon_{cr}} - 1\right)^2\right] + \alpha f_t \qquad (2)$$

Where ε_r and σ_r are the strain and the stress of the tension element, respectively. ε_{cr} is the strain at the cracking of concrete and E_c is the modulus of elasticity of concrete. The stress-strain behavior of concrete in tension is taken elastic as shown in Eq. 1 and gradual softening is taken after that as shown in Eq. (2). In Eq. (2), "m" value can be varied to simulate appropriate softening slope and the value of α can be appropriately assumed to simulate the appropriate residual stress [3), 10)] . Here in this calculation m =0.5 and a = 0.0 are taken based on fracture energy concept, taking the length of each member as the characteristic length.

The diagonal compression member of concrete and the arch member shall resist the diagonal compression caused by shear. To consider the compression-softening behavior of crushed concrete, the model proposed by Collins et al.[18)] is adopted. In that model the softening coefficient was proposed as a function of the transverse tensile strain. So, the tension and compression members are considered as a pair together. Eq. (3) shows the compressive stress-strain relationship of concrete in this study. The stress-strain relationship for reinforcing bars is assumed to be elasto-plastic for the case of tension and compression members.

$$\sigma_c = -\eta f_c' \left(2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right) \qquad (3a)$$

Where, the peak softening coefficient

$$\eta = \frac{1.0}{0.8 - 0.34\left(\frac{\varepsilon_r}{\varepsilon_o}\right)} \leq 1.0 \qquad (3b)$$

And the strain at the peak stress $\varepsilon_o = -0.002$.

3. ADOPTION OF MINIMUM TOTAL POTENTIAL ENERGY

The effect of the total potential energy in the Modified Lattice Model during the calculation has a significant effect in the final results. It is found that, there is a relation between the area of the arch element and the corresponding total potential energy of the structure. Niwa et al. [14)] showed that if the ratio of the width of the arch element is assumed to be "t", the value of "t" is determined by minimizing the total potential energy for the whole structure. But in this work, it is found that this thickness is increasing gradually during the loading from the elastic stage up to the complete failure of the beam. It means that the area of the diagonal members is decreased gradually during the progress of different loading stages.

The physical explanation for the adoption of minimum total potential energy may be given firstly using a very simple spring model as shown in **Fig. 4**. In this model, the cross-section area of the spring consists of two different materials with different

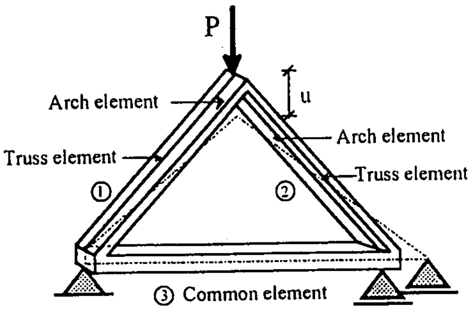


Fig. 5 Triangular model

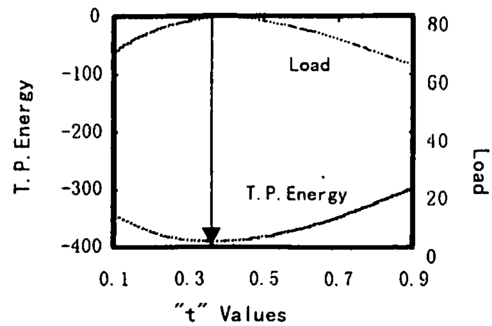


Fig. 6 Behaviour of the T.P. Energy and the applied Load with "t" values

modulus of elasticity E_1 and E_2 under a concentrated load P. Assume the stiffest portion lies in the middle of the cross section with a stiffness value E_1. The total potential energy for this model is obtained by Eq. (4). Substituting for the values of σ and ε, we can get the total potential energy as a parameter, dependent with the area of each material part inside the cross-section as shown in Eq. (5).

$$\pi = 1/2\int \sigma_1 \varepsilon dv_1 + 1/2\int \sigma_2 \varepsilon dv_2 - Pu \tag{4}$$

$$\pi = -Au^2(1/2)\left[E_1 t/l + E_2(1-t)/l\right] \tag{5}$$

From Eq. (5) the total potential energy is decreasing monotonically with the increase of the area of the stiffer portion. Therefore, the stiffer portion should occupy the total area to make the potential minimum. However, our beam element is not exactly the same category.

So, we show the real situation using the model shown in **Fig.5**. This model is a triangular shape under a concentrated load P. The cross-section area of each of the side 1 and 2 has been divided into two different materials with two different modulus of elasticity E_1 and E_2 representing the truss element and the arch element, respectively. The ratio of the width of the arch element is assumed to be "t" from the total width of the member. The member 3 is a common material with a definite modulus of elasticity. The total potential energy of the structure is calculated from Eq. (6).

$$\pi = 1/2\int \sigma\varepsilon dv - Pu \tag{6}$$

Where, u is the vertical displacement at the loaded point of the structure under the applied load "P". Take $\partial\pi/\partial t = 0$ to get "t" value corresponding to the minimum total potential energy and substitute it in the energy equation. **Fig.6** shows the relation of the total potential energy and the applied load "P" for the different values of "t". From this figure we find

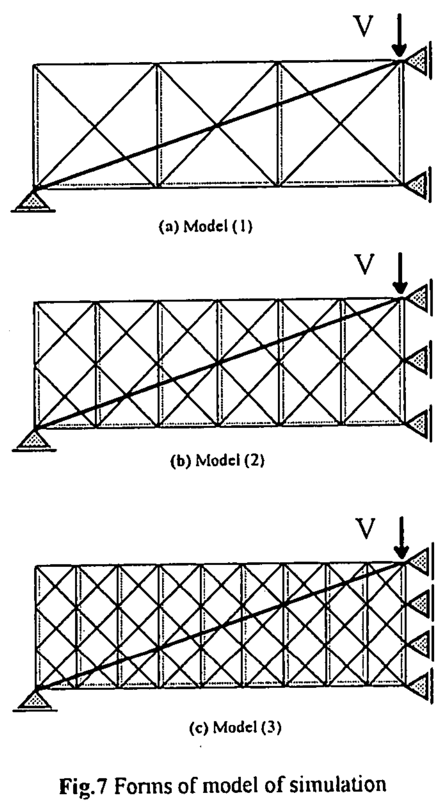


Fig.7 Forms of model of simulation

that the point corresponding to minimum total potential energy corresponds to the maximum applied load at a particular value of "t". It means that, if this value of "t" is used, we get the stiffest case of the beam with a minimum potential energy. So, in the Modified Lattice Model the total potential energy by applying Eq. (7), is calculated for different values of "t" starting from 0.1 ~ 0.9 with a very small increment.

Table 1 Outline of the experimental data.

No	Cross Sec.	b cm	h cm	d cm	a/d	f_c MPa	A_s cm^2	f_y MPa	A_w cm^2	f_{wy} MPa	s cm
1	R	20.3	50.8	42.5	2.15	31.0	23.1	530	1.42	530	13.3
2	R	20.3	45.7	38.9	2.00	24.6	24.5	320	1.42	320	18.3
3	R	30.0	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0
4	T	30.0 15.0	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0
5	R	45.0	60.0	52.5	2.86	43.9	95.7	383	1.43	355	25.0
6	R	45.0	60.0	52.5	2.86	66.2	95.7	383	1.43	355	15.0

*In this table R means rectangular section and T means T-shaped section. No.4: flange width=30.0cm, flange depth=7.5cm and web width =15cm.

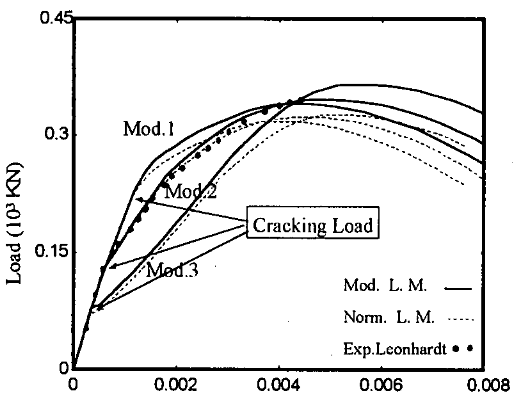


Fig.8 Comparison with experiment (No. 4)

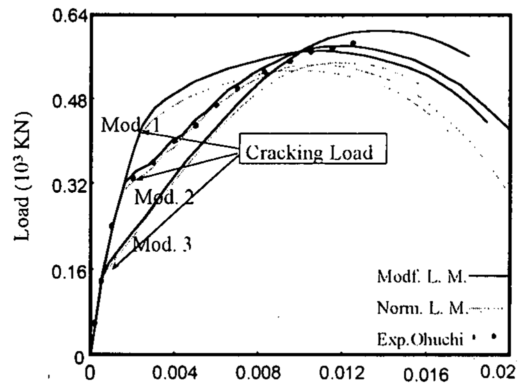


Fig.9 Comparison with experiment (No.5)

$$T.P.E. = \sum_{i=1}^{n} f_i \delta_i - p\Delta \qquad (7)$$

In Eq. (7), T.P.E. is the total potential energy of the structure, where f_i is the internal force inside each member of the structure, δ is the displacement of each individual member, n is the number of the truss members, p is the value of the external concentrated load and Δ is the displacement at the loaded point. By minimising these values of the total potential energy we can get the corresponding "t" value. From this value we can calculate the area of the arch element and the subdiagonal elements at each step of the calculation. So, when we consider the total potential energy and the corresponding area of the different elements we get more stiffer calculated results with the real response of the beam which becomes almost close to the experimental results along the different loading stages as we will see in the next section.

4. APPROPRIATE DISCRETIZATION METHOD FOR TRUSS MEMBER

In this section a clarification of the appropriate discretization of lattice members which angle may be firstly predetermined as 45 degrees is given. To investigate the extent of the discretization, three different truss models depending on the number of diagonal pairs along the depth of the beam are investigated. The three different forms are as shown in **Fig.7**. To determine the most appropriate discretization model among the three previous forms of truss model in **Fig.7**, six different reinforced concrete beams are investigated as an example, to compare results of the Modified Lattice Model with the experiment results. The outline of experimental data utilized to examine the Modified Lattice Model using the above mentioned three different forms of truss models in **Fig.7** are presented in **Table 1**.

The comparisons of the calculated results using the Modified Lattice Model and the normal lattice model are shown clearly in **Fig.8** (Leonhardt's experiment [12] No.4 in **Table 1**) and **Fig.9** (Ohuchi's experiment [16] No.5 in **Table 1**). The number of subsection diagonal members is increased from model (1) to model (2) and to model (3). After cracking, the neutral axis of the beam starts to move upward during the development of the cracks. The height of the cracks depends on the cross-section of the beam and the value of the steel reinforcement ratio. So, in case of model (1) if a crack occurs, it means the depth of the crack equals the whole depth of the beam. In this case, the complete failure takes place suddenly. However, it is not realistic because experimentally failure does not occur suddenly. In the case of model (2), if cracks happen, it means the depth of the crack equals half of the depth of the beam and the failure is not found suddenly like the previous case. This case looks logical and close to the experimental behavior of reinforced concrete beams. In the case of model (3), the depth of the first crack equals 1/3 of the total depth of the beam. In this case the development of the cracks is not similar to the experimental behavior of the beam. That is why we find that the numerical results are very close to the experimental results in case of model (2) as shown in **Fig.8** and **Fig.9**. That is in addition to the effect of change the distance of shear reinforcement among the three different models. So, it is found that, model (2) is the preferable model to implement the Modified Lattice Model. Comparing the results of the three models, we find the cracking load has been decreased starting from model (1) to model (3). In case of model (1), the elastic energy of the failure elements is much higher than that in model (2). Also in the case of model (2) it is much higher than that in the case of model (3). The reason of that is the increasing of number of subsection diagonal members. The strain energy has been decreased with the decrease in the original length of failure elements. However, the ultimate loads using these three models are almost same because of the similarity of the fracture energy for the three different models. From these experimental data, we can say that the Modified Lattice Model can capture the displacement behavior adequately and reach to almost same response of the original beam, especially the displacement at the peak is similar to the experimental results more than any other truss model.

The change of the thickness of the arch element is drawn in **Fig.10** (Clark's experiment [5]) and **Fig. 11** (Ohuchi's experiment [16]) for beams of No.2 and

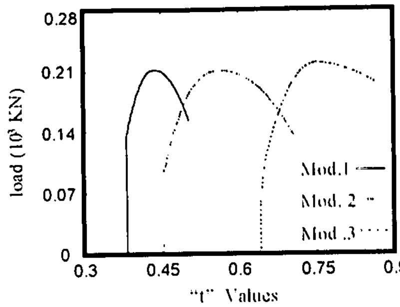


Fig.10 Comparison of "t" values for the three models (No.2)

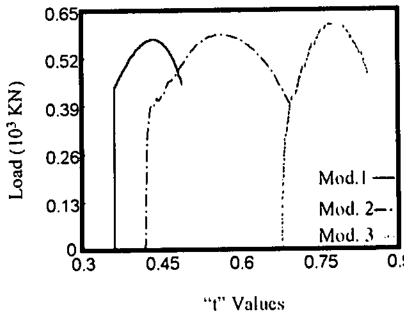


Fig.11 Comparison of "t" values for the three models (No. 5)

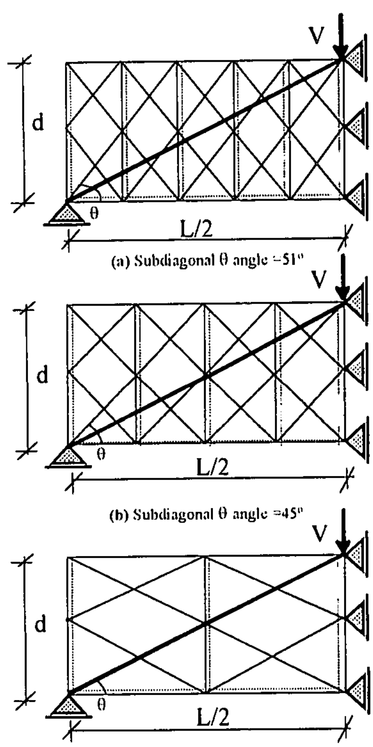


Fig.12 Three different subdiagonal angles

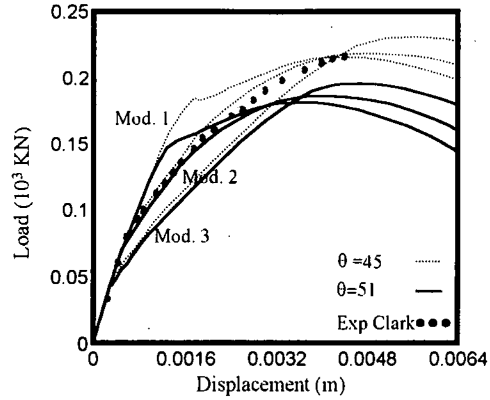


Fig.13 Load-displacement diagram in case of θ=51° and 45° (No.2)

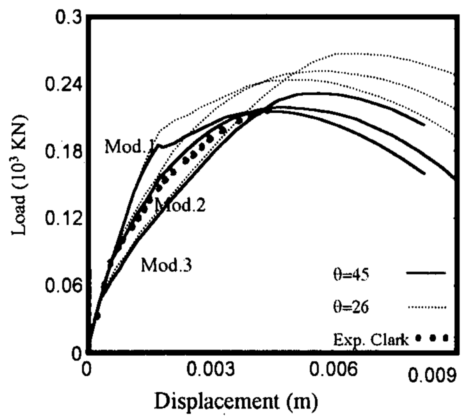


Fig.14 Load-displacement diagram in case of θ=26° and 45° (No.2)

No. 5 in **Table 1**, respectively. According to the Modified Lattice Model analysis, the thickness of the arch element is increasing gradually from the elastic stage, in which it remains constant, up to the complete failure of the beam. After the peak point, the depth of the arch element is decreased due to the extension of the cracks. So, the thickness of the arch element has increased gradually in order to maintain the effect of the arch element up to the failure point. As have been discussed above, most appropriate truss discretization is model (2). This suggests that the probable arch width is around 0.4b in the early loading stage and increased with the load up to 0.7b.

5. APROPRIATE SUBDIAGONAL ANGLE FOR THE MODIFIED LATTICE MODEL

To study the direction of initial cracking in each of the solved beam under the three different models in **Fig.7**, three deferent values of angle of inclination of the subdiagonals in the Modified Lattice Model are suggested. The suggested values are 51, 45 and 26 degrees as shown in **Fig.12 (a)**, **(b)** and **(c)** respectively. **Fig.13** and **Fig.14** show the load-displacement diagram for beams No.2 [5] in **Table 1** for the different subdiagonal angle and also for the different models, which are mentioned in **Fig.7**. The numerical results using the Modified Lattice Model are compared with the experimental results. It has been found that the results using model (2) with diagonal angle 45 degrees are very close to the experimental results. Under any other inclination of diagonal angle, the relation of the load-displacement goes diverging from the experimental results up or down. In the case of angle 51 degree the length of the diagonal members are decreased comparing with the case of 45 degree, so the relation of load-displacement is kept lower than the relation using 45 degree diagonal angle as shown in **Fig.13**. But in the case of the same models using diagonal angle 26 degree the results become upper than the relation using diagonal angle 45 degree as in **Fig.14**. Actually this happens because the increasing of the diagonal member length increases the elastic energy and the stiffness of the structure. This behavior was the same using the three different models, and also for each angle of inclination for the diagonal members.

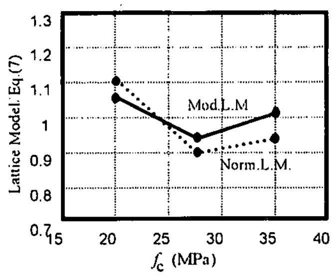
6. EXAMINATION OF THE APPLICABILITY OF THE MODIFIED LATTICE MAODEL

To examine the applicability of the Modified Lattice Model, many different beams are calculated numerically using the second model in **Fig.7 (b)**. The shear strength is calculated for each beam and compared with the basic concept for shear strength equation.

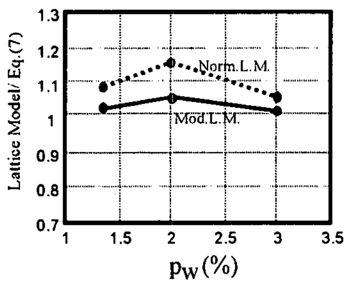
(1) For beams with web reinforcement

Different reinforced concrete beams with different parameters are analysed using the Modified Lattice Model. The value of the shear strength of each beam is compared with Eq. (8), which is considered a basic concept to calculate the shear strength for the truss analogy [17].

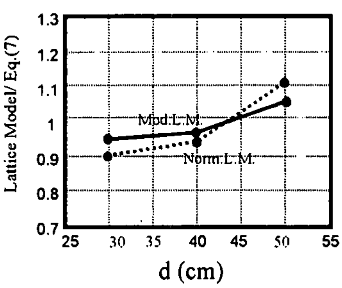
$$V_y = V_c + V_s \tag{8}$$



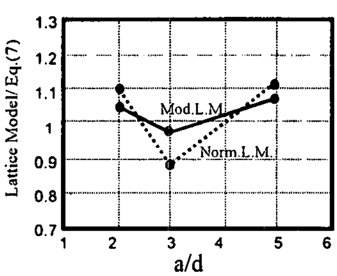
(a) Shear carrying capacity with f_c (MPa) for beams with web R.



(b) Shear carrying capacity with p_w (%) for beams with web R.



(c) Shear carrying capacity with the depth for beams with web R.



(d) Shear carrying capacity with a/d Ratio

Fig.15 Change of shear carrying capacity for beams with web reinforcement

Where, V_c is the shear capacity of linear members without shear reinforcement, obtained by Eq. (9) [15] and V_s is the shear capacity carried by shear reinforcing steel and obtained by Eq. (10) [17]

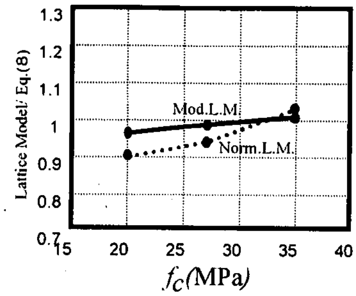
$$V_c = 0.20 f_c'^{1/3} p_w^{1/3} d^{-0.25}\left[0.75 + \frac{1.4}{a/d}\right] b_w d \qquad (9)$$

$$V_s = A_w f_{wy} d \frac{z}{s} \qquad (10)$$

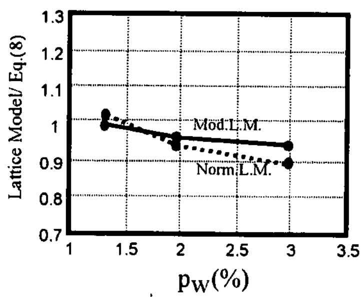
Where, f_c is the compressive strength of concrete (MPa), p_w is the reinforcement ratio $(=100A_s/(b_w d))$, d is the effective depth of a concrete beam (m), a/d is the shear span -effective depth ratio, A_w is the area of shear reinforcement over the interval s, f_{wy} is the yield strength of the shear reinforcement and $z = d/1.15$. To examine the applicability of the Modified Lattice Model for beams with web reinforcement, the comparisons are carried out using Eq. (8) and also with the normal Lattice Model [14]. **Fig.15** shows the change of ratio of the results of predicted shear carrying capacity between the normal Lattice Model, Modified Lattice Model and Eq. (8) with the variation of different parameters taking beam No.1 [2] in **Table 1** as a definite example for the cross-section area. The parameters, which are selected and combined, are the concrete strength, reinforcement ratio, effective depth and also the shear span-depth ratio of the beam. As seen from **Fig.15**, the shear carrying capacity by the normal Lattice Model is generally smaller than that by Eq. (8), but the predicted results using the Modified Lattice Model is much closer to Eq. (8) and admissible. Also from **Fig.15**, the tendency of the prediction by the Modified Lattice Model is not necessarily similar to Eq (8). The ratio is varied from 0.96 to 1.08, but in the case of the normal Lattice Model the ratio was almost from 0.88 to 1.17.

(2) For beams without web reinforcement

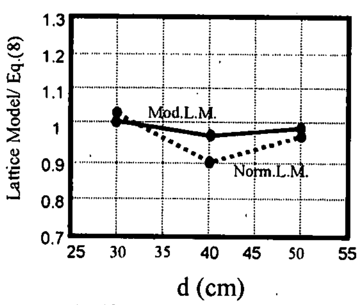
To examine the applicability of the suggested Modified Lattice Model for concrete beams without web reinforcement, numerical calculation is performed and compared with Eq. (9) which [15] has been accepted as a basis of the design equation in the JSCE. The comparisons are carried out using Eq. (9) and also with the normal Lattice Model [14]. **Fig.16** shows the change of the ratio of the results for predicted shear carrying capacity between the normal Lattice Model, Modified Lattice Model and Eq. (9), with the variation of different parameters. Taking the dimension of the cross-section area of the beam No. 6 [16] in **Table 1** as an example. The parameters, which are selected and combined, are concrete strength, reinforcement ratio, effective depth and the shear span-depth ratio. As seen from **Fig.16**, the predicted shear carrying capacity by the



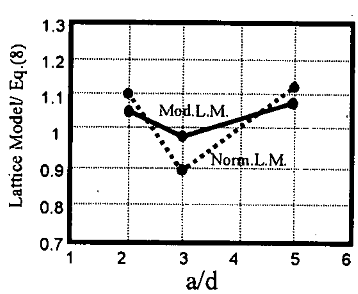
(a) Shear capacity with f'_c (MPa)



(b) Shear capacity with p_w (%)



(c) Shear capacity with the



(d) Shear capacity with the ratio (a/d)

Fig. 16 The change of shear carrying capacity for beams without web reinforcement

normal Lattice Model is smaller than that by Eq. (9), but the variation for the Modified Lattice Model is much smaller and admissible. Also from **Fig.16** the tendency of the prediction by the Modified Lattice Model is not necessarily similar to

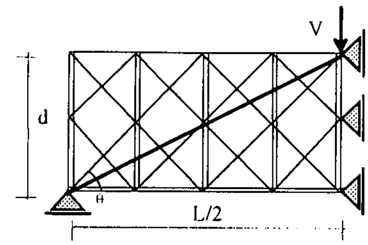
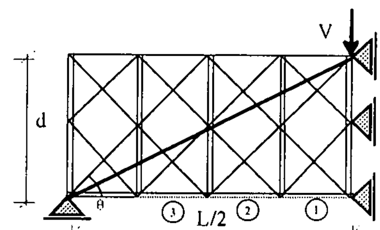
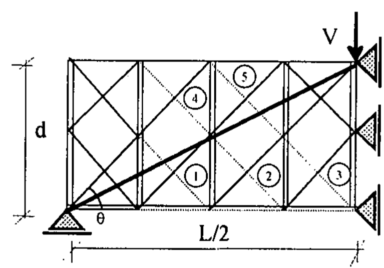


Fig.17 Illustrative beam example (No.2)



(a) Cracks in the bottom members



(d) Cracks in the diagonal members

Fig.18 The propagation of cracks

Eq. (9). The ratio is varied from 0.96 to 1.03, but in the case of normal Lattice Model the ratio is almost from 0.88 to 1.1 [14]. Predicted shear failure mode by the Modified Lattice Model is the failure of the diagonal tension member, which is corresponding, to the experimental results. Consequently, it can be considered that the prediction of the shear carrying capacity by the Modified Lattice Model is adequate.

7. APPLICATION OF THE MODIFIED LATTICE MODEL FOR SHEAR FAILURE SIMULATION

To investigate the change of the stress states in each member inside the reinforced concrete beam using the Modified Lattice Model, Clark's experiment (No.2 in **Table 1**) was chosen as a subject for a solved example. **Fig.17** shows the Modified Lattice Model for the reinforced concrete beam No. 2 in **Table 1**. The stresses in diagonal members of concrete and stirrups and the stress of the arch member are examined. From the output results of the simulation of this beam using the Modified Lattice Model, it is found that at the primary cracking stage, the concrete elements in the bottom cord start to crack firstly as shown in **Fig.18 (a)**. Then the initiation of the diagonal cracking happens as shown in **Fig.18 (b)**. The initiation of the yielding of stirrups starts to take place. Although the stirrups start yielding and the diagonal tension elements have cracked, but the beam still continue to be loaded up to the complete failure. That is because of the existence of the arch element, which continues up to the end of loading with some stirrups. At the final stage, all the stirrups yielded. At that time the arch element crushed immediately. From this simulation for the failure of that beam we can consider it as a shear failure.

According to this simulation and considering the objectivity of the post processing for calculated results and the simple representation for the shear resisting mechanism, the Modified Lattice Model can simulate the shear failure mode with a very smart way. Although the Modified Lattice Model in which the compatibility condition, the equilibrium condition and the used constitutive equations are more simplified methods comparing with the normal FEM, the Modified Lattice Model can capture the shear behaviour of concrete beams reasonably throughout the change of the shear resisting mechanism.

8. CONCLUSIONS

In the proposed Modified Lattice Model, a reinforced concrete beam subjected to shear force is converted into a simple truss and arch members by the consideration of the minimum total potential energy for the structure at each step of loading. A nonlinear incremental analysis is performed. The conclusions obtained from this research are as follows:

1. By minimizing the total potential energy of the reinforced concrete beam, we get only one value for the thickness of the arch element, which corresponds to the stiffest case for the structure, which is quite similar to the original response of the experimental analysis.
2. The Modified Lattice Model has the tendency to estimate the stiffness of the beam closer to the experimental results. Furthermore, the predicted displacement at the peak is almost similar to the experimental results.
3. The thickness of the arch member, which plays a very important role in the Modified Lattice Model, is increasing gradually with the increase in displacement of the loading point after the initiation of diagonal cracks up to the complete failure of the beam.
4. The applicability of the Modified Lattice Model is examined for beams with and without web reinforcement under different parametric conditions. The tendency of the prediction of shear strength by the Modified Lattice Model is very close to the basic shear strength equations, which are accepted by the Standard Specification of JSCE. Also comparing with the experimental results, it gives the satisfied accuracy.
5. In case of Model (2) with 45 degrees of subdiagonal members, the position of the neutral axis is reasonable agreement with the case of experimental work. So, model (2) with two pairs of subdiagonal members and with 45 degrees for the angle of inclination is the appropriate discretization to implement the Modified Lattice Model analysis.
6. Using the different forms of the Modified Lattice Model, the ultimate load is almost kept constant but the cracking load is decreasing depending on the strain energy of the cracked element.

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鉄筋コンクリートはり解析についての格子モデルの基本的特性に関する研究

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格子モデルを修正した新しい手法が著者によって述べられている．この新手法は，構造物が弾性域から始まって破壊に至るまで各増分を求め最小総合ポテンシャルエネルギーを計算することによるものである．最小総合ポテンシャルエネルギーを構造物に採用しようというものである．トラスの斜材の角度と適切な形状は格子モデルの結果に影響を及ぼすとても重要なパラメータとなる．それらについて本論文において研究している．既存のせん断耐荷力算定式と実験データを用い，修正格子モデルの適用性を検討する．