J. Materials, Conc. Struct., Pavements, JSCE, No. 592/V-39, 169-180, 1998 May

3D ANALYSIS OF RC MEMBERS BY UNIFIED CONCRETE PLASTICITY MODEL

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Three Dimensional finite element analysis of *reinforced concrete* (RC) members is a very complicated matter. Many researchers have proposed various ways of analyzing RC members. Most of them present separate models for tension and compression. Recently Tanabe et al. proposed a plasticity model named Unified Concrete Plasticity Model[1)] which can be applied in multi-axial stress-strain situation. As a part of this research, this model was further developed[2) ~ 5)] and in this paper, the feasibility of application of this modified model is tested before further development and implementation of advanced features like inelastic unloading of concrete, advanced reinforcement models etc. This is done by three dimensional finite element analysis of simple problems like beam and cantilever. In this analysis, the details of longitudinal reinforcement and lateral reinforcement or stirrups are included.

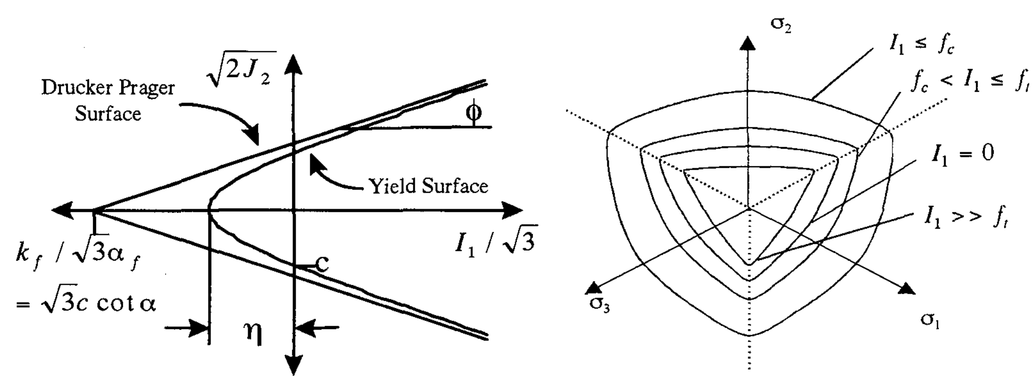
***Key Words** : Unified Concrete Plasticity Model, reinforced concrete members*

1. INTRODUCTION

The behavior of reinforced concrete structures has been extensively studied from various approaches both experimentally and numerically for a long time. The behavior of concrete in presence of reinforcement is quite different from that in absence of reinforcement. As it is very difficult to carry out experiment on full scale structures, one has to either analytically predict and simulate the failure phenomenon or carry out experiments of scaled down structures. Size effect creates serious drawback in the interpretation of experimental results on scaled down structures. For analytically predicting or simulating the failure phenomenon, one needs to know the stress-strain relationship of concrete and reinforcement in any type of stress, strain or damage situations. Many type of damage phenomenon like concrete in presence of stirrups are three dimensional in nature and need three dimensional analysis to understand the failure process even for case like reinforced concrete beam.

On the other hand, many researchers have pointed out that concrete in tension in presence and absence of reinforcement show quite different behavior and is governed by tension stiffening effect and fracture energy respectively. There are different aspects that need to be taken into consideration for realistic analysis of a RC members and is still a matter that need further research and attention. For realistic implementation of tension stiffening effect, many researchers have proposed different methods as it plays an important role in the behavior of RC members. These details are discussed later in details in Sec. 4.

Depending on the type RC members, researchers have adopted different strategies in a wide spectrum ranging from discrete - smeared crack approach, stress-strain relationship for concrete by direct or plasticity based approach etc. for the analysis. All these approaches have their relative merits and demerits. Often mixed strategies of applying discrete crack and smeared crack approach has also been adopted to differentiate between dominant crack and smeared cracks. Most of the researchers have applied *different,* though *interdependent stress-strain relationship* for concrete in *tension* and *compression.* For concrete in compression, compression softening behavior (first developed by Vechico and Collins[6)] and later modified by others) is generally adopted. For concrete in tension, tension stiffening effect and fracture energy concept is adopted for concrete with and without reinforcement respectively. These concepts are developed using experimental results on simplified elements and



a) Meridian Plane b) Diviatoric Plane

Fig.1 The Initial Yield Surface for Unified Concrete Plasticity Model

loading cases for easy interpretation and the applicability of these results to more general cases is still a matter of further research.

Another set of researchers have attempted to understand the behavior of reinforced concrete structures by plasticity approach. Advanced features taking care of non-local approach by Integral formulation or more successfully by Gradient plasticity have overcome problems of mesh sensitivity and could solve problems of concrete under uniaxial tension quite successfully. In spite of these success, problems still remains about the applicability of the approach to more general three dimensional stress-strain conditions. Even in simpler cases of classical plasticity, e.g. Drucker-Prager approach, one has to adopt different formulation for concrete in tension and compression.

Chen[7] deals in details the different available classical plasticity approaches. CEB reports[8),9)] deals with all available types of approaches that can be adopted for analysis of reinforced concrete members.

Recently a three dimensional classical plasticity model, named Unified Concrete Plasticity Model was proposed by Tanabe et al.[1]. It claimed to be able to capture the concrete behavior in *tension* and *compression* by an *unified approach*. This model is a modified Drucker-Prager model, such that Mohr-Coulomb core matches both at tensile and compressive meridian. This model was later modified by Gupta and Tanabe[2)~5)]. The applicability of the Unified Concrete Plasticity Model in three dimensional analysis of reinforced concrete structure has not been analyzed till now. Without going into the comparison of merits and demerits of different existing approaches, the main aim of this paper is to check the feasibility of application of the Unified Concrete Plasticity Model in three dimensional analysis of reinforced concrete structure before further development and implementation of advanced features like inelastic unloading of concrete, advanced reinforcement models etc. The authors are now working on the implementation of these advanced feature.

As a first stage of implementation and checking the applicability of the Unified Concrete Plasticity Model, the tension stiffening effect is implemented in a very approximate manner to check the feasibility of the application of the concrete model so that steps can be taken for further development of the advanced features. At this stage, only elastic unloading is implemented.

For this reason, two simple examples of beam under two point loading and column under top lateral loading are adopted so that more complicated cases can be dealt after implementation of more advanced features.

The above two cases are analyzed and compared with experimental results. The crack pattern obtained in the beam analytically and experimentally is also compared. Though some interesting results are obtained in the comparison of the above two cases, the authors understand that the results obtained at this first stage application of this new model needs further attention.

2. CONSTITUTIVE MODEL: UNIFIED CONCRETE PLASTICITY MODEL

Fig. 1 shows the initial shape of the yield surface of Unified Concrete Plasticity Model. To take care of the problem of selection of negative and positive sign in Drucker-Prager formulation for compressive and tensile condition, Tanabe et al.[1] introduced a parameter y in the denominator of Eq. 2, which is dependent on I_1 and θ (Eq. 4) . As a result we get triangular shape in the tensile region and more circular shape in the compressive region. As a result, this model can take care of stress-strain situation in both tensile and compressive zone in an ***unified*** manner. It is in this sense that Tanabe et al.[1] named the model *Unified Concrete Plasticity Model.*

Similarity with the name Unified Theory proposed by Hsu. and his colleagues[10)~13)] is accidental.

Gupta and Tanabe[2)] showed that various variations of the model proposed by Tanabe et al.[1)] based on the order of yield function and definition of damage are possible without significant change of behavior[3)~5)]. In this paper, the first order yield function and the simple damage law are used because it was found to be sufficient and better as it creates less deviation from the yield surface in comparison to the originally proposed second order yield surface[2)~5)]. The modified model is presented here in brief. The yield function is given as

$$g(\sigma,\omega)=\sqrt{J_2+k_f(1-AA^*/\eta_0)^2}-(k-\alpha_f I_1)=0 \quad (1)$$

$$k_f=\frac{6C\cos\phi}{\sqrt{3}\left(3+y\sin\phi_1\right)},\alpha_f=\frac{2\sin\phi}{\sqrt{3}\left(3+y\sin\phi_1\right)} \quad (2)$$

$$AA^*=\sqrt{3}c_o\cot\phi_o/\eta_o \quad (3)$$

$$y=\sqrt{a\left(\cos 3\theta+1.00\right)+0.01}-1.10$$

$$a=0.5r^2+2.1r+2.2,$$

$$r=\begin{cases}3.14 & I\le f_c' \\ 6.07-2.93\cos\left(\dfrac{f_t-I_1}{f_t-f_c'}\pi\right) & f_c'<I\le f_t \\ 9.0 & I>f_t\end{cases} \quad (4)$$

where σ is stress tensor; I_1, J_2 and J_3 are the stress invariants; $\cos 3\theta=(3\sqrt{3}J_3)/(2J_2^{1.5})$ and $\phi_1=14°$ is a material constant. Gupta and Tanabe[2)~5)] had proposed to make h, the distance of the tip of the yield surface a dependent variable unlike Tanabe et al.[1)]. On the other hand, Gupta and Tanabe[2)~5)] had proposed to use cohesion c and friction angle ϕ as the most important parameters depending on the damage ω. Gupta and Tanabe[2)~5)] pointed out that the cohesive and frictional property of concrete should be very different in the case of tensile and compressive zone. Hence it was proposed that the parameters c and ϕ should also reflect the real cohesive and tensile properties of the material, depending of the damage state, stress-strain condition etc. As a first step towards simulating the real cohesive and frictional properties of the material, the initial material parameters defining them are assumed to be constant at all stress conditions. The change of frictional angle ϕ and cohesion c are assumed to be different in tension and compression zone.

In this model, it was assumed that hardening of stress-strain is primarily caused due to the increase of friction angle ϕ and the softening occurs because of decrease of cohesion c. Since we don't see hardening in uniaxial tension or in parts of tension-compression zone, ϕ is assumed to be constant in this range. On the other hand, since we find clear hardening in compression-tension zone or in compression zone even in Kupfer's experiment, ϕ is assumed to change from ϕ_o to ϕ_f in compression - compression zone. The sharp drop in cohesion in tension zone is to create the sharp drop in stress-strain curve depending on fracture energy or tension stiffening effect, as the case may be. In compression zone, the gradual drop of cohesion c is to create gradual drop in stress in compression. An appropriate variation between the two zones was proposed based on parameter $X(=I_1/\sqrt{3J_2})$[2)~5)] (**Fig. 2**).

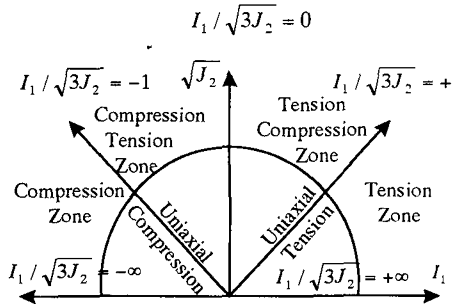


Fig. 2 Different Zones in (I_1,J_2) Space

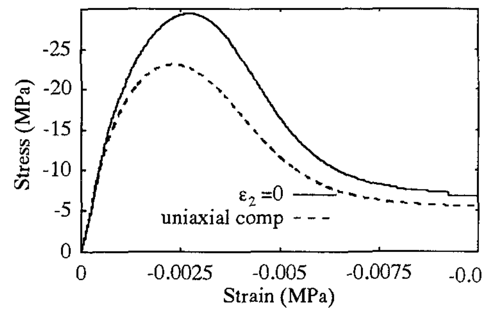


Fig. 3 Stress Strain in Uniaxial Compression

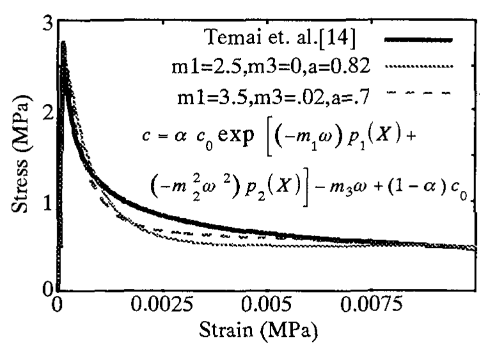


Fig. 4 Stress Strain in Uniaxial Tension

Tension stiffening effect requires stress to remain non-zero for RC members, unlike in plain concrete. Even in compression, it has often been suggested that concrete stress-strain curve actually does not become zero with a constant value after 1.75 ε_0 (ε_0 - strain at peak stress point for uniaxial compression). Based on this requirement, the value of cohesion c is not dropped to zero in the final stage.

Based on material constants ϕ_0, ϕ_f , c_0, η_0 the authors propose the following relationship for c and ϕ for appropriate use in analysis of RC members as though other relations can also be considered as shown in **Fig. 2**. If necessary, more general relationship for cohesion [3)~5)] can adopted.

$$c=\alpha c_0 \exp\left[(-m_1\omega)p_1(X)+(-m_2^2\omega^2)p_2(X)\right]+(1-\alpha)c_0,$$

$$\phi=\begin{cases}\phi_0+\left(\phi_f-\phi_0\right)\sqrt{(\omega+k)(2-\omega-k)}\;p_2(X) & \omega\le 1\\ \phi_0+\left(\phi_f-\phi_0\right)p_2(X) & \omega>1\end{cases} \tag{5}$$

where $\kappa=10^{-3}$ (small value) is introduced to get rid of the singularity caused by atω=0 and

$$\{p_1(X)\quad p_2(X)\}=\begin{Bmatrix}0 & 1\\ \frac{1}{2}\cos\left(\frac{X-a1}{a2-a1}\pi\right)+\frac{1}{2} & \frac{-1}{2}\cos\left(\frac{X-a1}{a2-a1}\pi\right)+\frac{1}{2}\\ 1 & 0\end{Bmatrix}\quad \begin{matrix}X\le a1\\ a1<X\le a2\\ X>a2\end{matrix} \tag{6}$$

where a1= -1 and a2= -0.15 was found to be appropriate[3)~5)] for concrete stress-strain curve. The *simple damage* model is defined such that scalar plastic multiplier$d\lambda$ is equal to plastic strain

$$d\omega=\beta' d\varepsilon^p=\beta' d\lambda \tag{7}$$

Fig. 3 shows the uniaxial compression behavior with and without lateral strain (ε_2 =0) and show quite reasonable stress-strain behavior with increased strength and ductility. **Fig. 4** shows stress strain response using different variation for cohesion c for uniaxial tension in comparison to the one proposed by Temai et al.[14].

3. MATERIAL PARAMETER SELECTION

This model has many input parameters. We can get similar set of uniaxial tension and compression stress-strain curve using different sets of input parameters. Hence it is not enough to match the uniaxial tension and uniaxial compression only. Hence the following steps should be followed to fix up the material parameter as shown in **Fig. 5**. These graphs are drawn with appropriate parameter for the analysis of the beam in such a way that the following conditions are fulfilled.

a) Kupfer's[15] biaxial peak stress envelope **(Fig. 5a)** is satisfied.

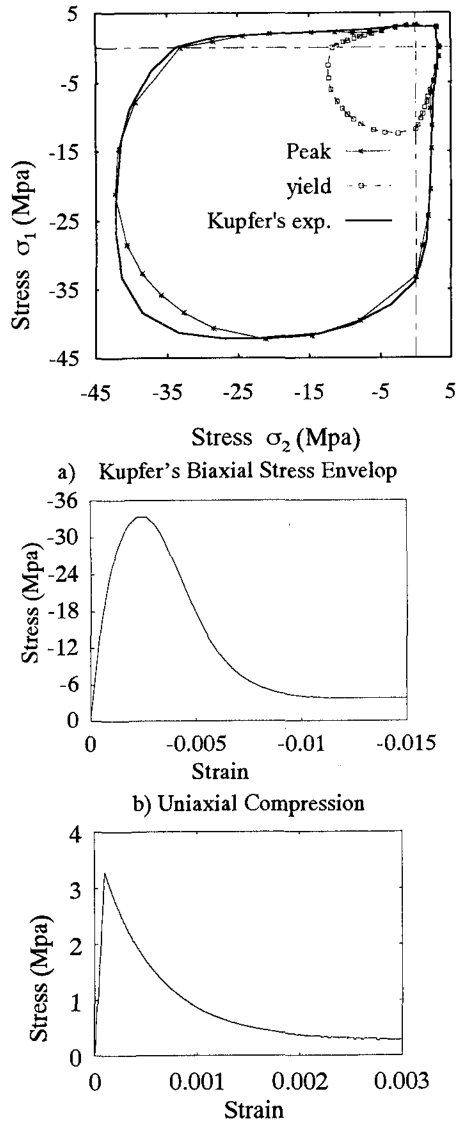


Fig. 5 Matching of Various Stress-Strain Relationship for Determination of Material Parameters

b) For uniaxial compression, peak stress, peak strain and appropriate hardening followed by appropriate softening behavior **Fig. 5b)**

c) Peak stress and appropriate softening slope for uniaxial tension**(Fig. 5c)**

For easy determination of the material parameter one should remember the following characteristics of the yield surface.

a) Since cohesion c and friction angle ϕ is controlled by the damage ω, which is a function of β_1 (Eq. 4.36). Therefore we can control the overall ductility of the stress-strain behavior by changing β_1 only.

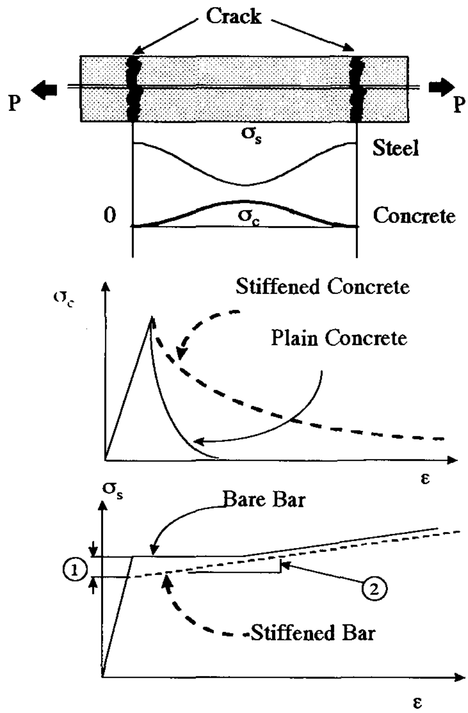


Fig. 6 Tension Stiffening Effect in RC Panels

This change will effect all stress-strain behavior after yielding equally.

b) One has to change c_o, ϕ_t and ϕ_f to match the Kupfer's biaxial peak strength envelope.

c) Changing m_1 effects the uniaxial compression behavior without effecting the uniaxial tension. Changing m_2 effects the uniaxial tension softening behavior without effecting the uniaxial compression behavior. η_o can be changed to control the peak strength of concrete in tension.

4. REINFORCEMENT AND TENSION STIFFENING EFFECT

For concrete in absence of reinforcement, the behavior highly depends on the fracture energy released due to the formation of cracks. Size effect in beams without stirrups failing in shear is a direct effect of such phenomenon and are being studied by different approaches.

On the other hand, Rizkalla et al.[16),17)], Vecchio and Collins[6)], and other researchers[10)~14),18)~20)] have pointed out by experimental and numerical research on thin reinforced concrete panel under uniaxial tension that concrete in presence of reinforcement acts very differently in comparison to that of the unreinforced concrete specimen[15)]. In these cases, the force is generally applied through the reinforcements. A schematic diagram of this problem is given in

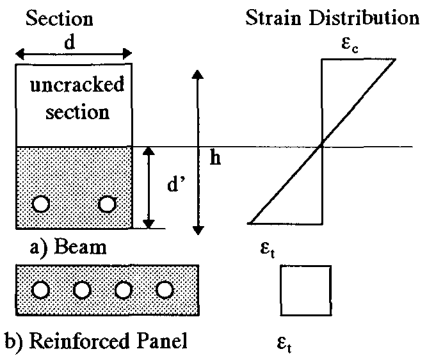


Fig. 7 Distribution of Tensile Strain and Cracked Section

Fig. 4, where the stress and strain distribution is also provided between the cracks.

In these uniaxial tension studies on thin reinforced concrete panel with both lateral and longitudinal reinforcement, multiple cracks are generated. When the concrete cracks, the steel is generally in elastic condition. The surface cracks generally coincides approximately with the location of the transverse reinforcement or at places where weak zone exists. Though the number of cracks increases in the initial stage, a stable crack configuration is generally achieved before the reinforcement yields at some cracked section. Concrete between the cracks take significant amount of stress. This phenomenon is called ***tension stiffening*** effect.

Since many researchers adopt smeared crack approach, it is often important to calculate the average stress of concrete and steel. Many researchers have pointed out that when it is reasonable to assume average concrete stress does not become zero at large strain because concrete between the crack take significant amount of stress, it is perhaps also reasonable to assume that average steel stress (called as apparent yield stress) at the yield point is lowered because steel yields initially only at the cracked points.

Hsu and his colleagues[10)~13)] have proposed the use of bilinear curve (or single equivalent curve) with two important features: *lower apparent yield stress* (marked ① in **Fig. 6**) and *higher post yield slope* (marked ② in **Fig. 6**) for proper simulation of the average stress-strain relationship in a reinforced concrete member due to tension stiffening effect. Okamura, Maekawa and his colleagues[14),18)~20)] are also studying these effects in more detailed manner. They found that the *apparent yield stress* and *post yield slope* to be dependent on the reinforcement ratio. In these experiments, multiple cracks are generated and followed by a stage with stable crack configuration.

However, in real cases, cracks do not occur in such simplistic manner. For example, in the case of the beam under two point loading, the bottom sections is cracked under tension. Though multiple cracks are generated in equal interval, each crack propagates(with increasing area of cracked surface) with a varying distribution of strain along the crack(**Fig. 6**). However in the case of thin panels, the cracks were generally through with uniform strain distribution cracks across the cross-section. Hence, the applicability of the models, that are derived based on research of thin panels, to more complicated structure needs more attention. Hence one important question remains:

"Would these models that were derived based on study of thin RC panels be applicable to RC members like columns and beams where the situations are quite different?"

About the two problems (beam under two point loading and column under top lateral loading) selected for analysis in this paper, one important difference should be noted. In the case the beam under two point loading, cracks appear in regular interval in the bottom very similar manner to the panels under uniform tension. However in the case of column under top lateral loading, even though more then one crack appear, the bottom crack is always predominant and cracks are not in regular interval. Hence we can expect that tension stiffening effect would be more dominantly reflected in the load-displacement behavior of beam under two point loading then the case of column. But since more then one crack appear, some sort of effect due to tension stiffening effect should be reflected in the load-displacement behavior of the column.

Recently Okamura, Maekawa and his colleagues[14),18)~20)], had proposed the implementation of tension stiffening effect by dividing the concrete volume into RC and non-RC volume[19),20)], where they have implemented tension stiffening effect in RC volume for both concrete and reinforcement. This is a good approach and the prospects looks promising.

Since the main aim of this paper is to check the feasibility of application of Unified Concrete Plasticity Model, the tension stiffening effect is implemented in a very simple manner. In concrete, it is reflected in the softening slope of the stress-strain in uniaxial tension as shown in **Fig. 6**. The effect of implementation of different stress-strain behavior for reinforcement in tension with lowered apparent yield stress and higher post yield slope is tested. Though the authors understand that it is worth dividing the concrete in RC and non-RC zones, as a first stage of implementation of the Unified Concrete Plasticity Model, is implemented by adopting similar behavior of concrete in all parts.

5. RC BEAM UNDER TWO POINT AND RC COLUMN UNDER LATERAL LOADING

In both the analysis presented in this paper, 8 nodded three dimensional elements are used for concrete and reinforcement elements are modeled as two nodded truss element.

Fig. 8a) shows the dimensions of the beam under two point loading with enough stirrups to cause bending failure carried out in Nagoya University, Japan as a part of regular yearly educational activity. **Fig. 9** shows a column under lateral load with controlled displacement[21)]. Based on the average of two cylinder strength (f_c' =33.75 MPa and 35 MPa), E_c and f_t (mean value) are calculated according to the CEB-FIP Model Code -90[22)]. Material parameters are chosen to satisfy Kupfer's experimental results[15)] and stress strain of concrete under tension to simulate tension stiffening effect. The selected parameters are shown in **Table 1** and rest of the parameters which were assumed equal in the two cases are μ=0.22, $\phi_o = 5^\circ$, $\phi_f = 36^\circ$, m_1=4.0, m_2=0.83, η_o =7.0 MPa, k = 1.0 x 10^{-3}, ω_f=1.0, $\beta' = 35$, γ = 0.92, a1= -1.0 and a2= -0.15. **Fig. 5** shows three steps for the material parameter selection for the beam under two point loading. Similar steps were taken for the column and is given in more details in the doctoral thesis of Gupta[5)].

For the beam, in drawing the load-displacement diagram of the experiment, the displacement at the support due to support shrinkage is also taken into consideration. **Fig. 8b)** shows the finite element mesh of the quarter section of the beam that is taken for the analysis. As necessary, X or Y direction displacements are restrained for all points in the two symmetry section. In this case, all sections are assumed to have uniform lateral reinforcement. Three cases, with the three different stress-strain relationship for reinforcement with modulus of elasticity E_s=210000 MPa, are considered in the analysis as shown in **Table 2** or **Fig.10** where E_s is the modulus of elasticity in the elastic zone. In case A, we take a bilinear elastic - perfectly plastic curve (post - yield slope = 0.01% of E_s is taken for stability of the analysis). In case B, we take a bilinear curve with approximate apparent yield stress and higher slope to simulate tension stiffening effect. In case C, we take trilinear curve with case B type bilinear

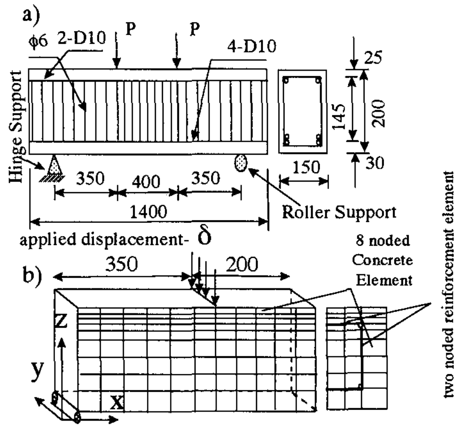


Fig. 8 Dimension and Mesh of the Beam for Bending Failure (in mm)

Table 1 Common Parameters for the Concrete

	E_c (MPa)	f_t (MPa)	f_c' (MPa)	c_o (MPa)	ϕ_1
Beam	34600	3.15	33.75	22.58	14°
Column	34960	3.23	35	24.58	14.3

Table 2: Parameters for Reinforcements for the Beam

Case	f_y (MPa)	slope (x Es)	f_{y1} (MPa)	slope (xEs)
A	370.0	0.0001	-	-
B	323.4	0.00785	-	-
C	323.4	0.00785	370.0	0.0001

Table 3: Parameter for Reinforcement for Column

Case	f_y^* (MPa)	slope (xEs)
A	380.0	0.0001
B	332.2	0.00785
C	380.0	0.00785

curve in the beginning and change to case A type curve (with yield stress of f_{y1} and post yield slope of 0.01% of E_s) at the intersection point.

For the column, the dimension and details of the specimen and the mesh selected are shown in **Fig. 9**. The base is not taken into consideration due to computer limitation. Half section is taken for analysis. Hence the y direction is restrained for all points in the symmetry section. Three cases with different stress strain behavior of reinforcement are considered as shown in **Table 3** or **Fig.11** where E_s is the modulus of elasticity in the elastic zone. For case A, yield strength with post yield slope of $0.0001E_y$ was used to maintain stability in the calculation. In Case B, the tension stiffening effect is incorporated approximately by adopting appropriate lower apparent yield stress and higher apparent post yield slope. In Case C, we just incorporate the slope as case B , but use f_y = 380 MPa in place of f_y^*.

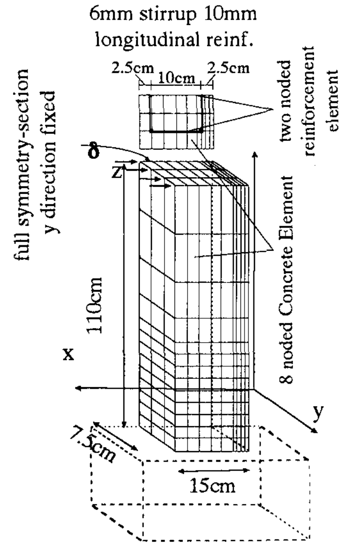


Fig. 9 Dimension and Mesh Used for Column

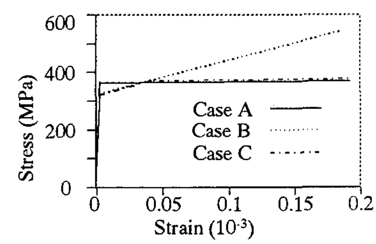


Fig. 10 Reinforcement Model for the Beam

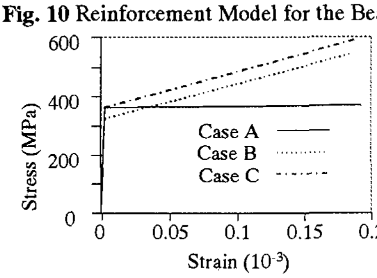


Fig. 11 Reinforcement Model for the Column

Reasonable match in load displacement diagram between the experiment and analysis was observed for both beam and the column. The details will be discussed details in later section. In the case of the column, since the bottom was assumed fixed, the effect or reinforcement pullout (in other words, the displacement of the node of the reinforcement in tension inside the footing) is ignored. The order of top displacement is 5.5 mm at the yield point of the steel due of this effect by approximate method provided by Ishibayashi, T. and Yoshino, S[23]. The effect of this is also reflected in the difference in deflection of the analysis and experiment in the elastic stage. However the difference between analysis and experiment is of the order of 1.8 mm. Since this is the initial stage of analysis, the authors

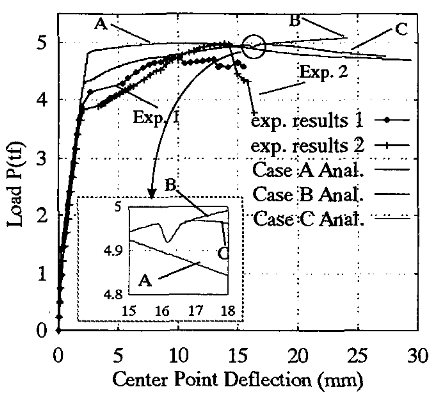


Fig. 12 Load Deflection Behavior for the Beam Under Two Point Loading

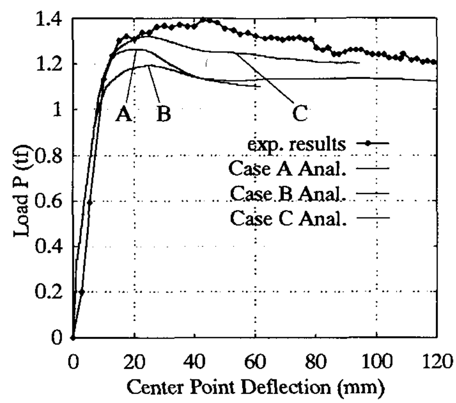


Fig. 13 Load Deflection Behavior for the Column Under Top Lateral Loading

will check this in future when more detailed checking for material parameters determination for different strength of concrete is considered. This order of deflection of the beam at the ultimate load is about 6.5 mm[23] and is small compared to the actual defection at this stage.

(1) Evaluation of Implementation of Tension Stiffening Effect

From the above calculations for the beam under two point loading or the column under lateral loading, a few interesting observations can be made.

First let us study the results as obtained when the beam is analyzed when the elastic-perfectly plastic (case A) behavior is assumed in the analysis. In this analysis(with case A) and experiment (**Fig. 12**). The load-displacement showed a stiff behavior after that first concrete crack appeared. In case of the analysis, the load displacement diagram first deviate to a more ductile behavior at the point when the reinforcement first yields. However, in the experiment, the load-displacement diagram deviate at a much lower load level. In these experiments, the authors did not check yield point of the reinforcement.

After the deviation, load-displacement behavior in the experiment showed considerable stiffness with considerable increase of load with increase in displacement where as almost no increase of strength was noticed for the beam in case of the case A analysis. The peak strength in the experiment was reached at a later stage of the load-displacement diagram. It should be noted that the peak load in the experiment matched well with that case A analysis also matched with peak strength or ultimate strength by equivalent stress block method(P=4.75 tf).

This behavior is correctly predicted when the tension stiffening effect (case B and C) with *lowered apparent yield stress* and *higher post yield slope* was implemented for the reinforcement. As a result, the point of deviation of the load displacement diagram from the stiff behavior to more ductile behavior was lowered and considerable increase of load after this point was noticed.

In the *case of column*, even though case B and C resembled the shape of experimental load deflection diagram better than the elastic-perfectly plastic case A. However, case C without lowered apparent yield stress provided better load-deflection diagram than case B with lowered apparent yield stress at the point of deviation of the load displacement diagram from the stiff behavior to more ductile behavior. The peak strength or ultimate strength by equivalent stress block method is 1.12 tf.

The *possible* reason between the difference of the beam and column lies in the fracture process in the cases. In the case of the beam(**Fig. 14**), the crack zone was wide and cracks appear at regular interval. Hence reinforcement might have indeed yielded in multiple positions in the beam simulating tension stiffening effect properly similar to that of the thin plate under uniaxial tension. However in the case of the column (**Fig. 13**), the crack zone was very small and reinforcement possibly yielded only in a small zone at the bottom.

The abrupt failure noted in the beam could not be simulated. Implementation of case B of the beam resulted in increasing slope. The decrease in load do not occur as the stress in the reinforcement keeps increasing. Use of intermediate relationship (case C) resulted in decrease in load. The authors checked the effect of imposing symmetry of quarter beam analysis by adopting half beam. Similar results(not shown here) were found.

(In Fig.12, value of P is mentioned in tf)

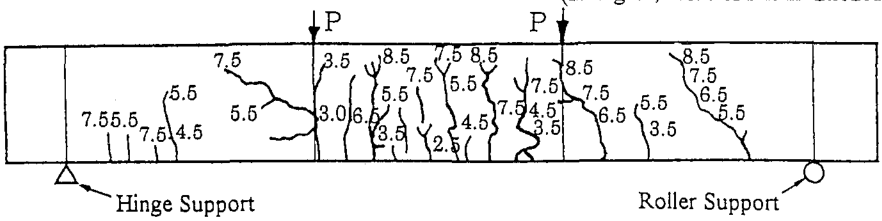


Fig. 14 The Crack Pattern for the Beam

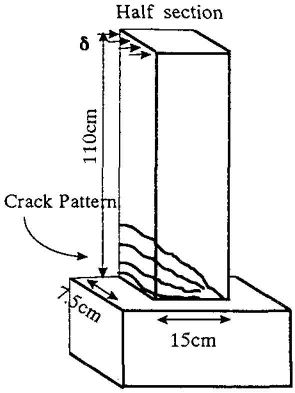


Fig. 15 Crack Pattern for the Column

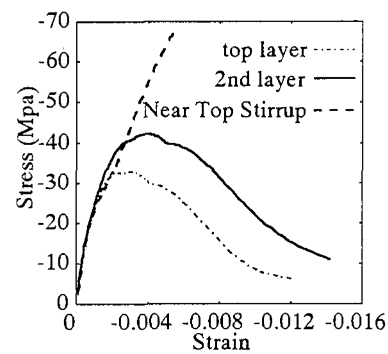


Fig. 16 Confinement Effect in σ_x-ε_x Relationship: Stress-Strain at the Center Section of the Beam

We can see in the magnified zone of **Fig. 9** that there is a sharp drop in load. The reason behind this was found when we studied the incremental strain $\Delta\varepsilon_x$ in the localized crack that developed as shown in **Fig. 17.1.3.3**. This is described in details later.

The difference in the load deflection behavior in the experimental result can be noted. The column showed ductile behavior where as the beam showed brittle behavior in the last stage is strange. The reason could be because the column was tested in displacement controlled manner where the beam was tested in load controlled (hand control) manner.

The authors intends to implement large deflection small strain theory and beam element for reinforcement in compression to simulate buckling of the reinforcement in future so that the reinforcement in the compression zone buckles. This can help possibly simulate the failure of beams in the final stage and provide a more realistic model for the reinforcement.

(2) Damage Process in the Beam

The concrete stress distribution at the central section at a later *Stage e* is shown in **Fig. 17.4** for Case C of the beam. We can see that bottom part of the section has cracked, where as top part has already crushed. The top layer provided a stress-strain behavior quite similar to the uniaxial compression case. The effect of confinement on the stress-strain behavior in the second layer and in the layer near the stirrup is shown in **Fig. 16** which shows increased peak and ductility. The part near the stirrup showed high confinement effect. This confinement effect near the stirrup looks unrealistic and needs more attention and possibly can be rectified by implementation of reinforcement element that can take can buckle in compression between the stirrups.

To understand the condition of the cracks, **Fig. 17.1** shows the stress σ_x, strain ε_x and incremental strain $\Delta\varepsilon_x$ distribution in x-z plane (Section II) at different stages for the beam. Strain ε_x (also in incremental strain $\Delta\varepsilon_x$) distribution of **Fig. 17.1.3** shows concentration of higher strain occurrence at multiple points. They represent multiple discrete crack in the analysis. This occurred because the eight nodded element can only show linear strain. This problem can be removed by using higher order element. However, looking from another point of view, this simulation of multiple crack can be called more realistic. In the experiment, we also get similar distinct multiple cracks.

If we notice the crack pattern of the experiment (**Fig. 11**), it can be observed that the central cracks grows at a later stage. Finally, strain localization occurs and the central crack opened up. This results in closing of other cracks. A phenomenon, quite similar to this, was observed in the analysis also.

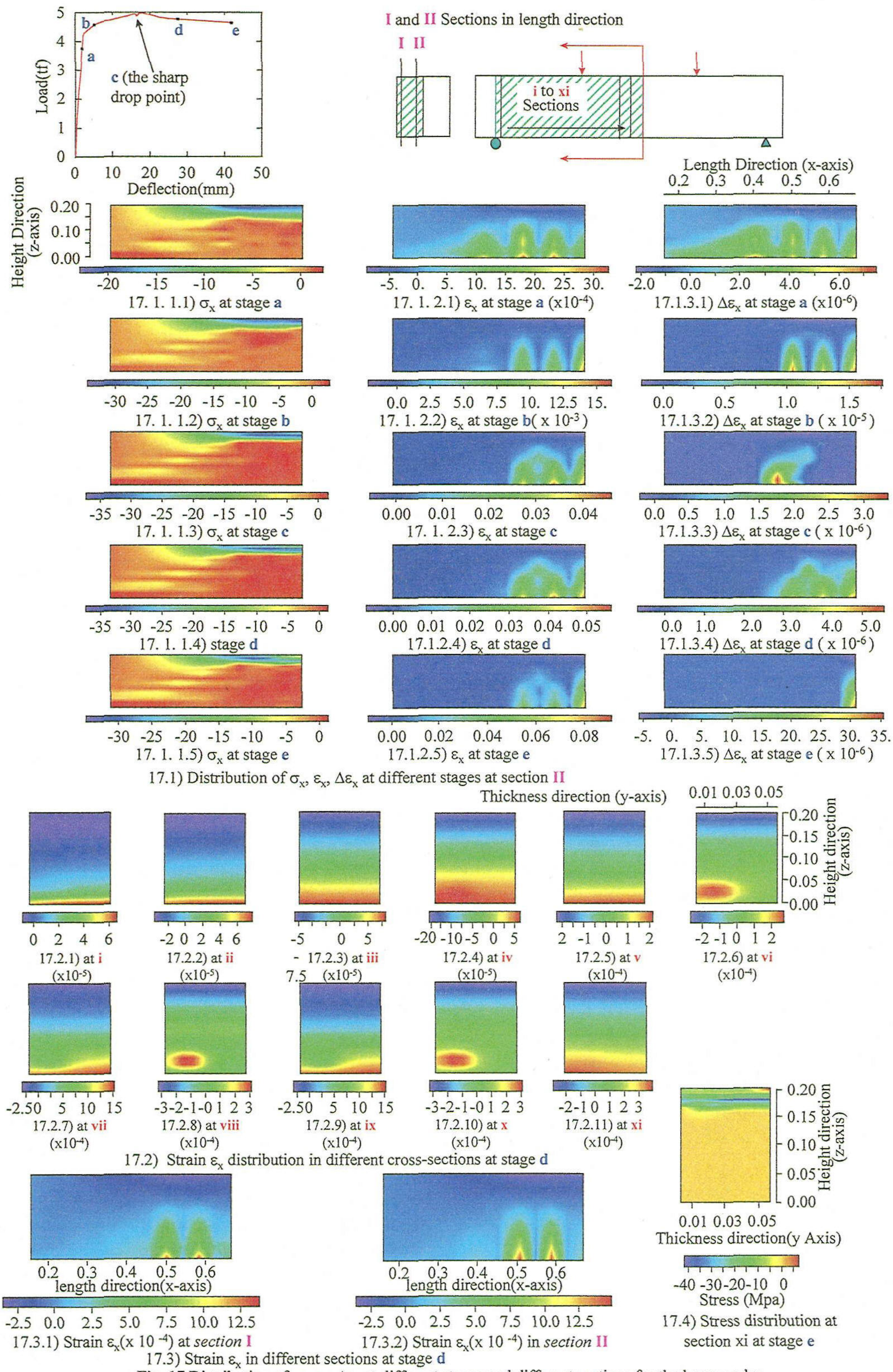


Fig. 17 Distribution of σ_x, ε_x, $\Delta\varepsilon_x$ at different stages and different sections for the beam under two point loading at stage d of Model R-3

Fig. 17.1.3 of strain $\Delta\varepsilon_x$ distribution shows four major crack. The forth crack is outside the central zone(**Fig. 17.1.3.1**) as is observed in the experiment. This represent the shear crack. In the beginning, the central crack is not the dominating crack. However in the final stage(**Fig. 17.1.2.5** or **17.1.3.5**), stain localization occurs at the center of the beam and the central crack open up. The fourth crack or the shear crack closed at this stage. The strain in the other two crack also decreases. **Fig. 17.2** and **17.3** shows the distribution of strain at stage a to show that the strain is quite three dimensional in nature.

6. CONCLUSION

In this paper, three dimensional finite element analysis of two cases of a beam under two point loading and column under lateral loading were carried out using Unified Concrete Plasticity Model. The aim of the paper was to check the feasibility of applicability of Unified Concrete Plasticity Model in three dimensional analysis of RC members so that further steps can be taken to develop the advanced features of the model and reinforcement. After carrying out these calculations, we can make the following conclusion.

a) The Unified Concrete Plasticity Model is able to satisfactorily simulate the experimental results.
b) The stress-strain relationship was simulated properly and it reflected well in the comparison of the development of the crack pattern between the experiment and the analysis.
c) The confinement effect was simulated well as shown in the stress in the inner layer.
d) Implementation of tension stiffening effect in the reinforcement effects the load deflection diagram considerably.
e) Implementation of the higher post-yield slope for the reinforcement provide better results for both the cases, however in case of the column, implementation without lowered apparent yield stress provided better results.

f) One of the possible reason can be the fact that there was a wide range of uniform multiple crack pattern in the case of beam providing a proper tension stiffening effect where as the crack zone in the column was very narrow.

There are also some other interesting results of beam failing in shear that were also successfully carried out[5] where diagonal sudden failure is noticed. More detailed analysis will be made and will be presented in future.

Since unexpectedly high confinement effect was noticed near the stirrup at the top, there may be necessity to introduce beam element for the reinforcement to simulate the buckling that occurs in compression zone at the final stage of failure. It is also important to implement better methods so that we get more realistic unloading paths. At present, the authors are presently working on these topics.

Finally there exists a critical point that needs further attention. The authors understands that strength of the beam under two point loading at yield point of the reinforcement should be controlled by the sectional properties at the crack point. Now generally the reinforcement yields at these crack points at the actual yield stress of the reinforcement. Based on this logic, the strength should not be lowered due to the lowering of the average yield point of the reinforcement under tension.

On the other hand, the implementation of tension stiffening effect is also logical if one does not implement bond slip between concrete and steel is more detailed manner.

However one thing that is important is that analysis of the beam (with case A), experiment and the sectional properties showed equal peak strength. The authors is not sure if in the experiment, the point of deviation of the load-deflection diagram from stiff zone to more ductile zone represent the yielding of steel or not. However, difference between analysis and theory at this are also witnessed by other researchers also[24]. Tension stiffening effect with *lowered apparent yield strength* was also implemented by An et al.[19),20)], even though they did not study the difference from the same point of view as is done in this paper. If *lowered apparent yield strength* for tension reinforcement is implemented, the strength of the beam under two point loading at yield point will inevitably be lowered. If indeed the lowering of strength due to lowering of apparent yield strength of the reinforcement in tension is a real phenomenon or not is a matter that needs more attention in future.

Finally, this is to note that the authors faced a lot of problems due to the higher tendency of deviation from the yield surface when the original yield surface with second order stress terms[1] was used. All the calculations presented in this paper calculations were carried out using yield surface with first order stress and similar problem was not witnessed. However no quantitative documentation is offered because no results using the original yield surface exists with the authors. A qualitative proof of this was presented by Gupta and Tanabe[4] using constitutive level calculations.

ACKNOWLEDGEMENT: The first author wants to acknowledge and thank Prof. H. Umehara of

Nagoya Institute of Technology, Prof. Ikuo Hirasawa of Chubu University, Prof. Hsu, T.T.C. of Houston University, USA and Prof. K. Maekawa of Tokyo University for all the communications. The first author also wants to thank Yoneyama Rotary of Japan for supporting the last two years of his doctoral degree of which this paper is a part.

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(Received December 16,1997)

統一化塑性モデルによる鉄筋コンクリート（ＲＣ）部材の三次元解析

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鉄筋コンクリート（ＲＣ）部材の3次元有限要素解析は極めて複雑であり,各種の考慮を必要とするが,中でも3次元応力空間におけるコンクリートの構成則が重要である.今まで様々な構成則が発表されているが,その多くが引張側と圧縮側で別々のモデルを用いている.本研究は,多軸応力ひずみ状態で適応可能な「統一可塑性モデル1)」を更に発展させ2)~5),この論文ではこの発展したモデルの3次元解析への実用性,を実構造物について検証した．その際に,主鉄筋と帯鉄筋を有する柱•はりについて3次元解析を行なった.