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SHEAR-FATIGUE BEHAVIOR OF STEEL-CONCRETE SANDWICH BEAMS WITH SHEAR REINFORCEMENT

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Fatigue tests were carried out for steel-concrete sandwich beams with shear reinforcement. Under fatigue loading, the sandwich beams indicated three failure modes, namely shear compression failure due to crushing of concrete between diagonal cracks and the fatigue fracture of the steel plates either the lower flange plate or the shear reinforcing steel plates. The *S-N* relationship for the different failure modes is presented. The fatigue strength of the sandwich beams was also predicted by the finite element method. A simple method to predict the strains in the shear reinforcing plates under fatigue loading is presented. A fatigue design proposal for the sandwich beams is also presented.

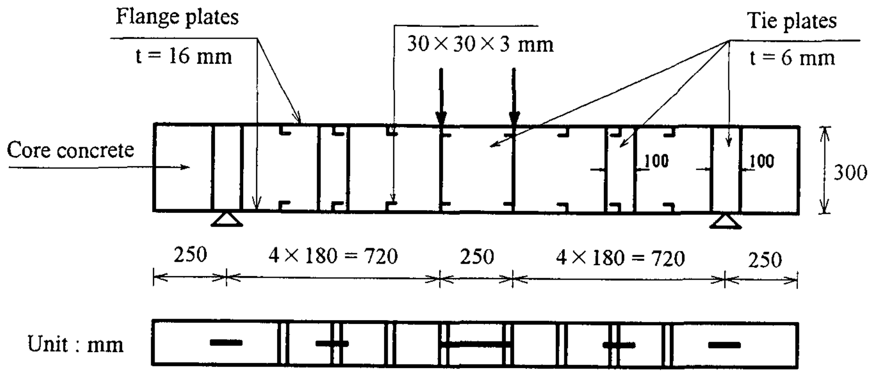
Key Words *: fatigue strength, finite element analysis, shear reinforcement, steel-concrete sandwich beam*

1. INTRODUCTION

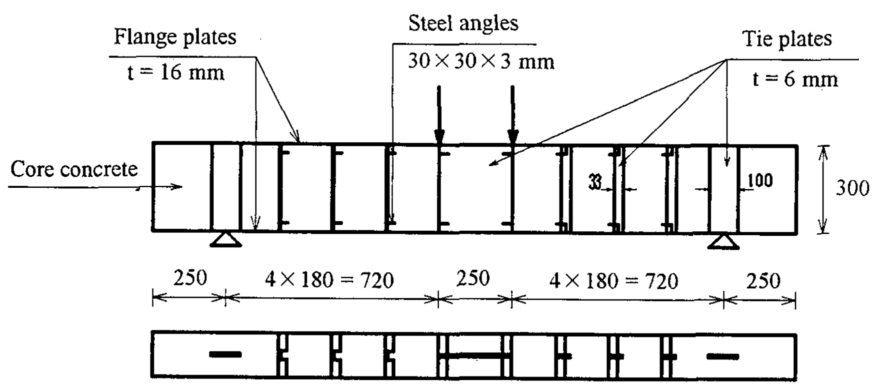
Recently, the applications of composite structures have become increasingly popular. A new type of composite structures, which is the steel-concrete sandwich structure, has been developed to fulfill complicated structural requirements. As shown in **Fig.1**, the steel-concrete sandwich member is composed of core concrete, flange steel plates, shear reinforcing steel plates, and shear connectors (i.e., steel angles) to transfer the shear between the concrete and the flange plates. The sandwich member has proved higher load carrying capacity and higher ductility than comparable ordinary RC member. The sandwich member has also proved good constructibility since the cost of formwork and the period of construction can be considerably saved. However, the fatigue strength of the sandwich member appears to be a weak point because of the welding parts between the steel components. Under fatigue loading, fatigue cracks originate at these welding parts. Then, these cracks propagate gradually through the steel plates until complete fracture occurs. The flexural capacity and the shear capacity of this type of members have been extensively investigated under static loading conditions. Therefore, the design code for the steel-concrete sandwich structures has been proposed recently[1)]. However, there is a lack of the experimental data regarding the strength of this type of members under fatigue loading conditions. A previous paper was presented by the authors studying the shear-fatigue behavior of steel-concrete sandwich beams without shear reinforcement[2)]. This paper presents experimentally and analytically the fatigue strength of the steel-concrete sandwich beams with shear reinforcement.

2. EXPERIMENTAL WORK

Experimental works were carried out for the steel-concrete sandwich beams shown in **Fig.1**, which had a span length of 1.69 m and a cross section of 150×300 mm. The sandwich beams were tested by two symmetrical concentrated loads, and the shear span to effective depth ratio (***a/d***) was equal to 2.4. The thickness of the upper and lower flange plates was 16 mm. As shown in **Fig.1**,



a) Sandwich beam type A



b) Sandwich beam type B

Fig.1 Geometry and loading configuration of the sandwich beams

the sandwich beams were provided with shear reinforcing steel plates. For the sandwich beam type A, the right and the left shear spans were reinforced with a vertical steel plate of 100 mm in width and 6 mm in thickness. This vertical plate was placed at the center of the shear span parallel to the member axis. For the sandwich beam type B, the right and the left shear spans were reinforced with three vertical steel plates each of 33 mm in width and 6 mm in thickness. In the right shear span, the vertical plates were placed parallel to the member axis with a spacing of 180 mm. However, in the left shear span, the vertical plates were placed normal to the member axis with a spacing of 180 mm. These vertical steel plates were welded at their ends to the upper and lower flange plates. These vertical steel plates will be called hereafter as the ' tie plates '. Note that the ratio of the shear reinforcement is identical in all the shear spans for the sandwich beams under investigation. The flange plates as well as the tie plates were of steel type SM 490A which had a yielding point of 400 MPa and a tensile strength of 550 MPa. The average compressive strength of the concrete was 15.4 MPa. The maximum size of aggregate used in the concrete mix was equal to 25 mm. Steel angles of 30 × 30 × 3 mm size were used as shear connectors which were welded to the flange plates. V-shaped groove was prepared for welding with full penetration. The welding material is for high strength steel with a tensile strength greater than 490 MPa.

Tests were carried out for six sandwich beams of type A, which are identified as specimens A1, A2, A3, A4, A5, and A6. Tests were carried out also for two sandwich beams of type B, which are identified as specimens B1 and B2. At first, specimen A1 was tested under static monotonic loading in order to know the static load carrying capacity of the sandwich beam type A. The sandwich beams type A and type B were assumed to have the same static load carrying capacity since the ratio of the shear reinforcement was identical. Thereafter, the specimens A2, A3, A4, A5, A6, B1, and B2 were tested under fatigue loading. The

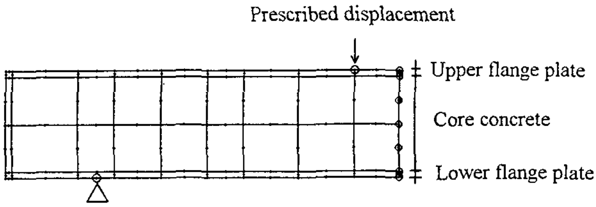


Fig.2 The finite element mesh

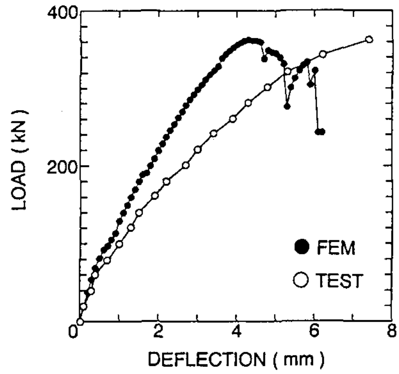


Fig.3 Load-deflection curves under static loading

minimum fatigue load (P_{min}) was kept constant at 20 kN which is about 5.5% of the static load carrying capacity of the beam ($P_{u,exp}$). For specimens A2, A3, A4, A5, A6, B1, and B2, the maximum fatigue load (P_{max}) was equal to 52.5%, 68.3%, 81.1%, 90.4%, 96.1%, 54.3%, and 41.8%, respectively of the static load carrying capacity of the beam ($P_{u,exp}$). In the fatigue tests, the specimens were loaded dynamically with 240 cycles per minute in a sinusoidal waveform until failure. Only in the fatigue test with P_{max}= 96.1% of $P_{u,exp}$, the loading frequency was equal to 60 cycles per minute. During the fatigue tests, the strain measurements of the tie plates were observed to check whether the tie plates fractured or not. Also, at the end of each fatigue test, the concrete was removed to confirm the fatigue fracture of the tie plates.

3. FINITE ELEMENT ANALYSIS

A computer program for nonlinear finite element method (WCOMR)[3] was used to analyze the steel-concrete sandwich beam shown in **Fig.1a** (sandwich beam type A). The finite element mesh of the beam is shown in **Fig.2**. For the concrete and steel elements, eight-node quadratic elements were used. Each concrete and steel element contains (3×3) Gauss points. The constitutive models for the concrete and steel elements are given in references 3) and 4). Bond elements were provided to simulate the shear connectors and the interface between the concrete and the lower flange plate. A linear bond stress-slip relationship was adopted as a constitutive law for the bond elements[4]. Prescribed displacements were given at the loading point as shown in **Fig.2**.

4. RESULTS AND DISCUSSION

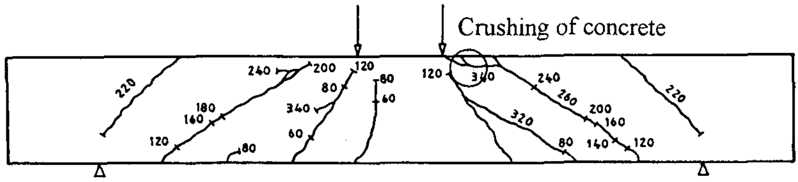
(1) Static monotonic loading

At first, the sandwich beam (specimen A1) was tested under static monotonic loading. The sandwich beam was also analyzed under static monotonic loading by using the finite element method. The experimental as well as the analytical load-deflection curves are shown in **Fig.3**. The experimental ultimate failure load ($P_{u,exp}$) was equal to 362.7 kN, while the analytical ultimate failure load ($P_{u,FEM}$) was equal to 362.5 kN. In both the experiment and the analysis, the failure mode of the beam was a shear compression failure which is characterized by diagonal cracking and crushing of concrete as shown in **Fig.4**. There is an excellent agreement between the analytical and the experimental ultimate failure loads as shown in **Fig.3**. However, the analytical load-deflection curve indicates higher stiffness compared to the experimental one which may be caused by the overestimation for the stiffness of the bond elements. Unfortunately, there is no available experimental data from which the stiffness of the bond elements could be estimated precisely. Furthermore, some instability was observed in the analytical load- deflection curve once the concrete in some elements goes into the softening range as shown in **Fig.3**.

(2) Fatigue loading

a) Results of the fatigue tests

Fatigue tests were carried out for five sandwich beams of type A (specimens A2, A3, A4, A5, and A6). For specimens A2, A3, A4, A5, and A6, the maximum fatigue load (P_{max}) was equal to 52.5%, 68.3%, 81.1%, 90.4%, and 96.1%, respectively of the experimental static strength of the beam ($P_{u,exp}$).



(numbers indicate the load value in kN)

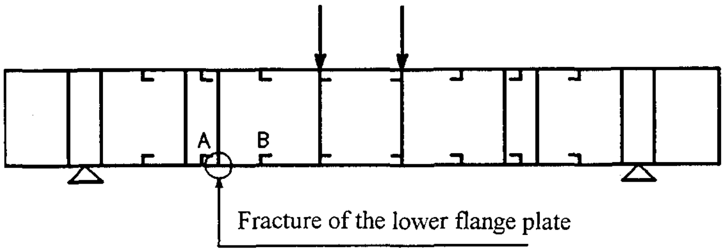
Fig.4 Crack pattern under static monotonic loading

Table 1 Results of the fatigue tests

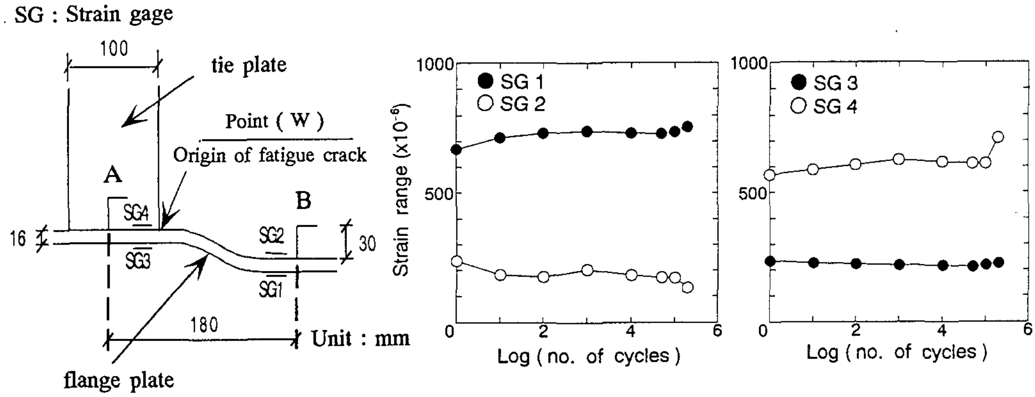
Fatigue test no.	Specimen	f_c' 1) (MPa)	$P_{min} / P_{u,exp}$ (%)	$P_{max} / P_{u,exp}$ (%)	Fatigue life (cycles)	Failure mode
-	A1 2)	16.7	-	-	-	CC 5)
1	A2	18.1	5.5	52.5	310,799	FP 3)
2	A3	13.8	5.5	68.3	13,724	TP 4)
3	A4	14.5	5.5	81.1	4662	TP 4)
4	A5	15.3	14.3 5.5	86.0 90.4	6000 2100	TP 4)
5	A6	14.5	5.5	96.1	70	CC 5)
6	B1	17.1	5.5	54.3	51,389	TP 4)
7	B2	13.0	5.5	41.8	526,000	TP 4)

1) The compressive strength of the concrete 2) Specimen for the static loading test
3) FP : Fracture of the lower flange plate 4) TP : Fracture of the tie plate 5) CC : Crushing of the concrete

The results of the fatigue tests are illustrated in **Table 1**. In fatigue test no.1 (P_{max} = 52.5% of $P_{u,exp}$), fatigue failure occurred due to fracture of the lower flange plate as shown in **Fig.5a**. The strain measurements have indicated that the part AB of the lower flange plate was subjected to local bending deformations (see **Figs.5a** and **5b**). These local bending deformations result in a concentration of the tensile stresses at point W which is the edge of welding line between the tie plate and the lower flange plate (see **Fig.5b**). Therefore, a fatigue crack originated from point W and then propagated in the lower flange plate with increasing the number of cycles (N) until complete fracture occurred. In fatigue test no.2 (P_{max}=68.3% of $P_{u,exp}$), the sandwich beam failed due to crushing of concrete in the vicinity of the loading point as shown in **Fig.6a**.Thereafter, the concrete was removed and a fatigue crack of 50 mm in length was found to propagate in the tie plate (see **Fig.6b**). In fatigue test no.3 (P_{max}= 81.1% of $P_{u,exp}$), the sandwich beam failed due to crushing of concrete in the vicinity of the loading point as shown in **Fig.6a**. Thereafter, the concrete was removed and the tie plate was found to be completely fractured (i.e., completely separated from the lower flange plate) (see **Fig.6c**). In fatigue test no.4, the sandwich beam was subjected to 6000 cycles during which the fatigue load was fluctuating between a minimum value of P_{min}=14.3% of $P_{u,exp}$ and a maximum value of P_{max}=86% of $P_{u,exp}$. Thereafter, the load range was increased to be fluctuating between a minimum value of P_{min}=5.5% of $P_{u,exp}$ and a maximum value of P_{max}=90.4% of $P_{u,exp}$. Then, fatigue failure occurred after 2100 cycles with the higher load range (i.e., P_{min}=5.5% and P_{max}=90.4% of $P_{u,exp}$). In this test, the sandwich beam failed due to crushing of concrete in the vicinity of the loading point as shown in **Fig.6a**. Then, the concrete was removed and a fatigue crack of 10 mm in length was found to propagate in the tie plate (see **Fig.6d**). In fatigue tests no.2, no.3, and no.4, the failure mode of the sandwich beam is considered to be fracture of the tie plate. That is because the fatigue crack occurs in the tie plate at first and then new diagonal crack

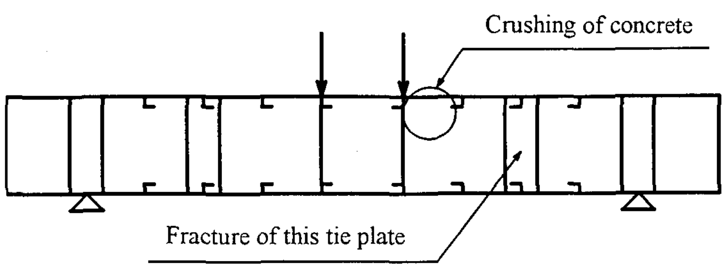


a) Failure mode of the sandwich beam

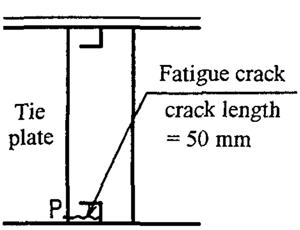


b) Local bending deformations of the lower flange plate

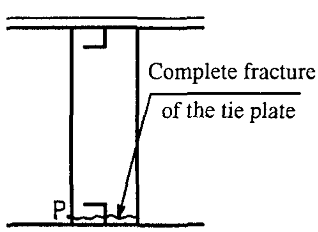
Fig.5 The failure mode in fatigue test no.1 (P_{max} / P_u = 52.5%)



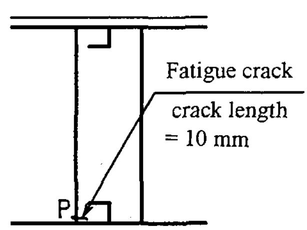
a) Failure mode of the sandwich beam



b) Fracture of tie plate
(fatigue test no.2)

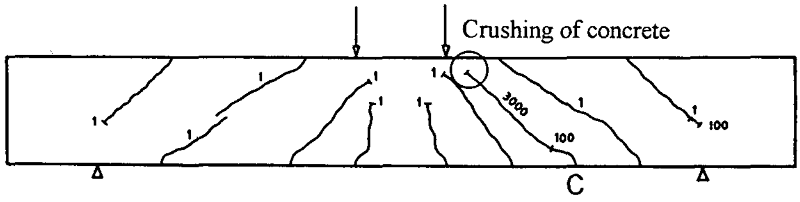


c) Fracture of tie plate
(fatigue test no.3)



d) Fracture of tie plate
(fatigue test no.4)

Fig.6 The failure mode in fatigue tests no.2, no.3, and no.4



(numbers indicate the number of loading cycles)

Fig.7 Crack pattern of the sandwich beam in fatigue test no.3

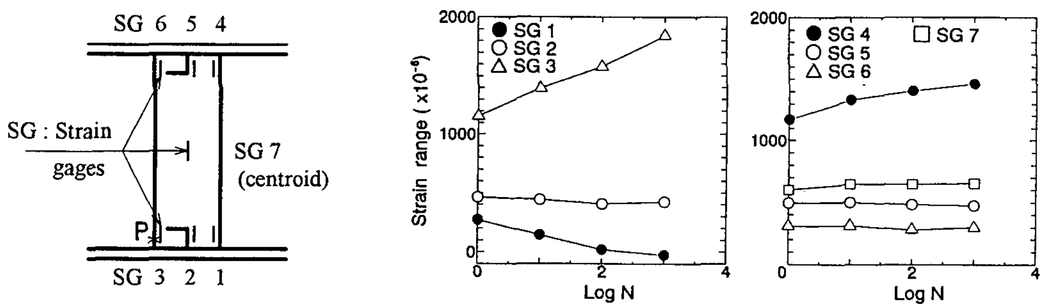


Fig.8 The relationship between *logN* and the strain ranges in the fractured tie plate in fatigue test no.3

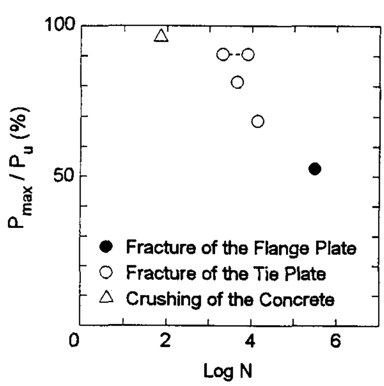


Fig.9 The *S-N* relationship of the beam type A

originates from the shear connector at point C (see **Fig.7**) and propagates towards the loading point. Therefore, the deflection of the beam increases and finally the crushing of the concrete takes place. It is observed that the fracture of the tie plate occurs always at the welding part between the tie plate and the lower flange plate. Also, the fatigue crack starts always from point P which is the most tensioned point in the tie plate (see **Figs.6b**, **6c**, and **6d**). This could be illustrated by the relationship between the number of loading cycles (N) and the strain ranges in the tie plate as shown in **Fig.8**. In fatigue test no.5 (P_{max}=96.1% of $P_{u,exp}$), the sandwich beam failed due to crushing of concrete in the vicinity of the loading point. Then, the concrete was removed and the tie plate was found to be sound (i.e., no fatigue crack has occurred in the tie plate). Therefore, in fatigue test no.5, the failure mode of the beam is considered to be crushing of concrete. The *S-N* relationship for fatigue failure of the sandwich beam type A is shown in **Fig.9**. Note that the fatigue life of the beam at $P_{max}/P_{u,exp}$=90.4% is plotted by two points connected with a dotted line, the first point at 2100 cycles and the second point at 8100 cycles. The first point neglects the effect of the 6000 cycles with P_{max}=86% of $P_{u,exp}$ and P_{min}= 14.3% of $P_{u,exp}$. On the other hand, the second point considers the effect of the small load range (P_{max}= 86% and P_{min}=14.3% of $P_{u,exp}$) exactly the same as the effect of the big load range (P_{max}=90.4% and P_{min}=5.5% of $P_{u,exp}$). Hence, the actual fatigue life of the beam is located between 2100 cycles and 8100 cycles.

Fatigue tests were carried out for two sandwich beams of type B (specimens B1 and B2). For specimens B1 and B2, the maximum fatigue load (P_{max}) was equal to 54.3% and 41.8%, respectively of the experimental static strength of the beam ($P_{u,exp}$). The results of the fatigue tests are illustrated in **Table 1**. In fatigue test no.6 (P_{max}= 54.3% of $P_{u,exp}$), the sandwich beam failed due to crushing of concrete in the vicinity of the loading point as shown in **Fig.10**. The fatigue life of the beam was equal to 51,389 cycles. The failure

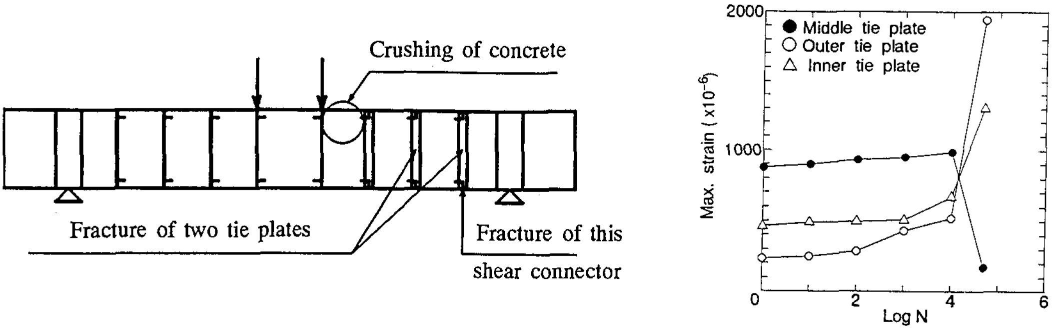


Fig.10 The failure mode in fatigue test no.6 ($P_{max} / P_{u,exp}$ = 54.3%)

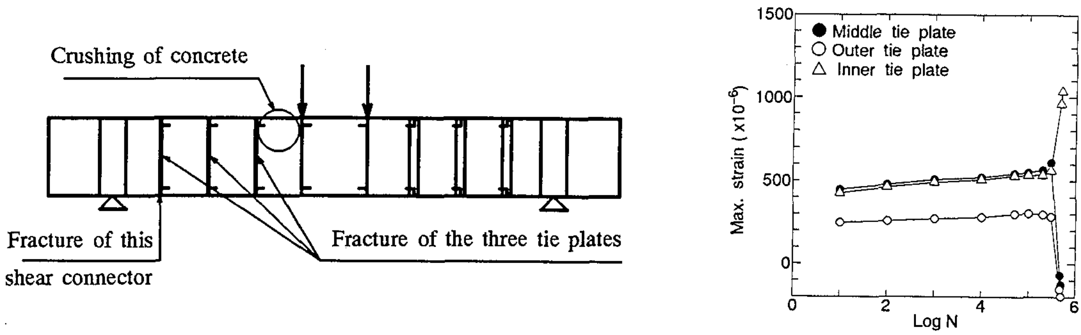


Fig.11 The failure mode in fatigue test no.7 ($P_{max} / P_{u,exp}$ = 41.8%)

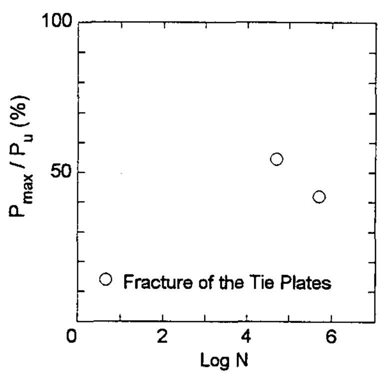


Fig.12 The *S-N* relationship of the beam type B

occurred for the shear span in which the tie plates were placed parallel to the member axis (see **Fig.10**). Thereafter, the concrete was removed and the middle and the outer tie plates were found to be completely fractured (i.e., completely separated from the lower flange plate) as shown in **Fig.10**. Also, the outer shear connector was completely separated from the lower flange plate (see **Fig.10**). The strain measurements have indicated that the middle tie plate was fractured between 10^4 cycles and 5×10^4 cycles. That is because the maximum strain of the middle tie plate drops suddenly between 10^4 cycles and 5×10^4 cycles, while the maximum strains of the inner and the outer tie plates increase (see **Fig.10**). Hence, the contribution of the inner and the outer tie plates increases after the fracture of the middle tie plate. Although the middle tie plate was fractured, the sandwich beam could still sustain the maximum load and the fatigue test was continued until 51,389 cycles. At 51,389 cycles, the outer tie plate was fractured and the crushing of concrete occurred. In fatigue test no.7 (P_{max}=41.8% of $P_{u,exp}$), the sandwich beam failed due to crushing of concrete in the vicinity of the loading point as shown in **Fig.11**. The fatigue life of the beam was equal to 526,000 cycles. The failure occurred for the shear span in which the tie plates were placed normal to the member axis (see **Fig.11**). Thereafter, the concrete was removed and all the three tie plates were found to be fractured as shown in **Fig.11**. The middle and the outer tie plates were completely separated from the lower flange plate, while the inner tie plate was completely separated from the upper flange plate. Also, the outer shear connector was completely separated from the lower flange plate (see **Fig.11**). The strain measurements have indicated that the middle and the outer tie plates were fractured between 300,000 cycles and 482,000 cycles. That is because the maximum

Table 2 Results of the fatigue analysis

Beam No.	P_{min} (kN)	P_{max} (kN)	$P_{max} / P_{u,FEM}$ (%)	Fatigue life (cycles)	Failure mode
1	20	290	80.0	1000	CC[1)]
2	20	322	88.8	100	CC[1)]
3	20	200	55.1	939,000	TP[2)]
4	20	251.2	69.3	241,300	TP[2)]
5	20	288.6	79.6	221,000	TP[2)]
6	20	200	55.1	293,600	FP[3)]
7	20	251.2	69.3	86,300	FP[3)]
8	20	288.6	79.6	50,100	FP[3)]

1) CC : Crushing of the concrete   2) TP : Fracture of the tie plate   3) FP : Fracture of the lower flange plate

strains of the middle and the outer tie plates drop suddenly between 300,000 cycles and 482,000 cycles, while the maximum strain of the inner tie plate increases (see **Fig.11**). Hence, the contribution of the inner tie plate increases after the fracture of the middle and the outer tie plates. Although the middle and the outer tie plates were fractured, the sandwich beam could still sustain the maximum load and the fatigue test was continued until 526,000 cycles. At 526,000 cycles, the inner tie plate was fractured and the crushing of concrete occurred. In fatigue tests no.6 and no.7, the failure mode of the sandwich beam is considered to be fracture of the tie plates. That is because the fracture of the tie plates occurs at first and then the crushing of the concrete takes place. Note that the fracture of the tie plates occurs always at the welding part between the tie plate and the flange plate. The ***S-N*** relationship for fatigue failure of the sandwich beam type B is shown in **Fig.12**.

b) Results of the fatigue analysis

The fatigue strength of the sandwich beam with the crushing of concrete failure was predicted by using the finite element method. The analytical models used for the fatigue analysis as well as the analysis procedure are explained in detail in reference 2). The fatigue analysis was based upon reducing the compressive strength (f'_c), the tensile strength (f_t), and the stiffness of the concrete (E_c) with increasing the number of loading cycles (N) or increasing the concrete stress range (S_r). The sandwich beam was analyzed for two different external load ranges. The minimum fatigue load (P_{min}) was kept constant at 20 kN. The maximum fatigue load (P_{max}) was chosen to be 290 kN and 322 kN which is 80.0% and 88.8%, respectively of the analytical static strength of the beam ($P_{u,FEM}$=362.5 kN). The analytical results are illustrated by beams no.1 and no.2 in **Table 2**. The output load-deflection curve for beam no.2 is shown in **Fig.13**. As shown in **Fig.13**, the beam was loaded until the maximum fatigue load (P_{max}) and then unloaded until the minimum fatigue load (P_{min}) in order to calculate the maximum and minimum stresses at every concrete Gauss point. The maximum and minimum stresses at every concrete Gauss point were updated with increasing the crack propagation. These maximum and minimum stresses were used together with an input number of loading cycles (N=100 cycles) to make some reduction for the strength and the stiffness of the concrete Gauss points. The reduction of the strength and the stiffness of the concrete Gauss points results in decreasing the overall stiffness of the beam and finally failure of the beam by crushing of concrete at the maximum fatigue load as shown in **Fig.13**. Some instability was observed in the analytical load-deflection curve once the concrete in some elements goes into the softening

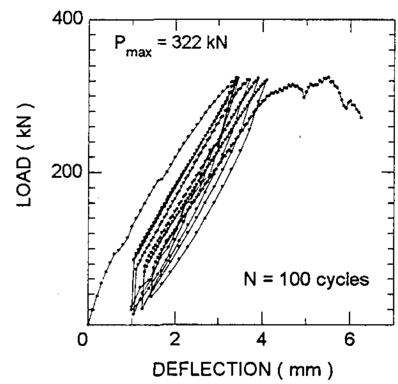


Fig.13 The analytical load-deflection curve

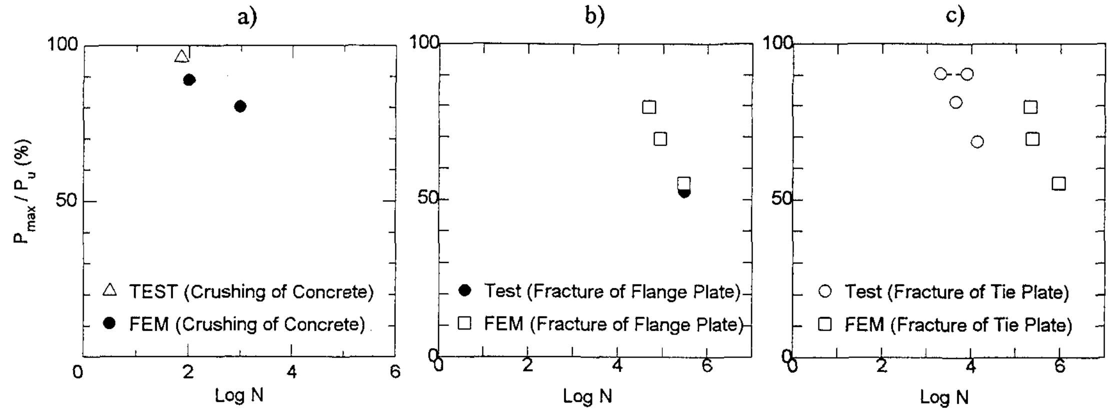


Fig.14 Comparison between the experimental and the analytical *S-N* relationships

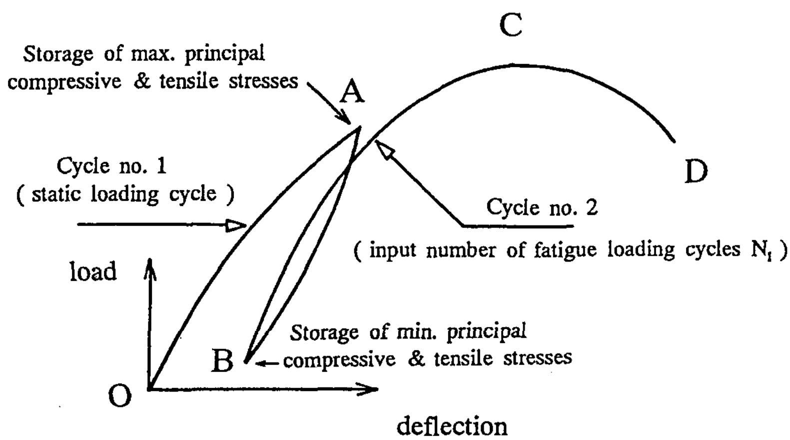


Fig.15 Analysis procedure

range as shown in **Fig.13**. Hence, for a percentage of P_{max}/P_{uFEM}=88.8%, the fatigue life of the beam was considered to be 100 cycles. The analytical *S-N* relationship of the sandwich beam is shown in **Fig.14a**, and also compared with the experimental one for the same failure mode (i.e., the crushing of concrete failure mode).

The fatigue strength of the sandwich beam with the fracture of flange plate failure was also predicted by using the finite element method. The sandwich beam was analyzed for different external load ranges. The minimum fatigue load (P_{min}) was kept constant at 20 kN. The maximum fatigue load (P_{max}) was chosen to be 55.1%, 69.3%, and 79.6% of the analytical static strength of the beam ($P_{u,FEM}$=362.5 kN). The fatigue analysis was carried out by the following procedure:

- At first, a static loading cycle (OAB) is applied as shown in **Fig.15**.
- The maximum tensile stress in the lower flange plate at the critical point is stored at point (A). Similarly, the minimum tensile stress in the lower flange plate at the critical point is stored at point (B). Hence, the stress range in the lower flange plate at the critical point is calculated. In this case, the critical point is the location of the flange plate fracture in the fatigue test (see **Fig.5**).
- Then, the calculated stress range is multiplied by an amplification factor to account for the effect of the local bending deformations of the flange plate as well as the effect of the shear transfer between the concrete and the lower flange plate. Unfortunately, the strain measurements just at the fractured point were not available so as the value of the amplification factor can be calculated precisely. Therefore, the amplification factor was calculated by comparing the stress range of the flange plate calculated by the finite element method with the stress range measured in the fatigue test in air[5]. This amplification factor was equal to 3.6.
- Then, using the S_r-N relationship of the flange

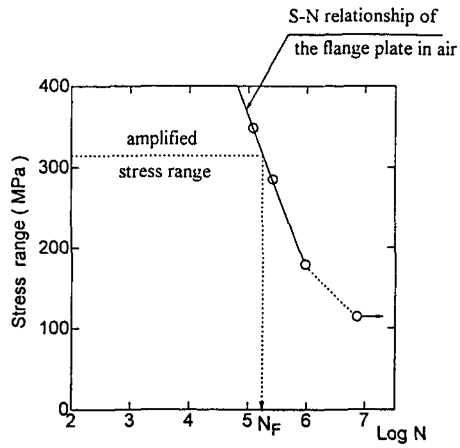


Fig.16 Prediction for the fatigue life of the beam failing due to fracture of the flange plate

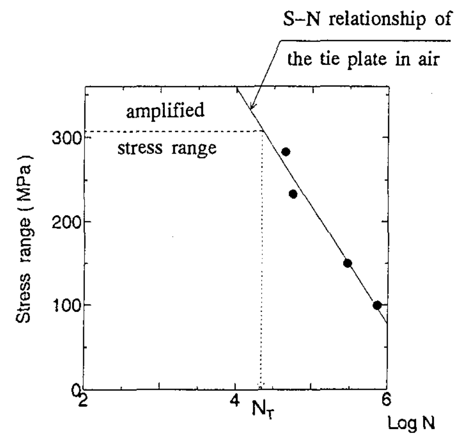


Fig.17 Prediction for the fatigue life of the beam failing due to fracture of the tie plate

plate in air[5)] and knowing the value of the amplified stress range, the number of loading cycles (N_F) can be calculated as shown in **Fig.16**.
- If the input number of cycles (N_I) (see **Fig.15**) is equal to the calculated one (N_F), the sandwich beam is considered to fail due to fracture of the lower flange plate after the loading cycles (N_I).
The analytical results are illustrated by beams no.6, no.7, and no.8 in **Table 2**. The analytical *S-N* relationship of the sandwich beam is shown in **Fig.14b**, and also compared with the experimental one for the same failure mode (i.e., the fracture of flange plate failure mode). Excellent agreement is observed between the experimental and the analytical results.

The fatigue strength of the sandwich beam with the fracture of tie plate failure was also predicted by using the finite element method. The sandwich beam was analyzed for different external load ranges. The minimum fatigue load (P_{min}) was kept constant at 20 kN. The maximum fatigue load (P_{max}) was chosen to be 55.1%, 69.3%, and 79.6% of the analytical static strength of the beam ($P_{u,FEM}$=362.5 kN). The analysis procedure used herein is similar to the procedure explained previously to predict the fracture of flange plate failure mode. However, the stress range is calculated at the centroid of the tie plate. Then, this calculated stress range is multiplied by an amplification factor to account for the effect of the shear deformations of the tie plate (see **Fig.8**). In this study, the amplification factor was equal to 3.0. This amplification factor was obtained by comparing the stress range at the centroid of the tie plate measured in fatigue tests no.2, no.3, and no.4 (see **Table 1**) with the stress range measured in the fatigue test in air[6)]. Then, using the S_r-N relationship of the tie plate in air[6)], the number of loading cycles (N_T) is calculated as shown in **Fig.17**. If the input number of cycles (N_I) is equal to the calculated one (N_T), the beam is considered to fail due to fracture of the tie plate.

The analytical results are illustrated by beams no.3, no.4, and no.5 in **Table 2**. The analytical *S-N* relationship of the sandwich beam is shown in **Fig.14c**, and also compared with the experimental one for the same failure mode (i.e., the fracture of tie plate failure mode). Poor agreement is obtained between the experimental and the analytical results. It was observed that the maximum stresses of the tie plate can be predicted reasonably by the finite element method. However, the minimum stresses or the stress ranges cannot. Therefore, the predicted fatigue life of the beam is longer than the experimental one.

5. SIMPLE METHOD TO PREDICT THE STRAINS IN THE TIE PLATES OF THE SANDWICH BEAMS UNDER FATIGUE LOADING

One of the significant aims in this study is to propose a method to predict the strains in the tie plates of the sandwich beams under fatigue loading. Studying the shear-fatigue behavior of reinforced concrete beams, Ueda and Okamura[7)] proposed a method to calculate the strains in the stirrups under fatigue loading. This method can be used to calculate the maximum strain in the stirrups (i.e.,

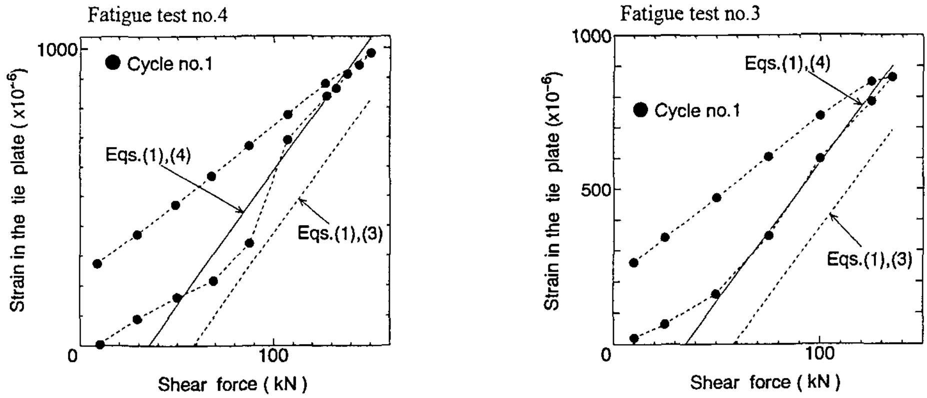


Fig.18 Relationship between the applied shear force and the strains in the tie plates

strain at maximum fatigue load), as well as the strain range. In this study, the same method is used to calculate the strains in the tie plates of the sandwich beam shown in **Fig.1a**. According to this method, the strain in the tie plate at the applied maximum shear force is calculated by the following equation,

$$\varepsilon_{wmax} = \frac{(V_{max} - V_{co} \times 10^{-0.036(1-|r|)\log N})}{A_w E_w} \qquad (1)$$

Also, the strain range in the tie plate is calculated by the following equation,

$$\varepsilon_{wr} = \varepsilon_{wmax} (V_{max} - V_{min}) / (V_{max} + V_{co}) \qquad (2)$$

where :

ε_{wmax}: strain at the centroid of the tie plate at the applied maximum shear force
ε_{wr} : strain range at the centroid of the tie plate
V_{max} : the applied maximum shear force
V_{min} : the applied minimum shear force
$r = V_{min} / V_{max}$
N : the number of fatigue loading cycles
A_W : cross-sectional area of the tie plate
E_W : Young's modulus of the tie plate

The value V_{co} in Eqs.(1) and (2) is the shear force carried by concrete at initial loading (i.e., in the first loading cycle of the fatigue test). In the case of reinforced concrete beams, the value of V_{co} can be calculated by the following equation[7],

$$V_{co} = 0.2 f_c'^{1/3} (1 + \beta_p + \beta_d)\, b_W d \qquad (3)$$

$\beta_p = (100A_s / b_W d)^{1/2} - 1$
$\beta_d = (1000/d)^{1/4} - 1$
A_s : cross-sectional area of the tensile bars
b_W : the width of the beam
d : the effective depth of the beam (mm)
f_c' : the compressive strength of the concrete (MPa)

However, in the case of the sandwich beams, the value of V_{co} was found to be around 0.6 of the value calculated by Eq.(3). This could be illustrated by the relationships between the applied shear force and the strains in the tie plates as shown in **Fig.18**. In this figure, the strain measurements in the first loading cycle in two of the fatigue tests are plotted. The strains in the tie plates calculated by Eq.(1) are also plotted by the dotted straight lines, using N=1 and the value of V_{co} calculated by Eq.(3). For the solid lines, the value of V_{co} was equal to 0.6 of the value calculated by Eq.(3). There is a good agreement between the solid lines and the strain measurements in the first loading cycle. Hence, for the sandwich beams investigated in this study, the value of V_{co} is about 0.6 of the corresponding value for RC beams and defined as follows,

$$V_{co} = 0.6 \times 0.2 f_c'^{1/3} (1 + \beta_p + \beta_d)\, b_W d \qquad (4)$$

where:

$\beta_p = (100A_s / b_W d)^{1/2} - 1$
$\beta_d = (1000/d)^{1/4} - 1$
A_s: cross-sectional area of the lower flange plate
b_W : the width of the sandwich beam
d : the effective depth of the sandwich beam (mm)
f_c' : the compressive strength of the concrete (MPa)

Thereafter, knowing the value of V_{co}, Eq.(1) was used to calculate the maximum strains at the

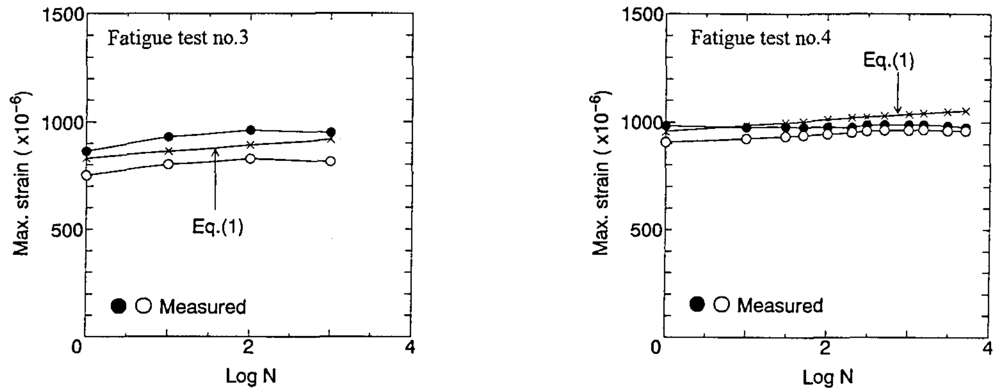


Fig.19 Comparison between the maximum strains calculated by Eq.(1) and the maximum strains measured in the fatigue tests

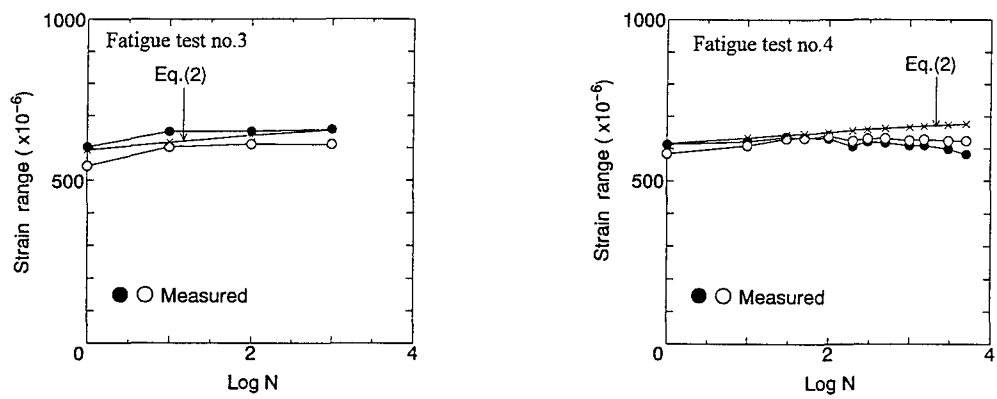


Fig.20 Comparison between the strain ranges calculated by Eq.(2) and the strain ranges measured in the fatigue tests

centroid of the tie plates during the fatigue tests. Also, Eq.(2) was used to calculate the strain ranges at the centroid of the tie plates during the fatigue tests. **Figure 19** illustrates a comparison between the maximum strains calculated by Eq.(1) and the maximum strains measured in two of the fatigue tests. Also, **Fig.20** illustrates a comparison between the strain ranges calculated by Eq.(2) and the strain ranges measured in two of the fatigue tests. There is a very good agreement between the strain measurements in the fatigue tests and the strains calculated by Eqs.(1) and (2) although the calculated values indicate greater increase than the measured. Hence, it is concluded that Eqs.(1) and (2) can be used to predict the maximum strains and the strain ranges in the tie plates of the sandwich beams under fatigue loading. Note that these predicted strains are the strains at the centroid of the tie plate (i.e., excluding the effect of the shear deformations of the tie plate).

Another significant point is to estimate the maximum strain and the strain range at point P which is the most tensioned point in the tie plate (see **Fig.8**). In the fatigue tests of the sandwich beam, point P was the origin of the fatigue crack in the tie plate. The strain at point P depends on the width of the tie plate. If the width of the tie plate is increased, the shear deformations of the tie plate will be increased which in turn will result in increasing the strain at point P. In the present study, the maximum strains and the strain ranges at point P were approximately three times bigger than the corresponding values at the centroid of the tie plate.

6. DESIGN PROPOSAL FOR SANDWICH BEAMS

This chapter presents a procedure which can be used to design steel-concrete sandwich beams under fatigue loading. The design procedure is explained by the chart in **Fig.21**. The design procedure can be summarized as follows,

1- Decide the values of the maximum and

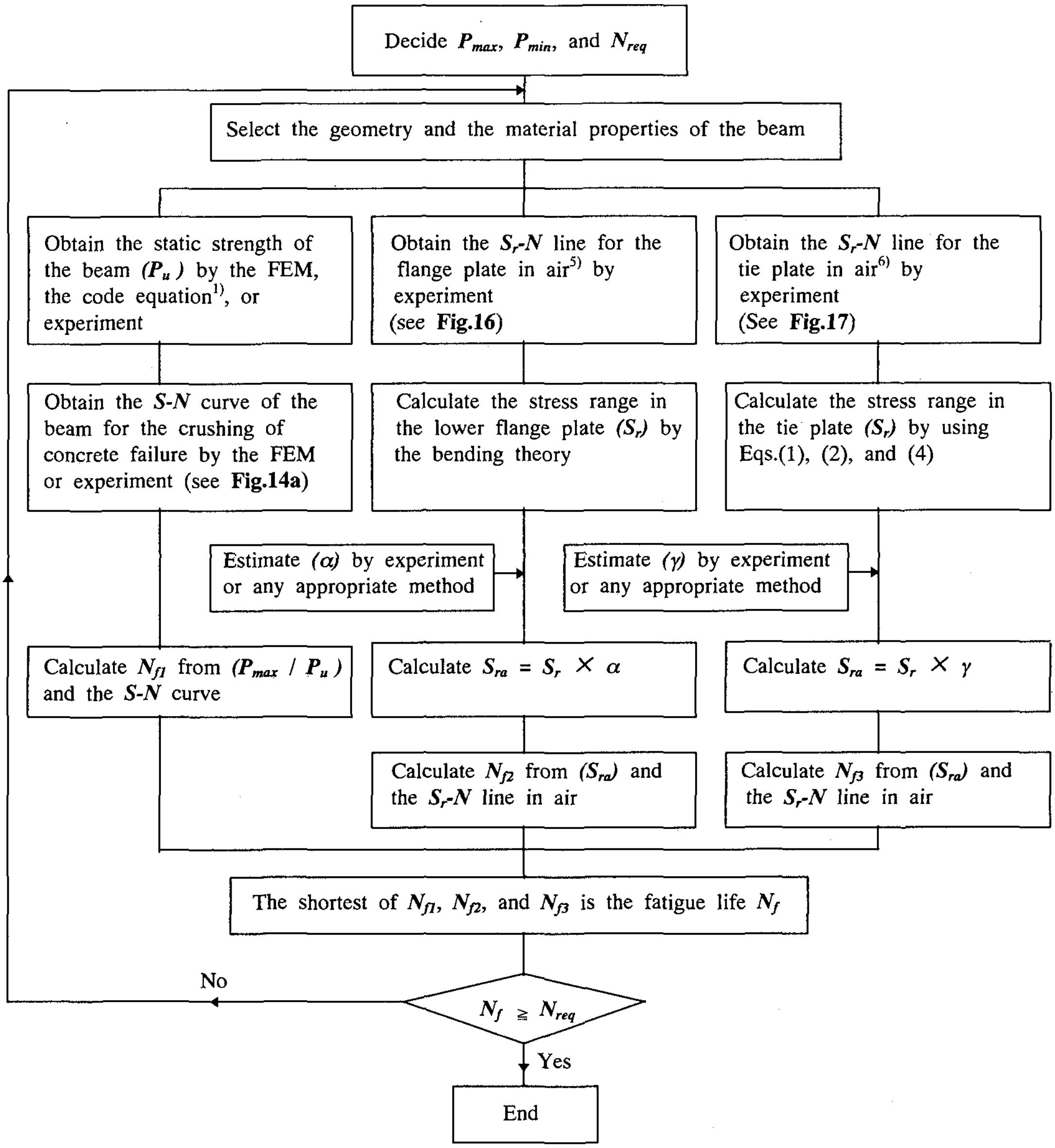


Fig.21 Design procedure for sandwich beams

minimum fatigue loads (P_{max} and P_{min}) considering actual loading conditions and decide the required fatigue life of the sandwich beam (N_{req}).

2- Select the geometry of the sandwich beam and the material properties (i.e., the compressive strength of the concrete, the yielding point, and the tensile strength of the steel plates).

3- Evaluate the static load carrying capacity of the beam (P_u) by the finite element method, by carrying out a static loading test, or by using a design code equation[1].

4- Knowing the percentage of (P_{max} /P_u), the S-N relationship which can be obtained by the finite element method presented in the present study as shown in **Fig.14a** can be used to predict the fatigue life of the sandwich beam failing due to crushing of concrete.

5- Using the conventional bending theory, the stress range in the lower flange plate at point W can be calculated (see **Figs.5a** and **5b**).

6- This calculated stress range at point W should be multiplied by an amplification factor (α) to account for the influence of the local bending deformations of the flange plate (see **Fig.5b**) as well as the influence of the shear transfer between the concrete and the lower flange plate. In the present study, this amplification factor (α) was approximately equal to 5.0.

7- Then, using the S_r-N relationship of the flange plate in air[5] (see **Fig.16**) and the stress range calculated in step 6, the fatigue life of the sandwich beam failing due to fracture of the lower flange plate can be calculated as shown in **Fig.16**.

8- Using Eqs.(1), (2), and (4), the stress range at the centroid of the tie plate can be calculated.

9- This calculated stress range at the centroid of the tie plate should be multiplied by an amplification factor (γ) to account for the effect of the shear deformations of the tie plate. In the present study, this amplification factor (γ) was approximately equal to 3.0.

10- Then, using the S_r-N relationship of the tie plate in air[6] and the stress range calculated in step 9, the fatigue life of the sandwich beam failing due to fracture of the tie plate can be calculated as shown in **Fig.22**.

11- The shortest of the fatigue lives calculated in steps 4, 7, and 10 should be selected as the fatigue life of the sandwich beam (N_f).

12- If N_f is less than the required fatigue life (N_{req}), the dimensions and the material properties of the beam should be changed until N_f becomes equal to or longer than N_{req}.

The design procedure described here is applicable to not only the case in this study but also other cases of the sandwich beams with tie plate. At this moment, it is necessary to conduct either the FEM analysis or the fatigue test due to lack of the data on the *S-N* curve and the amplification factors. The FEM analysis, however, can save time and cost to get the *S-N* curve compared with the fatigue test.

7. CONCLUSIONS

(1) The sandwich beam investigated in this study indicates a shear compression failure mode under static monotonic loading. This failure mode is characterized by diagonal cracking and crushing of concrete.

(2) For a maximum fatigue load (P_{max}) ranging between 41.8% and 90.4% of the static strength of the sandwich beam, the failure mode of the beam is fracture of the steel plates either the lower flange plate or the shear reinforcing steel plates. However, for a very large maximum load (P_{max}=96.1% of the static strength), the failure mode of the beam is crushing of concrete.

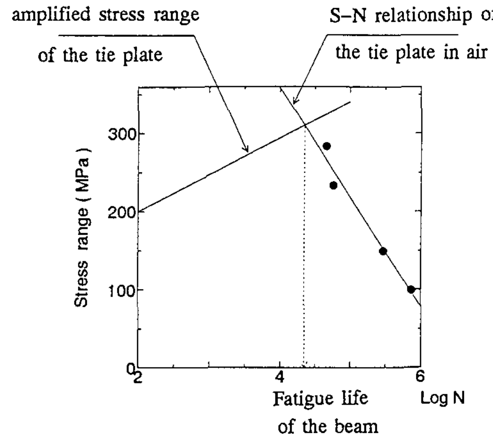


Fig.22 Prediction for the fatigue life of the sandwich beam failing due to fracture of the tie plate

(3) The fatigue strength of the beam with the crushing of concrete failure could be predicted by using the finite element method in which the compressive strength, the tensile strength, and the stiffness of the concrete are reduced with increasing the number of cycles (N) or increasing the stress range.

(4) The fatigue strength of the beam with the fracture of flange plate failure could be predicted by using the finite element method presented in this study. The stress range in the lower flange plate calculated by the finite element method is multiplied by an amplification factor to account for the effect of the local bending deformations of the flange plate as well as the effect of the shear transfer between the concrete and the lower flange plate. Then, for an input number of cycles, the S_r-N relationship of the flange plate in air[5] is used to check whether the flange plate is fractured or not.

(5) Under fatigue loading, the maximum stresses of the tie plate can be predicted by the finite element method presented in this study. The minimum stresses or the stress ranges, however, cannot be predicted reasonably by the finite element method. The further study is necessary.

(6) The fatigue strength of the beam with the fracture of tie plate failure could be predicted by using the simple method presented in **Sec.5**. The stress range in the tie plate calculated by using Eqs.(1), (2), and (4) is multiplied by an amplification factor to account for the effect of the shear deformations of the tie plate. Then, for an input number of cycles, the S_r-N relationship of the tie plate in air[6] is used to check whether the tie plate is fractured or not.

(7) A design proposal to evaluate the fatigue

strength of the sandwich beams with web reinforcement is presented based on the results of this study.

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せん断補強された鋼コンクリートサンドイッチ梁のせん断疲労

Mohab ZAHRAN・上田　多門・角田　與史雄

せん断補強鋼板を有する鋼コンクリートサンドイッチ梁の疲労実験を行った．3種類の梁のせん断疲労破壊モードが観察された．せん断ひび割れ間のコンクリートの圧縮破壊，引張フランジ鋼板もしくはせん断補強鋼板の疲労破断によるせん断圧縮破壊である．各々の破壊モードに対する *S-N* 関係を示すとともに，有限要素解析によるせん断疲労強度の推定法を示した．また，疲労荷重下のせん断補強鋼板の歪みを求める簡便法，および，疲労荷重下の鋼コンクリートサンドイッチ梁の設計法に対する提案を示した．