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MODIFICATION OF THE UNIFIED CONCRETE PLASTICITY MODEL AND ITS CHARACTERISTICS

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In this paper, strain controlled calculations in constitutive level are carried out to investigate the characteristics of the plasticity model named Unified Concrete Plasticity Model (proposed by Tanabe et al.[1]) and modifications are proposed studying the basic characteristics of the effects of the material parameters, namely the characteristics of the variation of cohesion and friction angle with increasing damage in various stress zones in the plane of ($I_1, \sqrt{J_2}$).

Key Words : *plasticity, constitutive relations, Unified Concrete Plasticity Model*

1. INTRODUCTION

Unlike metal, the behavior of concrete under tension and compression is significantly different. Most of the constitutive models for concrete adopt separate constitutive modeling for tension and compression. In a real structure, except in special cases of uniaxial tension or compression or beam under two point loading which is a case of pure bending, the stress-strain behavior is generally multi-axial in nature. Hence for a general structural analysis of reinforced concrete structures, one need a constitutive relation that is valid under multi-axial situation. Recently the Unified Concrete Plasticity Model was presented by Tanabe et al.[1], which was a modified Drucker-Prager approach such that Mohr-Coulumb core match both at the tensile and compressive meridians, aiming at tension stiffening effects and compressive strength reduction effects due to tensile strength incorporated into a single plasticity model. This model was numerically able to simulate various experimental results of proportional loading by Kufer[2] and uniaxial tension experiment by Petersson[3] within reasonable limits.

Though the model looked very promising, all the calculations carried out to validate this model was carried out by constitutive level calculations. When this model was implemented in the finite element analysis, problems were encountered even in the case of uniaxial compression. It was realized that carrying out finite element analysis is impossible when the chosen constitutive model exhibits material instability, like snapback in stress-strain relation etc. due to the inappropriate choice of material parameters. This inspired the author to further investigate the characteristics of this model and the reasons for the defects of this model.

The main aim of this paper is to clarify the characteristics of the unified concrete plasticity model in depth and propose the necessary modifications for the mode based on characteristics of the variations of the material parameter cohesion and friction angle in various stress zones in the plane of ($I_1, \sqrt{J_2}$). Moreover, it is possible to have various variations of this type of model based on the order of stress terms for the equation defining the yield surface and the way damage is defined. The modified model based on these variations are compared.

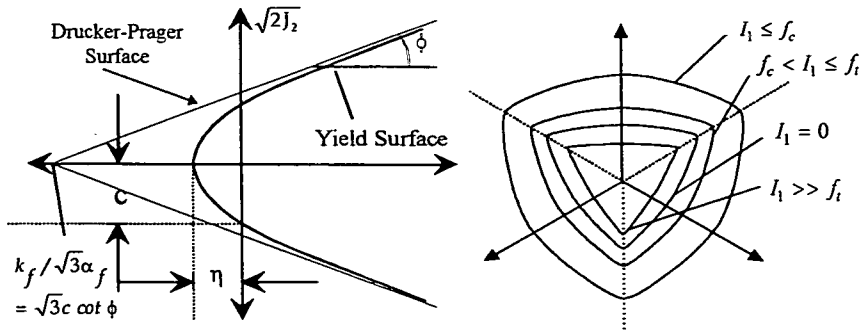
2. THE CONSTITUTIVE MODEL

The Unified Concrete Plasticity Model follows essentially the basic concept of classical plasticity. The subsequent failure surface is assumed to change its size continuously depending on the damage $\omega(W^p)$ accumulated in the concrete material. Associated flow rule is assumed for simplicity as the primary aim of this paper is to study the characteristics of this model. The yield surface (**Fig. 1**) is given by

$$\begin{aligned} f &= f(\sigma_{ij}, \omega(W^p)) \\ &= J_2 - \left(k_f - \alpha_f I_1\right)^2 + \left(k_f - \alpha_f \eta\right)^2 = 0 \end{aligned} \tag{1}$$

where I_1, J_2 and J_3 are stress invariants and

$$k_f = \frac{6c\cos\phi}{\sqrt{3}\left(3 + y\sin\phi_1\right)}, \alpha_f = \frac{2\sin\phi}{\sqrt{3}\left(3 + y\sin\phi_1\right)} \tag{2}$$



a) Meridian Plane b) Diviatoric Plane

Fig.1 The Unified Concrete Yield Surface

where $\phi_1 = 14°$ is a material constant. Cohesion c, friction angle ϕ and η depends on damage ω and are defined by material constants $\phi_0, \phi_f, c_0, \eta_0$, b, m as

$$c = c_0 \exp\left[-(m\omega)^2\right], \eta = \eta_0 \exp(-\omega / b),$$
$$\phi = \begin{cases} \phi_0 + (\phi_f - \phi_0)\sqrt{2\omega - \omega^2} & \omega \le 1 \\ \phi_f & \omega > 1 \end{cases} \quad (3)$$

and y is a parameter that is used to match the Drucker-Prager based surface with Mohr-Coulomb core both at the tensile and compressive meridian

$$y = \sqrt{a(\cos 3\theta + 1.00) + 0.01} - 1.10, a = 0.5r^2 + 2.1r + 2.2,$$
$$r = \begin{cases} 3.14 & I \le f_c' \\ 6.07 - 2.93\cos(I_1\pi / f_c') & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases} \quad (4)$$

where $\cos 3\theta = (3\sqrt{3}J_3 / (2J_2^{1.5})$ .The shape of the yield surface(**Fig. 1**) depends of material parameters c, ϕ and η. These parameters changes as the damage ω increases incrementally. The stress-strain relation depends on the rate of change of these parameters. The ductility of the stress-strain curve depends on the relation between plastic strain and damage ω.

3. IMPORTANCE OF THE CONSTITUTIVE LEVEL CALCULATIONS

Adjustment of the material parameter to simulate the appropriate concrete strength and ductility is an important step in finite element analysis. If the constitutive model has defects such as material instability, finite element analysis does not provide rational results and can lead to loss of convergence. To overcome this problem, constitutive level calculation is found to be very effective. The basic calculation carried out in the constitutive level is the calculation of tangent modulus D_{ep}. Stress is calculated from strain. This is more like displacement controlled process where instability

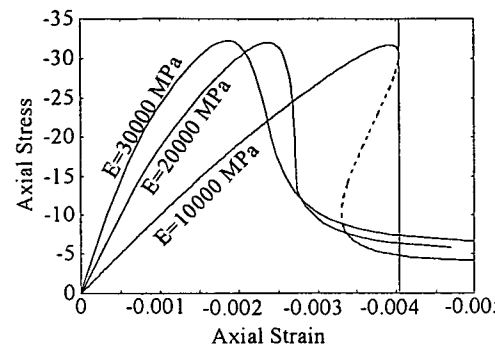


Fig.2 Snapback- example of ill conditioned stress-strain relation

occurs when softening curve is ill conditioned because the stress-strain curve shows snapback (**Fig. 2**). In this model, constitutive level check is a very important step as this model is very sensitive if the parameters are not chosen after full consideration.

This method is applied to understand and modify the Unified Concrete Plasticity Model. When finite element analysis gives undesired results, the strain history can be economically used to understand the problem. All results presented in this paper are results of constitutive level calculation.

4. CHARACTERISTICS OF THE MODEL

(1) Uniaxial compression

Fig. 3 presents a parametric study for uniaxial compression. Obviously, we get a elastic perfectly plastic curve when c, ϕ and η are assumed constant. The shape of the stress-strain softening slope when only c changes while keeping ϕ and η constant, resemble the c-ω curve. The effect of ϕ is to make the peak of the *curve a little round*. The uniaxial behavior under changing c and ϕ falls between the previous two curves. η has less effect of the shape of the stress-strain curve of uniaxial compression.

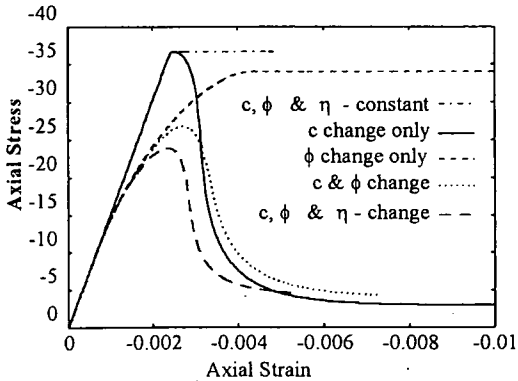


Fig. 3 Uniaxial Compression- parametric study

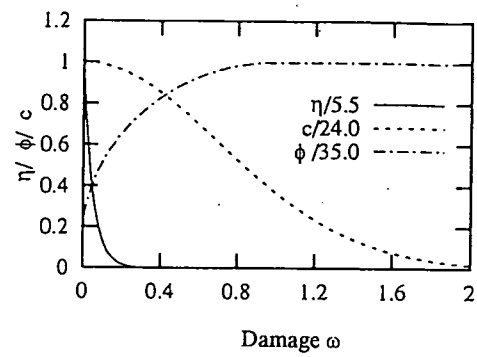


Fig. 4 Typical variation of c, ϕ and η

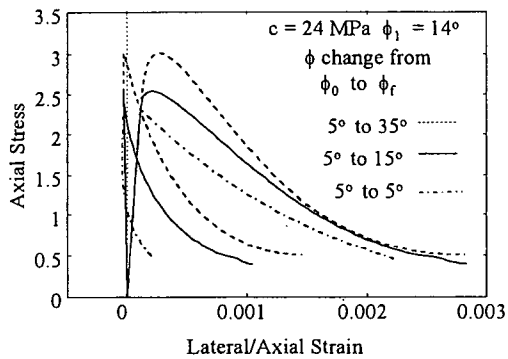


Fig. 5 Parametric study of ϕ for uniaxial tension showing lateral and axial strain

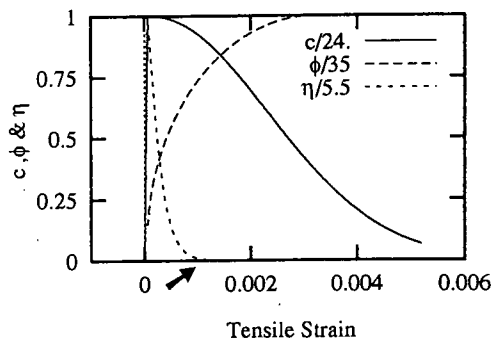


Fig. 7a Variation of c, ϕ and η for S:T=-1:1

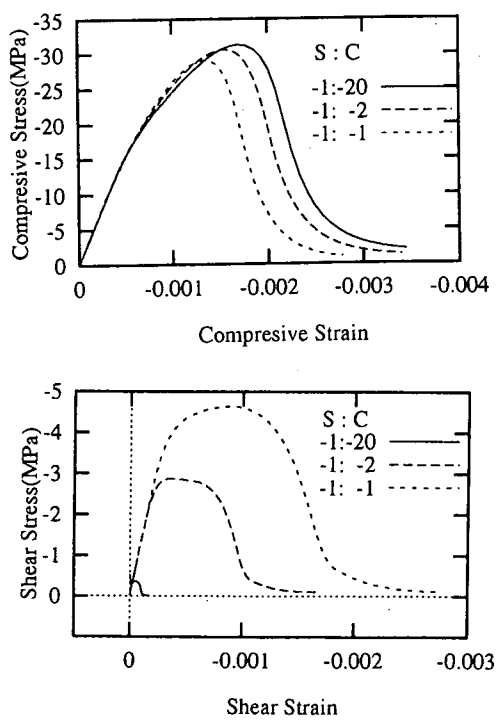


Fig. 6 Stress strain relation when Shear strain: Compressive strain = S:C is applied

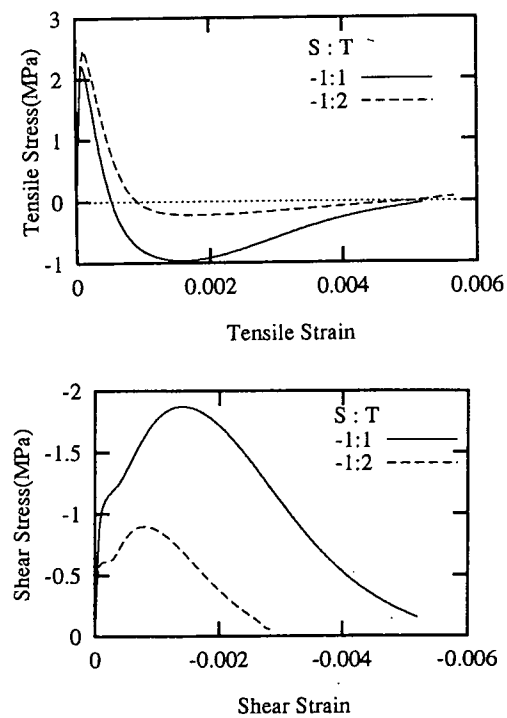


Fig. 7b Stress strain relation when Shear strain: Tensile strain = S:T is applied

(2) Uniaxial tension

The rate of change of η(the distance of the tip of the surface from the origin (**Fig. 1.a**)) occurs much faster then rate of change of c or ϕ as can be seen in **Fig. 4**, drawn using Eq. 3 where η_o=5.5 MPa, b=0.06, c_o=24 MPa, m=1.0, ϕ_o=5°, ϕ_f=25°. η become almost zero rapidly approximately at ω=0.2. However c become zero at about ω=2, and ϕ changes from ϕ_o to ϕ_f when ω=1. As a result, the uniaxial tension behavior depend more on how fast the tip of the yield surface moves towards the origin due to the rate of change of η. The softening slope found to be quite independent of the variation of c and ϕ; and depends on parameter b[1]. It was stated that ϕ has less effect on the uniaxial tensile behavior. This statement was found to be true only in case of the axial behavior. It has significant effect on the lateral behavior. **Fig. 5** shows the lateral and axial behavior of concrete under tension with constant initial friction angle ϕ_o and different values of final friction angle ϕ_f. In the original paper[1], these results (**Fig. 5**) are not mentioned. It means concrete under uniaxial tension will expand laterally. This looks unrealistic as concrete cannot expand laterally while cracking and can be called the first defect of the existing model.

(3) Pure shear and mixed shear calculations

Pure or mixed shear stress situation occur very frequently in real structures, like cantilever. Though the cases of compression-shear(**Fig. 6**) looks fine, hunch back like curve (**Fig. 7b** lower figure) is noticed in case of shear stress for shear-tension calculation where the point takes additional shear after tension failure has occurred. Though there is no experimental basis to check the validity of these results, this type of results looks unrealistic. **Fig. 7b** shows two cases with different ratios of applied tension : shear strain. It was noticed that the hunch back occurs when the value of η approximately becomes zero(**Fig. 7a** arrow). This hunch back like curve in proportional shear-tension calculation creates numerical problems while carrying out FEM analysis. It is unrealistic too because concrete that has failed under tension cannot take increasing amount of shear stress. This can be called the second defect of the existing model.

5. MODIFYING THE EXISTING MODEL

The state of stress-strain, or even the shape of the yield surface highly depends on the cohesion and friction angle. But Tanabe et al.[1] in an attempt to create a unified theory which can be applied to all cases, introduced a material parameter η which is the distance between the tip of the yield surface in tension zone from the origin. By making the rate of change of η independent of the rate of change of c and ϕ, the tip of the yield surface and hence the tension zone is reduced to zero at a faster rate, even though the value of c is not yet zero. This gave excellent results in tension zone and compression zone. This also made the behavior of tension zone quite independent of behavior of compression zone.

The present author finds the assumption that the range of ϕ to be constant in all cases to be illogical because it presents illogical results for lateral behavior in case of uniaxial tension. The present author also finds the fact that rate of change of η to be independent of rate of change of c and ϕ as not realistic. Assuming the same η_o or the initial distance of the tip of the yield surface, η or the distance of the tip of the yield surface is proposed to be proportional to the distance of tip of the Drucker-Prager surface from the origin($= \sqrt{3}c\cot\phi = k_f / \sqrt{3}\alpha_f$)

$$\eta = AA^* \sqrt{3}c\cot\phi = AA^* k_f / \sqrt{3}\alpha_f \tag{5}$$

$$AA^* = \sqrt{3}c_0 \cot\phi_0 / \eta_0 . \tag{6}$$

The modified yield surface equation looks like

$$\begin{aligned} f &= J_2 - \left(k - \alpha_f I_1\right)^2 + \left(k_f - \alpha_f \eta\right)^2 \\ &= J_2 - \left(k - \alpha_f I_1\right)^2 + k_f\left(1 - AA^* / \sqrt{3}\right)^2 = 0 \end{aligned} \tag{7}$$

The following assumptions are made to determine the material parameter c and ϕ

a) Initial parameters are constant in all stress conditions

b) For compression zone, rate of change of c and ϕ similar to the original model

$$c = c_0 \exp\left(-m\omega^2\right)$$

$$\phi = \begin{cases} \phi_0 + \left(\phi_f - \phi_0\right)\sqrt{(\omega+k)(2-\omega-k)} & \omega \le 1 \\ \phi_f & \omega > 1 \end{cases} \tag{8}$$

a) For tension zone, constant ϕ and first order variation for c is assumed

$$\phi = \phi_0, \quad c = c_0 \exp\left(-m\omega\right) \tag{9}$$

In the previous model, no physical significance was offered for the material parameter η. With the above assumption, since friction angle remains constant for uniaxial tension, it is clear that by adopting first order variation of cohesion c in uniaxial tension, the rate of change of η or the distance of the tip of the yield surface in tension remains similar to the original model. Hence rate of change of η in the original model represented the rate of change of cohesive property of concrete in tension. For cohesion c and friction angle ϕ in between the

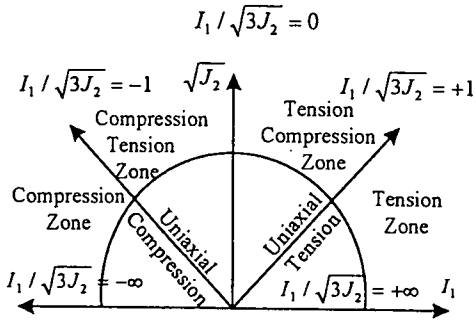


Fig. 8 Different Zones in $(I_1, \sqrt{J_2})$ space

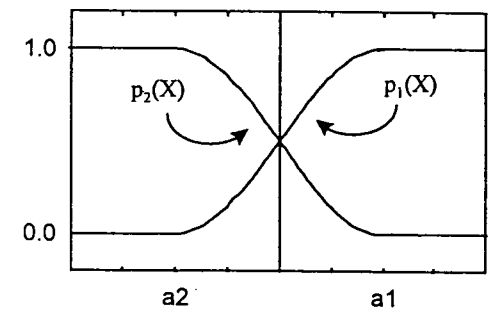


Fig. 9 Variation of $p_1(X)$ & $p_2(X)$ with X

tension to compression zone, it is proposed to use variation based on the parameter $X = I_1/\sqrt{3J_2}$, whose variation is shown in **Fig. 8**. $X = I_1/\sqrt{3J_2}$ is based on well established way of defining zones in stress space in the plane of $(I_1, \sqrt{J_2})$[4]. X=1 for uniaxial tension and X = -1 for uniaxial compression (**Fig. 8**). Variation of c and ϕ is given as

$$c = c_0 \exp\left[\left(-m_1\omega\right)p_1(X) + \left(-m_2^2\omega^2\right)p_2(X)\right]$$

$$\phi = \begin{cases} \phi_0 + \left(\phi_f - \phi_0\right)\sqrt{(\omega+k)(2-\omega-k)}\, p_2(X) & \omega \le 1 \\ \phi_0 + \left(\phi_f - \phi_0\right)p_2(X) & \omega > 1 \end{cases} \quad (10)$$

where $\kappa=10^{-3}$ (small value) is introduced to get rid of the singularity at $\omega=0$ when $\partial\phi/\partial\omega = \infty$. The following condition must be satisfied

$$\{p_1(X) \quad p_2(X)\} = \begin{cases} 0 \quad 1 & for \quad X=-1 \\ 1 \quad 0 & for \quad X=1 \end{cases} \quad (11)$$

If $X=a1$ and $X=a2$ represent the boundary of compression and tension zone respectively, the following relation is proposed for intermediate range, though this can be a topic of further research

$$\{p_1(X) \quad p_2(X)\} = \begin{cases} 0 & X \le a1 \\ \frac{1}{2}\cos\left(\frac{X-a1}{a2-a1}\right) + \frac{1}{2} & a1 < X \le a2 \\ 1 & X > a2 \end{cases}$$

$$\begin{cases} 1 & X \le a1 \\ -\frac{1}{2}\cos\left(\frac{X-a1}{a2-a1}\right) + \frac{1}{2} & a1 < X \le a2 \\ 0 & X > a2 \end{cases} \quad (12)$$

It can be noted that the initial yield point coincide with the Kupfer's peak strength envelope up to the tension-compression zone(approximately X=-0.15) (**Fig. 11** and **13**). In Kupfer's experiment[2], it was also noted that brittle failure was noticed in tension-tension zone and tension-compression zone. However hardening was noticed in compression-tension zone and in compression-compression zone. Since the effect of increase in ϕ is to make the peak round as seen in Sec. 4.1, it should be noted that if Eq. 8 is assumed (the material parameter for uniaxial compression), the stress-strain relation has rounded peak and Eq. 9 is assumed (the material parameter for uniaxial tension), the stress-strain relation has sharp peak. However the post peak slope can easily be controlled in either of the case by controlling the c-ω relationship. Therefore, it is thought logical to assume the tension zone up to the point initial yield strength match to provide a brittle failure. Parametric study is carried out with a1=-1 and a2 as a variable with a2 =1 (**Fig. 10**) and a2=-0.15 (**Fig. 13**). Both the cases provided acceptable results for Kupfer's experiment (**Fig. 10,13**). However when shear calculations were checked, better results were obtained for a2=-0.15 (**Fig. 15,17**).

When we use Eq. 10 for c-ω relation, the stress in the final stage is expected to become zero as can be seen in Sec. 7.2. If however it is necessary to maintain non-zero stress at the final stage of damage for compression or tension (due to tension stiffening effect) one can use more appropriate c-ω relation where cohesion c does not become zero in the final stage. The details of the necessity and applicability of the general c-ω relation will be discuss in the companion paper[5], where more details related to reinforced concrete structure is discussed.

$$c = c_0 \exp\left[\left(-m_1\omega\right)p_1(X) + \left(-m_2^2\omega^2\right)p_2(X)\right] + F_1(X)\omega + F_2(X) \quad (13)$$

Eq. 4 is modified to make it continuous in all ranges:

$$y = \sqrt{a(\cos 3\theta + 1.00) + 0.01} - 1.10,$$

$$a = 0.5r^2 + 2.1r + 2.2, \quad (14)$$

$$r = \begin{cases} 3.14 & I \le f_c' \\ 6.07 - 2.93\cos\left(\frac{f_t - I_1}{f_t - f_c'}\pi\right) & f_c' < I \le f_t \\ 9.0 & I > f_t \end{cases}$$

Table 1 Different types of variations

Name	C(MPa)	ϕ_f(deg)	β^1	a1	a2	Eq. No.
1A	28	32.7	30	−1	+1	12,16
1B	28	32.7	66	−1	+1	12,15
2A	28	32.7	1700	−1	+1	10,16
2B	28	32.7	66	−1	+1	10,15
3	24	35	40	−1	−0.15	12,16
4	24	35	85	−1	−0.15	12,15
5	24	35	85	−1	−0.15	10,15

6. VARIATIONS OF THE MODIFIED UNIFIED CONCRETE PLASTICITY MODEL

In this section, variations based on the order of yield surface and definition of damage are presented. The purpose is to check the applicability of these variations and choose the most convenient set of equations for further application to reinforced concrete members for future research[5].

(1) In constitutive equation

The yield function for Unified Concrete Plasticity Model is a second order variation of stress terms.

$$f_2(\sigma,\omega) = J_2 - \left(k_f - \alpha_f I_1\right)^2 + k_f\left(1 - AA^* / \eta\right)^2 = 0 \quad (15)$$

However the above equation can also be rewritten as follows in the first order version

$$f_1(\sigma,\omega) = \sqrt{J_2 + k_f\left(1 - AA^* / \eta\right)^2} - \left(k_f - \alpha_f I_1\right) = 0 \quad (16)$$

where α_f and k_f as defined in Eq. 2. In the original paper[1], the reason for selecting the second order variation was not specified. In present research, behavior of both the versions are compared.

(2) In the definition of damage

In the original model[1], classical plasticity approach was adopted. $\omega(dW^p)$ was defined as

$$\omega = \frac{\beta}{\sigma_e \varepsilon_0}\int dW^p = \frac{\beta^1}{\sigma_e}\int dW^p \quad (17)$$

where $dW^p = \sigma_{ij} d\varepsilon_{ij}^p = \sigma_e d\varepsilon^p$ and $d\varepsilon_{ij}^p$ is the plastic strain, dW^p is the plastic work, σ_e and $d\varepsilon^p$ is effective plastic stress and strain, β and ε_o ($\beta' = \beta / \varepsilon_0$) are material constants. Though in the original paper, the basic method used is a little different, basically the effective plastic strain $d\varepsilon^p$ can be defined as

$$d\varepsilon^p = \sqrt{d\varepsilon_{ij}^p . d\varepsilon_{ij}^p} = d\lambda\sqrt{\frac{df}{d\sigma_{ij}} . \frac{df}{d\sigma_{ij}}} \quad (18)$$

Therefore Eq. 5 can be rewritten as

$$d\omega = \beta' d\varepsilon^p = d\lambda\beta'\sqrt{\frac{df}{d\sigma_{ij}} . \frac{df}{d\sigma_{ij}}} \quad (19)$$

Table 2 Variations based on order and damage

Name	Order	Damage
Case 1A or 3	First (Eq. 16)	Simple (Eq. 20)
Case 1B or 4	First (Eq. 16)	SH (Eq. 19)
Case 2A	Second (Eq. 15)	Simple (Eq. 20)
Case 2B or 5	Second (Eq. 15)	SH (Eq. 19)

This is the typical definition of *strain hardening* damage. Simple definition(Eq. 20) is also acceptable. Hereafter, Eq. 19 is called *strain hardening*(SH) *damage* and Eq. 20 as *simple damage* model.

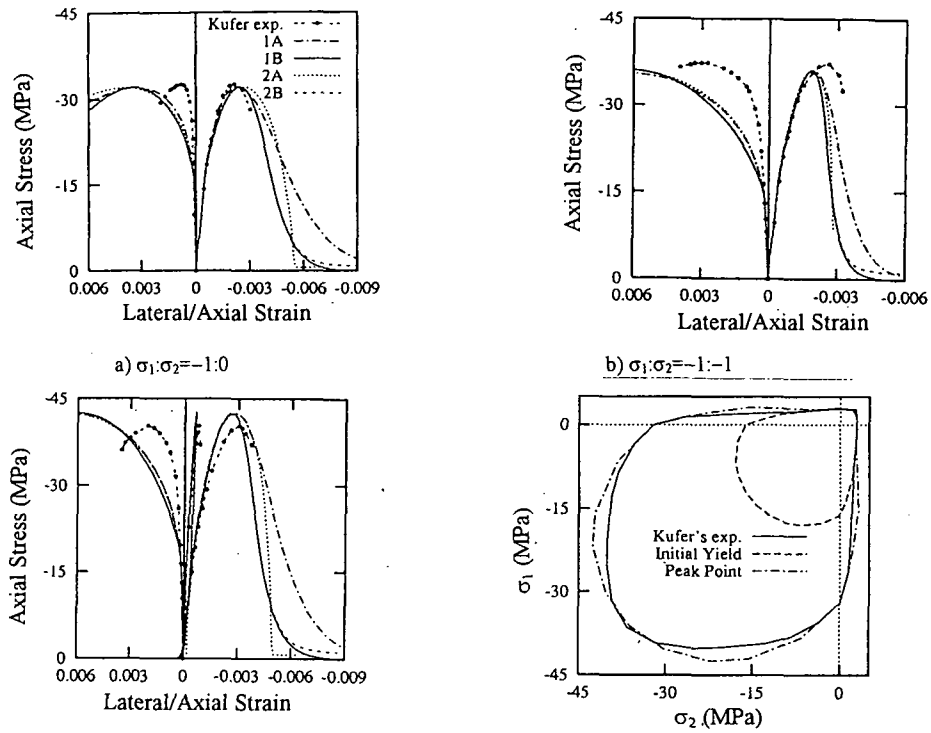
$$d\omega = \beta' d\varepsilon^p = \beta' d\lambda \quad (20)$$

8. ANALYSIS AND DISCUSSION

As we can see above , we can get 4 (Eq. 15/16 x Eq. 19/20) type of variations. Two variations based on the choice of variation of $\{p_1(X) \quad p_2(X)\}$ is also considered. It was found enough to change only the β' to get similar results in all the cases for the β' to get similar results in all the cases for the experimental results of Kupfer's biaxial peak strength envelop. The parameters that are constant are: E=32500 MPa, μ=0.22, f_t'=3.25 MPa, f_c= 32.5 MPa, $\phi_1 = 14°$, $\phi_o = 5°$, m_1=10.0, m_2=1.0, η_o =5.52 MPa, k = 1.0 x 10^{-3}, ω_f=1.0. **Table 1** shows the material parameters β^1. The seven cases considered are as shown in **Table 2** .

(1) Comparison with Kupfer's experimental results

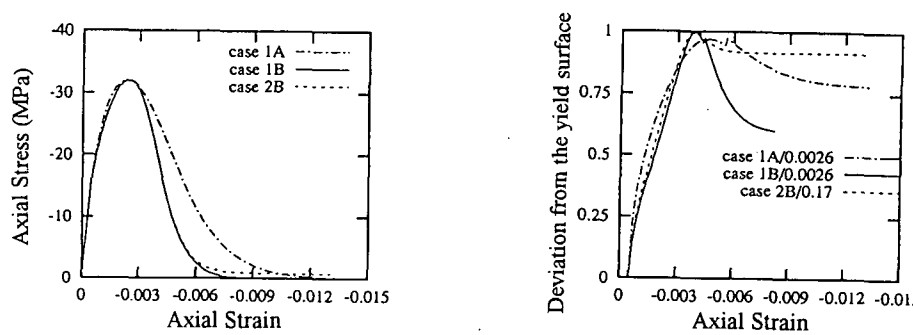
Fig. 10 and **13** shows the comparison of first four cases with that of Kupfer's experimental results of the first four cases with a2=+1.0 and the later three with a2=-0.15 respectively. In the first three (a, b, c) of both these figures shows the comparison with three specific cases of biaxial stress-strain relationship. It might look strange, but all these results were obtained by changing only the value of β', which controls the rate of development of damage. Since post-peak behavior depends of the rate of formation of the damage, each of the four cases act independently. Though cases 1A,1B and 2B showed reasonable softening diagram, case 2A



a) $\sigma_1:\sigma_2=-1:0$ b) $\sigma_1:\sigma_2=-1:-1$

c) $\sigma_1:\sigma_2=-1:0.-52$ d)Peak and yield stress:

Fig. 10 Comparison with Kupfer's Biaxial Experiment using a2= +1.0



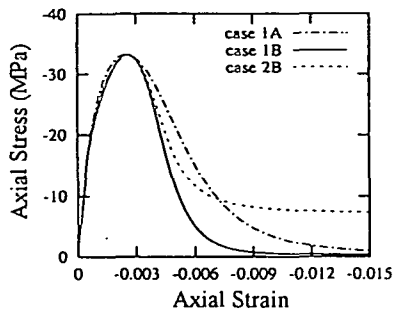
a) Uniaxial Stress-strain b) Deviation from the yield surface

Fig. 11 Accuracy of calculation using small steps

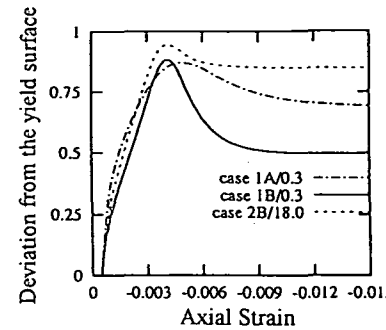
shows very sharp decline, which look not reasonable to be used for concrete. **Fig. 10d** and **13d** shows the comparison with the peak strength envelop of Kupfer's experiment for case 1A and 3 respectively. The results of the other three cases were not presented because results of the four case were almost exactly similar with a maximum difference of about 5%. Therefore, if they are drawn, it would be indistinguishable and hence are not drawn. **Fig. 10d** and **13d** also show the initial yield point envelop from the analysis. **Fig. 10** and **13** has small difference in the assumed values of c_o and ϕ_f. This is to show that we can get almost similar results with different combination of the parameters.

(2) Relative deviation from the yield surface

The constitutive level calculation shown in **Fig. 10-13** are conducted with very small steps so that the error is minimized. Return mapping to maintain f=0 is not done. To check the applicability of the model, only results of uniaxial compression are used for simplicity to exhibit the relative tendency of deviation from the yield surface. Similar tendency can be found in other cases too. **Fig. 11a** shows the

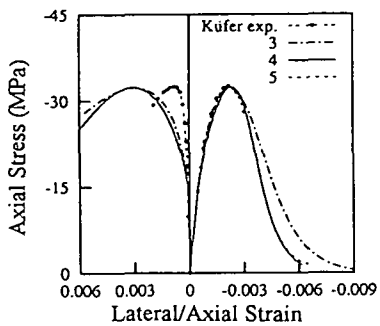


a) Uniaxial Stress-strain

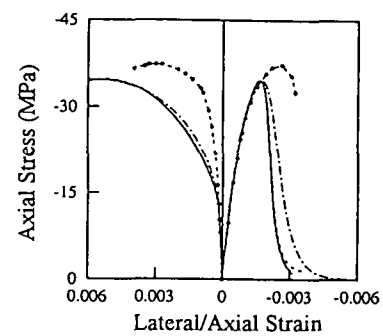


b) Deviation from the yield surface

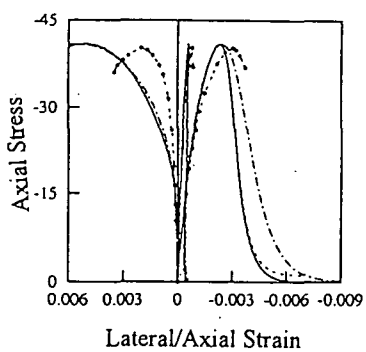
Fig. 12 Accuracy of calculation using large steps



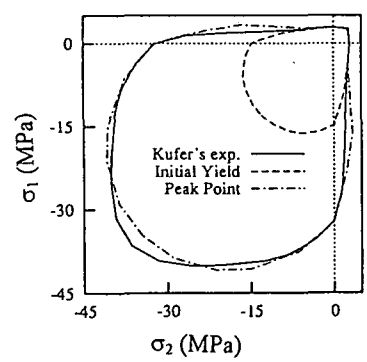
a) σ_1:σ_2=−1:0



b) σ_1:σ_2=−1:−1



c) σ_1:σ_2= −1: 0.−52



d)Peak and yield stress:

Fig. 13 Comparison with Kupfer's Biaxial Experiment using a2= -0.15

uniaxial stress-strain, which is same as that of **Fig. 10a**. In this calculation, 0.0152 axial strain is applied in 14705 equal increment. In this case(**Fig. 11a**) we can note that all stresses comes down to almost zero value. **Fig. 12** shows results using bigger increment 0.0152 axial strain is applied in equal increment so that the deviation becomes prominent. **Fig. 11b** and **12b** shows the normalized error(deviation from the yield surface) with increase in lateral strain. Error in all the 3 cases increased (0.0026 → 0.3 for case 1A and 1B, and 0.17 → 18.0 for case 2B) with increase in step size. Case 1A and 1B showed similar error behavior. Direct comparison between case 1A and 1B with that of case 2B is not possible because there is a difference in order of the yield surface. However, they can be compared by checking the residual stress which is supposed to drop to zero value as we can in the case of fine steps (**Fig. 11a**). However in case of large steps(**Fig. 12a**), the stress in case of 2B remains significantly high where as stress in case of

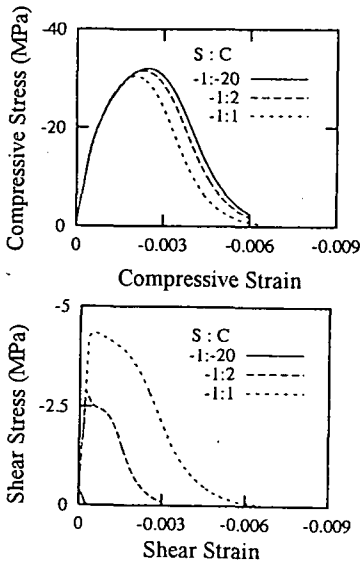


Fig. 14 Stress Strain relation when Shear strain: Compressive strain = S:C is applied

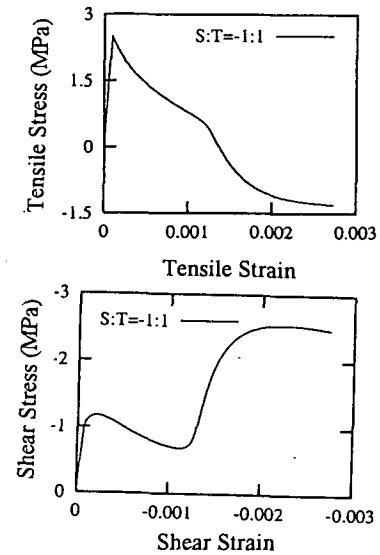


Fig. 15 Stress Strain relation when Shear strain: Compressive strain =S:T is applied

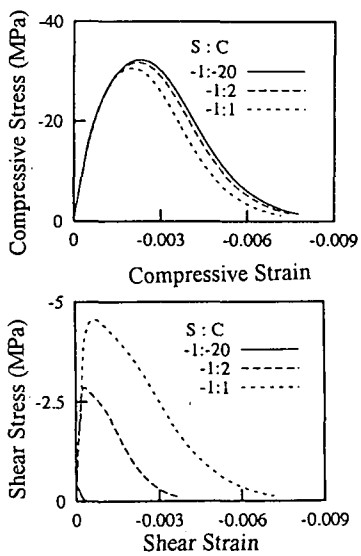


Fig. 16 Stress Strain relation when Shear strain: Compressive strain =S:C is applied

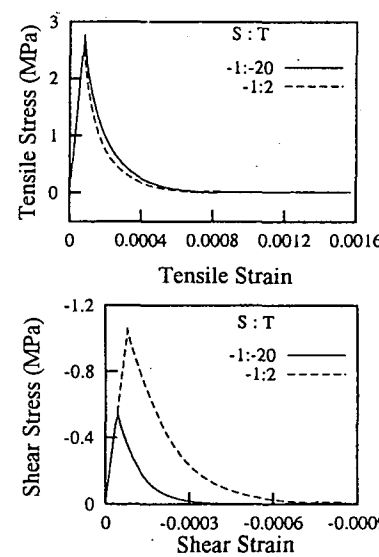


Fig. 17 Stress Strain relation when Shear strain: Compressive strain =S:T is applied

cases 1A and 1B tended to zero. The reason for the residual stress is because of the higher deviation of the yield surface. Similar residual stress can be seen in the original paper for compression cases. Hence it can be concluded that yield surface with first order equation has less tendency to deviate from the yield surface and is numerically more efficient.

(3) Softening behavior of concrete in tension

In FEM, getting a mesh independent result is an important criteria. Various attempts are being made in this direction by various authors. Though limited success have been reported by various authors, it has not been yet fully successful. In a constitutive model, it is important to be able to get different softening slopes for tension without effecting the stress-strain relationship in compression for incorporation of fracture energy in unreinforced concrete members and tension stiffening effect in reinforced concrete members. The softening slope can be changed by controlling η_o of Eq. 10 or adopting more general c-ω relation. This is possible because of the introduction of independent variation of c and ϕ

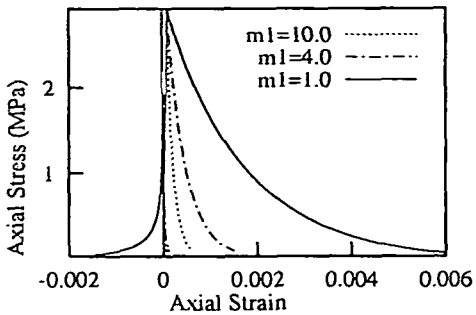


Fig. 18 Effect of m1 on uniaxial tension of case 1A

based on parameter $X = I_1 / \sqrt{3J_2}$ in tension and compression zone. The results can be seen in **Fig. 18**. The peak of the uniaxial tension can be directly controlled by changing η_o.

(4) Pure shear and mixed shear calculations

It has been mentioned before that the hunch back like curve in proportional shear-tension calculation creates numerical problems in FEM analysis . This looks unrealistic too because concrete that has failed under tension, cannot take an increasing amount of shear stress. When similar calculations were repeated for all the cases, it showed similar problems in cases of 1A, 1B and 2B. However, they did not show similar problem in cases 3-5. Here only case 1A in **Fig. 15** and case 3 in **Fig. 17** are shown . **Fig. 14** and **16** shows the behavior in when proportional shear-compression is applied.

10. CONCLUSIONS

The Unified Concrete Plasticity Model has been analyzed, modified and compared with Kupfer's experimental results. We can conclude the following:

a) For this constitutive model, detailed constitutive level calculations are very necessary before conducting any finite element analysis. By this method we can avoid frustrating experience due to material instability.

b) The uniaxial softening curve is very similar in shape as the c-ω curve. So it is important to recognize appropriate variation of c and ϕ at various range. In tension it is recognized that first order (negative) exponential curve as more appropriate where as higher order is more appropriate in compressive zone

c) The initial value of friction angle is assumed to be constant. It was found that constant ϕ for uniaxial tension and gradually increasing ϕ for uniaxial compression provides better results for lateral strain.

d) First order yield surface with simple damage rule(case A or B) can also simulate the concrete behavior equally well as that of second order yield surface with strain hardening damage model.

e) Since the comparison of Kupfer's experimental results were proved successful in intermediate range, hence it was proved that the variation of c and ϕ based $X = I_1 / \sqrt{3J_2}$ to be logical.

In this paper, the actual finite element analysis is not shown because the application to finite element analysis requires various other considerations related to reinforced concrete members. Since unified concrete plasticity model is a new model to be used for concrete, detailed explanation was given about how this model was developed. The actual finite element analysis is included in the companion paper[5].

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統一化塑性モデルの修正提案とその特徴

Supratic GUPTA・田辺忠顕

田辺ら[1]によって提案された統一化塑性モデルについて，詳細な検討を行った結果，いくつかの欠陥が認められた．この欠陥は，ＦＥＭ解析を行う際に，大きな障害となることが判明し，構成則の特徴を保ちながら，修正を行い，新たな統一化塑性モデルを提案した．新たな提案を（ $I_1, \sqrt{J_2}$ ）平面上の領域を区分して，適切なパラメダーを割り振ることが基本となっている．