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INVITED PAPER

ENFORCEMENT GAMES IN ENVIRONMENTAL REGULATION : THE CASE OF MULTIPLE POLLUTANTS

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A model of environmental compliance is developed to determine the optimal enforcement strategy of an environmental agency when an operator may discharge more than one kind of pollutant. Specifically, an extensive game model is constructed, and the complete set of Nash equilibria is found. Examining the relative values of model parameters for Nash equilibria that produce desirable environmental results permits effective environmental policies to be identified and characterized.

1. INTRODUCTION

The waste products of human activities pollute the environment. Some pollutants decompose to harmless substances quickly, but, unfortunately, many wastes persist for long periods of time, and adversely affect water, air, and land quality. As population increases, the variety and quantity of pollutants may increase, deepening the current environmental crisis and perhaps reaching a critical position. Fortunately, this problem is becoming better known and understood. For instance, government leaders from around the world met at an international conference in Rio de Janeiro, Brazil, in June, 1992, to discuss major environmental problems and draw up international agreements to control them. Many saw this event as a milestone on the route toward environmental protection. As *Haimes* (1992) proclaimed, "a worldwide environmental awakening is gathering force to save Planet Earth from humanity's harmful actions."

To achieve sustainable development, that is, to develop economically within desirable and stable environmental conditions, or simply to preserve their natural resources, many countries have put in place extensive environmental laws and regulations. Nonetheless, the mere existence of laws and regulations does not guarantee compliance. As emphasized in Canada's Green Plan (*Government of Canada*, 1990, p. 156), "Legislation and regulation are only as good as their enforcement". In other words, the enforcement of environmental laws and regulations is essential to ensure environmental integrity.

The objective of this paper is to develop a formal enforcement model to study enforcement systematically in one case where its efficiency may be improved – when an operator can release more than one pollutant in its effluent. Because the inspection/enforcement process involves interactive decision making by the firm (operator) and the enforcement agency, and because they have conflicting objectives, non-cooperative game theory models are employed. In particular, an extensive form game is used to identify effective enforcement strategies against discharges of multiple pollutants.

First, the situation of enforcement against one polutant will be reviewed. The problem has been studied using several game models (*Kilgour et al.*, 1992; *Fukuyama et al.*, 1994; *Kilgour*, 1997). In general, it has been shown that when the subjects of a regulation decide to violate or comply based only on their own interest, and when they prefer to violate if and only if certain that the violation will not be detected, then enforcement can be difficult and expensive. This conclusion applies, in particular, to environmental regulations, because of the relatively high costs of monitoring and inspection, the high variability of data, and, frequently, uncertainty concerning the nature and severity of sanctions against violators.

The problem addressed here is an extension of these studies: When an enforcement agency must guard against several different pollutants, and these pollutants can be identified only in separate and independent tests, is it sometimes useful to apply one test and use it as a trigger for others (in other words, carrying out no further testing unless the first test produces evidence of violation), or is it better to carry out all tests, regardless of the results on the first? Note that the operator is assumed to be using a process that produces several different kinds of waste, and therefore has the options of releasing none, some, or all of its wastes without proper treatment. Clearly, the sequential testing strategy assessed here would reduce costs. The important question is whether it would also lead to more violation, whether its net benefits would be negative. The objective is simply to identify cost–effective strategies that encourage compliance when there may be multiple pollutants.

A number of simplications are employed throughout. First, questions of magnitude of pollution will be ignored – either a pollutant is released in sufficient quantity to violate the regulation, or its environmental damage is negligable. Second, testing is assumed perfect. If a pollutant has been released, and if a test for the presence of this pollutant is applied, then the test indicates the presence of the pollutant with certainty. As well, the test never gives "false positives", so that whenever the test indicates the presence of the pollutant, then there has certainly been a violation. Third, testing is assumed to be costless to the operator, though not to the agency. Fourth, only the case of two pollutants will be addressed here – more complex problems, involving three or more pollutants, must await future research.

Mathematical models developed for application to environmental enforcement problems can be considered to be part of verification theory, which involves the enforcement of social norms through inspection. *Fang et al.* (1994) provide a literature review of verification theory and its application to arms control, crime and punishment, auditing, and environmental problems. The situation for enforcement problems involving multiple pollutants is modeled for the first time here, using an extensive form model. In addition to the references above, *Fang et al.* (1997), *Hipel et al.* (1994, 1995), *Kilgour* (1994) and *Yin et al.* (1995) use extensive games to analyse environmental enforcement problems.

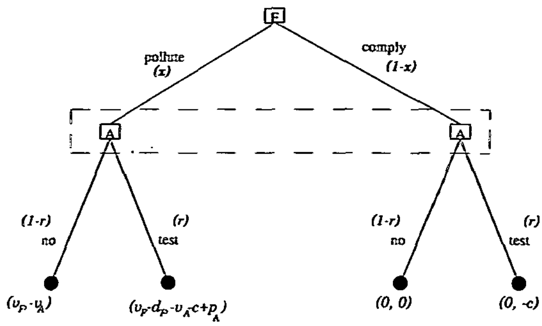

Fig. 1 Game model of enforcement against one pollutant

2. GAME MODEL OF INSPECTION AGAINST ONE POLLUTANT

Suppose that any possible pollution due to the activity of a regulated firm can be detected by a single test. **Fig. 1** shows an extensive game that models the joint decision problem of the firm (F) and the environmental enforcement agency (A). The firm's decision is whether to pollute or comply, and the agency's is whether to test or not. Even though **Fig. 1** shows the firm's decision as taking place before the agency's decision, the agency must take its choice without knowledge of the decision already made by the firm, as indicated by the agency's two decision nodes within the same information set.

In the model shown in **Fig. 2**, the players' payoffs at each terminal node are shown as a 2–vector. The first component is F's payoff [technically a von Neumann–Morgenstern (1953) utility], and the second is A's. The payoff parameters are as follows: v_F is F's gain for violating; d_F is the loss to F due to the punishment it receives if the violation is detected; c is the cost to the agency of carrying out an inspection, or testing; v_A is the loss to the agency if F violates; and p_A is the amount by which A's loss is reduced if the violation is detected. All of these payoff parameters are assumed to be positive.

Games such as the simple game of **Fig. 2** have been analysed elsewhere (*Kilgour et al.*, 1992; *Fukuyama et al.*, 1994; *Kilgour*, 1997). To explain the results, some strategic variables are needed. Let x denote the probability that F chooses its Pollute action (so F chooses its Comply action with probability $1-x$), and let r denote the probability that A chooses its Test action (so A chooses not to test with probability $1-r$). These probabilities are shown in **Fig. 1**.

The Nash (1951) equilibria of the game of **Fig. 1** are as follows:

1. If $p_A < c$, the unique equilibrium is $x=1$, $r=0$.
2. If $p_A > c$ and $d_F < v_F$, the unique equilibrium is $x=1$, $r=1$.
3. If $p_A > c$ and $d_F > v_F$, the unique equilibrium is x

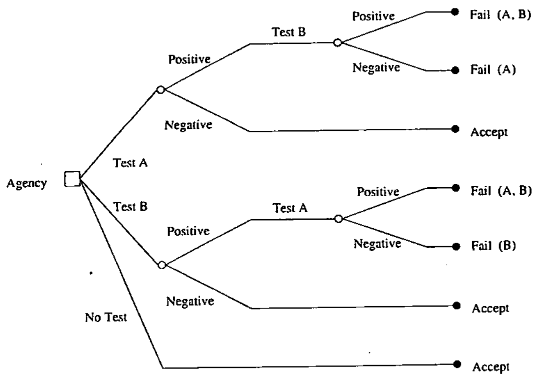

Fig. 2 Test procedure against pollutants A and B.

$= c/p_A, r = v_F/d_F$.

Other possibilities, where $p_A = c_A$ or $d_F = v_F$, will not be considered here. As characterized by *Kilgour et al.* (1992), these cases represent three possibilities, as follows:

1. Testing is not cost–effective, and is never used. As there are never any inspections, violation occurs for certain.
2. Testing is cost–effective, but sanctions against violations are inadequate. Both violation and testing occur for certain.
3. Testing is cost–effective, and sanctions are adequate. Both violation and testing occur probabilistically. The rate of compliance is $(p_A - c)/p_A$.

Thus, the best that can be hoped for – with cost–effective testing and adequate sanctions – is partial compliance.

According to the model of **Fig. 1**, it is impossible to prevent violations using the possibility of inspection. In fact, the situation has another undesirable aspect. At equilibrium (3), F's expected utility is 0 and A's is $-(cv_A)/p_A$. Thus the outcome at equilibrium is Pareto–inferior, as both players would achieve the value 0 at the Comply/Not Test outcome. Other models (such as *Kilgour*, 1997) take an even bleaker view of this situation. If there is a testing cost for F as well as for A, then both sides are actually worse off at equilibrium (3) than at the Comply/Not Test outcome.

Thus the Pareto–superior outcome, with payoffs (0, 0), is unstable for strategic reasons, even in the most favourable circumstance. This explains the need to improve procedures for enforcement – to take measures to alter the game of **Fig. 1** to produce a more socially useful outcome. See *Kilgour* (1997) for further discussion of this point.

In the next section, an extensive game is used to model a situation in which a firm can discharge two types of pollutant, which can be identified reliably, but only by independent tests. By examining the relative values of parameters for the Nash equilibria of this game, and by identifying the equilibria that achieve desirable levels of environmental protection, effective environmental policies in the presence of multiple pollutants will be determined.

3. GAME MODEL OF INSPECTION AGAINST TWO POLLUTANTS

(1) Test Procedure and Game Model

If the firm has two different pollutants, A and B, then it has four alternative choices. It can pollute with both A and B, or pollute with only A, or only B, or comply with the regulations, that is, not pollute. In the test procedure under consideration, the agency has three choices. It can test for pollutant A, or for B, or not to conduct any test. In this test procedure, whenever the initial test result is positive, the other pollutant is to be tested; otherwise, the test is to be stopped. This test procedure is shown in **Fig. 2**

An environmental enforcement model against two pollutants is shown in **Fig. 3**. To interpret this extensive game, start from the top of the figure and read downwards. Each box containing a capital letter represents a decision point for the decision maker designated by that letter. For instance, at the top of the game model decision maker F, the firm (or operator), chooses to violate with pollutant A, or B, or A and B, or to comply. Decision maker A, the agency, is not aware of decision maker F's choice and, thus, corresponding nodes labeled A are enclosed by dotted lines to indicate that they are in the same information set. For the information set shown in **Fig. 3**, the agency must choose to test A first, or test B first, or not test. Following the test procedure of **Fig. 2**, the game must end at one of the termination points shown as solid circles in **Fig. 3**. For example, if the firm discharges both A and B, and the agency decides not to test, the game ends at the leftmost termination. Each possible termination point has a value or utility for each decision maker; these are indicated by a 2–vector in which the first and second entries are the utilities (payoffs) for the firm and the agency, respectively. Again, these payoffs are measured in *von Neumann–Morgenstern* (1953) utilities.

Table 1 lists the utility parameters used to express the payoffs for the termination points of the game model, where the subscript designates the decision maker and the superscript the pollutant. Since it is assumed that the value of each parameter is positive, a negative sign applied to a parameter indicates a negative value. The use of only ten parameters to describe the two decision makers' utilities at twelve different outcomes implies that the utilities must satisfy certain functional relationships, but these requirements are in fact quite reasonable in the con-

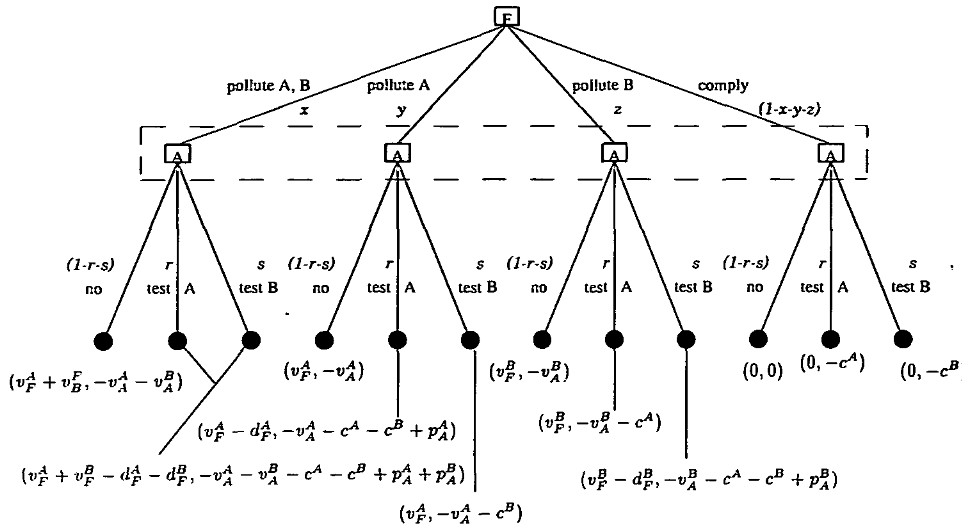

Fig. 3 Game model of enforcement against pollutants A and B

Table 1 Payoff Parameters for Game Model In Fig. 3

	Decision Maker F Firm	Decision Maker A Agency
Values for violation with A	v_F^A	$-v_A^A$
Values for violation with B	v_F^B	$-v_A^B$
Cost of testing A	-	$-c^A$
Cost of testing B	-	$-c^B$
Penalty for conviction with A	$-d_F^A$	-
Penalty for conviction with B	$-d_F^B$	-
Value for stopping violation with A	-	p_A^A
Value for stopping violation with B	-	p_A^B

text of the problem being modelled.

Consider the payoffs for the leftmost termination point of **Fig. 3**, as an illustration of the payoff calculation. The firm's utility at this outcome is $v_F^A + v_F^B$, where v_F^A and v_F^B are the values gained by violating with pollutants A and B respectively. The utility of the agency is $-v_A^A - v_A^B$. The values contain negative signs because these factors are costs to the agency, relative to the status quo. The x, y and z variables shown in **Fig. 3** are strategic variables: x represents the probability that the firm violates with both A and B, y the probability that the firm violates with only A, and z is the probability that the firm violates with only B. Thus the probability of not violating is $(1-x-y-z)$. The strategic variable r is the probability that the agency tests A first and the strategic variable s is the probability that the agency tests B first; thus the probability of not testing is $(1-r-s)$.

(2) Expected Utilities

Prior to determining the Nash Equilibria of an extensive game such as the one shown in **Fig. 3**, the expected utility for each decision maker must be calculated. The expected utility for the firm can be shown to be

$$\begin{aligned} E_F(x, y, z; r, s) &= x(1-r-s)(v_F^A + v_F^B) + x(r+s) \\ &\quad (v_F^A + v_F^B - d_F^A - d_F^B) + y(1-r-s)v_F^A \\ &\quad + yr(v_F^A - d_F^A) + ysv_F^A + z(1-r-s)v_F^B \\ &\quad + zrv_F^B + zs(v_F^B - d_F^B) \\ &= [(v_F^A + v_F^B) - (d_F^A + d_F^B)(r+s)]x \\ &\quad + (v_F^A - d_F^A r)y + (v_F^B - d_F^B s)z \end{aligned} \tag{1}$$

while the expected utility for the agency is

$$\begin{aligned} E_A(x, y, z; r, s) &= x(1-r-s)(-v_A^A - v_A^B) + x(r+s) \\ &\quad (-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B) \\ &\quad + y(1-r-s)(-v_A^A) + yr(-v_A^A - c^A \\ &\quad - c^B + p_A^A) + ys(-v_A^A - c^B) + z(1-r-s) \\ &\quad (-v_A^B) + zr(-v_A^B - c^A) + zs(-v_A^B - c^A \\ &\quad - c^B + p_A^B) + (1-x-y-z)r(-c^A) \\ &\quad + (1-x-y-z)s(-c^B) \\ &= -[(v_A^A + v_A^B)x + v_A^A y + v_A^B z] \\ &\quad + [(p_A^A + p_A^B - c^B)x + (p_A^A - c^B)y - c^A]r \\ &\quad + [(p_A^A + p_A^B - c^A)x + (p_A^B - c^A)z - c^B]s \end{aligned} \tag{2}$$

Recall the assumptions that

Table 2 Strategic Form Game Model

	T_A	T_B	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_A	$-v_A^A - c^A - c^B + p_A^A$ $v_F^A - d_F^A$	$-v_A^A - c^B$ v_F^A	$-v_A^A$ v_F^A	y
V_B	$-v_A^B - c^A$ v_F^B	$-v_A^B - c^A - c^B + p_A^B$ $v_F^B - d_F^B$	$-v_A^B$ v_F^B	z
C	$-c^A$ 0	$-c^B$ 0	0 0	$(1-x-y-z)$
	r	s	$(1-r-s)$	

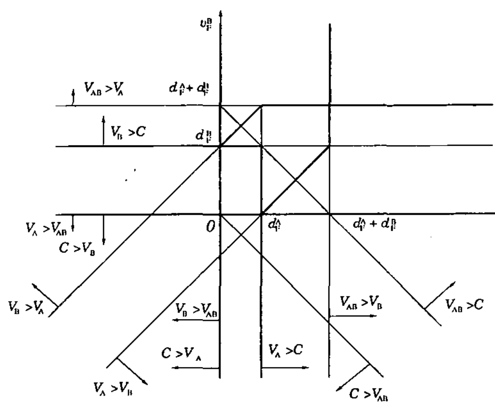

Fig. 4 Domination relationships among the firm's strategies

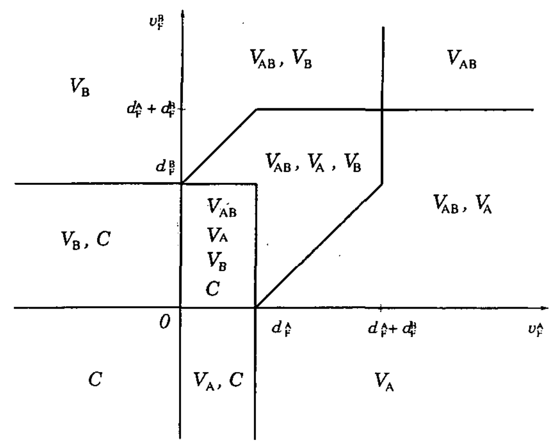

Fig. 5 Strategy regions of the firm

$$v_F^A > 0 \quad v_A^A > 0 \quad d_F^A > 0 \quad c^A > 0 \quad p_A^A > 0$$
$$v_F^B > 0;\ v_A^B > 0;\ d_F^B > 0;\ c^B > 0;\ p_A^B > 0. \qquad (3)$$

4. EQUILIBRIUM ANALYSIS

To analyse an extensive form game such as the one in **Fig. 3**, one finds all Nash equilibria (*Nash*, 1951). Generally, an equilibrium is a strategy vector relative to which no decision maker can gain in expected utility by departing unilaterally. Here, $(x^*, y^*, z^*; r^*, s^*)$ will represent the values of the strategic variables at a Nash equilibrium. All Nash equilibria of the game are found in the following analysis.

The extensive form game of **Fig. 3** can be converted to the strategic form game of **Table 2**. In addition to the parameters in **Table 1**, the following notation is used in **Table 2** to denote (pure) strategies:

V_{AB}—the firm violates with both A and B,
V_A—the firm violates with A only,
V_B—the firm violates with B only,
C—the firm complies,
T_A—the agency tests A first,
T_B—the agency tests B first,
T_N—the agency does not test.

(1) The Possible Equilibria

In this game, the firm has four strategies, V_{AB}, V_A, V_B, and C. According to the relative values of parameters v_F^A, v_F^B, d_F^A, and d_F^B, one strategy may dominate another in that the first always yields a greater utility than the second, no matter what choice the opponent makes. Hence, a strategy will never be chosen at equilibrium if it is dominated. All domination relationships and the regions where they occur are shown in **Fig. 4**.

The strategy regions of the firm with dominated strategies eliminated are shown in **Fig. 5**. It can be clearly seen that all strategies are dominated in some regions and not in others.

Strictly dominated strategies are never chosen at Nash equilibrium. Thus, to find all equilibria, the analysis is carried region by region. Since $v_F^A > 0$ and $v_F^B > 0$, only regions $[V_{AB}]$, $[V_{AB}, V_A]$, $[V_{AB}, V_B]$, $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$ will be considered.

First, note that when $p_A^A + p_A^B < c^A + c^B$, T_N dominates both T_A and T_B. V_{AB} is the firm's best response to the agency's only undominated strategy T_N. The only equilibrium is

$$Q_1 = (1, 0, 0; 0, 0) \qquad (4)$$

It will now be assumed that $p_A^A + p_A^B > c^A + c^B$, so that

Table 3

	T_A	T_N	
V_{AB}	$-v_A^A - v_A^B - c_A - c_B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_A	$-v_A^A - c^A - c_B + p_A^A$ $v_F^A - d_F^A$	$-v_A^A$ v_F^A	y
	r	$(1-r)$	

Table 4

	T_B	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_A	$-v_A^A - c^B$ v_F^A	$-v_A^A$ v_F^A	y
	s	$(1-s)$	

Table 5

	T_B	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_B	$-v_A^B - c^A - c^B + p_A^B$ $v_F^B - d_F^B$	$-v_A^B$ v_F^B	z
	s	$(1-s)$	

Table 6

	T_A	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_B	$-v_A^B - c^A$ v_F^B	$-v_A^B$ v_F^B	z
	r	$(1-r)$	

agency's strategy T_N does not dominate T_A and T_B. The analyses will be conducted according to the strategy region of the firm.

Region $[V_{AB}]$

From the row corresponding to V_{AB} in **Table 2**, it is clear that T_A and T_B result in identical outcomes, and both are preferred to T_N. Hence, there are two (pure) equilibria,

$$Q_{2.a}=(1, 0, 0; 1, 0) \quad (5)$$

$$Q_{2.b}=(1, 0, 0; 0, 1) \quad (6)$$

Region $[V_{AB}, V_A]$

Refer to the two rows corresponding to V_{AB} and V_A in **Table 2**. There are now two cases:

1. If $p_A^A > c_A$, T_A dominates T_B. The game can be simplified to **Table 3.**
   (1) If $p_A^A > c^A + c^B$, T_A dominates T_N.
      (A) If $v_F^B > d_F^B$, the only equilibrium is
      $$Q_{2.a}=(1, 0, 0; 1, 0) \quad (7)$$
      (B) If $v_F^B < d_F^B$, the only equilibrium is
      $$Q_3=(0, 1, 0; 1, 0) \quad (8)$$
   (2) If $p_A^A < c^A + c^B$
      (A) If $v_F^B > d_F^B$, V_{AB} dominates V_A. The only equilibrium is
      $$Q_{2.a}=(1, 0, 0; 1, 0) \quad (9)$$
      (B) If $v_F^B < d_F^B$, the only equilibrium is a mixed strategy equilibrium,
      $$Q_4=(x^*, y^*, 0; r^*, 0), \quad x^*=\frac{c^A+c^B-p_A^A}{p_A^B},$$
      $$y^*=\frac{p_A^A+p_A^B-c^A-c^B}{p_A^B}, \quad r^*=\frac{v_F^B}{d_F^B}. \quad (10)$$
2. If $p_A^A < c^A$, T_B dominates T_A. The game can be reduced to **Table 4.** The only equilibrium is
$$Q_5=(x^*, y^*, 0; 0, s^*), \quad x^*=\frac{c^B}{p_A^A+p_A^B-c^A},$$
$$y^*=\frac{p_A^A+p_A^B-c^A-c^B}{p_A^A+p_A^B-c^A}, \quad s^*=\frac{v_F^B}{d_F^A+d_F^B}. \quad (11)$$

Region $[V_{AB}, V_B]$

This case is similar to the previous. Only two rows remain, corresponding to V_{AB} and V_B.

1. If $p_A^B > c^B$, T_B dominates T_A. The game can be simplified to **Table 5.**
   (1) If $p_A^B > c^A + c^B$, T_B dominates T_N
      (A) If $v_F^A > d_F^A$, the only equilibrium is
      $$Q_{2.b}=(1, 0, 0; 0, 1) \quad (12)$$
      (B) If $v_F^A < d_F^A$, the only equilibrium is
      $$Q_6=(0, 0, 1; 0, 1) \quad (13)$$
   (2) If $p_A^B < c^A + c^B$
      (A) If $v_F^A > d_F^A$, V_{AB} dominates V_B. The only equilibrium is
      $$Q_{2.b}=(1, 0, 0; 0, 1) \quad (14)$$
      (B) If $v_F^A < d_F^A$, the only equilibrium is the mixed equilibrium,
      $$Q_7=(x^*, 0, z^*; 0, s^*), \quad x^*=\frac{c^A+c^B-p_A^B}{p_A^A},$$
      $$z^*=\frac{p_A^A+p_A^B-c^A-c^B}{p_A^A}, \quad s^*=\frac{v_F^A}{d_F^A}. \quad (15)$$
2. If $p_A^B < c^B$, T_A dominates T_B. The game can be simplified to **Table 6.** The only equilibrium is
$$Q_8=(x^*, 0, z^*; r^*, 0), \quad x^*=\frac{c^A}{p_A^A+p_A^B-c^B},$$
$$z^*=\frac{p_A^A+p_A^B-c^A-c^B}{p_A^A+p_A^B-c^B}, \quad r^*=\frac{v_F^A}{d_F^A+d_F^B}. \quad (16)$$

Region $[V_{AB}, V_A, V_B]$

In **Table 2**, only the three rows corresponding to V_{AB}, V_A, and V_B remain.

1. If $p_A^A > c^A$ and $p_A^B < c^B$, T_A dominates T_B. The game can be simplified to **Table 7.**
   (1) If $v_F^B > d_F^B$, V_{AB} dominates V_A (or, if $v_F^A < v_F^B$, V_B dominates V_A), and the game model can be further reduced to **Table 6.** The only equilibrium is
   $$Q_8=(x^*, 0, z^*; r^*, 0) \quad (17)$$

Table 7

	T_A	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_A	$-v_A^A - c^A - c^B + p_A^A$ $v_F^A - d_F^A$	$-v_A^A$ v_F^A	y
V_B	$-v_A^B - c^A$ v_F^B	$-v_A^B$ v_F^B	z
	r	$(1-r)$	

Table 8

	T_A	T_N	
V_A	$-v_A^A - c^A - c^B + p_A^A$ $v_F^A - d_F^A$	$-v_A^A$ v_F^A	y
V_B	$-v_A^B - c^A$ v_F^B	$-v_A^B$ v_F^B	z
	r	$(1-r)$	

(2) If $v_F^B < d_F^B$ and $v_F^A > v_F^B$, this becomes a 3×2 game (**Table 7**). The fundamental equilibria can be found by analyzing three 2×2 games, since any fundamental equilibrium which is not pure must involve two pure strategies for each player.

(A) When row V_{AB} is removed, the game is reduced to **Table 8.**

(a) If $p_A^A < c^A + c^B$, T_N dominates T_A. A possible equilibrium is (0, 1, 0 ; 0, 0). However, the expected utility of the firm for these strategies is $E_F = v_F^A$, whereas, if the firm chooses strategy V_{AB}, the corresponding expected utility would be $E'_F = v_F^A + v_F^B$. Since $E_F < E'_F$, this is not an equilibrium.

(b) If $p_A^A > c^A + c^B$, then

$$Q_9 = (0, y^*, z^*; r^*, 0),\quad y^* = \frac{c^A}{p_A^A - c^B},\quad z^* = \frac{p_A^A - c^A - c^B}{p_A^A - c^B},\quad r^* = \frac{v_F^A - v_F^B}{d_F^A}. \tag{18}$$

is an equilibrium.

(B) When row V_A is removed, the game is reduced to **Table 6.** There is always an equilibrium

$$Q_8 = (x^*, 0, z^*; r^*, 0) \tag{19}$$

in this case.

(C) When row V_B is deleted, the game becomes **Table 3.**

(a) If $p_A^A > c^A + c^B$, T_A dominates T_N. A possible equilibrium is (0, 1, 0 ; 1, 0). The resulting expected utility of the firm is $E_F = v_F^A - d_F^A$, but if the firm were to choose strategy V_B, the corresponding expected utility would be $E'_F = v_F^B$. Suppose $E_F > E'_F$. Then $v_F^A - d_F^A > v_F^B$, which leads to $(v_F^A - v_F^B)/d_F^A > 1$. However, this is impossible, since it is outside the region $[V_{AB}, V_A, V_B]$. Hence, this possibility is not an equilibrium.

(b) If $p_A^A < c^A + c^B$, then

$$Q_4 = (x^*, y^*, 0; r^*, 0) \tag{20}$$

is an equilibrium.

In summary, the equilibria when $p_A^A > c^A$ and $p_A^B < c^B$ are

$$Q_4 = (x^*, y^*, 0; r^*, 0), \tag{21}$$
$$Q_8 = (x^*, 0, z^*; r^*, 0), \tag{22}$$
$$Q_9 = (0, y^*, z^*; r^*, 0). \tag{23}$$

2. If $p_A^B > c^B$ and $p_A^A < c^A$, T_B dominates T_A. The game model is simplified to as **Table 9.** By a similar method, the equilibria when $p_A^B > c^B$ and $p_A^A < c^A$ are

Table 9

	T_B	T_N	
V_{AB}	$-v_A^A - v_A^B - c^A - c^B + p_A^A + p_A^B$ $v_F^A + v_F^B - d_F^A - d_F^B$	$-v_A^A - v_A^B$ $v_F^A + v_F^B$	x
V_A	$-v_A^A - c^B$ v_F^A	$-v_A^A$ v_F^A	y
V_B	$-v_A^B - c^A - c^B + p_A^B$ $v_F^B - d_F^B$	$-v_A^B$ v_F^B	z
	s	$(1-s)$	

$$Q_5 = (x^*, y^*, 0; 0, s^*), \tag{24}$$
$$Q_7 = (x^*, 0, z^*; 0, s^*), \tag{25}$$

$$Q_{10} = (0, y^*, z^*; 0, s^*),\quad y^* = \frac{p_A^B - c^A - c^B}{p_A^B - c^A},\quad z^* = \frac{c^B}{p_A^B - c^A},\quad s^* = \frac{v_F^B - v_F^A}{d_F^B}. \tag{26}$$

3. If $p_A^A > c^A$ and $p_A^B > c^B$, this is a 3×3 game. The equilibria are found by analyzing the nine 2×2 games which are contained in the 3×3 game, as well as the 3×3 game. six equilibria were found in the nine 2×2 games, and one in the 3×3 game as follows :

$$Q_{2,c} = (1, 0, 0; r^*, s^*), \tag{27}$$
$$Q_4 = (x^*, y^*, 0; r^*, 0), \tag{28}$$
$$Q_7 = (x^*, 0, z^*; 0, s^*), \tag{29}$$
$$Q_9 = (0, y^*, z^*; r^*, 0), \tag{30}$$
$$Q_{10} = (0, y^*, z^*; 0, s^*), \tag{31}$$

$$Q_{11} = (x^*, y^*, z^*; r^*, s^*),\quad x^* = 1 - y^* - z^*,$$
$$y^* = \frac{(p_A^A + p_A^B - c^A - c^B)(p_A^B - c^B)}{(p_A^A)^2 + (p_A^B)^2 + p_A^A p_A^B - (p_A^A + p_A^B)(c^A + c^B) + c^A c^B},$$
$$z^* = \frac{(p_A^A + p_A^B - c^A - c^B)(p_A^A - c^A)}{(p_A^A)^2 + (p_A^B)^2 + p_A^A p_A^B - (p_A^A + p_A^B)(c^A + c^B) + c^A c^B}, \tag{32}$$

$$Q_{12}=(0, y^*, z^*; r^*, s^*),\ \ y^*+z^*=1,\ \ r^*+s^*=1,$$

$$y^*=\frac{p_A^B-c^B}{p_A^A+p_A^B-c^A-c^B},\ \ z^*=\frac{p_A^A-c^A}{p_A^A+p_A^B-c^A-c^B}. \quad (33)$$

Region $[V_{AB}, V_A, V_B, C]$

From **Table 2**, this is a 4×3 game. The fundamental equilibria can be found by analyzing four 3×3 games.

1. When row C is removed, the game is reduced to a 3×3 game. It is obvious that the analysis procedure in this case is similar to that conducted for region $[V_{AB}, V_A, V_B]$. However, there are some differences between the two cases. When $p_A^A>c^A$ and $p_A^B>c^B$, as in the region $[V_{AB}, V_A, V_B]$, the equilibria are found by analysing nine 2×2 games and one 3×3 game. In region $[V_{AB}, V_A, V_B, C]$, since $v_F^A<d_F^A$ and $v_F^B<d_F^B$, $(1, 0, 0; r^*, s^*)$ is not an equilibrium. When $p_A^A>c^A$ and $p_A^B>c^B$, only the following six equilibria are possible

$$Q_4=(x^*, y^*, 0; r^*, 0), \quad (34)$$
$$Q_7=(x^*, 0, z^*; 0, s^*), \quad (35)$$
$$Q_9=(0, y^*, z^*; r^*, 0), \quad (36)$$
$$Q_{10}=(0, y^*, z^*; 0, s^*), \quad (37)$$
$$Q_{11}=(x^*, y^*, z^*; r^*, s^*), \quad (38)$$
$$Q_{12}=(0, y^*, z^*; r^*, s^*),\ \ y^*+z^*=1,\ \ r^*+s^*=1. \quad (39)$$

2. When row V_{AB} is removed, the game is reduced to a 3×3 game. It is easy to prove that no new equilibrium is found in the nine resulting 2×2 games; as for the 3×3 game, the only equilibrium is

$$Q_{13}=(0, y^*, z^*; r^*, s^*),\ \ y^*+z^*<1,\ \ r^*+s^*<1,$$

$$y^*=\frac{c^A}{p_A^A-c^B},\ \ z^*=\frac{c^B}{p_A^B-c^A},\ \ r^*=\frac{v_F^A}{d_F^A},\ \ s^*=\frac{v_F^B}{d_F^B}. \quad (40)$$

3. When row V_B is removed, no new equilibrium is found in either the nine 2×2 games, or the 3×3 games.
4. Similarly when row V_A is removed, no new equilibrium can be identified.

In summary, thirteen strategy combinations are equilibria somewhere in these regions.

$$Q_1=(1, 0, 0; 0, 0), \quad (41)$$
$$Q_{2.a}=(1, 0, 0; 1, 0), \quad (42)$$
$$Q_{2.b}=(1, 0, 0; 0, 1), \quad (43)$$
$$Q_{2.c}=(1, 0, 0; r^*, s^*), \quad (44)$$
$$Q_3=(0, 1, 0; 1, 0), \quad (45)$$
$$Q_4=(x^*, y^*, 0; r^*, 0), \quad (46)$$
$$Q_5=(x^*, y^*, 0; 0, s^*), \quad (47)$$
$$Q_6=(0, 0, 1; 0, 1), \quad (48)$$
$$Q_7=(x^*, 0, z^*; 0, s^*), \quad (49)$$
$$Q_8=(x^*, 0, z^*; r^*, 0), \quad (50)$$
$$Q_9=(0, y^*, z^*; r^*, 0), \quad (51)$$
$$Q_{10}=(0, y^*, z^*; 0, s^*), \quad (52)$$
$$Q_{11}=(x^*, y^*, z^*; r^*, s^*), \quad (53)$$
$$Q_{12}=(0, y^*, z^*; r^*, s^*),\ \ y^*+z^*=1,\ \ r^*+s^*=1, \quad (54)$$
$$Q_{13}=(0, y^*, z^*; r^*, s^*),\ \ y^*+z^*<1,\ \ r^*+s^*<1. \quad (55)$$

Note that the existence conditions for an equilibrium may be different in different regions.

(2) Existence Conditions for the Equilibria

Each of the equilibria can exist under some specific conditions, which are listed here.

Q_1: (1, 0, 0; 0, 0)

This equilibrium exists in all regions. It exists if and only if

$$p_A^A+p_A^B<c^A+c^B \quad (56)$$

The region of existence is shown in **Fig. 8.**

Q_{2a}: (1, 0, 0; 1, 0), Q_{2b}: (1, 0, 0; 0, 1), and Q_{2c}: (1, 0, 0; r^*, s^*)

These equilibria exist in regions $[V_{AB}]$, $[V_{AB}, V_A]$, $[V_{AB}, V_B]$ and $[V_{AB}, V_A, V_B]$. They exist if and only if

$$p_A^A+p_A^B>c^A+c^B, \quad (57)$$

The regions of existence are shown in **Figs. 9, 10** and **11.**

Q_3: (0, 1, 0; 1, 0),

This equilibrium exists in region $[V_{AB}, V_A]$. It exists if and only if

$$v_F^B<d_F^B,\ \ \frac{v_F^A-v_F^B}{d_F^A}>1;\ \ p_A^A>c^A+c^B. \quad (58)$$

The region of existence is shown in **Fig. 12.**

Q_4: (x^*, y^*, 0; r^*, 0)

This equilibrium can occur in regions $[V_{AB}, V_A]$, $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$v_F^B<d_F^B,\ \ \frac{v_F^A}{v_F^B}>\frac{d_F^A+d_F^B}{d_F^B};\ \ p_A^A>c^A,\ \ p_A^A<c^A+c^B,$$
$$p_A^A+p_A^B>c^A+c^B. \quad (59)$$

The region of existence is shown in **Fig. 13.**

Q_5: (x^*, y^*, 0; 0, s^*)

This equilibrium can occur in regions $[V_{AB}, V_A]$, $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$v_F^B<d_F^A+d_F^B,\ \ \frac{v_F^A}{v_F^B}>\frac{d_F^A}{d_F^A+d_F^B};\ \ p_A^A<c^A,$$
$$p_A^A+p_A^B>c^A+c^B. \quad (60)$$

The region of existence is shown in **Fig. 14.**

Q_6: (0, 0, 1; 0, 1)

This equilibrium exists in region $[V_{AB}, V_B]$. It exists if and only if

$$v_F^A<d_F^A,\quad \frac{v_F^B-v_F^A}{d_F^B}>1;\quad p_A^B>c^A+c^B. \tag{61}$$

The region of existence is shown in **Fig. 15.**

$Q_7:(x^*, 0, z^*; 0, s^*)$

This equilibrium can occur in regions $[V_{AB}, V_B]$, $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$v_F^A<d_F^A,\quad \frac{v_F^B}{v_F^A}>\frac{d_F^A+d_F^B}{d_F^A};\quad p_A^B>c^B,\quad p_A^B<c^A+c^B,$$
$$p_A^A+p_A^B>c^A+c^B. \tag{62}$$

The region of existence is shown in **Fig. 16.**

$Q_8:(x^*, 0, z^*; r^*, 0)$

This equilibrium can occur in regions $[V_{AB}, V_B]$, $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$v_F^A<d_F^A+d_F^B,\quad \frac{v_F^B}{v_F^A}>\frac{d_F^B}{d_F^A+d_F^B};\quad p_A^B<c^B,$$
$$p_A^A+p_A^B>c^A+c^B. \tag{63}$$

The region of existence is shown in **Fig. 17.**

$Q_9:(0, y^*, z^*; r^*, 0)$

It can exist in regions $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$, It exists if and only if

$$\frac{v_F^A-v_F^B}{d_F^A}<1,\quad \frac{v_F^A}{v_F^B}>\frac{d_F^A+d_F^B}{d_F^B};$$
$$p_A^A>c^A+c^B,\quad \frac{c^A}{p_A^A-c^B}+\frac{c^B}{p_A^B-c^A}>1. \tag{64}$$

The region of existence is shown in **Fig. 18.**

$Q_{10}:(0, y^*, z^*; 0, s^*)$

This equilibrium exists in regions $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$\frac{v_F^B-v_F^A}{d_F^B}<1,\quad \frac{v_F^B}{v_F^A}>\frac{d_F^A+d_F^B}{d_F^A};$$
$$p_A^B>c^A+c^B,\quad \frac{c^A}{p_A^A-c^B}+\frac{c^B}{p_A^B-c^A}>1. \tag{65}$$

The region of existence is shown in **Fig. 19.**

$Q_{11}:(x^*, y^*, z^*; r^*, s^*)$

This equilibrium can occur in regions $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

$$v_F^A<d_F^A+d_F^B,\quad v_F^B<d_F^A+d_F^B,\quad \frac{v_F^A-v_F^B}{d_F^A}<1,\quad \frac{v_F^B-v_F^A}{d_F^B}<1;$$
$$p_A^A>c^A,\quad p_A^B>c^B,\quad \frac{c^A}{p_A^A-c^B}+\frac{c^B}{p_A^B-c^A}>1. \tag{66}$$

The region of existence is shown in **Fig. 20.**

$Q_{12}:(0, y^*, z^*; r^*, s^*), y^*+z^*=1, r^*+s^*=1$

Q_{12} exists in regions $[V_{AB}, V_A, V_B]$ and $[V_{AB}, V_A, V_B, C]$. It exists if and only if

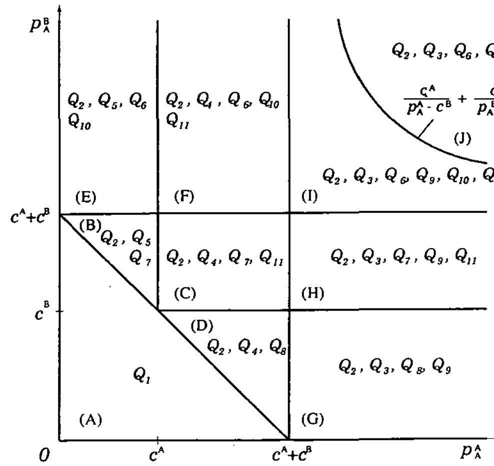

Fig. 6 The existence regions of the equilibria

$$\frac{v_F^A-v_F^B}{d_F^A}<1,\quad \frac{v_F^B-v_F^A}{d_F^B}<1,$$
$$v_F^Ad_F^A+v_F^Bd_F^B<(d_F^A)^2+d_F^Ad_F^B+(d_F^B)^2;$$
$$p_A^A>c^A,\quad p_A^B>c^B,\quad \frac{c^A}{p_A^A-c^B}+\frac{c^B}{p_A^B-c^A}<1. \tag{67}$$

The region of existence is shown in **Fig. 21.**

$Q_{13}:(0, y^*, z^*; r^*, s^*), y^*+z^*<1, r^*+s^*<1$

This equilibrium exists in region $[V_{AB}, V_A, V_B, C]$ only. It exists if and only if

$$\frac{v_F^A}{d_F^A}+\frac{v_F^B}{d_F^B}<1;\quad p_A^A>c^A+c^B,\quad p_A^B>c^A+c^B,$$
$$\frac{c^A}{p_A^A-c^B}+\frac{c^B}{p_A^B-c^A}<1. \tag{68}$$

The region of existence is shown in **Fig. 22.**

Combining **Fig. 8–22**, the existence regions for all equilibria from the agency's viewpoint are shown in **Fig. 6**

5. THE DESIRABLE EQUILIBRIA

From **Fig. 6**, it can be seen that there is more than one equilibrium in each area except area (A). For instance, in area (J) equilibria Q_2, Q_3, Q_6, Q_{12}, and Q_{13} can occur. Which is the most desirable equilibrium from the viewpoint of agency? What should agency do to achieve this desirable equilibrium? In order to answer these questions, further analysis is carried out in this section. Obviously, the agency prefers the equilibrium which maximizes its expected utility. Thus, the first task is to calculate the agency's expected utility for all thirteen equilibria.

(1) The Expected Utility of the Agency

For convenience, the formula for the agency's expected utility is repeated here.

$$E_A(x,y,z;r,s) = -[(v_A^A+v_A^B)x+v_A^Ay+v_A^Bz] + [(p_A^A+p_A^B-c^B)x+(p_A^A-c^B)y-c^A]r + [(p_A^A+p_A^B-c^A)x+(p_A^B-c^A)z-c^B]s \quad (69)$$

The expected utilities of the agency at the various equilibria are as follows:

$$E_A(Q_1) = -(v_A^A+v_A^B) \quad (70)$$

$$E_A(Q_2) = -(v_A^A+v_A^B)+(p_A^A+p_A^B-c^A-c^B) \quad (71)$$

$$E_A(Q_3) = -v_A^A+(p_A^A-c^A-c^B) \quad (72)$$

$$E_A(Q_4) = -v_A^A-v_A^B\frac{(c^A+c^B-p_A^A)}{p_A^B} \quad (73)$$

$$E_A(Q_5) = -v_A^A-v_A^B\frac{c^B}{p_A^A+p_A^B-c^A} \quad (74)$$

$$E_A(Q_6) = -v_A^B+(p_A^B-c^A-c^B) \quad (75)$$

$$E_A(Q_7) = -v_A^B-v_A^A\frac{(c^A+c^B-p_A^B)}{p_A^A} \quad (76)$$

$$E_A(Q_8) = -v_A^B-v_A^A\frac{c^A}{p_A^A+p_A^B-c^B} \quad (77)$$

$$E_A(Q_9) = -v_A^B-(v_A^A-v_A^B)\frac{c^A}{p_A^A-c^B} \quad (78)$$

$$E_A(Q_{10}) = -v_A^A-(v_A^B-v_A^A)\frac{c^B}{p_A^B-c^A} \quad (79)$$

$$E_A(Q_{11}) = -(v_A^A+v_A^B)+(p_A^A+p_A^B-c^A-c^B)\frac{(p_A^B-c^B)v_A^B+(p_A^A-c^A)v_A^A}{(p_A^A)^2+(p_A^B)^2+p_A^Ap_A^B-(p_A^A+p_A^B)(c^A+c^B)+c^Ac^B} \quad (80)$$

$$E_A(Q_{12}) = -(v_A^Ay^*+v_A^Bz^*)+[(p_A^A-c^B)y^*-c^A]r^* + [(p_A^B-c^A)z^*-c^B]s^* \quad (81)$$

Recall that $y^*+z^*=1$ and $r^*+s^*=1$ at Q_{12}. It can be proved that if $c^A/(p_A^A-c^B)+c^B/(p_A^B-c^A)<1$,

$$\frac{p_A^B-c^B}{p_A^A+p_A^B-c^A-c^B} \geq \frac{c^A}{p_A^A-c^B} \quad (82)$$

$$\frac{p_A^A-c^A}{p_A^A+p_A^B-c^A-c^B} \geq \frac{c^B}{p_A^B-c^A} \quad (83)$$

Let $y'=c^A/(p_A^A-c^B)$ and $z'=c^B/(p_A^B-c^A)$. Then,

$$\frac{p_A^B-c^B}{p_A^A+p_A^B-c^A-c^B} = y'+\triangle y, \quad \triangle y \geq 0 \quad (84)$$

$$\frac{p_A^A-c^A}{p_A^A+p_A^B-c^A-c^B} = z'+\triangle z, \quad \triangle z \geq 0 \quad (85)$$

Hence,

$$\begin{aligned} E_A(Q_{12}) &= -[v_A^A(y'+\triangle y)+v_A^B(z'+\triangle z)] \\ &\quad + [(p_A^A-c^B)(y'+\triangle y)-c^A]r \\ &\quad + [(p_A^B-c^A)(z'+\triangle z)-c^B]s \\ &= -(v_A^Ay'+v_A^Bz')-[v_A^A-(p_A^A-c^B)r]\triangle y \\ &\quad -[v_A^B-(p_A^B-c^A)s]\triangle z \end{aligned} \quad (86)$$

$$\begin{aligned} E_A(Q_{13}) &= -(v_A^Ay^*+v_A^Bz^*)+[(p_A^A-c^B)y^*-c^A]r^* \\ &\quad +[(p_A^B-c^A)z^*-c^B]s^* \\ &= -(v_A^Ay'+v_A^Bz') \end{aligned} \quad (87)$$

Recall that $y^*+z^*<1$ and $r^*+s^*<1$ at Q_{13}.

(2) The Desirable Equilibria in Each Area

In each area, we now compare the expected utilities of the agency for all possible equilibria.

It can be shown that $E_A(Q_4) \geq E_A(Q_2)$ iff (if and only if)

$$v_A^B+c^A+c^B-p_A^A-p_A^B \geq \frac{c^A+c^B-p_A^A}{p_A^B}v_A^B. \quad (88)$$

which is true iff $v_A^B \geq p_A^B$. This can be assumed to be true, because otherwise the agency would prefer not to stop violation involving pollutant B. Hence, $E_A(Q_4) \geq E_A(Q_2)$. Similarly, it is easy to prove that

$$\text{if } v_F^A > v_F^B, \quad E_A(Q_3) \geq E_A(Q_2), \quad (89)$$

$$\text{if } v_F^B > v_F^A, \quad E_A(Q_6) \geq E_A(Q_2), \quad (90)$$

$$\text{if } v_F^B > v_F^A, \quad E_A(Q_7) \geq E_A(Q_2). \quad (91)$$

1. In area (A), the total cost for testing the pollutants A and B is greater than the benefits for stopping pollution. The only equilibrium is Q_1.
2. In area (B), there are three equilibria, Q_2, Q_5, and Q_7. When parameters $p_A^A, p_A^B, c^A, c^B, v_A^A, v_A^B, v_F^A$ and v_F^B are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_2), E_A(Q_5)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_7), E_A(Q_5)\}$ if $v_F^B > v_F^A$.
3. In area (C), there are four equilibria, Q_2, Q_4, Q_7, and Q_{11}. When the parameters are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_{11}), E_A(Q_4)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_{11}), E_A(Q_7)\}$ if $v_F^B > v_F^A$.
4. In area (D), there are three equilibria, Q_2, Q_4, and Q_8. When the parameters are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_4), E_A(Q_8)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_2), E_A(Q_8)\}$ if $v_F^B > v_F^A$.
5. In area (E), there are four equilibria, Q_2, Q_5, Q_6, and Q_{10}. When the parameters are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_5), E_A(Q_2)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_5), E_A(Q_6), E_A(Q_{10})\}$ if $v_F^B > v_F^A$.
6. In area (F), there are five equilibria, Q_2, Q_4, Q_6, Q_{10}, and Q_{11}. When the parameters are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_{11}), E_A(Q_4)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_{11}), E_A(Q_6), E_A(Q_{10})\}$ if $v_F^B > v_F^A$.
7. In area (G), there are four equilibria, Q_2, Q_3, Q_8, and Q_9. When the parameters are fixed, the desirable equilibrium corresponds to Max $\{E_A(Q_8), E_A(Q_3), E_A(Q_9)\}$ if $v_F^A > v_F^B$, and to Max $\{E_A(Q_8), E_A(Q_2)\}$ if $v_F^B > v_F^A$.
8. In area (H), there are five equilibria, Q_2, Q_3, Q_7, Q_9,

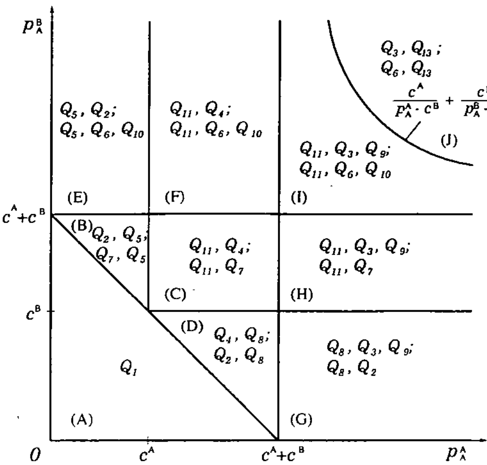

Fig. 7 The possible desirable equilibria in different regions

and Q_{11}. When the parameters are fixed, the desirable equilibrium corresponds to $Max\ \{E_A(Q_{11}), E_A(Q_3), E_A(Q_9)\}$ if $v_F^A > v_F^B$, and to $Max\ \{E_A(Q_{11}), E_A(Q_7)\}$ if $v_F^B > v_F^A$.

9. In area (I), there are six equilibria, Q_2, Q_3, Q_6, Q_9, Q_{10}, and Q_{11}. When the parameters are fixed, the desirable equilibrium corresponds to $Max\ \{E_A(Q_{11}), E_A(Q_3), E_A(Q_9)\}$ if $v_F^A > v_F^B$, and to $Max\ \{E_A(Q_{11}), E_A(Q_6), E_A(Q_{10})\}$ if $v_F^B > v_F^A$.
10. In area (J), there are five equilibria, Q_2, Q_3, Q_6, Q_{12}, and Q_{13}. The agency's expected utilities at the latter two are

$$E_A(Q_{13}) = -(v_A^A y' + v_A^B z') \tag{92}$$

$$E_A(Q_{12}) = -(v_A^A y' + v_A^B z') - [v_A^A - (p_A^A - c^B)r]\triangle y - [v_A^B - (p_A^B - c^A)s]\triangle z. \tag{93}$$

Note that $v_A^A \geq p_A^A$ and $v_A^B \geq p_A^B$, leading to $[v_A^A - (p_A^A - c^B) r] > 0$ and $[v_A^B - (p_A^B - c^B)s] > 0$. Also $\triangle y \geq 0$, $\triangle z \geq 0$. Hence, $E_A(Q_{13}) \geq E_A(Q_{12})$. Thus it can be shown that when the parameters are fixed, the desirable equilibrium corresponds to $Max\ \{E_A(Q_3), E_A(Q_{13})\}$ if $v_F^A > v_F^B$, and to $Max\ \{E_A(Q_6), E_A(Q_{13})\}$ if $v_F^B > v_F^A$.

All possible desirable equilibria are shown in **Fig. 7**.

6. DISCUSSION

From the analyses above, it is clear that when parameters p_A^A, p_A^B, c^A, c^B, v_A^A, v_A^B, v_F^A and v_F^B have been determined, a unique desirable equilibrium can be found. Then in order to achieve the desirable equilibrium, the agency should choose suitable values for d_F^A and d_F^B to meet the existence conditions for this equilibrium. For example, if in area (J), the desirable equilibrium has been determined to be equilibrium Q_{13}, then the agency should set the values of d_F^A and d_F^B so that $v_F^A/d_F^A < 1$, $v_F^B/d_F^B < 1$, and $v_F^A/d_F^A + v_F^B/d_F^B < 1$ in order to achieve this equilibrium.

From **Fig. 7**, it can be seen that an equilibrium can exist in different areas. For instance, equilibrium Q_4 appears in areas (C), (D) and (F), and equilibrium Q_3 in areas (G), (H), (I) and (J). When p_A^A and p_A^B are fixed, equilibrium Q_4 may occur in area (C), (D), or (F), as determined by the values of c^A and c^B. A larger value of $c^A + c^B$ leads to $p_A^B < c^A + c^B$, making equilibrium Q_4 appear in areas (C) or (D) only. A smaller value of $c^A + c^B$ results in $p_A^B > c^A + c^B$, making equilibrium Q_4 exist in area (F) only. Similarly, when $c^A + c^B$ diminishes, the location of equilibrium Q_3 moves from area (G) to (H), and from (H) to (I). Similar statements hold for the other equilibria. Notice that for a specific equilibrium, the agency's expected utility E_A increases when $c^A + c^B$ decreases. This means that the agency's expected utility at an equilibrium in a higher area is greater than in a lower area. For example, $E_A(Q_3)$ in area (J) is greater than $E_A(Q_3)$ in area (I), which is greater than $E_A(Q_3)$ in area (H), which is greater than $E_A(Q_3)$ in area (G).

Combining the above results, another conclusion follows. The agency's expected utility for the desirable equilibrium in a higher area is greater than for the desirable equilibrium in a lower area. That is, the value of E_A for the desirable equilibrium in area (J) is greater than for the desirable equilibrium in area (I), which is greater than for the desirable equilibrium in area (H) or (F), which is greater than for the desirable equilibrium in area (C), which is greater than for the desirable equilibrium in area (A). Hence, reducing the sum of the costs for testing is always important for enforcement against multiple pollutants.

Finally, the conclusions for the two pollutant model can be usefully compared to those for the one pollutant model of section 2, above. In the one pollutant case, the conditions $p_A > c$ and $d_F > v_F$ were found to be most favourable for enforcement effectiveness. When there are two pollutants, A and B, the corresponding conditions are

$$\frac{c^A}{p_A^A - c_B} + \frac{c^B}{p_A^B - c_A} < 1 \tag{94}$$

to produce area (J), and

$$\frac{v_F^A}{d_F^A} + \frac{v_F^A}{d_F^A} < 1 \tag{95}$$

to achieve equilibrium Q_{13}. Thus (94) and (95) generalize the condition that (a) testing be cost–effective, and (b) sanctions be adequate. If these conditions are not met, a lower level of enforcement effectiveness results. For instance, it is possible to deter the release of one of the pollutants, sometimes or always, in areas (C), (F), or (H). One can readily guess the form

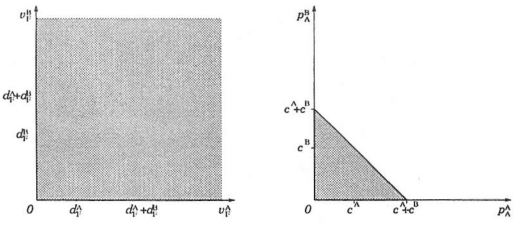

Fig. 8 $Q_1=(1, 0, 0; 0, 0)$

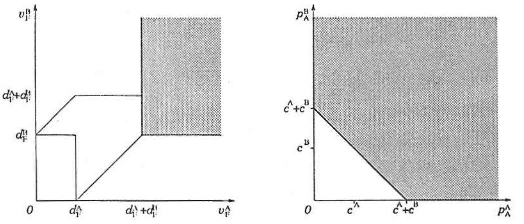

Fig. 9 $Q_{2.a}=(1, 0, 0; 1, 0)$

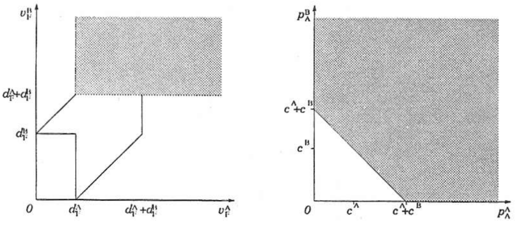

Fig. 10 $Q_{2.b}=(1, 0, 0; 0, 1)$

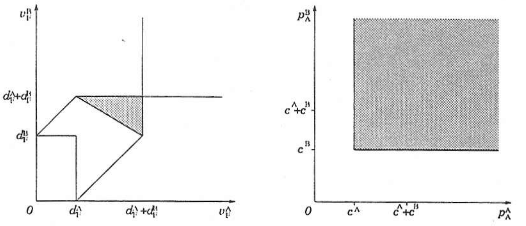

Fig. 11 $Q_{2.c}=(1, 0, 0; r^*, s^*)$

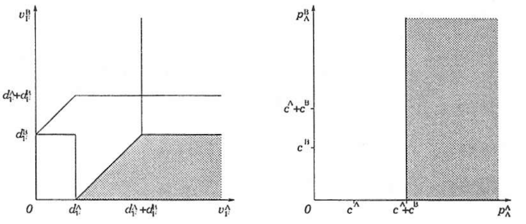

Fig. 12 $Q_3=(0, 1, 0; 1, 0)$

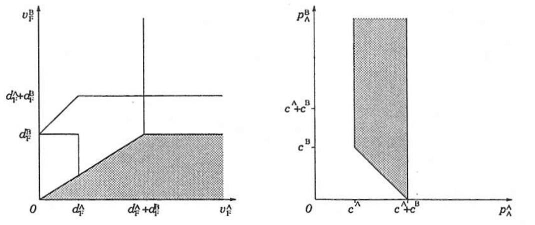

Fig. 13 $Q_4=(x^*, y^*, 0; r^*, 0)$

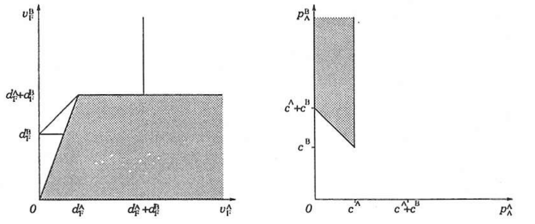

Fig. 14 $Q_5=(x^*, y^*, 0; 0, s^*)$

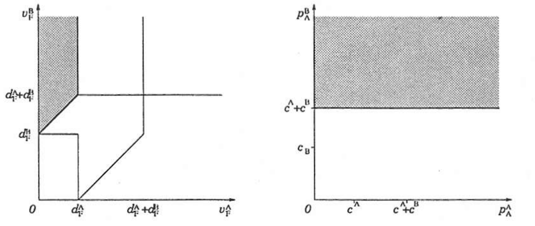

Fig. 15 $Q_6=(0, 0, 1; 0, 1)$

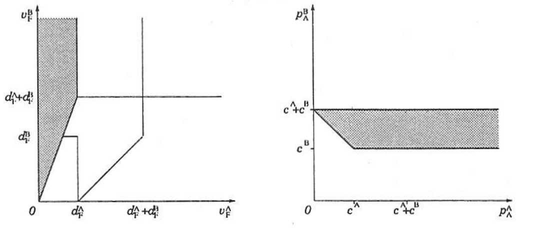

Fig. 16 $Q_7=(x^*, 0, z^*; 0, s^*)$

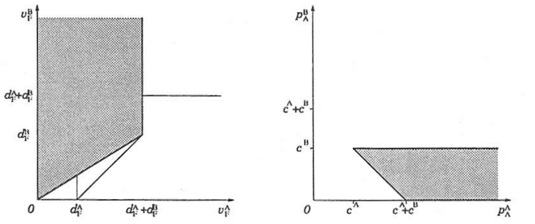

Fig. 17 $Q_8=(x^*, 0, z^*; r^*, 0)$

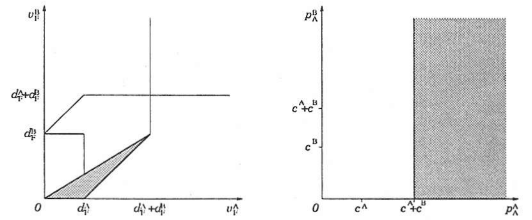

Fig. 18 $Q_9=(0,\ y^*,\ z^*\ ;\ r^*,\ 0)$

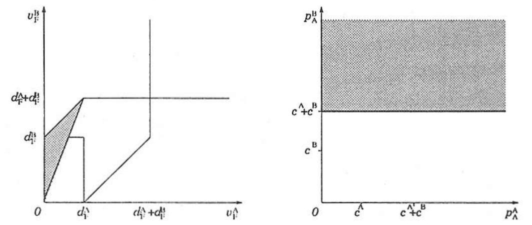

Fig. 19 $Q_{10}=(0,\ y^*,\ z^*\ ;\ 0,\ s^*)$

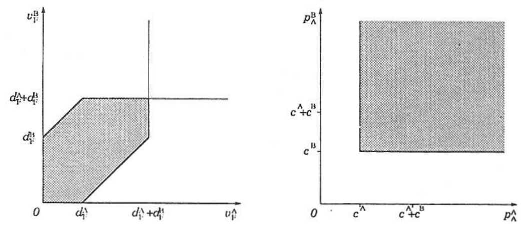

Fig. 20 $Q_{11}=(x^*,\ y^*,\ z^*\ ;\ r^*,\ s^*)$

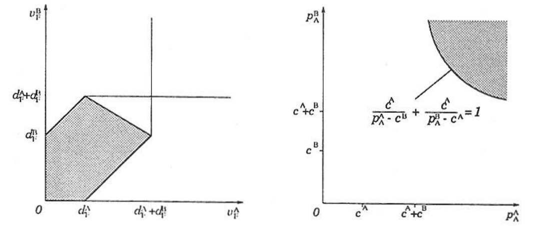

Fig. 21 $Q_{12}=(0,\ y^*,\ z^*\ ;\ r^*,\ s^*,\ )$

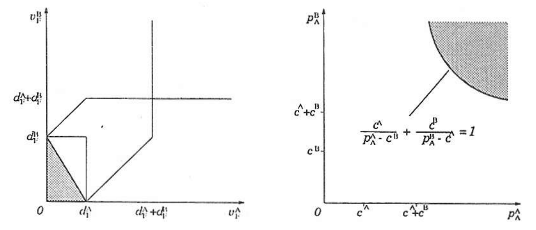

Fig. 22 $Q_{13}=(0,\ y^*,\ z^*\ ;\ r^*,\ s^*)$

of (94) and (95) when there are more than two pollutants, but confirmation must await further research.

7. CONCLUSIONS

The compliance model in **Fig. 3** has been successfully developed in order to determine optimal enforcement policies to deter the discharge of two pollutants by a firm. As shown in the analyses of this model in Sections 3 and 4, all of the Nash equilibria involve at least some level of violation. This is characteristic of one–stage enforcement – for instance, *Hipel et al.* (1995) observe a very similar phenomenom in the analysis of environmental·enforcement using a reporting system.

When the objective is to deter two types of pollution efficiently, the most desirable equilibrium, in terms of maintaining environmental integrity, is equilibrium Q_{13} which is defined in equation (40) in Section 3 and displayed in **Fig. 7**. In the discussion in Section 5, it is shown that the desirable equilibrium can be reached in practice by selecting appropriate penalty levels. More specifically, when the testing costs are low enough compared to the benefits gained for stopping violations for the two pollutants, Q_{13} can occur when penalties are suitably selected. For equilibrium Q_{13}, there is never a violation for both pollutants at the same time, and full compliance can sometimes be reached. Also, the agency need not always test for either pollutant.

In order to improve compliance beyond that given in the desirable equilibrium, one approach may be to consider repeated testing for a firm which has violated in the past. *Fukuyama et al.* (1994) use a repeated game to investigate the problem of how to penalize a firm for violations when the agency interacts with the firm over a period of time. However, they do not consider multiple pollutants in their research. Future research could focus upon the use of repeated games and associated review and punishment terms in the presence of multiple pollutants in order to discover desirable equilibria, which may include full compliance. Additionally, one could develop compliance models for more than two pollutants.

The literature contains other uses of extensive games to study optimal enforcement policies systematically, but not in the case of multiple pollutants. In particular, *Kilgour et al.* (1992) investigate the effectiveness of enforcement when an agency can issue a control order for dealing with a suspected violation by a firm and the firm, in turn, can appeal the control order. *Hipel et al.* (1994) and *Yin et al.* (1994) study how the efficacy of a reporting system such as "whistle–blowing" for improving environmental compliance depends on the cost of reporting system. *Fang et al.* (1997) discuss the consequences for enforcement of varying the level of violation penalty (corresponding to p_F^A and p_F^B in the models above).

The compliance models constructed here, as well as in the references cited above, are useful for improving and developing effective enforcement policies. In practice, when a violation actually takes place, bargaining may occur among an agency and a firm, and perhaps other interested parties, in order to achieve a suitable resolution. The Graph Model for Conflict Resolution (*Fang et al.*, 1993) can be employed to study this bargaining process systematically. For example, a decision support system (*Sage*, 1991) for the Graph Model for Conflict Resolution, GMCR, was used to model and analyse negotiations that took place when Uniroyal Chemicals Ltd. appealed a control order regarding the discharge of a carcinogen that had polluted an underground aquifer in Elmira, Ontario, Canada (*Hipel et al.*, 1993 ; *Kilgour et al.*, 1994). The decision support system GMCR is included on diskette with the book of *Fang et al.* (1993). *Hipel and McLeod* (1994, Ch. 1) discuss how conflict resolution and other techniques from operations research and statistics can be used for enhancing environmental decision making.

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