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DYNAMIC VEHICLE DISPATCHING IN A TRANSPORTATION SYSTEM

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ABSTRACT

This study is an operational research approach towards the effective design of advanced ground transportation systems. To this effect, a stochastic model was constructed over a network with M stations connected arbitrarily by routes and the relationships between parameters were simulated by this model.

The model is based on two important postulates. 1) the assumption of complete randomness of passenger demand (homogeneous or non-homogeneous Poisson process) 2) the adoption of a dynamic vehicle dispatching policy. "Dynamic" is a term used here in contrast to the term "fixed". Fixed dispatching policy is a policy which schedules vehicles according to predetermined time-table. Dynamic dispatching policy is a strategy to dispatch vehicles without any predetermined time-table, dynamically responding to the continuous change of passenger demand.

Although there are many conceivable kinds of dynamic dispatching policies, two of them, gc-when-fill policy and go-when-fill-with-time-constraint policy were studied.

First, the dynamic dispatching policy which plays a central role in this study was examined, and the probability density function of vehicle departure interval, average passenger waiting time, and average vehicle loading factor were derived. Next, the stochastic behaviours of vehicle flows in the network such as distribution of vehicles as the function of time in the network were examined for the vehicles flows dispatched by the dynamic dispatching policy. Lastly, it was studied how to control the distribution of vehicles over the network to realize a smooth operation of the systems.

CHAPTER I. INTRODUCTION

This study is an operational research to-wards the effective design of advanced ground transportation systems. To this effect, a stochastic model was constructed and the relationships between parameters were simulated by this model.

The model is based on two important postulates. 1) the assumption of complete randomness of passenger demand (Homogeneous or non-homogeneous poisson process) 2) the adoption of a dynamic vehicle dispatching policy. In the following paragraphs, the author attempts to justify these two premises.

Although the real transportation demand might be between the extremes of completely random and completely deterministic, this study is forcussed on the case of completely random passenger demand and tries to give some insight into the interior structure of a transportation system which involves probabilistic passenger demand. Furthermore, it should be mentioned here that the exposed passenger demand is a result of potential passenger demand distorted by the existing vehicle dispatching policy. When a trip is motivated in a customer's mind (call this a potential passenger demand), this original motivation to trip is in most cases distorted by the existing vehicle dispatching scheduling (e.g. time-table). The dynamic dispatching policy treated here will less distort the potential passenger demand.

It should be recalled that the customer's demand has been proved to be very similar to a Poisson process in the case of telephone calls in an exchange center, where potential customer's demands are exposed almost as they are.

Dynamic dispatching policy is a policy which dispatches vehicles dynamically responding to passenger demand. "Dynamic" is a term used here in contrast to the term "fixed". Fixed dispatching policy is a policy which schedules vehicles according to predetermined time-table. Dynamic dispatching policy is a strategy to dispatch vehicles without any predetermined time-table, dynamically responding to the continuous change of passenger demand, analogous to the dispatching of taxis or airport limousine.

Guided ground transpotation systems have traditionally used fixed dispatching policies and with substantial success. However, dynamic dispatching

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policy has been used very little and the efficiency of this policy is not so much known. This is one reason why the author is so interested in how the transportation network systems might respond to the dynamic dispatching policy. Furthermore, the introduction of this policy gives the ground transportation systems the attraction of flexible service that the customer can go just when he wishes to go. The dynamic dispatching policy is, so to speak, very much oriented towards the customer's trip motivation.

Although there are many conceivable kinds of dynamic dispatching policies, two of them, go-when-fill policy and go-when-fill-with-time-constraint policy were studied. By the former we mean a policy to dispatch a vehicle as soon as the seats of a vehicle are filled by customers. By the latter we mean a policy to dispatch a vehicle when the seats of the vehicles are filled or when the first arrival to the vehicle waits X minutes, waiting time limit.

In the chapter II, the stochastic model is described in detail. In the chapter III, the dynamic dispatching policy which plays a central role in this study was studied, and the probability density function of vehicle departure interval, average passenger waiting time, and average vehicle loading factor were derived. Next in chapter IV, the stochastic behaviours of vehicle flows in the network such as distribution of vehicles as the function of time in the network were examined for the vehiche flows dispatched by the dynamic dispatching policy. In chapter V, it was studied how to control the distribution of vehicles over the network to realize a smooth operation of the systems.

Although this study should go to the stage of the optimization problem, the optimization problem was left as a future research topic and here in this paper, it was mainly tried to show the possible approach to the effective design of the transportation systems by theoretical analysis and by showing the calculation method required to obtain the relationships between parameters involved.

CHAPTER II. MODEL DESCRIPTION

In this chapter, the stochastic model treated in this study will be described in detail.

Network

We assume a transportation system with M stations and routes connecting them arbitrarily. (Fig. 2.1)

Passenger Demand Pattern

It is assumed that at an arbitrary station, say, i-th station, $(M-1)$ queuing channels are formed associated with $(M-1)$ destinations, in the system. Passenger arrivals to each channel occur independ-

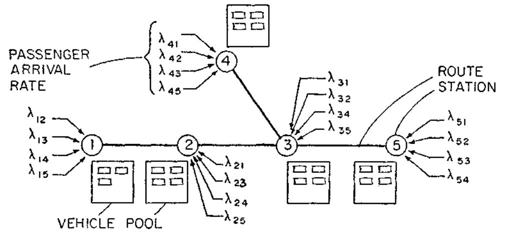

Fig. 2.1 An Example of Network & Passenger Demand.

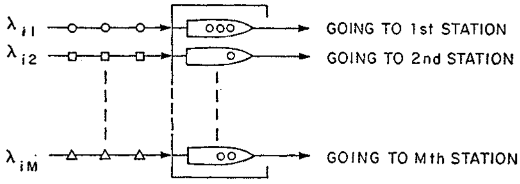

Fig. 2.2 Passenger Arrival and Vehicle Dispatching at i-th Station.

ently with Poisson distribution law of parameter $\lambda_{ij}(t)$, i and j respectively referring to the station concerned and destination. (Fig. 2.1) For the whole system, one has a demand rate matrix Λ as follows:

$$\Lambda=\|\lambda_{ij}\|=\text{From}\ \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ M \end{array}\!\!\begin{array}{c} \qquad T_0 \\ \begin{array}{ccc} 1 & 2\cdots\cdots & M \end{array} \\ \begin{bmatrix} 0 & \lambda_{12}\cdots\cdots & \lambda_{1M} \\ \lambda_{21} & 0\cdots\cdots & \lambda_{2M} \\ \vdots & \vdots & \vdots \\ \lambda_{M1} & \lambda_{M2}\cdots\cdots & 0 \end{bmatrix} \end{array}$$

For the almost whole part of this study, the case of homogeneous Poisson is discussed. However, in the study of vehicle dispatching policy, the time dependent demand rate case, *i.e.*, non-homogeneous Poisson case will also be discussed in the appendix.

Vehicle Operating Policy (Fig. 2.2)

Consider an arbitrary station, say, i-th station. We assume that vehicles are dispatched independently in each channel according to the dynamic dispatching policy and travellers are sent directly to their destination, through a pre-determined route, without stopping in the way.

The vehicles which have arrived at the destination become empty there, enter into a vehicle pool of that station and wait for the future demand there.

Vehicle Pool (Fig. 2.3)

It is assumed that each station has its own vehicle pool. Vehicles demanded by arriving passengers at that station are supplied from this pool. Unloaded vehicles which have just finished their service enter into this pool and wait for the future demand.

Vehicle Dispatching Policy

For this model, the dynamic dispatching policy is used which was described in Chapter I. Among many conceivable kinds of dynamic dispatching policies, here are studied two kinds of them : go-

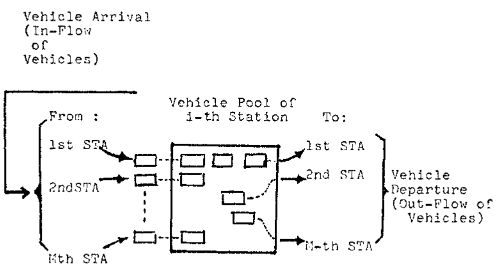

Fig. 2.3 Vehicle Pool, In-Flow and Out-Flow of Vehicles.

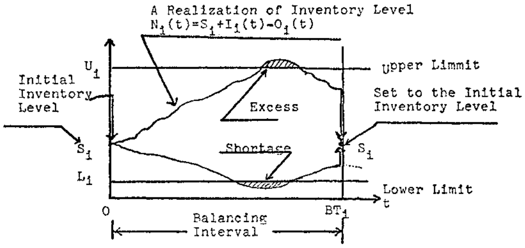

Fig. 2.4 Fluctuation of Vehicle Inventory Level at Vehicle Pool of Each Station.

when-fill policy and go-when-fill-with-time-constraint policy.

Go-When-Fill Policy

Suppose C seats for each vehicle. As soon as passengers going to the same destination make a group of size C, a vehicle is dispatched to that destination, *i.e.*, a vehicle goes when its seats are filled.

Go-When-Fill-With-Time-Contraint Policy

In the case of go-when-fill policy, the first arrival of a group must wait a long time if the last *i.e.* the C-th arrival comes very late. To avoid such an un-favorable situation, a vehicle is dispatched when the first arrival of a group waits X minutes, waiting time limit, even when C seats are not filled. Of course, if C passengers going to the same destination arrive before the first arrival passenger waits X minutes, a vehicle is dispatched according to the go-when-fill policy.

Vehicle Fleet Size and Vehicle Capacity

In this model it is assumed that all vehicles have the same number of seats, C. One of the objectives of the study is to control the value of C, optimizing the whole system operation.

As for the fleet size of vehicles, it is closely related with the proposed inventory control policy described below. The inventory level of vehicles at each pool is controled so that vehicles are supplied almost everytime demands occur.

Vehicle Inventory Control

Observe an arbitrary station, say, i-th station. We can observe here the in-flow and the out-flow of vehicles. The vehicles carrying passengers from other stations to the i-th station constitute the in-flow of vehicles into the vehicle poll of the i-th station and the vehicles leaving the i-th station for other stations constitute the outflow of vehicles from the pool of i-th station. Refer to Fig. 2.3. As the result of in-flow and out-flow of vehicles, the inventory level of vehicles at each station pool fluctuates stochastically (Fig. 2.4). As the fleet size of vehicles and the size of vehicle pools in the system are limited, a suitable control must be made over the vehicle inventory level and the probability of the event that the number of vehicles at each pool becomes in excess or in shortage must be kept under a specified small value.

The proposed inventory control policy is as follows : choose an initial inventory S_i, inventory upper limit U_i, lower limit L_i, vehicle balancing interval BT_i and inventory control confidence level α for each station, the subscript i referring to the i-th station. At each beginning of interval BT_i, the inventory level is set to S_i. During the interval BT_i, the inventory level fluctuates as a result of superposition of in-flow and out-flow of vehicles. The maximum value is chosen for BT_i, BT_i satisfying the following relations :

$$\max_{0\le t\le BT_i} P[N_i(t)=S_i+I_i(t)-O_i(t)>U_i] + P[N_i(t)=S_i+I_i(t)-O_i(t)<L_i]\le\alpha$$

where $I_i(t)$ = number of vehicles of in-flow in time interval t,

$O_i(t)$ = number of vehicles of out-flow in time interval t,

$N_i(t)$ = number of vehicles in a pool at time point t.

In this way, for any realization of stochastic process of vehicle inventory level, the unfavorable situation for the vehicle inventory that vehicles are in excess or in shortage is made almost impossible (Fig. 2.4).

Optimization

In this model, control variables are C, capacity of a vehicle; X, passenger waiting time limit; U_i, S_i, L_i inventory control variables. Optimum values being chosen for these variables, the value of a ccst function constructed suitably from various considerations on the whole system could be minimized.

CHAPTER III. STUDY ON THE DYNAMIC VEHICLE DISPATCHING POLICY

As described in Chap. II, proposed model assumes that for each channel of $(M-1)$ queuing channels of each station, vehicles are dispatched independently. Thus it is suffice to investigate the vehicle departure behavior for one arbitrary channel. After the behavior of vehicle departure of one channel is studied, we could deduce the other results by superposing the independent processes. Thus in this chapter the discussion is forcussed on the case where passengers arrive at a station by the Poisson distribution law of parameter λ, going to the same destination.

In the following sections, first, the go-when-fill-with-time-constraint policy will be examined and next, go-when-fill policy will be discussed. In this chapter it is assumed that a vehicle can be supplied without delay as soon as a demand occurs. For each policy the vehicle departure interval distribution, average vehicle departure interval, average passenger waiting time and average loading factor would be examined. Finally, it is studied how these quantities are affected by the values of capacity of vehicle C, waiting time limit X, and passenger arrival rate λ, with the aid of computer calculations.

(1) Go-When-Fill-With-Time-Constraint Policy

a) Vehicle Departure Interval Distribution Function

Vehicles will be dispatched in two ways.

(a) Vehicles are dispatched with fill. (Fig. 3.1.1)

(b) Vehicles are dispatched without fill. (Fig. 3.1.2)

Let's define the following notations : (Refer to Fig. 3.1.1, 3.1.2)

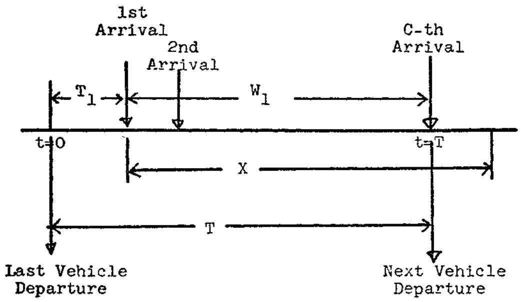

Fig. 3.1.1 Vehicle Departure with Fill.

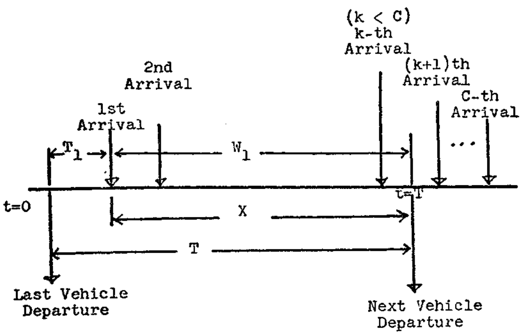

Fig. 3.1.2 Vehicle Departure without Fill.

T_1=time from the last departure ($t=0$) to the next first passenger arrival.

W_1=time from the first passenger arrival to the next vehicle departure epoch.

T=vehicle departure interval$=T_1+W_1$

λ=passenger arrival rate.

C=number of seats of a vehicle.

X=waiting time limit for the first arrival passenger.

First $f_{W_1}(t)$ and $F_{W_1}(t)$ can be obtained as follows.

Noting that for the case $0<t<X$ $(C\geq 2)$

$$F_{W_1}(t)=P[W_1\leq t]$$
$$=P(C-1 \text{ or more passengers arrive within } t)$$

and for the case $t=X$, $(C\geq 2)$

$$f_{W_1}(X)=P(W_1=X)=P(C-2 \text{ or less passengers arrive within } X)$$

$$f_{W_1}(t)=\begin{bmatrix} \dfrac{\lambda(\lambda t)^{C-2}e^{-\lambda t}}{(C-2)!} & 0<t<X & C\geq 2 \\ \displaystyle\sum_{k=0}^{C-2}\frac{e^{-\lambda X}(\lambda X)^k}{k!} & t=X & C\geq 2 \\ 0 & \text{elsewhere} & C\geq 2 \end{bmatrix}$$

$$F_{W_1}(t)=\begin{bmatrix} 1-\displaystyle\sum_{k=0}^{C-2}\frac{e^{-\lambda t}(\lambda t)^k}{k!} & 0<t<X & C\geq 2 \\ 1 & t\geq X & C\geq 2 \end{bmatrix} \qquad \cdots\cdots(\text{III-1})$$

Next $f_{T_1}(t)$ and $F_{T_1}(t)$ can be obtained as follows :

$$\left.\begin{array}{ll} f_{T_1}(t)=\lambda e^{-\lambda t} & 0<t<\infty \\ F_{T_1}(t)=1-e^{-\lambda t} & 0<t<\infty \end{array}\right\}\cdots\cdots(\text{III-1})'$$

Noting that $T=W_1+T_1$, and that T_1 and W_1 are mutually independent, we can obtain $f_T(t)$, $F_T(t)$, convoluting $f_{T_1}(t)$, $f_{W_1}(t)$ and $F_{T_1}(t)$, $F_{W_1}(t)$. Note that $f_{W_1}(t)$ has a spike at X for the case $t\geq X$.

$$f_T(t)=\begin{bmatrix} \dfrac{\lambda(\lambda t)^{C-1}e^{-\lambda t}}{(C-1)!} & 0<t<X & C\geq 2 \\ \lambda\displaystyle\sum_{k=0}^{C-1}\frac{(\lambda X)^k e^{-\lambda t}}{k!} & X\leq t & C\geq 2 \\ \lambda e^{-\lambda t} & 0<t<\infty & C=1 \end{bmatrix}$$

$$F_T(t)=\begin{bmatrix} \displaystyle\sum_{k=C}^{\infty}\frac{(\lambda t)^k e^{-\lambda t}}{k!} & 0<t<X & C\geq 2 \\ 1-\displaystyle\sum_{k=0}^{C-1}\frac{e^{-\lambda t}(\lambda X)^k}{k!} & X\leq t & C\geq 2 \\ 1-e^{-\lambda t} & t>0 & C=1 \end{bmatrix} \qquad \cdots\cdots(\text{III-2})$$

b) Expected Value and Variance of Departure Interval

Let $E(T)$=expected value of T

$V(T)$=variance of T

As $T=W_1+T_1$, and T_1 and W_1 are mutually independent,

$$E(T)=E(T_1)+E(W_1)$$
$$V(T)=V(T_1)+V(W_1)$$

From (III-1) and (III-1)',

$$\left.\begin{aligned} E(T) &= \frac{C}{\lambda} - \frac{C-1}{\lambda} P(\lambda X, C-1)^* \\ &\quad + X \cdot P(\lambda X, C-2) \\ V(T) &= \frac{1}{\lambda^2} + \frac{C(C-1)}{\lambda^2}\{1-P(\lambda X, C)\} \\ &\quad + X^2 \cdot P(\lambda X, C-2) - \left\{ \frac{C-1}{\lambda}[1-P \right. \\ &\quad \left. \cdot(\lambda X, C-1)] + X \cdot P(\lambda X, C-1) \right\}^2 \end{aligned}\right\} \quad \cdots\cdots(\text{III-3})$$

c) Average Loading Factor

Let's define the average loading factor η:

$$\eta = \frac{E(B)}{C}$$

where B=random variable of number of passengers in a vehicle

$E(B)$=expected value of B.

For the case $1 \leq k \leq C-1$,

Event $[B=k] \rightleftarrows$ Event $[(k-1)$ arrivals in time X after the first arrival$]$

and for the case $k=C$,

Event $[B=C] \rightleftarrows$ Event $[(C-1)$ or more arrivals in time X after the first arrival$]$

Then $P[B=k] = \dfrac{e^{-\lambda X}(\lambda X)^{(k-1)}}{(k-1)!} \quad 1 \leq k \leq C-1$

$$P[B=C] = \sum_{k=C-1}^{\infty} \frac{e^{-\lambda X}(\lambda X)^k}{k!}$$

Thus,

$$E(B) = \begin{bmatrix} \lambda X \cdot P(\lambda X, C-3) + (1-C) & \\ \cdot P(\lambda X, C-2) + C & C \geq 3 \\ (1-C)e^{-\lambda X} + C & C=2 \\ 1 & C=1 \end{bmatrix} \quad \cdots\cdots(\text{III-4})$$

Therefore,

$$\eta = \frac{E(B)}{C} = \begin{bmatrix} \frac{\lambda X}{C} P(\lambda X, C-3) + \frac{1-C}{C} & \\ \cdot P(\lambda X, C-2) + 1 & C \geq 3 \\ (2-e^{-\lambda X})/2 = 1 - \frac{1}{2} e^{-\lambda X} & C=2 \\ 1 & C=1 \end{bmatrix} \quad \cdots\cdots(\text{III-5})$$

d) Passenger Waiting Time

Let's define random variables MW and MW_k.

$$MW_k = \frac{1}{k}(W_1 + W_2 + \cdots + W_k) \quad 1 \leq k \leq C,$$

where k=Number of passengers in an arbitrary vehicle dispatched.

W_i=waiting time of i-th passenger.

MW_k=waiting time per passenger of an arbitrary vehicle, given that the vehicle contains k passengers.

MW=unconditional waiting time per passenger of an arbitrary vehicle.

(a) The Case Vehicle Departs Without Fill

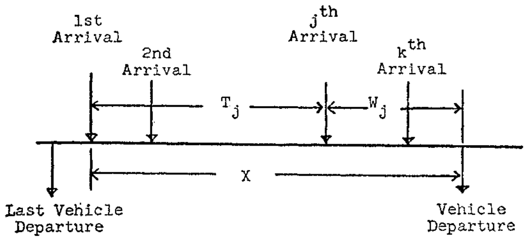

Fig. 3.1.3 Vehicle Departure without Fill.

Let k passengers be in the vehicle. $(k<C)$

Referring to the Fig. 3.1.3, define

T_j=the time between the first arrival and j^{th} arrival.

W_j=the waiting time of j^{th} passenger $=X-T_j$.

In this case, we note that $(k-1)$ events of Poisson type with parameter λ have occured in time X. The j^{th} passenger, in this case, becomes the $(j-1)^{\text{th}}$ event.

Let $E(W_{j/k})$=expected value of W_j, given that the vehicle departs with k passengers

P_k=probability that a vehicle departs with k passengers.

P_k can be calculated as follows:

$$P_k = \frac{e^{-\lambda X}(\lambda X)^{k-1}}{(k-1)!} \quad (1 \leq k < C) \cdots\cdots(\text{III-6})$$

Noting that if the k events of Poisson type have occured in time T, the k times $T_1 < T_2 < \cdots T_k$ in the interval O to T at which events occur are random variables having the same distributions as if they are the order statistics corresponding to k independent variable $U_1, U_2, \cdots, U_k$ uniformly distributed on the interval O to T (Parzen: Stochastic Process, pp. 139~144),

we can derive,

$$\left.\begin{aligned} E(W_{j/k}) = X \cdot \left(1 - \frac{j-1}{k}\right) \quad & (1 \leq j \leq k) \\ & (1 \leq k \leq C-1) \end{aligned}\right\} \cdots(\text{III-7})$$

However, $MW_k = \dfrac{1}{k}(W_1 + W_2 + \cdots + W_k)$

$$\begin{aligned} \text{Then, } E(MW_k) &= \frac{1}{k}\sum_{j=1}^{k} E(W_{j/k}) \\ &= \frac{1}{k}\sum_{j=1}^{k}\left(1 - \frac{j-1}{k}\right)X \\ &= \frac{k+1}{2k} X \quad (1 \leq k \leq C-1) \cdots(\text{III-8}) \end{aligned}$$

(b) The Case The Vehicle Departs With Fill

This case is decomposed into the following events:

Event $E_k = (k-1)$ arrivals within X after the first arrival.

$k = C, C+1, C+2, \cdots, \infty$

In the event E_k, $(k-1)$ events of Poisson type happened in time X, where $k=C, C+1, C+2, \cdots$.

Using the same notations as in (a),

define $E(T_{j/k})$=expected value of T_j, given that Event E_k happened.

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* $P(z, k) = \sum_{j=0}^{k} \dfrac{e^{-z}(z)^i}{i!}$

$E(W_{j/k})$ = expected value of W_j, given that Event E_k happened. $(W_j = T_c - T_j)$

Then, noting the relationships between k events of Poisson type and order statistics as before, we get

$$E(T_{j/k}) = \frac{j-1}{k} X$$

$$E(T_{C/k}) = \frac{C-1}{k} X$$

$$\therefore \quad E(W_{j/k}) = E(T_C - T_{j/k}) = E(T_{C/k}) - E(T_{j/k}) = \left(\frac{C-j}{k}\right) X \quad (k \geq C,\ 1 \leq j \leq C) \quad \cdots\cdots (\text{III-9})$$

As for P_k = probability that Event E_k happens,

$$P_k = \frac{e^{-\lambda X}(\lambda X)^{k-1}}{(k-1)!} \quad \cdots\cdots (\text{III-10})$$

Define $MWC_k = \frac{1}{C}(W_1 + \cdots + W_C)$

MWC_k is the waiting time per passenger in a vehicle dispatched with fill, given that Event E_k happens.

$$E(MWC_k) = \frac{1}{C}\sum_{j=1}^{C} E(W_{j/k}) = \frac{1}{C}\sum_{j=1}^{C}\left(\frac{C-j}{k}\right) X = \frac{C-1}{2k} X \quad (k \geq C) \cdots\cdots (\text{III-11})$$

Thus, from (III-6), (III-8), (III-10), (III-11),

$$E(MW) = \sum_{k=1}^{C-1} E(MW_k) \cdot P_k + \sum_{k=C}^{\infty} E(MWC_k) \cdot P_k$$

$$\therefore \quad E(MW) \begin{cases} = \dfrac{X}{2} P(\lambda X,\ C-2) + \dfrac{2-C}{2\lambda} \cdot P(\lambda X,\ C-1) + \dfrac{C-1-e^{-\lambda X}}{2\lambda} & (C \geq 2) \\ = 0 & (C = 1) \end{cases} \quad \cdots\cdots (\text{III-12})$$

In closing this section, the author wishes to mention that the go-when-fill-with-time constraint policy presents a new renewal model. It can be rephrased, for example, as follows : replace the material if it received C blows or if X time units elapsed after the first blow. This kind of renewal model has not been studied so far and may find many applications in the future.

(2) Go-When-Fill-Policy

This policy is equivalent to the go-when-fill-with-time-constraint policy with time constraint $X = \infty$.

The notations are the same as those in the section 1.

a) Vehicle Departure Interval Distribution Function

$$\left.\begin{aligned} f_T(t) &= \frac{\lambda(\lambda t)^{C-1} e^{-\lambda t}}{(C-1)!} \quad 0 \leq t \leq \infty \\ F_T(t) &= \sum_{k=C}^{\infty} \frac{e^{-\lambda t}(\lambda t)^k}{k!} \quad C \geq 1 \end{aligned}\right\} \cdots\cdots (\text{III-13})$$

b) Expected Value and Variance of T

$$\left.\begin{aligned} E(T) &= C/\lambda \\ V(T) &= C/\lambda^2 \end{aligned}\right\} \cdots\cdots (\text{III-14})$$

c) Passenger Waiting Time

Letting $X \to \infty$ in (III-12), we get

$$E(MW) = \frac{C-1}{2\lambda} \quad \cdots\cdots (\text{III-15})$$

(3) Numerical Calculations and Discussion

For the rather small range of $\lambda = 0.5 \sim 5.0$ and for the rather small range of $C = 1 \sim 30$, the relations between $E(T)$, $E(W)$, η and C, X are given in Fig. 3.3.1 to Fig. 3.3.6.

It should be mentioned that for each graph, the case $X = \infty$ corresponds to the go-when-fill policy.

As shown in Fig. 3.3.1 to 3.3.3, for high intensity of passenger arrival ($\lambda = 5.0$), the mean waiting time per passenger is not so much sensitive to the value of X, but linearly proportionally increases with the value of C. However for the low intensity of passenger arrival ($\lambda = 0.5$) the mean waiting time is more sensitive to X than to C. Figures 3.3.4 to 3.3.6 show that for high passeger demand ($\lambda = 5.0$), the average loading factor is insensitive to both the value of X and that of C. However, for the low passenger demand case ($\lambda = 0.5$), η is very sensitive to both of X and C. We can observe the relationships between C, X and $E(T)$, similar to those between C, X and $E(\overline{W})$.

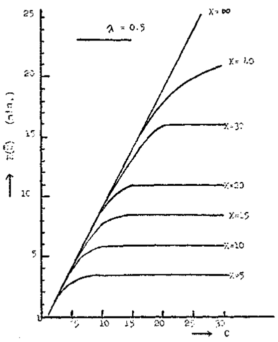

Fig. 3.3.1 $E(\overline{W})$ vs. C.

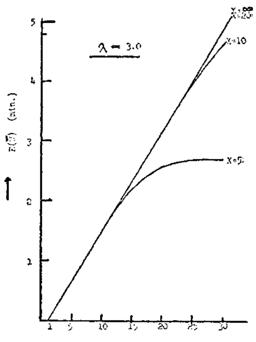

Fig. 3.3.2 $E(\overline{W})$ vs. C.

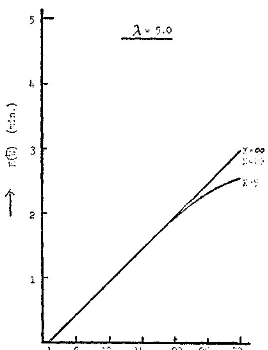

Fig. 3.3.3 $E(\overline{W})$ vs. C.

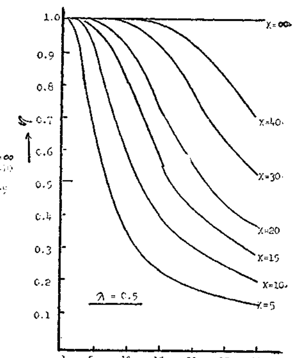

Fig. 3.3.4 η vs. C.

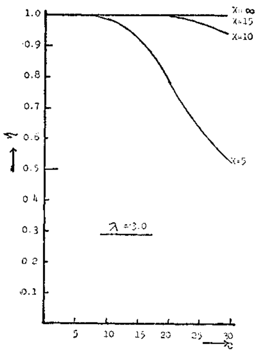

Fig. 3.3.5 η vs. C.

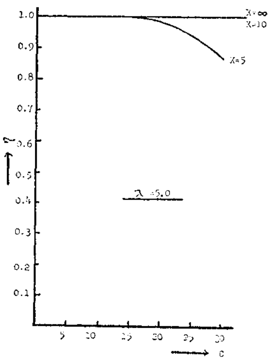

Fig. 3.3.6 η vs. C.

These results suggest us an approach to the effective design of a transportation system to which the proposed stochastic model is applicable, if a suitable cost function is constructed for the whole system operation. (unit of λ, arrivals/minutes : unit of X, minutes)

CHAPTER IV. STOCHASTIC BEHAVIOUR OF VEHICLE INVENTORY LEVEL IN THE POOL

As described in Chapter II, the in-flow and out-flow of vehicles are formed at each station pool in this model. The objective of this chapter is to study the stochastic behaviour of vehicle inventory level in an arbitrary vehicle pool.

In the following discussion, the author makes a reasonable assumption that the whole system starts at the same time at a certain epoch and begins to accept passengers as demands from that epoch on. Let's call this epoch as the system-start-epoch. The study will be forcussed on the equilibrium state of the system operation, where equilibrium state means the state of the system which can be observed after a suitably long time from the system-start-epoch.*

(1) Out-Flow of Vehicles in a Pool

a) General Discussion

Let an arbitrary station be i-th station. Vehicles depart to j-th station at intervals distributed by $f_{T_{ij}}(t)$, where $f_{T_{ij}}(t)$ is p.d.f. of departure interval from i-th station to j-th station, as given in Chapter III. After the system-start-epoch, the departure of vehicles forms an ordinary renewal process. The ordinary renewal process is meant by the process in which all the renewal time intervals have the same p.d.f. If an arbitrary time point θ is picked up after a very long time from the system-start-epoch, the departures of vehicles in the interval $(\theta, \theta+t)$ can be considered as an equilibrium renewal process, where the equilibrium renewal process is meant by the one in which the first renewal time has the excess life distribution and all other renewal intervals have the same distribution. (Cox : Renewal Theory, pp. 27~28, pp. 61~66)

* Even if each station starts the operation at different time epochs, the system can be in equilibrium state if we observe the system after a long time from all of different starting time points.

Consider this time interval $(\theta, \theta+t)$ and set the beginning of the interval θ to the time origin, 0. We can calculate the distribution of number of vehicle departures in time $(0, t)$ as follows : (Fig. 4.1.1)

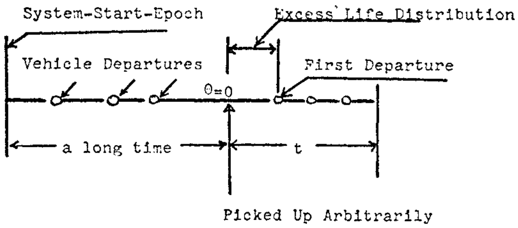

Fig. 4.1.1 Equilibrium Renewal Process For Out-Flow.

Let T_{ij} = interval between vehicle departure from (i) to (j)

$N_{ij}(t)$ = number of vehicle departure in interval $(0, t)$ from (i) to (j)

$K_r(t)$ = C.D.F. of time to r-th departure

$G_{ij}(t, Z)$ = p.g.f. of random variable $\{N_{ij}(t) : t \geq 0\}$

$f_{T_{ij}}(t)$ = p.d.f. of $T_{ij'}$ as given in Chapter III

$F_{T_{ij}}(t)$ = C.D.F. of $T_{ij'}$ as given in Chapter III

However, $K_r(t)$ is given by

$$K_0(t) = 1$$

$$\left.\begin{aligned} K_1(t) &= \frac{1}{E(T_{ij})}\int_0^t (1-F_{T_{ij}}(u))du \\ K_r(t) &= \int_0^t K_{r-1}(t-u) f_{T_{ij}}(u)du \quad (r \geq 2) \end{aligned}\right\} \quad \cdots\cdots (\text{IV-1})$$

Then $P(N_{ij}(t)=r) = K_r(t) - K_{r+1}(t)$

$$G_{ij}(t, Z) = \sum_{r=0}^{\infty} P(N_{ij}(t)=r)Z^r = 1 + \sum_{r=1}^{\infty} Z^{r-1}(Z-1)K_r(t) \cdots\cdots (\text{IV-2})$$

Now let's call the vehicle departure from (i) to (j) as a process $(i \to j)$. At (i), we can observe $(M-1)$ mutually independent processes $(i \to 1)$, $\cdots$, $(i \to M)$.

For the superposed process of $(i \to M)$, let

$O_i(t) = \sum_{j \neq i} N_{ij}(t)$

$P_{O_i}(t, r) = P(O_i(t) = r)$

$G_i(t, Z)$ = p.g.f. of $\{O_i(t), t \geq 0\}$

Then,

$$G_i(t, Z) = \sum_{r=0}^{\infty} P(O_i(t)=r)Z^r = \prod_{j \neq i} G_{ij}(Z, t) \quad \cdots\cdots (\text{IV-3})$$

(Refer to Fig. 4.1.2)

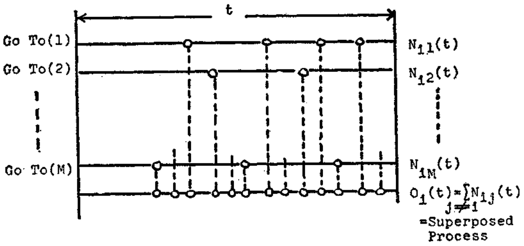

Fig. 4.1.2 Superposed Vehicle Departure From (i) To All Other Stations.

As we have already obtained $f_{T_{ij}}(t)$, $F_{T_{ij}}(t)$ in Chapter III, we can derive $K_r(t)$ by (IV-1) and $G_{ij}(t, Z)$ by (IV-2) and finally $G_i(Z, t)$ by (IV-3). Thus we can know the distribution of number of superposed vehicle departure in time t from i-th station to all other stations.

b) Dynamic Dispatching Policy

Applying the previous formulae given in "(1) General Discussion", the necessary probabilities are calculated using $f_T(t)$, $F_T(t)$ and $E(T)$ given in (III-2), (III-3), (III-13) and (III-14).

(2) In-Flow of Vehicle in a Pool

a) General Discussion

Observe at the i-th station the vehicle arrivals from k-th station. The arrival of vehicle will form a modified renewal process. The distribution of time to the first arrival from the system-start-epoch will differ from those of all other following arrival intervals, due to the effect of travel time from the k-th station to the i-th station. Other arrival intervals have the same distribution $f_{T_{Ki}}(t)$ as given in Chapter III. However, if an arbitrary time point θ is picked up after a long time from the system-start-epoch, the arrivals of vehicles from (k) to (i) in interval $(\theta, \theta+t)$ will be an equilibrium renewal process. (Cox : Renewal Theory, pp. 27~28, pp. 61~66)

Consider the time interval $(\theta, \theta+t)$ and set θ to the time origin, 0.

Let $N_{Ki}(t)$ = number of vehicle arrival in interval $(0, t)$ from (k) to (i)

We can calculate the distribution of number of vehicle departure in time $(0, t)$ in the same way as in section 1.

For the superposed vehicle arrival from all other stations to i-th station,

let $I_i(t)=\sum_{K\neq i} N_{Ki}(t)$

$P_{I_i}(t, r)=P[I_i(t)=r]$

$G_{Ki}(t, Z)$ = p.g.f. of $\{I_i(t),\ t\geq 0\}$

Then

$$\left.\begin{aligned}\hat{G}_i(t, Z)&=\sum_{r=0}^{\infty} P(I_i(t)=r)Z^r\\ &=\prod_{K\neq i} G_{Ki}(t, Z)\end{aligned}\right\}\cdots\cdots\cdots\cdots(\text{IV}-4)$$

b) Dynamic Dispatching policy

The necessary probabilities can be calculated in the same way as in the section 1.

(3) Fluctuation of Inventory Level

—Superposition of Out-Flow and In-Flow of Vehicles—

a) General Discussion

Observe the vehicle pool of i-th station. Due to the superposition of out-flow and in-flow of vehicles, the inventory level fluctuates stochastically.

Let $S_i(t)=S_i+I_i(t)-O_i(t)$

where S_i = initial inventory at time 0.

$I_i(t)$, $O_i(t)$ as given in the section 1 and 2 of this chapter.

$S_i(t)$ = superposed inventory level at time t

$$\begin{aligned}P(S_i(t)=n)&=\sum_{k=0}^{\infty} P(S_i+I_i(t)-O_i(t)\\ &=n/O_i(t)=k)\\ &\qquad\cdot P(O_i(t)=k)\\ &=\sum_{k=0}^{\infty} P(I_i(t)=n+k-S_i|O_i(t)\\ &=k)P(O_i(t)=k)\end{aligned}$$

Independency of $I_i(t)$ and $O_i(t)$

$$\begin{aligned}&=\sum_{k=0}^{\infty} P(I_i(t)=n+k-S_i)P(O_i(t)=k)\\ &=\sum_{k=0}^{\infty} P_{I_i}(t,\ n+k-S_i)P_{O_i}(t,\ k)\end{aligned}$$

Because $P_{O_i}(t, k)$ and $P_{I_i}(t, k)$ are zero for $k<0$,

$$P(S_i(t)=n)=\begin{cases}\sum_{k=0}^{\infty} P_{I_i}(t,\ n+k-S_i)P_{O_i}(t,\ k), & \text{if } n-S_i\geq 0\\ \sum_{k=S_i-n}^{\infty} P_{I_i}(t,\ n+k-S_i)P_{O_i}(t,\ k) & \text{if } n-S_i<0\end{cases}\cdots\cdots\cdots(\text{IV}-5)$$

Thus $P(S_i(t)=n)$ is given in terms of $P_{I_i}(t, k)$ and $P_{O_i}(t, k)$ which are already obtained in previous sections.

b) Numerical Calculations and Discussion

As for the go-when-fill policy, the calculation of. $P(S_i(t)=n)$, was programmed for the electronic computer according to the formula (IV-5). Typical numerical results are given in Fig. 4.3.1 to Fig. 4.3.4.

For these examples, the following Λ was chosen:

$$\Lambda=\|\lambda_{ij}\|=\text{From}\ \begin{array}{c|cccc} & \multicolumn{4}{c}{T_0}\\ & 1 & 2 & 3 & 4\\ \hline 1 & 0 & 1.0 & 0.4 & 1.0\\ 2 & 0.5 & 0 & 0.4 & 0.5\\ 3 & 0.5 & 1.0 & 0 & 0.5\\ 4 & 0.4 & 1.0 & 0.4 & 0\end{array}$$

(unit of λ_{ij} : arrivals/minutes)

Fig. 4.3.1 to Fig. 4.3.3 show the probabilits of different inventory levels for $C=5$ at different time points. Fig. 4.3.4 is for $C=10$. Due to the structure of the demand matrix Λ, the station 2 has the tendency to collect vehicles as time goes on and the

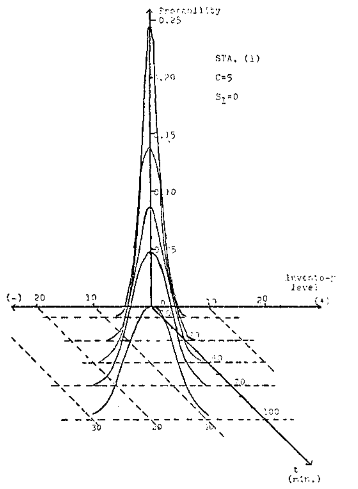

Fig. 4.3.1 Probability of Inventory Level ar STA. (1).

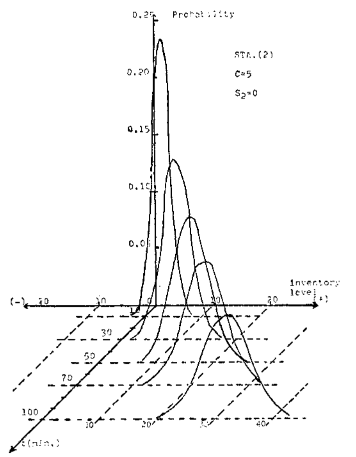

Fig. 4.3.2 Probability of Inventory Level at STA. (2).

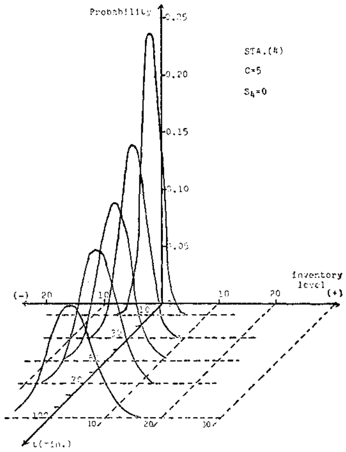

Fig. 4.3.3 Probability of Inventory Level at STA. (4).

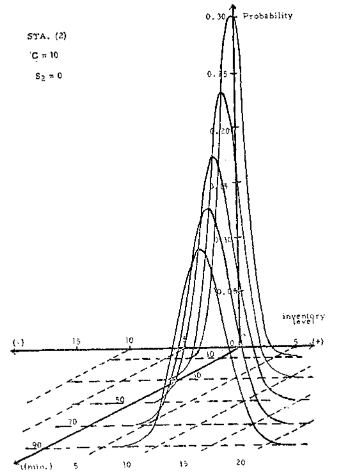

Fig. 4.3.4 Probability of Inventory Level at STA. (2).

station 1 and 3 have the tendency to lose vehicles, while the station 4 has a good property that the in-flow and out-flow of vehicles are well balanced. The effect of vehicle capacity C on the probability of inventory level can be observed by comparing, for example, Fig. 4.3.2 with Fig. 4.3.4. Variance of probability and the tendency of collecting vehicles are decreased by increasing the value of C. These figures were shown to visualize the stochastic behavior of inventory level at each station of the network for the given value of C and Λ. How to apply these results to the effective design of a transportation system will be discussed in Chapter V, section 2.

CHAPTER V. INVENTORY CONTROL OF VEHICLES IN POOLS

So far it has been assumed that vehicles can be supplied without delay from vehicle pools as soon as demands occur. This assumption implicitly requires the infinite number of vehicles in each pool. To remove this rather unrealistic situation and to make the model closer to a practical one, an inventory control is carried out over vehicles i each pool so that, with finite size of vehicle fleet and finite size of vehicle pools, the whole systern may operate as if there were infinite vehicle supplies and every demand may be satisfied without delay.

Besides the objectives described above, this inventory control has another important end that the distribution of vehicles is smoothed over the whole system periodically.

(1) Inventory Control Technique

The proposed inventory control policy is given in chapter II.

Note that in this vehicle inventory model, both the quantity of demand and that of supply are random variables. Furthermore the demand can occur at every time point continuously and so can the supply. These special characteristics to this inventory model have rejected the direct application of the present technique of inventory control already established and required a control technique described in chapter II.

As mentioned, the inventory control technique makes almost impossible the unfavorable situation that the vehicles are in excess or in shortage in each pool. Besides this, the inventory control strategy has another important objective. As studied in chapter IV, fluctuation of inventory level depends on the rate of passenger flow, λ_{ij}. Thus, some station has the tendency to collect vehicles, leaving other stations in shortage of vehicles. The inventory control technique balances and smoothes vehicle

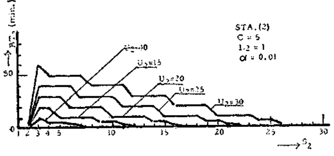

Fig. 5.2.1 BT_2 vs. S_2.

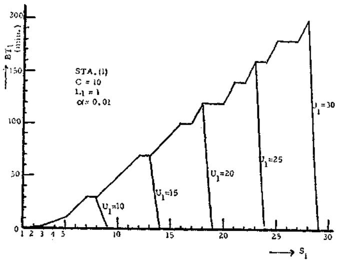

Fig. 5.2.2 BT_1 vs. S_1.

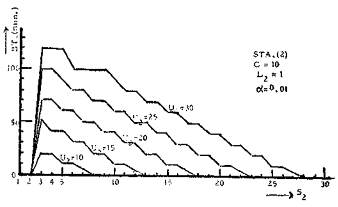

Fig. 5.2.3 BT_2 vs. S_2.

distribution over the whole system by resetting the inventory level of each station to S_i, initial inventory level, periodically at each end of the balancing period BT_i.

Here the author wishes to spend a few words about the relationship between the inventory control model and the optimization of the system operation. Upper limit U_i of inventory level determines the size of each pool (*i.e.* construction cost of pool). Initial inventory level S_i will be a factor to determine the fleet size of vehicles along with U_i (vehicle capital cost). Balancing interval BT_i will be associated with balancing cost of vehicles because balancing vehicles at each BT_i requires the transportation cost of vehicles. Thus, U_i, L_i, S_i and BT_i will be control variables in the model in an optimization problem of the whole system along with the capacity of vehicle C and passenger waiting time limit X.

(2) Numerical Calculation and Discussion

For the go-when-fill policy, a subroutine EXCSHT was programmed. The subroutine EXCHST calculates

$$P[N_i(t)>U_i]+P[N_i(t)<L_i]$$
$$=\sum_{n=U_i+1}^{\infty}P[N_i(t)=n]+\sum_{n=L_i-1}^{-\infty}P[N_i(t)=n]$$

Using the subroutine EXCSHT, maximum BT_i which satisfies the following relation was obtained for different values of C, U_i, S_i, L_i and given α.

$$\operatorname*{Max}_{0<t<BT_i} P[N_i(t)>U_i]+P[N_i(t)<L_i]\leq\alpha$$

Fig. 5.2.1 to Fig. 5.2.7 were obtained for the same demand matrix Λ as given in chapter IV, section 3, (b) and C=5, 10, 15, 20; L_i=1; U_i=10, 15, 20, 25, 30; S_i=1 to U_i; α=0.0.1.

As the relation $P[N_i(t)>U_i]+P[N_i(t)<L_i]\leq\alpha$ was examined at every 10 minutes for t=0 to 100 minutes and at every 20 minutes for t=100 to 400 minutes, rugged lines were got in these figures.

From these figures, several interesting observations can be made.

(1) There are some optimum values for S_i which maximize BT_i for given Λ, C, U_i, L_i α and station.

(2) As described in chapter IV, for the demand rate matrix Λ which was used here, station (1) and (3) have the tendency to lose vehicles, while station (2) and (4) have the tendency to collect vehicles : besides, station (4) has a nice property that the in-flow and out-flow of vehicles are almost balanced. These things are reflected in these figures. For example, in Fig. 5.2.2 to Fig. 5.2.5, station (1) and

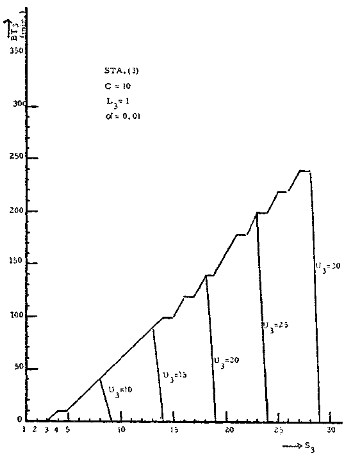

Fig. 5.2.4 BT_3 vs. S_3.

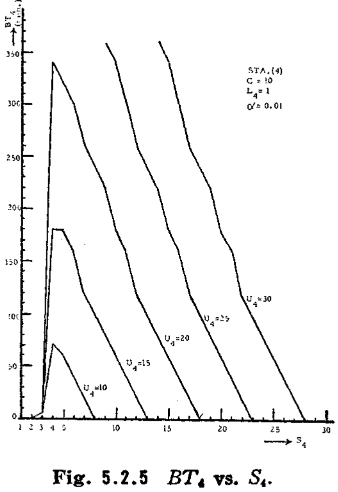

Fig. 5.2.5 BT_4 vs. S_4.

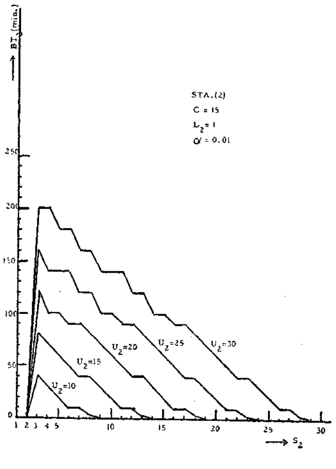

Fig. 5.2.6 BT_2 vs. S_2.

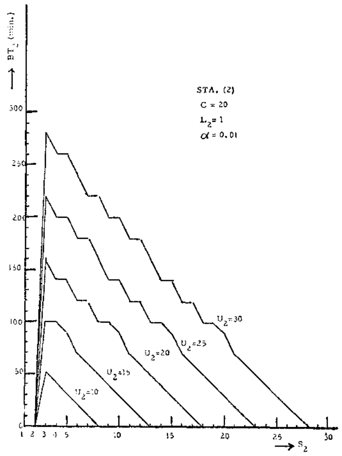

Fig. 5.2.7 BT_2 vs. S_2.

(3) have the optimum values of S_i near the upper limit of inventory level because these stations are apt to lose vehicles, while station (2) and (4) have the optimum values of S_i near the lower limit of inventory level because they are apt to collect vehicles. Station (4) has more than 400 minutes for BT_4 at $S_4=4$ to 6 and $U_4=30$ because of the nice property that the in-flow and out-flow of vehicles are well balanced, whereas at other stations BT_i are 250 or less minutes because of their more conspicuous tendency to lose or to collect vehicles.

(3) The values of S_i are sensitive to the value of BT_i in such stations as station (4) where the in flow and out-flow of vehicles are well balanced.

(4) BT_i can be increased almost linearly proportionally to the value of U_i with the values of all other parameters being not changed.

(5) The optimum values of S_i for given U_i, L_i, α and at given station are not changed by changing the value of C. However, the value of BT_i can be increased almost linearly proportionally to the value of C. For example, at station (2), for $U_2=30$, $L_2=1$, $BT_2=60$ minutes for $S_2=3$ and $C=5$: $BT_2=120$ min. for $S_2=3$ to 5 and $C=10$: $BT_2=200$ min. for $S_2=3$, 4 and $C=15$: $BT_2=280$ min. for $S_2=3$ and $C=20$.
etc.

These figures can be used in various ways as an aid to the effective design of transportation system for which the model was constructed. For example, the following questions can be answered :

(1) For given A, C, U_i and α, and at the given station, at every how many minutes must the vehicles be balanced and at what value of the optimum initial inventory level S_i?

(2) For given A, C, α and at given station, what value of U_i is necessary in order to secure the value of BT_i larger than some specified value?. Or, for given value of A, α, U_i and at the given station, what value of C is necessary in order to secure the value of BT_i larger than some specific value?
etc.

CHAPTER VI. CONCLUDING REMARKS

In concluding this study, the author wishes to spend a few pages in discussing optimization of the whole system operation and a few future research topics which lie in the direction of developing this study further.

In the optimization problem; input is the passenger arrival rate matrix A; control variables are individual vehicle capacity C, passenger waiting timit X, inventory upper limit U_i, lower limit L_i, initial inventory level S_i, inventory balancing interval BT_i; the major outputs will include (1) revenue from passengers, (2) penalty due to average passenger waiting time, (3) vehicle running cost, *i.e.*, fuel cost, vehicle maintenance cost and drivers' cost, (4) vehicle capital cost, *i.e.*, production cost and depreciation cost, (5) construction cost of vehicle pool, (6) vehicle balancing cost, etc.

Choosing optimal values for C, X, U_i, L_i, S_i, BT_i, the cost function couled be minimized. This is, in general, reduced to a non-linear mathematical programming problem.

As for a few future research topics which would extend the present study further, the following items would be listed :

(1) Time-dependent arrival rate

In this study, time-independent arrival rate (homogeneous Poisson) is treated. Because the real transportation demand rate is time-dependent, the whole discussion in this study must be made also for time dependent arrival rate, although in the appendix, the study of dynamic dispatching is given for the non-homogeneous Poisson case.

(2) Go-when-fill-with-time-constraint policy

Numerical calculations for vehicle in-flow and outflow and vehicle inventory level.

In this study, these kinds of numerical calculations were note done for go-when-fill-with-time-constraint policy. The analytical calculation requires complicated inversion of Laplace transform of irrational functions. However, by repeated use of a subroutine which calculates the integral value of any function by Simpson's formula, the desired probabilities can be calculated by electronic computer.

If, in this way, a subroutine is programmed which calculates $p[N_{ij}(t)=n]$ for the go-when-fill-with-time-constraint policy, the same quantities as those presented in this paper can be calculated using this program.

Furthermore, it should be mentioned here that referring to Fig. 4.3.1 to Fig. 4.3.4, the distributions of number of vehicles dispatched or arriving in a given time are very symmetric even in rather short intervals for the demand matrix Λ which was used. This observation leads us to the study of possible application of limit theorems for renewal theory which ensure us to treat the distributions of renewals in a given time as normal distributions. The approximations by normal distribution would make various calculations significantly easier.

(3) Vehicle control model

In the study of inventory control, the vehicle inventory control of each station was studied independently of those of other stations. However, one must proceed further to the investigation of inventory control of vehicles taking into considerations the interaction between station pools of the whole system, *i.e.*, as a multiechelon inventory system. In this case two possible types of multiechelon inventory systems may be considered. One type is to redistribute vehicles through a central vehicle pool which smooths the distribution of vehicles acting as an adjustment pool to all station pools of the system. The other type is to balance vehicles over the whole system through moving vehicles directly between stations, *i.e.*, from stations with excess inventory level to stations with low inventory level. The latter type requires to solve a classical transportation problem in choosing an optimal assignment of vehicle transhipment among stations of the network, if the vehicle disribution is carried out at the same instant for all the stations of the system.

(4) In this study the same passenger waiting time limit X was used for all pairs of stations of the system.

It will be interesting, if different passenger waiting time limits were chosen for different pairs of stations of the system (for example, X_{ij} for the travelers' flow from (i) to (j) of intensity λ_{ij}). This being done the number of control variables will be increased significantly large and the model would be made more flexible, although the optimization problem would be found more complicated.

ACKNOWLEDGMENT

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The author would like to thank Professor E. Farnsworth Bisbee who guided and encouraged him during the completion of this work.

DEFINITION OF SYMBOLS

p.d.f. = probability density function

C.D.F. = commulative distribution function

$f_T(t)$ = p.d.f. of random variable T evaluated at t

$F_T(t)$ = C.D.F. of random variable T evaluated at t

p.g.f. = probability generating function

(i) = i-th station

$$P(z, k)=\sum_{j=0}^{k} e^{-z}z^i/i\,!$$

$$f^*(s)=\int_0^\infty f(t)e^{-st}dt = \text{Laplace transform of } f(t)$$

$\mathfrak{L}^{-1}\{f^*(s)\}$ = inverse transform of $f^*(s)$

APPENDIX

STUDY ON THE DYNAMIC VEHICLE DISPATCHING POLICY FOR NON-HOMOGENEOUS POISSON CASE

Because the real transportation mean demand rate is time-dependent, the study of the transportation systems should be made for mean arrival rate. Although the whole discussions given in this paper should be repeated for time-dependent arrival rate only the study of the dynamic vehicle dispatching policy for non-homogeneous Poisson case is given in this appendix, other discussions left as a further study.

1. GO-WHEN-FILL-WITH-TIME-CONSTRAINT-POLICY

In this section, the case where λ_{ij} is time-dependent, *i.e.*, non-homogeneous Poisson case will be investigated, and the results of the same kind as in chapter III would be obtained.

Dispatching policy adopted is the same as given in the chapter III, that is,

Dispatch vehicle if C passengers arrive or if the first arrival waits X-minutes.

Where C = number of seats in a vehicle $(C \geq 1)$

Passenger arrival is of the non-homogeneous Poisson type with the continuous mean value function of $m_\tau(t)=\int_\tau^{t+\tau}\lambda(u)\,du$.

What we wish to investigate is,

(1) Conditional vehicle dispatching interval distribution, given the last vehicle dispatched at time point θ. We are going to answer the question at what time the next vehicle will be dispatched, given the last vehicle was dispatched at time θ.

(2) Conditional average passenger waiting time, given the last vehicle was dispatched at time θ. With a prior knowledge that the last vehicle departed at time θ, what will be the average waiting time of the passengers departing in the next vehicle?

(1) Some Properties of Non-homogeneous Poisson Process and Related Useful Theorems.

Here several theorems on non-homogeneous Poisson process will be presented. They are important to the present study. These theorems will be referred to by their ordered numbers in the later discussion. Proofs given by the author himself except for theorem 2 will be omitted here for the limitation of the amount of pages.

Theorem 1.

For non-homogeneous Poisson process with continuous mean value function $m(t)=\int_0^t \lambda(u)du$, we have the following probability generating function,

$$G(Z,t,\tau)=\sum_{n=0}^{\infty} P(N(t+\tau)-N(\tau)=n)\cdot Z^n$$
$$=E(Z^{N(t+\tau)-N(\tau)})=\exp[(Z-1)\int_\tau^{t+\tau}\lambda(u)du]$$

where $N(t+\tau)-N(\tau)=$Number of occurrence of events in the time interval $[\tau, t+\tau]$

and $P[N(t+\tau)-N(\tau)=n]=\frac{1}{n!}e^{-m_\tau(t)}(m_\tau(t))^n$

where $m_\tau(t)=\int_\tau^{t+\tau}\lambda(u)du$

Theorem 2.

Define a stochastic process $\{M(u),\ u\geq 0\}$ by

$$M(u)=N(m_\tau^{-1}(u)) \quad u\geq 0$$

where $\{N_\tau(t),\ t\geq 0\}$ is a non-homogeneous Poisson process with continuous mean value function $m_\tau(t)=\int_\tau^{t+\tau}\lambda(u)du$ and $u=\int_\tau^{t+\tau}\lambda(u)du=m_\tau(t)$.

$$E(M(u))=E(N(m_\tau^{-1}(t)))=m(m_\tau^{-1}(t))$$
$$=m_\tau(t)=u.$$

$\{M(u),\ u\geq 0\}$ is a homogeneous Poisson process with intensity 1 (Parzen : "Stochastic Process" page 124-126).

Theorem 3.

Let $\{N(t),\ t\geq 0\}$ be a non-homogeneous Poisson process with continuous mean value function $m_\tau(t)=\int_0^t\lambda(u)du$.

Define a stochastic process $\{N_\tau(t),\ t\geq 0\}$ by $N_\tau(t)=N(t+\tau)-N(\tau)$. Given that $N_\tau(t)=k$, k times $\tau_1<\tau_2<\cdots<\tau_k$ in the interval $(\tau, \tau+t)$ at which events occur are random variables having the same distribution as if they were the order statistics corresponding to k independent random variables $U_1, U_2, \cdots, U_k$ with common distribution function,

$$F_U(u)=\frac{m_\tau(u)}{m_\tau(t)} \quad 0\leq u\leq t$$

where $m_\tau(u)=\int_\tau^{\tau+u}\lambda(s)ds$.

Theorem 4.

Given that k events of non-homogeneous Poisson with continuous mean value function $m_\tau(t)$ have happened in time $[\tau, \tau+t]$, the time between τ and the time of i-th arrival has the following probability density function.

$$P(U_i=u)=\frac{k!}{(i-1)!\,(k-i)!}\left(\frac{\lambda(\tau+u)}{m_\tau(X)}\right)\cdot\left(\frac{m_\tau(u)}{m_\tau(X)}\right)^{i-1}\left(1-\frac{m_\tau(u)}{m_\tau(X)}\right)^{k-i}$$

and

$$P[W_i=w]=\frac{k!}{(i-1)!\,(k-i)!}\left(\frac{\lambda(\tau+X-w)}{m_\tau(X)}\right)\cdot\left(\frac{m_\tau(X-w)}{m_\tau(X)}\right)^{i-1}\left(1-\frac{m_\tau(X-w)}{m_\tau(X)}\right)^{k-i}$$

This is the direct result from theorem 4.

Theorem 5.

In the same situation as theorem 4, *i.e.*, given that k events of non-homogeneous Poisson have happened in time $(\tau, \tau+X)$,

$$E(W_i|N_\tau(X)=k)=\int_0^X P(W_i=w|N_\tau(X)=k)wdw$$
$$=\int_0^X\frac{k!}{(i-1)!\,(k-i)!}\left(\frac{m_\tau(X-w)}{m_\tau(X)}\right)^{i-1}\cdot\left(1-\frac{m_\tau(X-w)}{m_\tau(X)}\right)^{k-i}\cdot\frac{\lambda(\tau+X-w)}{m_\tau(X)}wdw$$

Define $MW_k=\frac{1}{k}(W_1+W_2+\cdots+W_k)$

then $E(MW_k|N_\tau(X)=k)=\frac{1}{k}\sum_{j=1}^{k}E(W_i|N_\tau(X)=k)$
$$=\frac{1}{m_\tau(X)}\int_0^X m_\tau(w)dw$$

(2) Vehicle Dispatching Interval Distribution Function

Vehicle will be dispatched in two ways :

Case Ⅰ……dispatched with fill (Fig. A-1)

Case Ⅱ……dispatched without fill (Fig. A-2).

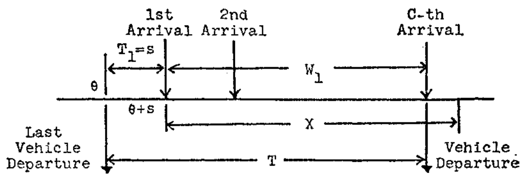


Fig. A 1 Case Ⅰ For Vehicle Departure.

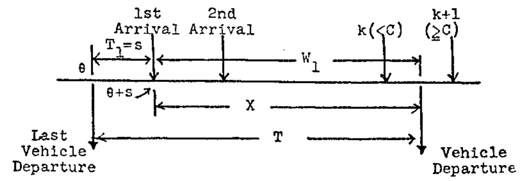


Fig. A 2 Case Ⅱ For Vehicle Departure.

Define

T_1=time from the last vehicle departure to the first passenger arrival.

W_1=time from the first passenger arrival to the departure time of the next vehicle.

T=departure interval.

θ=time point of the last vehicle departure. (Note that $T=T_1+W_1$)

X=waiting time limit.

$F_{W_1}(t|s)=P[W_1\leq t|$ the first arrival is at $s]$.

$f_{W_1}(t|s)$=p.d.f. corresponding to $F_{W_1}(t|s)$.

$F_{T_1}(t|\theta)=P[T_1\leq t|$ the last vehicle departs at time $\theta]$.

$f_{T_1}(t|\theta)$=corresponding p.d.f.

$F_T(t|\theta)=P[T\leq t|$ the last vehicle departs at time $\theta]$.

$f_T(t|\theta)$=corresponding p.d.f.

For $0<t<X$

$F_{W_1}(t|s)=P[C-1$ or more arrivals between s and $s+t]$

For $t=X$,

$f_{W_1}(X|s)=P[C-2$ or less arrivals in $(s, s+X)]$

From theorem 1

$$F_{W_1}(t/s)=\sum_{k=C-1}^{\infty}\frac{1}{k!}e^{-m_s(t)}(m_s(t))^k \quad 0<t<X$$

$$F_{W_1}(X|s)=1 \qquad t=X$$

$$f_{W_1}(t|s)=\frac{\partial F_{W_1}(t|s)}{\partial t}$$

$$=\frac{(m_s(t))^{C-2}}{(C-2)!}\lambda(t+s)e^{-m_s(t)} \quad (0<t<X)$$

$$f_{W_1}(X|s)=\sum_{k=0}^{C-2}\frac{1}{k!}e^{-m_s(X)}(m_s(X))^k \qquad (t=X) \qquad \cdots\cdots(A\text{-}1)$$

Next,

$$F_{T_1}(t|\theta)=1-P(T_1>t|\theta)$$

$$=1-e^{-m_\theta(t)}$$

$$\therefore \quad f_{T_1}(t|\theta)=\lambda(\theta+t)e^{-m_\theta(t)}\cdots\cdots(A\text{-}2)$$

Next,

$$F_T(t|\theta)=\int_0^\infty P(T\leq t|T_1=s,\ \theta)f_{T_1}(s|\theta)ds$$

$$=\int_0^t P(W_1\leq t-s|T_1=s,\ \theta)f_{T_1}(s|\theta)ds$$

(i) The case $0<t<X$

Then, $t-s<X$

From (A-1),

$$P(W_1\leq t-s/T_1=s,\ \theta)=F_{W_1}(t-s/s+\theta)$$

$$=\sum_{k=C-1}^{\infty}\frac{m_{s+\theta}(t-s)^k}{k!}e^{-m_{s+\theta}(t-s)}$$

From (A-2),

$$f_{T_1}(s|\theta)=\lambda(s+\theta)e^{-m_\theta(s)}$$

$$F_T(t|\theta)=\int_0^t\sum_{k=C-1}^{\infty}\frac{(m_{s+\theta}(t-s))^k}{k!}\cdot e^{-m_{s+\theta}(t-s)}\lambda(s+\theta)e^{-m_\theta(s)}ds$$

$$\therefore \quad f_T(t|\theta)=\frac{dF_T(t|\theta)}{dt}=\frac{\lambda(t+\theta)}{(C-1)!}e^{-m_\theta(t)}(m_\theta(t))^{C-1} \qquad \cdots\cdots(A\text{-}3)$$

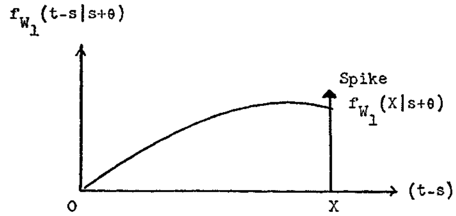
(ii) The case $t\geq X$

$$f_T(t/\theta)dt=\int_0^\infty P[t\leq T=T_1+W_1\leq t+dt|T_1=s,\theta)\cdot f_{T_1}(s/\theta)ds$$

$$=\left(\int_0^\infty f_{W_1}(t-s|s+\theta)f_{T_1}(s/\theta)ds\right)dt$$

$$\therefore \quad f_T(t/\theta)=\int_0^\infty f_{W_1}(t-s|\theta+s)f_{T_1}(s/\theta)ds$$

$f_{W_1}(t-s/s+\theta)$ has a mixed distribution as follows :



We interpret $f_{W_1}(t-s/s+\theta)$ as δ-function of magnitude $f_{W_1}(X/s+\theta)$ at the point $(t-s=X)$.

$$f_T(t|\theta)=\int_0^t f_{W_1}(t-s|s+\theta)f_{T_1}(s|\theta)ds$$

$$=\left\{\int_0^{t-x-\varepsilon}+\int_{t-x-\varepsilon}^{t-x}+\int_{t-x}^{t}\right\}\cdot f_{W_1}(t-s|s+\theta)f_{T_1}(s|\theta)ds$$

However $(0\leq s\leq t-x-\varepsilon)$ implies $(t-s\geq x+\varepsilon)$

Thus,

$$\int_0^{t-x-\varepsilon}f_{W_1}(t-s|s+\theta)f_{T_1}(s|\theta)ds=0$$

$(t-X-\varepsilon\leq s\leq t-X)$ implies $(X\leq t-s\leq X+\varepsilon)$, and $(t-X\leq s\leq t)$ implies $(t-s\leq X)$.

$$\therefore \quad f_T(t|\theta)=\int_{t-x-\varepsilon}^{t-x}\frac{1}{\varepsilon}f_{W_1}(X|\theta+s)f_{T_1}(s|\theta)ds$$

$$+\int_{t-x}^{t}\frac{\lambda(t+s)e^{-m_{s+\theta}(t-s)}}{(C-2)!}\cdot(m_{s+\theta}(t-s))^{C-2}f_{T_1}(s|\theta)ds$$

$$=\frac{\lambda(t+\theta)}{(C-2)!}e^{-m_\theta(t)}\int_{t-x}^{t}\lambda(s+\theta)\cdot(m_{s+\theta}(t-s))^{C-2}ds+\frac{1}{\varepsilon}\sum_{k=0}^{C-2}\frac{1}{k!}\cdot\int_{t-x-\varepsilon}^{t-x}e^{-m_\theta(s+x)}(m_{s+\theta}(x))^k\lambda(s+\theta)ds$$

Let $I=\frac{1}{\varepsilon}\int_{t-x-\varepsilon}^{t-x}e^{-m_\theta(x+s)}\lambda(s+\theta)(m_{s+\theta}(x))^k ds$

$$\lim_{\varepsilon\to0}I=\lim_{\varepsilon\to0}\frac{d}{d\varepsilon}\int_{t-x-\varepsilon}^{t-x}e^{-m_\theta(s+x)}\lambda(s+\theta)\cdot(m_{s+\theta}(x))^k ds$$

$$=\lambda(t-x+\theta)e^{-m_\theta(t)}(m_{t-x+\theta}(x))^k$$

Thus,

$$f_T(t|\theta)=\frac{\lambda(t+\theta)}{(C-1)!}e^{-m_\theta(t)}(m_{t-x+\theta}(x))^{C-1}+\sum_{k=0}^{C-2}\frac{1}{k!}\lambda(t-x+\theta)e^{-m_\theta(t)}(m_{t-x+\theta}(x))^k \qquad \cdots\cdots(A\text{-}4)$$

(3) Expected Value of T

$$E(T/\theta)=\int_0^x\frac{\lambda(t+\theta)}{(C-1)!}e^{-m_\theta(t)}(m_\theta(t))^{C-1}tdt+\int_x^\infty\frac{\lambda(t+\theta)}{(C-1)!}e^{-m_\theta(t)}(m_{t-x+\theta}(x))^{C-1}tdt+\sum_{k=0}^{C-2}\frac{1}{k!}\int_x^\infty\lambda(\theta+t-x)e^{-m_\theta(t)}\cdot(m_{t-x+\theta}(x))^k tdt\cdots\cdots(A\text{-}5)$$

(4) Mean Loading Factor

$$E(B/\theta)=\sum_{B=1}^{C} B\cdot P(B/\theta),$$

where B=number of passengers in a dispatched vehicle

$P(B/\theta)$=Prob (next vehicle dispatched with B passengers, given the last vehicle dispatched at θ)

$$P(B/\theta)=\int_0^\infty P(B/s+\theta)f_{T1/\theta}(s/\theta)ds$$

where $f_{T1/\theta}(s/\theta)$=conditional p.d.f. of time to the first arrival from the last vehicle departure time, given that the last vehicle departure is at epoch θ.

In the case $B<C$

$$P(B/s+\theta)=P(B-1 \text{ arrivals in time}(s+\theta,\ s+\theta+x))$$
$$=\frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^{B-1}}{(B-1)!}$$

$$\therefore\ P(B/\theta)=\int_0^\infty \frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^{B-1}\lambda(s+\theta)e^{-m_\theta(s)}ds}{(B-1)!}$$

In the case $B=C$

$P(B/\theta+s)=P(C/\theta+s)=P(C-1$ or more arrivals in time $(\theta+s,\ \theta+s+x))$

$$=\sum_{k=C-1}^{\infty}\frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^k}{k!}$$

$$\therefore\ P(C/\theta)=\sum_{k=C-1}^{\infty}\int_0^\infty \frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^k}{k!}\cdot\lambda(s+\theta)e^{-m_\theta(s)}ds$$

$$\therefore\ E(B/\theta)=\sum_{B=1}^{C-1}B\cdot\int_0^\infty\frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^{B-1}}{(B-1)!}\cdot\lambda(s+\theta)e^{-m_\theta(s)}ds$$
$$+C\cdot\sum_{k=C-1}^{\infty}\int_0^\infty\frac{e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^k}{k!}\cdot\lambda(s+\theta)e^{-m_\theta(s)}ds \quad\cdots\cdots\text{(A-6)}$$

Let η=average loading factor.

$$\eta=E(B/\theta)/C$$

(5) Average Passenger Waiting Time

(i) Case I The vehicles are dispatched without fill. Suppose vehicles are dispatched with $k(<C)$ passengers. It means that $(k-1)$ passengers arrive with in X minutes after the first passenger arrival. Refer to Fig. A 2.

Define $E(\overline{W}_k/s)$=conditional expectation of $\overline{W}_k$, given the 1st passenger arrived at s time units after the last vehicle departure,

where $\overline{W}_k=\frac{1}{k}(W_1+W_2+\cdots+W_k)$

=waiting time per passenger in a vehicle

and W_i=waiting time of i-th passenger in a vehicle

$$E(\overline{W}_k)=\int_0^\infty E(\overline{W}_k/s)f_{T_1}(s)ds$$

$$E(\overline{W}_k/s)=E\left(\frac{W_1+W_2+\cdots+W_k}{k}\Big/s\right)$$
$$=\frac{k-1}{k}E\left(\frac{W_2+\cdots+W_k}{k-1}\Big/s\right)$$
$$+\frac{1}{k}E(W_1/s)$$

From theorem 5,

$$E\left(\frac{W_2+\cdots+W_k}{k-1}\Big/s\right)=\frac{1}{m_s(x)}\int_0^x m_s(w)dw$$

and $E(W_1/s)=X$

$$\therefore\ E(\overline{W}_k/s)=\frac{k-1}{k}\frac{1}{m_s(x)}\int_0^x m_s(w)dw+\frac{x}{k}$$
$$=1/k\left[x+\frac{(k-1)}{m_s(x)}\int_0^x m_s(w)dw\right]$$

As we are studying the problem with condition that the last vehicle departed at time θ, the above expression is rewritten as follows:

$$E(\overline{W}_k/s,\ \theta)=\frac{1}{k}\left(x+\frac{k-1}{m_{s+\theta}(x)}\int_0^x m_{s+\theta}(w)dw\right)$$

$$\therefore\ E(\overline{W}_k/\theta)=\int_0^\infty\frac{1}{k}\left(x+\frac{k-1}{m_{s+\theta}(x)}\int_0^x m_{s+\theta}(w)dw\right)\cdot\lambda(s+\theta)e^{-m_\theta(s)}ds\ \cdots\cdots\text{(A-7)}$$

where we defined $E(\overline{W}_k/\theta)$=conditional expectation of $\overline{W}_k$, given that the last vehicle departed at epoch θ.

(ii) Case II Vehicles dispatched with fill.

This case is decomposed into the following events.

Event $E_k=(k-1)$ arrivals within x minutes after the 1st arrival, $k=C,\ C+1,\ C+2,\ \cdots\infty$

Suppose event E_k happened. (Refer to Fig. A 3)

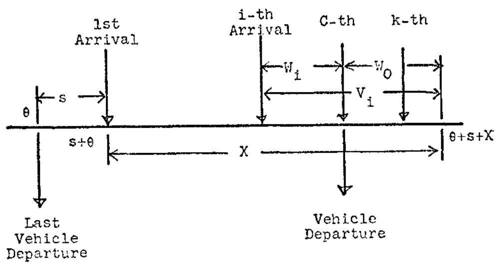


Fig. A 3 Event E_k.

Define W_i=i-th passenger waiting time

W_0=time between vehicle departure and time point $(\theta+s+x)$

$V_i=W_i+W_0$

Define $E(\overline{W}_k/s,\ \theta)$=expected value of $\overline{W}_k$, given s and θ,

where $\overline{W}_k=\frac{1}{C}(W_1+\cdots+W_C)$

$=\frac{1}{C}(W_1+\cdots+W_{C-1})$ (Note that $W_C=0$)

$$\therefore\ E(\overline{W}_k/\theta)=\int_0^\infty E(\overline{W}_k/\theta,\ s)f_{T/\theta}(s)ds$$
$$=\int_0^\infty E(\overline{W}_k/s,\ \theta)\lambda(s+\theta)e^{-m_\theta(s)}ds$$

As $W_i=V_i-W_0$,

$$\overline{W}_k=\frac{1}{C}(V_1+\cdots+V_{C-1})-\frac{C-1}{C}W_0$$

$$E(\overline{W}_k/s,\ \theta)=\frac{1}{C}[X+E(V_2+\cdots+V_{C-1}|s,\ \theta)]$$

$$-\frac{C-1}{C}E(W_0|s,\theta)$$

(Note that $V_1=X$)

From theorem 5,

$$E(W_0/s,\theta)=\int_0^x\frac{(k-1)!}{(C-2)!(k-C)!}\left(\frac{m_{\theta+s}(x-w)}{m_{s+\theta}(x)}\right)^{C-2}\cdot\left(1-\frac{m_{s+\theta}(x-w)}{m_{s+\theta}(x)}\right)^{k-C}\cdot\frac{\lambda(s+\theta+x-w)}{m_{s+\theta}(x)}wdw$$

$$E(V_i/s,\theta)=\int_0^x\frac{(k-1)!}{(i-2)!(k-i)!}\left(\frac{m_{\theta+s}(x-w)}{m_{s+\theta}(x)}\right)^{i-2}\cdot\left(1-\frac{m_{s+\theta}(x-w)}{m_{s+\theta}(x)}\right)^{k-i}\cdot\frac{\lambda(s+\theta+x-w)}{m_{s+\theta}(x)}wdw \quad \cdots\cdots(\text{A-8})$$

(iii) Define $P_{k/s}=P(k-1$ arrivals within x minutes after the 1st arrival, given the 1st arrival is s time units after the last departure)

$P_k=P(k-1$ arrivals within x minutes after the 1st arrival)

$$P_{k/s}=\frac{1}{(k-1)!}e^{-m_{s+\theta}(x)}(m_{s+\theta}(x))^{k-1}$$

(from Theorem 1)

$$\therefore\quad P_k=\frac{1}{(k-1)!}\int_0^\infty\lambda(s+\theta)e^{-m_\theta(s+x)}(m_{s+\theta}(x))^{k-1}ds$$

(for $k\geq1$) $\cdots\cdots$(A-9)

(iv) Define $E(\overline{W}/\theta)=\sum_{k=1}^{\infty}E(\overline{W}_k|\theta)P_k$,

where $E(\overline{W}/\theta)$ =conditional expected value of waiting time. per passenger in a vehicle, given that the last vehicle departed at time θ.

$$\therefore\quad E(\overline{W}/\theta)=\sum_{k=1}^{C-1}E(\overline{W}_k/\theta)P_k+\sum_{k=C}^{\infty}E(\overline{W}_k/\theta)P_k$$

From (A-7), (A-8) and (A-9)

$$E(\overline{W}|\theta)=\sum_{k=1}^{C-1}\left\{\int_0^\infty\left[\frac{1}{k}\left(x+\frac{k-1}{m_{s+\theta}(x)}\int_0^x m_{s+\theta}(w)dw\right)\right]\cdot\lambda(s+\theta)e^{-m_\theta(s)}ds\right\}\frac{1}{(k-1)!}\cdot\left.\int_0^\infty\lambda(s+\theta)e^{-m_\theta(s+x)}(m_{s+\theta}(x))^{k-1}ds\right]$$
$$+\sum_{k=C}^{\infty}\left\{\int_0^\infty\left[\frac{1}{C}\left(x+\sum_{i=2}^{C-1}E(V_i|s,\theta)\right)-\frac{C-1}{C}\cdot E(W_0|s,\theta)\right]\lambda(s+\theta)e^{-m_\theta}(s)ds\right\}\left\{\frac{1}{(k-1)!}\cdot\int_0^\infty\lambda(s+\theta)e^{-m_\theta(s+x)}(m_{s+\theta}(x))^{k-1}ds\right\}$$

$\cdots\cdots$(A-10)

where $E(V_i|s,\theta)$ is given in (A-9).

For the special case of $\lambda(t)$ given in the Fig. A-4, detailed calculation was carried out for $E(T/\theta)$. It will not be included in this paper because of limitation of the amount of pages.

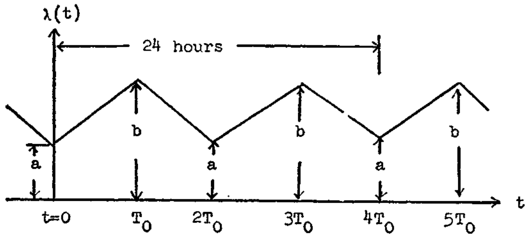


Fig. A 4 Periodical Demands Rate Function.

2. Go-When-Fill-Policy

In this case, with the same notation as before, and using the same theorems in the section 1. we have the following results.

(1) Vehicle Dispatching Interval Distribution.

$$f_T(t/\theta)=\frac{\lambda(t+\theta)e^{-m_\theta(t)}(m_\theta(t))^{C-1}}{(C-1)!}\quad 0<t<\infty C\geq1 \quad\cdots\cdots(\text{A-11})$$

(2) Expected Value of T.

$$E(T/\theta)=\int_0^\infty\frac{\lambda(t+\theta)e^{-m_\theta(t)}(m_\theta(t))^{C-1}}{(C-1)!}tdt \quad\cdots\cdots(\text{A-12})$$

(3) Average Passenger Waiting Time.

$$E(\overline{W}/\theta)=\frac{1}{C}\sum_{j=1}^{C}E(W_j|\theta)=\frac{1}{C}\int_0^\infty\left\{\lambda(\theta+t)e^{-m_\theta(t)}t\cdot\sum_{j=1}^{C}\left[\frac{(m_\theta(t))^{C-1}}{(C-1)!}-\frac{(m_\theta(t))^{j-1}}{(j-1)!}\right]\right\}dt \quad\cdots\cdots(\text{A-13})$$

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