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ON THE STATE SURFACE OF ANISOTROPICALLY CONSOLIDATED CLAYS

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1. INTRODUCTION

In the previous papers, the authors gave the consideration on the state surface and on the stress-strain resations for isotropically consolidated clays [Hata and Ohta (1968)[1], Hata and Ohta (1969)[2], Hata, Ohta and Yoshitani (1969)[3], Ohta and Hata (to appear)[4]]. The main subject of this paper is to expand the theory for anisotropically consolidated clays.

The foundations of the previous theory is introduced briefly as follows.

It is convenient to consider the volume change of clays to be consisting of two components : *consolidation* and *dilatancy* component. If a clay element. is sheared keeping the effective hydrostatic pressure σ_m' constant not to arise consolidation, the relation between the void ratio e and the stress ratio τ_{oct}/σ_m' is linear with the proportional constant μ.

This relation was found by Shibata (1963)[5], Shibata and Karube (1965)[6], Karube and Kurihara (1966)[7]. On the other hand, if one stresses a clay element keeping the stress ratio τ_{oct}/σ_m' constant not to induce dilatancy, one gets the linear relation between the void ratio e and $\ln \sigma_m'$ with the proportional constant λ. Summing up the differential forms of these relations, we get the fundamental state equation :

$$de=-\lambda\frac{d\sigma_m'}{\sigma_m'}+(1+e_0)\mu\left(\frac{\tau_{oct}}{\sigma_m'}\cdot\frac{d\sigma_m'}{\sigma_m'}-\frac{d\tau_{oct}}{\sigma_m'}\right) \quad \cdots\cdots(1)$$

where e_0 is the initial void ratio.

2. STATE SURFACE AND SWELLING WALL

Henkel and Sowa (1963)[8] showed that the obliquity of $e-\ln\sigma_m'$ plot in the case of anisotropic consolidation is consident with that in the case of isotropic consolidation. But it is not ascertained that the characteristic of dilatancy mentioned above is valid for anisotropically consolidated clays. Therefore, the validity of equation (1) is an assumption in the present stage. The coefficient μ may also be different from that of isotropically consolidated clays. It is note worthy that the volume change due to dilatancy does not take place during the anisotropic consolidation process which is defined as the loading with the constant stress ratio τ_{oct}/σ_m'. Let the void ratio be e_0 and applied stresses be τ_{octo}, σ_{m0}' at the final stage of anisotropic consolidation with the stress ratio $\tau_{oct}/\sigma_m'=k$.

Solving the fundamental equation (1) with the initial conditions given above, we get :

$$e-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{m0}'}+(1+e_0)\mu\left(\frac{\tau_{oct}}{\sigma_m'}-k\right)=0 \quad \cdots\cdots(2)$$

The surface represented by equation (2) is the *state surface* of anisotropically consolidated clays and it is reduced to be the state surface of isotropically consolidated clays when $k=0$. Equation (2) is the representation of the mechanical characteristics of the anisotropically consolidated clays. That the stress variables of equation (2) are the invariants is based on the tacit premise that the anisotropically consolidated clays are initially isotropic. In order to use equation (2) as the foundation of the theory, it should be assumed that anisotropically consolidated clays are the initially isotropic materials under the anisotropic stress state.

But this assumption is not supported by the experimental fact that the clay particles of anisotropically consolidated clays with no lateral deformation tend to be parallel.

Therefore the following discussion is based on the assumption that the microscopic anisotropy of clay skelton does not affect seriously on the macroscopic mechanical behavior of anisotropically consolidated clays.

Generally speaking, it is convenient to consider the deformation of materials under loading process

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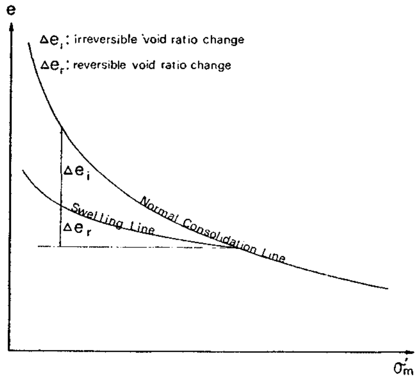

Fig. 1 Consolidation and swelling process of clay (τ_{oct}/σ_m'—constant)

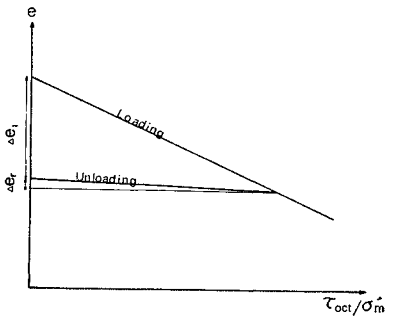

Fig. 2 Dilatancy of clay (σ_m'—constant)

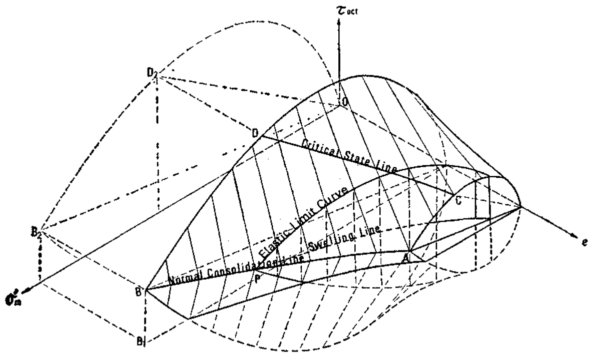

Fig. 3(a) State surface

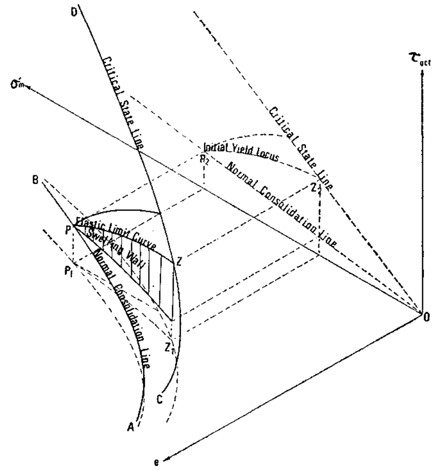

Fig. 3(b) Swelling wall

consists of *recoverable* (elastic) deformation and *irreversible* (plastic) deformation. Then it may be useful to divide the void ratio change due to consolidation and dilatancy into the two parts : elastic and plastic component of the void ratio change i.e. elastic and plastic component of volume strain. Fig. 1 shows the consolidation and the swelling processes of clay. The elastic and the plastic component of the void ratio change in the stress cycle of loading and unloading from the stress level σ_{m0}' to the same stress level σ_{m0}' are given in Fig. 1. It is well confirmed that the loading curve in the $e-\ln\sigma_m'$ diagram is plotted as a straight line with the obliquity λ. The unloading curve can be also approximated as a straight line with the obliquity κ. These two lines are respectively called normally consolidation line and swelling line. The linearity of the swelling line is not so complete as that of the normally consolidation line. The elastic and plastic component of the void ratio change in the stress cycle of loading and unloading from the stress level $\tau_{oct}/\sigma_m'=k$ to the same stress level τ_{oct}/σ_m' are given in Fig. 2. It may not be so extraordinary to assume the elastic component of dilatancy is negligiblly small. According to the considerations mentioned above, it is concluded that the characteristics of the reversible component of the void ratio change of clay are able to be represented by the equation of the swelling line :

$$e-e_y=-\kappa\ln\frac{\sigma_m'}{\sigma_{my}'} \qquad \cdots\cdots(3)$$

where (e_y, σ_{my}') is the intersection of the swelling line and the normal consolidation line in Fig. 1.

As mentioned above, the state surface is a surface in the τ_{oct}, σ_m', e space i.e. the *state space.* The swelling curve given by equation (3) is corresponding to a wall-like surface normal to the $e-\sigma_m'$ plane in the state space which is called *swelling wall* or *elastic surface.* The isometric views of the state surface and the swelling wall is given are Fig. 3(a) and (b).

3. STATE PATH

The mechanical state of clay is assumed to be represented by the three parameters τ_{oct}, σ_m' and e. In other words, the mechanical state of clay can be represented as a point in the state space called *state point.* When the mechanical state of a volume element of clay changes gradually, the correponding state point traverses in the state space. The locus of this state point is called *state path*. Equation (2) gives the void ratio of a volume element of clay under the stress state of (τ_{oct}, σ_m') and in the other sence, it gives the restriction on the mechanical state of the volume element. Then the state point of the volume element should be on the state surface given by equation (2). It is note worthy that equation (2) is derived from the characteristics of consolidation and dilatancy which is assumed as the purely irreversible plastic phenomenon. Therefore the state point on the state surface represented by equation (2) means that the volume element is yielding i.e. the mechanical state of the clay is in elastic-plastic range. Combining the concept of the state surface and swelling wall, it can be reasoned out that the volume element of the clay yields initially when the state point arrives at the intersection of the state surface and the swelling wall as shown in Fig. 3. The stress condition for initial yielding is given by the projection of this intersection which is called *elastic limit curve* onto the stress space i.e. $\tau_{oct}-\sigma_m'$ plane as proposed by Calladine (1963)[9].

The projection of the elastic limit curve onto the $\tau_{oct}-\sigma_m'$ plane is the initial yield locus. The term "*yield*" in this paper means merely the mechanical state accompanied by the irreversible deformation, but not the state commonly called "*failure state*" in the field of soil mechanics. Therefore a volume element of clay changing its volume due to dilatancy is said to be in the state of yielding. Judging from the stress-volumetric strain relations shown in Fig. 1 and 2, one can easily recognize that clays are not completely plastic materials, but stable work-hardening materials. However, the stress- shear strain relation of clay does not necessarily suggest that clays are stable*.

As mentioned above, the state point on the state surface represents the mechanical state of the clay on the yielding process, i.e., the elastic-plastic state. On the other hand, the state point on the swelling wall given by equation (3) represents the purely elastic state. Therefore, the state point representing the mechanical state of clay must be either on the swelling wall or on the state surface. After the consideration on purely elastic state and elastic-plastic state, it is necessary to consider on perfectly plastic state in which the elastic component of deformation does not increase at all. Such a perfectly plastic state is represented only by the state points on a curve lying on the state surface. This curve is named *critical state line* by Roscoe, Schofield and Wroth (1958)[10].

Summing up the above considerations, it can be concluded that the trace of the state point, state path, of a volume element of anisotropically consolidated clay is characterized generally as follows. With the advance of the loading process from the initial mechanical state to an arbitrary stress state, a volume element of clay is remained in the elastic state at the initial stage of the loading and becomes to be in the elastic-plastic state accompanied by the work-hardening effect after the initial yielding and finally arrives at the perfectly plastic state. The movement of the state point correspondent to this process is as follows. Starting from the initial state, the state point climbs up along the swelling wall and after the initial yielding, i.e., after arriving at the elastic limit curve it creeps over the state surface and finally reaches the critical state line lying on the state surface.

4. UNDRAINED SHEAR

The initial state of normally consolidated clay anisotropically compressed with the pressure of σ_{m0}' keeping the stress ratio τ_{oct}/σ_m' constant is as follows :

$$e=e_0,\ \sigma_m'=\sigma_{m0}',\ \tau_{oct}/\sigma_m'=\tau_{oct0}/\sigma_{m0}'=k$$

The initial state of over-consolidated clay rebounded from the normally consolidated state with the constant stress ratio $\tau_{oct}/\sigma_m'=k$ is given by

$$e=e_i \gtrsim e_0,\ \sigma_m'=\sigma_{mi}'\lesssim\sigma_{m0}',\ \tau_{oct}/\sigma_m' =\tau_{octi}/\sigma_{mi}'=k$$

Then the state points of normally and over consolidated clay should be on the swelling curve lying on the $\tau_{oct}/\sigma_m'=k$ plane as shown in Fig. 4. The initial state point on the swelling curve passing through the point (τ_{oct0}, σ_{m0}', e_0) is represented by (τ_{octi}, σ_{mi}', e_i) where

$$\tau_{octi}/\sigma_{mi}'=k \qquad (4)$$

and

$$e_i=e_0-\kappa\ln\frac{\sigma_{mi}'}{\sigma_{m0}'}\gtrsim e_0 \qquad (5)$$

For normally consolidated clay, it follows :

$$e_i=e_0,\ \sigma_{mi}'=\sigma_{m0}',\ \tau_{octi}=\tau_{oct0}$$

It is apparent that the state path of anisotropically consolidated clay stressed under undrained condition i.e., with the constant void ratio e_i is given by the intersection between $e=e_i$ plane and the swelling wall or the state surface. The equation representing the swelling wall is derived from equation (3) by substituting $e_y=e_0$ as follows :

$$e-e_0=-\kappa\ln\frac{\sigma_m'}{\sigma_{m0}'} \qquad (6)$$

and from equations (5) and (6), the intersection of the swelling wall and $e=e_i$ plane given as :

$$e=e_i,\ \sigma_m'=\sigma_{mi} \qquad (7)$$

* The definition of stability is $\dot\sigma_{ij}\dot\varepsilon_{ij}\geqq0$, i.e., $\dot\sigma_x\dot\varepsilon_x+\dot\sigma_y\dot\varepsilon_y+\dot\sigma_z\dot\varepsilon_z+\dot\tau_{xy}\dot\gamma_{xy}+\dot\tau_{yz}\dot\gamma_{yz}+\dot\tau_{zx}\dot\gamma_{zx}\geqq0$. Therefore to discuss on the stability separately for volumetric strain and for shear strain is not correct in the strict sense.

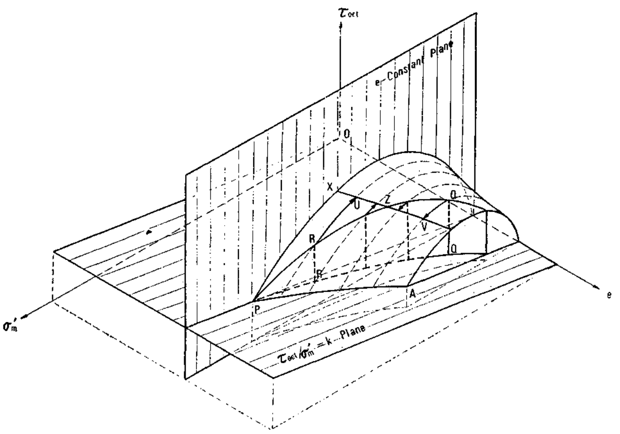

Fig. 4 State paths (Undrained shear)

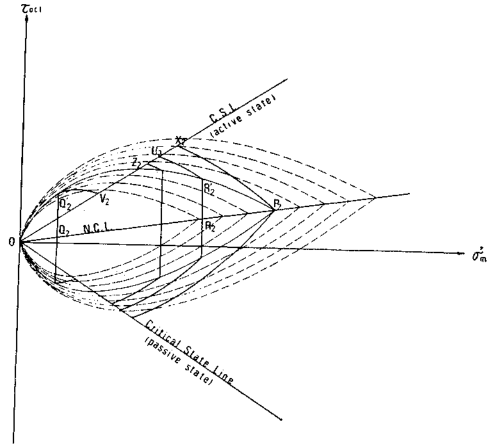

Fig. 5 Stress paths (Undrained shear)

Equations (7) show a line normal to the $e-\sigma_m'$ plane. As mentioned above, the state point of a volume element of clay in the purely elastic state is on the swelling wall and after it reaches the elastic limit curve which is given as the intersection of swelling wall and state surface the clay element begins to yield with work-hardening or work-softening effect. Then the line given by equations (7) must lead to the intersection of state surface and $e=e_i$ plane across the elastic limit curve. The initial yield locus given as the projection of the elastic limit curve onto the stress space is derived from equations (2) and (6) as follows :

$$\frac{\tau_{oct}}{\sigma_m'}-k=-\frac{\lambda-\kappa}{(1+e_0)\mu}\ln\frac{\sigma_m'}{\sigma_{m0}'} \quad \cdots\cdots\cdots\cdots\cdots(8)$$

The height of the line represented by equation (7) is given from equations (7) and (8) as follows :

$$\tau_{oct}=\sigma_{mi}'\left\{k-\frac{\lambda-\kappa}{(1+e_0)\mu}\ln\frac{\sigma_{mi}'}{\sigma_{m0}'}\right\} \quad \cdots\cdots\cdots\cdots(9)$$

and depends on the value of σ_{mi}'. In Fig. 4, the normal anisotropic consolidation curve is AP and the point P on the normal consolidation curve is ($e=e_0$, $\sigma_m'=\sigma_{m0}'$, $\tau_{oct}=\tau_{oct0}$). XZV lying on the state surface is the curve showing the perfectly plastic state and named as the critical void ratio line by Roscoe, Schofield and Wroth (1958)[10]. and renamed as the critical state line by Roscoe, Schofield and Thurairajah (1963)[11]. The projection of this critical state line onto $\tau_{oct}-\sigma_m'$ plane is the straight line passing the origin. Further consideration about the critical state line is given later. For normally consolidated clay, equation (9) becomes

$$\tau_{oct}=k\cdot\sigma_{m0}'=\tau_{oct0}$$

with $\sigma_{mi}'=\sigma_{m0}'$, then it can be concluded that there is no purely elastic state for normally consolidated clay stressed under the undrained condition. Now, the state path of the element of yielding clay is represented by the intersection between the state surface and $e=e_i$ plane under undrained condition. Such a state path can be derived from equation (2) by substituting $e=e_i$ as follows :

$$e_i-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{m0}'}+(1+e_0)\mu\left(\frac{\tau_{oct}}{\sigma_m'}-k\right)=0 \quad \cdots\cdots\cdots\cdots(10)$$

Equation (10) shows that the state path of clay stressed under undrained condition is dependent on the initial value of void ratio e_0. In Fig. 4, the state paths lying on the state surface are represented by PX, RU and VQ. For normally consolidated clay, $e_i=e_0$ and therefore equation (10) becomes

$$\frac{\tau_{oct}}{\sigma_m'}=-\frac{\lambda}{(1+e_0)\mu}\ln\frac{\sigma_m'}{\sigma_{m0}'}+k \quad \cdots\cdots\cdots\cdots(11)$$

giving the state path PX.

On the loading process, the elastic-plastic state path mentioned above is to reach the final point on the critical state line which represents the perfectly plastic state, i.e., the state parameter τ_{oct}, σ_m' and e do not change any more at the perfectly plastic state. Fig. 4 shows that the critical state line lying on the state surface intersects with the elastic limit curve at the point Z. An overconsolidated clay specimen whose initial state point is just under Z cannot experience the elastic-plastic state on the shearing process under undrained condition, because the state point arrives at the critical state line immediately when it reaches the elastic limit curve after experiencing the purely elastic state. The stress path is defined as the projection of the state path onto the stress space, for instance the $\tau_{oct}-\sigma_m'$ plane, as shown in Fig. 5.

In the above discussion, the domain enclosed by state surface, $\tau_{oct}=0$ plane, $\sigma_m'=0$ plane and $e=0$ plane is considered to show the purely elastic state. Then, the plastic deformation should not be found even in the repeated loading and unloading process so far as the state point is in the elastic domain, i.e., on such a process the loading and unloading state path should coincide each other. However, on the contrary, experimental data given by Murayama and Kurihara (1969)[12], Sangrey, Henkel and Esrig (1969)[13] show that the induced pore pressure in the normally consolidated clay increases monotonously on the undrined repeated shear process. This means that the state paths for loading and unloading process do not coincide each other and furthermore the state path shifts to the negative direction of σ_m' axis in accordance with the repetition. To eliminate such a difficulty, only the loading processes with monotonously increasing shear strain are to be treated in this paper.

The undrained state paths obtained in the above discussion are corresponding to the case of increasing shear stress from the initial state. Let us consider the case of decreasing octahedral stress from the initial state of $\tau_{oct}=\tau_{octi}$. Judging from the data on normally consolidated clay given by Skempton and Sowa (1963)[14], the state path for this case seems to be represented by equation (10) with the alternation of sign of the last term, i.e.,

$$e_i-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{m0}'}+(1+e_0)\mu\left|\frac{\tau_{oct}}{\sigma_m'}k\right|=0 \quad \cdots(12)$$

Then the state surface should be represented by

$$e-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{m0}'}+(1+e_0)\mu\left|\frac{\tau_{oct}}{\sigma_m'}-k\right|=0 \quad \cdots\cdots(13)$$

To keep the logical consistency, the equation of the initial yield locus (8) should be converted to the corresponding form as :

$$\left|\frac{\tau_{oct}}{\sigma_m'}-k\right|=-\frac{\lambda-\kappa}{(1+e_0)\mu}\ln\frac{\sigma_m'}{\sigma_{m0}'} \quad \cdots\cdots\cdots\cdots(14)$$

The initial yield locus given by the equation (14) intersects with the anisotropic consolidation line $\tau_{oct}/\sigma_m'=k$ at its vertex as shown in Fig. 5. Fig. 6 shows the state paths given by equation (12) on the $e-\sigma_m'$ plane.

5. DRAINED SHEAR

Generally speaking, the mean effective stress σ_m' and octahedral shear stress τ_{oct} are able to be applied to clay specimen independently from each other under drained condition. The state path of a clay element in elastic-plastic state can be obtained by substituting the appllied stresses σ_m' and τ_{oct} into equation (2). Here, let us consider a few cases corresponding to some special stress paths. The stress path for the conventional triaxial drained test carried on by changing the axial stres with constant radial stress is given as follows :

$$\tau_{oct}=\sqrt{2}\,(\sigma_m'-\sigma_{mi}')+k\,\sigma_{mi}' \quad \cdots\cdots\cdots\cdots(15)$$

And for the loading system of changing radial stress with constant axial stress, the stress path is represented as :

$$\tau_{oct}=-\frac{1}{\sqrt{2}}(\sigma_m'-\sigma_{mi}')+k\,\sigma_{mi}' \quad \cdots\cdots\cdots(16)$$

Equations (15) and (16) are both representing the planes in the state space. Then the state paths corresponding to such cases should be on these planes. From the above consideration, it can be concluded that : (a) the stress path in elastic state is given as the intersection of the plane defined by equation (15) or (16) with the swelling wall given by equation (6) and (b) the stress path in elastic-plastic state is given as the intersection of the plane defined by equation (15) or (16) with the state surface given by equation (13). These state paths are shown in Fig. 7 and 8 for the loading systems with the condition

$$\frac{\tau_{oct}}{\sigma_m'}\geqq k.$$

Fig. 9 shows the projections of these state paths onto the $e-\sigma_m'$ plane. It is the well accepted expe-

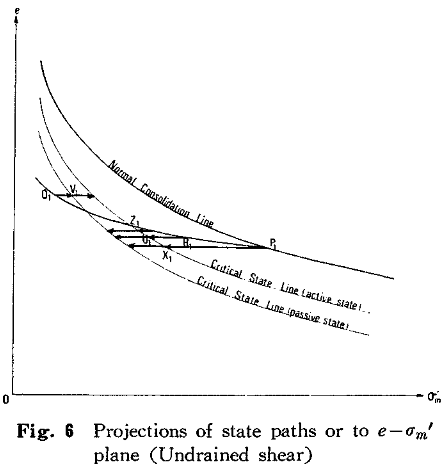

Fig. 6 Projections of state paths or to $e-\sigma_m'$ plane (Undrained shear)

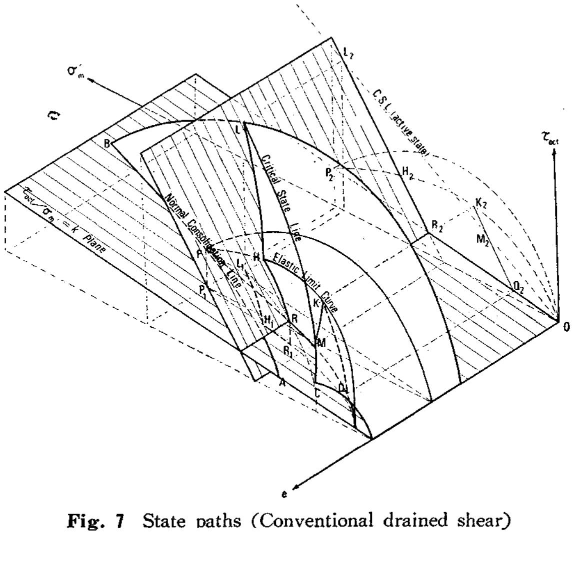

Fig. 7 State paths (Conventional drained shear)

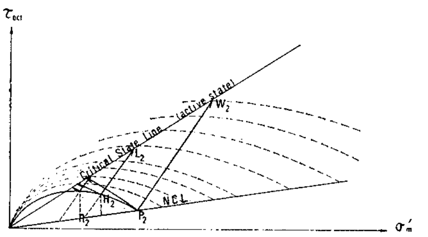

Fig. 8 Stress paths (conventional drined shear)

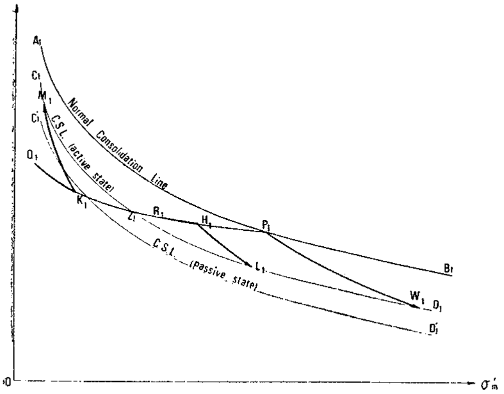

Fig. 9 Projections of state paths onto $e-\sigma_m'$ plane (Conventional drained shear)

rimental result that the volume of the heavily overconsolidated clay specimen or sand specimen decreases in the beginning of conventional drained compression test and then increases afterwards. Fig. 9 suggests that such a tendency is merely arising from the initial state and the stress path and is not the general mechanical behavior of such a soil stressed under the drained condition. In the textbooks of soil mechanics, the volume increase and volume decrease of soil induced by the shear stress are called positive and negative dilatancy respectively. But in the Authors' opinion, such a definition of dilatancy is not physically important. So called positive and negative dilatancy do not represent the essential mechanical behavior of soil. They can be controlled by the combination of the initial state and the stress condition of loading system.

Now let us consider the case of drained shear under constant effective mean stress σ_m' in which the stress path is confined on the plane defined as follows :

$$\sigma_m'=\sigma_{mi}' \quad \cdots\cdots(17)$$

where σ_{mi}' is the initial effective mean stress. In elastic state, the state path is given by the intersection of the swelling wall and the plane given by equation (17). And the state path in elastic-plastic state is given as the intersection between the state surface and $\sigma_m'=\sigma_{mi}'$ plane. The isometric view of

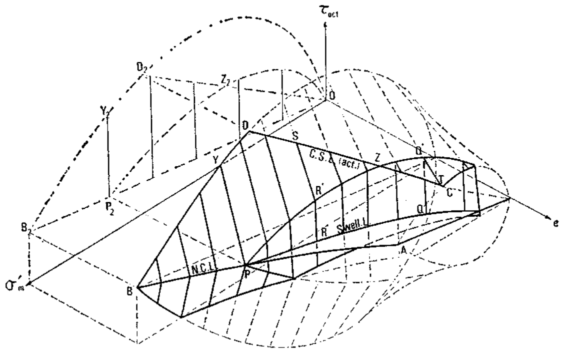

Fig. 10 State paths (σ_m'—constant test)

these state paths for the stress condition of $\tau_{oct}/\sigma_m' \geqq k$ is shown in Fig. 10. Substituting $\sigma_m'=\sigma_{mi}'$ into equation (13) for the stress condition $\tau_{oct}/\sigma_m' \geqq k$ leads to

$$e-e_0+\lambda \ln\frac{\sigma_{mi}'}{\sigma_{m0}'}+(1+e_0)\mu\left(\frac{\tau_{oct}}{\sigma_{mi}'}-k\right)=0 \quad \cdots\cdots(18)$$

and it can be concluded that the void ratio e and octahedral shear stress τ_{oct} are linearly related each other under constant effective mean stress σ_m'. Then we can see that the intersection of the state surface with the $\sigma_m'=\sigma_{mi}'$ plane is always a straight line regardless of σ_{mi}'. The intersecting point of $\tau_{oct}/\sigma_m'=k$ plane with this straight line is given from equation (18) by eliminating the third term as follows :

$$e-e_0+\lambda \ln\frac{\sigma_{mi}'}{\sigma_{m0}'}=0 \quad \cdots\cdots(19)$$

Equation (19) shows that the intersecting point is to be on the normal consolidation curve. In other words, the straight lines given by equation (18) intersect with the normal consolidation curve passing through the point (e_0, σ_{m0}') on $\tau_{oct}/\sigma_m'=k$ plane. Because the obliquities of these straight lines are represented by $\sigma_{mi}'/(1+e_0)\mu$, the obliquity of the line corresponding to $\sigma_{mi}'=0$ is to be zero. From the above consideration, it can be concluded that the state surface is constructed with infinite number of straight lines whose obliquities increase in order of the value of σ_m'.

6. CRITICAL STATE

The critical state is defined as the perfectly plastic state of soil. Roscoe, Schofield and Wroth (1958)[10] defined the critical state by

$$\frac{de}{d\gamma_{oct}}=\frac{d\sigma_m'}{d\gamma_{oct}}=\frac{d\tau_{oct}}{d\gamma_{oct}}=0 \quad \cdots\cdots(20)$$

where γ_{oct} is the octahedral shear strain. Assuming anisotropically consolidated clay to yield according to the associated flow rule, the critical state can be defined as the stationary points of the yield loci on the $\tau_{oct}-\sigma_m'$ diagram. The initial yield locus is

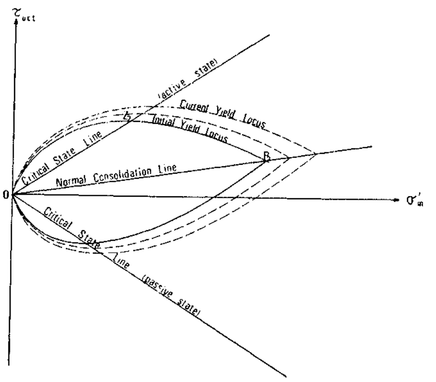

Fig. 11 Critical state line

already given by equation (14). After the initial yielding, the yield locus expands or shrinks according to the work-hardening or work-softening effect. It is natural to consider the yield loci to be the projections of some certain curves on the state surface onto the stress space, onto $\tau_{oct}-\sigma_m'$ plane, because the state points of the yielding clay are on the state surface. Calladine (1963)[9] suggested the yield loci to be given as the projections of the intersections between the state surface given by equation (13) and the swelling walls given by equation (3). Eliminating the void ratio from equations (3) and (13), we get

$$e_y-\kappa\ln\frac{\sigma_m'}{\sigma_{my}'}-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{mo}'}+(1+e_0)\mu\left|\frac{\tau_{oct}}{\sigma_m'}-k\right|=0 \quad\cdots\cdots(21)$$

The point (e_y, σ_{my}') is on the normal consolidation curve passing through the point (e_0,σ_{m0}') and then it follows :

$$e_y-e_0=-\lambda\ln\frac{\sigma_{my}'}{\sigma_{mo}'} \quad\cdots\cdots(22)$$

Substituting equation (22) into equation (21), the yield loci can be derived as follows :

$$\left|\frac{\tau_{oct}}{\sigma_m'}-k\right|=-\frac{\lambda-\kappa}{(1+e_0)\mu}\ln\frac{\sigma_m'}{\sigma_{my}'} \quad\cdots\cdots(23)$$

where σ_{my}' is the parameter representing the hardening effect.

Now the critical state, the stationary points of the yield loci, is given by differentiating equation (23) as follows :

$$\left(\frac{\tau_{oct}}{\sigma_m'}\right)_{crit}=\frac{\lambda-\kappa}{(1+e_0)\mu}$$

$$\text{for } \frac{\tau_{oct}}{\sigma_m'}\geqq k\ (\textit{active state})\quad\cdots\cdots(24)$$

$$\left(\frac{\tau_{oct}}{\sigma_m'}\right)_{crit}=-\frac{\lambda-\kappa}{(1+e_0)\mu}$$

$$\text{for } \frac{\tau_{oct}}{\sigma_m'}< k\ (\textit{passive state})\quad\cdots\cdots(25)$$

Fig. 11 shows the critical state line on $\tau_{oct}-\sigma_m'$ plane. It should be noted that the value of the stress ratio τ_{oct}/σ_m' at the critical state is independent of its initial value $(\tau_{oct}/\sigma_m')_{initial}=k$. This leads to the conclusion that the critical states of clays anisotropically consolidated with various stress ratio τ_{octi}/σ_{mi}' are uniquely determined by

$$\tau_{oct}/\sigma_m'=\frac{\lambda-\kappa}{(1+e_0)\mu}$$

if the coefficients λ, κ, μ do not vary according to the value of τ_{octi}/σ_{mi}'. In order to clarify the shape of the critical state line in the state space, equations (24) and (25) are to be substituted into equation (13) representing the state surface as follows :

$$e-e_0+\lambda\ln\frac{\sigma_m'}{\sigma_{mo}'}+\lambda-\kappa-k(1+e_0)\mu=0$$

$$\text{for } \frac{\tau_{oct}}{\sigma_m'}\geqq k\quad\cdots\cdots(26)$$

Equation (26) gives the projection of critical state line onto the $e-\sigma_m'$ plane. The critical state line on the $e-\sigma_m'$ plane coincides to the shifted normal consolidation curve be the amount of $\lambda-\kappa-k(1+e_0)\mu$ parallel to the e-axis downward.

7. CONCLUDING REMARKS

It has been shown that the mechanical behavior of anisotropically pre-consolidated clay could be reduced mathematically with the adequate initial conditions and the confining conditions from the concepts of state surface and swelling wall defined by three soil constants, compression index λ, swelling index κ, dilatancy index μ, and the initial stress ratio k and the initial void ratio e_0.

The basic assumptions of the theory are as follows :

a) Anisotropically consolidated clay can be assumed to be initially isotropic.
b) The volume change of anisotropically consolidated clay can be derived from the characteaistics of consolidation and dilatancy.
c) The anisotropically consolidated clay can be considered as the elastic-plastic material with work-hardening and work-softening effects.
d) The associated flow rule can be adaptable for the anisotropically consolidated clay.

At the present stage, the theory is adaptable only for the anisotropically pre-consolidated clay deforming with the monotonously increasing shear strain under the conditions of sufficiently slow strain rate and constant temperature.

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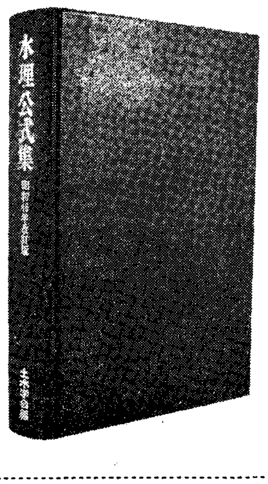

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