ON THE PRINCIPLES AND THE APPROXIMATE CALCULATION METHODS OF TRAFFIC ASSIGNMENT

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1. INTRODUCTION

A number of calculation methods in traffic assignment have been proposed and developed so far. But the author considers that they can be classified into the following three principles, the principle of travel time ratio[1], that of equal travel times[2] and that of minimizing total travel time[3]. The principle of travel time ratio is that, if our attention is paid to a certain OD traffic, the shorter the travel time of a route is, the higher the choice rate of it becomes. The principle of equal travel times is that travel times are identical on all routes used between two zones and less than (equal to) the travel times on all unused routes. And the principle of minimizing total travel time is that the total travel time of the flows over the road network is the smallest.

Among these principles there exist such relationships as follows.

Let us consider first the relationship between the principle of travel time ratio and that of equal travel times[1]. The assignment equation of the principle of travel time ratio is formulated such as choice rate of a route between an OD is in conversely proportion to n-th power of its travel time. In this equation it means that, the larger the value of the coefficient of travel time ratio n is, the more sharply travel time reacts to choice of a route, and that, against this, when n is achieved to zero routes are chosen equally in probability. Namely, it shows that in former case the solution comes near to that of the principle of equal travel times and in latter case, to that of uniformity assignment.

On the other side, the relationship between the principle of equal travel times and that of minimizing total travel time are explained by H. Inoue[4]. In short, it is concluded that these two assignment principle can be treated by a same method if only the capacity function which denotes relation between traffic volume and travel time on a road sec-

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tion is transformed.

Now the auther have investigated the uniqueness of the set of solutions on the principle of travel time ratio through a simple example in which routes between an OD do not overlap each other on any road section and proposed the calculation methods[1]. One is an iteration method: if the principle of travel time ratio of an OD is satisfied, the OD is exchanged for another OD and the procedure is continued until those of all OD are realized. The another is a division method. The division method in this principle is that: an OD table is sliced into a number of decomposed layers at the beginning. And whenever the assignment of the OD traffic demands of each layer to the given routes is over, the travel time of each road section is adjusted and the next sliced OD traffic demands are assigned on the basis of the travel time ratio adjusted just now. This procedure is continued till all OD traffic demands are assigned up.

After this through some examples, it is made sure that the results of these two methods are not so much defferent. From this, furthermore, the uniqueness of the set of solutions based on the principle of travel time ratio can be induced, if assigned routes are given, even though routes of some OD traffic overlap each other.

By the way, in the division method used very often up to now OD traffic demands of each layer are assigned to their shortest routes at any stage of the procedure[5]. Therefore it can be understood that the precedent division method is equivalent to the case that the coefficient of travel time ratio n is infinity and found that the assignment had been performed on the basis of the principle of equal travel times.

The result by the precedent division method, however, is not always consistent with that of the principle of equal travel times. Because it may sometimes, not so often, happen that the existing routes based on the principle of equal travel times vanish when the OD traffic demands increase under the constant OD pattern (which is the normalized

ratio of OD traffic demands)[6].

Accordingly, the precedent division method should be regarded as the method of an approximate calculation for the principle of equal travel times.

If it may be admitted to perform the assignment on the basis of the principle of equal travel times by the division method, the assignment based on the principle of minimizing total travel time can be carried out equally from the reason described above.

After all, the solution of the principle of travel time ratio, that of equal travel times and that of minimizing total travel time can be all obtained by the division method. But as above, these solutions do not always agree with their exact solutions even though the number of the division is so large. Nevertheless even in that case it may be considered that the solutions will not be so much different from their exact solutions.

There exist methods to obtain the exact solutions based on each traffic assignment principle respectively[1],[3],[7], but they are all theoretically advanced, and the computational procedure are considerably complicated. As compared with this, the division method can be easily understood by anyone and has a merit to be able to calculate briefly even if the capacity function is nonlinear. Besides, when we evaluate an actual road network, considering the irregularity and the fluctuation of traffic demands, even the approximate calculation by the division method may be sufficient. Accordingly, it can be said that the division method should be utilized as a practical and convenient method.

Meanwhile, the author considers that those assignment principle should be used as follows. First, as for the principle of minimizing total travel time, since the object of this principle is maximization of road network efficiency, so travellers are obliged to behave in such a way that they consider the overall situation and not just what suit themselves. And so the situation caused by this principle seems unlikely to be achieved in practice, as far as they choose subjectly their routes which seem to be more desirable. Therefore, this principle should be applied for problems of transportation planning for goods, for instance, those which are carried whithin a factory. That is, a traffic phenomenon on a road network can not be dealt with by this principle. Conversely, travellers' willingness for the choice of route are considered in the principle of travel time ratio and in that of equal travel times. Where, it is assumed that any driver chooses a root that seems to be quicker. Then, we can say that the principle of travel time ratio will come into being if information for their choice of root, or travel time, the degree of congestion and road condition, are vague.

And the principle of equal travel times will be realized if any driver gets information for that completely. Ina word, it can be said that the former principle is based on the standpoint of imperfect information and the latter principle is based on that of perfect information.

Therefore, if the distance of an OD traffic is very long the principle of travel time ratio may be applied and if short the principle of equal travel times may be used. But in the case of the principle of travel time ratio there exists a unique set of solutions respectively according to how to designate the routes to be assigned. It can be said, therefore, that the evaluation of the traffic flow over the whole road network based on this principle is wanting in objectivity in this aspect. On the other hand, because a set of solutions based on the principle of equal travel times, which is the set of traffic demands on road sections, are uniquely determined such a problem will never happen. Hence if we want to lay emphasis on objectivity in the evaluation of the road network, the assignment is obliged to carry out on the basis of the principle of equal travel times.

2. ASSIGNMENT EQUATION ON THE BASIS OF THE PRINCIPLE OF TRAVEL TIME RATIO[1]

Let us denote the ρ-th path flow of the k-th OD and its travel time as X_ρ^k and T_ρ^k respectively. Then, the choice rate of the ρ-th path between k-th OD, m_ρ^k is determined by their travel time ratio as below.

$$m_\rho^k = \frac{X_\rho^k}{Q_k} = \frac{(T_\rho^k)^{-n}}{\sum_{\rho=1}^{r_k} (T_\rho^k)^{-n}}, \quad \binom{\rho=1, 2, \cdots, r_k}{k=1, 2, \cdots, q}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

Where, we assume that there exist q OD pairs and that the k-th OD has r_k routes and the traffic demand is Q_k. In eq. (1), n is the coefficient of travel time ratio. If $r=2$ and $n=6$, eq. (1) is equivalent to the equation of conversion ratio shown by AASHO.

The travel time T_{ij} over the road section ij increses as traffic volume X_{ij} on it grows, and so let us express the relation between them as

$$T_{ij} = \phi(X_{ij}). \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

As X_{ij} is composed of the path flows passing through there, it is written as

$$X_{ij} = \sum_{k, \rho \in ij} X_\rho^k. \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

From this, eq. (2) is represented as follows.

$$T_{ij} = \phi(\sum_{k, \rho \in ij} X_\rho^k) \cdots\cdots\cdots\cdots\cdots(4)$$

And T_ρ^k is written from eq. (4) as

$$T_\rho^k = \sum_{ij \in k, \rho} T_{ij} = \sum_{ij \in k, \rho} \phi(\sum_{k, \rho \in ij} X_\rho^k). \cdots\cdots(5)$$

Accordingly, eq. (1) is

$$\frac{X_\rho^k}{Q_k} = \frac{\{\sum\limits_{ij\in k,\,\rho}\phi(\sum\limits_{k,\,\rho\in ij}X_\rho^k)\}^{-n}}{\sum\limits_{\rho=1}^{r_k}\{\sum\limits_{ij\in k,\,\rho}\phi(\sum\limits_{k,\,\rho\in ij}X_\rho^k)\}^{-n}},$$

$$\begin{pmatrix}\rho=1,\,2,\,\cdots,\,r_k-1\\k=1,\,2,\,\cdots,\,q\end{pmatrix}\cdots\cdots\cdots(6)$$

In eq. (6) the number of equation as to k-th OD is r_k-1, because X_ρ^k must always satisfy the OD condition. The OD condition is that the sum of the path flows of an OD is consistent with the traffic demand of the OD.

$$\sum_{\rho=1}^{r_k}X_\rho^k=Q_k \quad(k=1,\,2,\,\cdots,\,q)\cdots\cdots\cdots\cdots(7)$$

Consequently, the assignment can be performed by solving the higher order simultaneous equation composed of eq. (6) and eq. (7). Here, it is guaranteed that the set of solutions of this is determined uniquely[1].

3. ITERATION METHOD AND DIVISION METHOD IN THE ASSIGNMENT BASED ON THE PRINCIPLE OF TRAVEL TIME RATIO

(1) The iteration method[1]

Let us concider a simple example that there exists a single OD pair with paralleled routes. And let us simplify the notation and denote path flow and traffic demand of the OD as X_ρ and Q. Then, eq. (6) is equivalent to the following equation.

$$X_1\phi^n(X_1)=X_2\phi^n(X_2)=\cdots=X_\rho\phi^n(X_\rho)=\cdots$$
$$=X_r\phi^n(X_r)\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

Each term of eq. (8) is a monotone increasing function for positive X. If one of the variables X_ρ is given a certain positive constant, the other variables X_l ($l\neq\rho$) get a unique set of positive constants peculiar to X_ρ. This is evident from the characteristics of the function $F(X_l)$ shown in eq. (9). Therefore, if X_ρ is given, X_l can be obtained by searching the point satisfying $F(X_l)=0$.

$$F(X_l)=X_l\phi^n(X_l)-C=0\cdots\cdots\cdots\cdots\cdots(9)$$

where, $C=X_\rho\phi^n(X_\rho)$ is given.

Now, let us express the relation X_l and X_ρ as below.

$$X_l=h_l(X_\rho),\quad(l\neq\rho,\,l=1,\,2,\,\cdots,\,r)\cdots\cdots\cdots(10)$$

where,

$$\frac{dh_l(X_\rho)}{dX_\rho}>0,\text{ for }X_\rho>0\cdots\cdots\cdots\cdots\cdots(11)$$

Substituting eq. (10) for eq. (7) and transposing Q, let us define the following function $G(X_\rho)$ as

$$G(X_\rho)=X_\rho+\sum_{l\neq\rho}^r h_l(X_\rho)-Q\cdots\cdots\cdots\cdots(12)$$

If $G(X_\rho)=0$, it satisfies the OD condition.

Examining the characteristics of the function G

(X_ρ), it is found that

$$\left.\begin{aligned}&G(0)=-Q<0\quad\text{at }X_\rho=0\\&G(Q)=\sum_{l\neq\rho}^r h_l(Q)>0\quad\text{at }X_\rho=Q\\&\frac{dG(X_\rho)}{dX_\rho}=1+\sum_{l\neq\rho}^r\left[\frac{dh_l(X_\rho)}{dX_\rho}\right]>0,\\&\qquad\text{for }0<X_\rho<Q\end{aligned}\right\}\cdots\cdots(13)$$

Accordingly, X_ρ satisfying $G(X_\rho)=0$ is determined uniquely between zero and Q. At the same time this shows that the other variables X_l are gained uniquely also. Thus as for a single OD traffic it can be said that the set of solutions which satisfies the OD condition and of which components are all positive is determieed uniquely.

The method of this calculationis is as follows. First, as $F(X_l)$ is a monotone increasing and a convex function for $X_l>0$, so X_l which satisfies $F(X_l)=0$ can be obtained easily if only X_ρ is gixen. On the other hand, because it is known that $G(X_\rho)$ is not such a function as convex or concave but a monotone increasing function, the procedure of the calculation is performed in such a way that two points are connected by a straight line, of which one is positive and the another is negative, having the greatest number of iterations respectively.

And this method is also available in case of multiple OD pairs. Namely, since X_{ij} is generally composed of the path flows of various OD pairs, we may make path flows of all OD pairs fixed except for those of a certain OD which is now to be calculated. And the iteration procedure is carried on until travel time ratio of all OD pairs are realized, transferring the OD pair to be calculated successively.

(2) The division method

Against the above iteration method, the division method comes from the fact shown as below.

Observing how a pattern of social phenomenon chages, it can be said generally that a phenomenon in next time point will come into being, based on the information in present time point. For example, if there exist some traffic demands already on the road network, any traveller who is generating newly will choose his route which seems to be more desirable or quicker, judging from the traffic situation caused by the present traffic demands. Namely, such a thing repeats one after another and a pattern of social phenomenon will change gradually. The division method is the method into which this concept is introduced, and its computational procedure is as follows.

The unit OD table P (which is equivalent to OD pattern) is made by dividing the each given OD traffic demand by the total OD trips N. And N is

divided by m. Where it is better that m is as large as possible.

$$\frac{N}{m} = \Delta N \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

First, traffic demand $\Delta N \cdot P$ is assigned to the road network in the ratio of the travel time of each route on which there are no traffic demand. Then the traffic demand on each road section of the first time is gained. And the travel time is corrected to correspond to the traffic demand. And next using this adjusted travel time, the second $\Delta N \cdot P$ is assigned and the traffic demands of the first and the second assignments are summed up on each road section. After this the assignment calculation is similarly continued to the m-th time. Here it can be noticed that the solution approaches to the exact solution to a certain extent as the number of the division m becomes large.

Now let us compare the solution between these two methods.

(3) Numerical examples

In the road network shown in Fig. 1, let us consider there exist four OD traffics. And the traffic demands of them are given respectively as below.

OD 1–4 $Q=2\,000$ trips
OD 2–5 $Q=6\,000$ trips
OD 2–6 $Q=5\,000$ trips
OD 3–5 $Q=9\,000$ trips

And the routes of them are designated as shown in Fig. 1.

The relation between travel time and traffic volume on road section ij is assumed to be a linear function for the present as

$$T_{ij} = a_{ij}X_{ij} + b_{ij} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

where a and b are constants peculiar to the road section and are shown in Table 1. And the coefficient of travel time ratio is supposed to be $n=6$.

In the iteration method there had been a question whether different solutions exist or not if the order

of OD traffics in the computational procedure is changed. But it has been made sure through some numerical examples that a unique set of solutions is always obtained even though the order of them are different. The result is shown in the section of
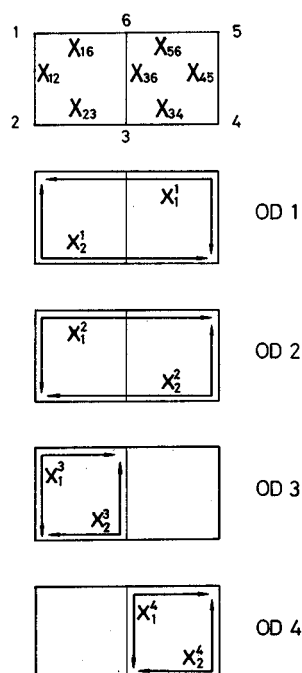

Fig. 1   Road Network, Traffic Demand on Road Section X_{ij} and Path Flow $X_\rho{}^k$.

Table 1   Values of a and b.

Link	a ($\times 10^{-3}$ minutes/cars)	b (minutes)
1–2	1.540	5
2–3	0.860	4
3–4	1.848	6
4–5	1.540	5
5–6	3.220	7
1–6	2.464	8
3–6	4.600	10

Table 2   Travel Time Ratio Assignment by Iteration Method and Division Method, Path Flows ($n=6$).

		OD Traffic Demand	$Q_1=2\,000$		$Q_2=6\,000$		$Q_3=5\,000$		$Q_4=9\,000$	
		Path Flow	$X_1{}^1$	$X_2{}^1$	$X_1{}^2$	$X_2{}^2$	$X_1{}^3$	$X_2{}^3$	$X_1{}^4$	$X_2{}^4$
Iteration Method		Traffic Demand	386	1 614	2 371	3 629	3 075	1 925	2 680	6 320
		Travel Time	67.8	53.5	62.8	58.5	38.3	41.4	55.7	48.3
Division Method	$m=5$	Traffic Demand	414	1 586	2 313	3 687	3 063	1 937	2 616	6 384
		Travel Time	67.6	53.4	62.2	58.8	38.0	41.4	55.1	48.6
	$m=10$	Traffic Demand	412	1 588	2 325	3 675	3 053	1 947	2 605	6 395
		Travel Time	67.6	53.4	62.2	58.8	38.0	41.1	55.1	48.6
	$m=100$	Traffic Demand	409	1 591	2 331	3 669	3 050	1 950	2 603	6 397
		Travel Time	67.6	53.4	62.2	58.8	38.0	41.1	55.1	48.6
	$m=1\,000$	Traffic Demand	408	1 592	2 331	3 669	3 050	1 950	2 603	6 397
		Travel Time	67.6	53.4	62.2	58.8	38.0	41.1	55.1	48.6

Traffic Demand : cars, Travel Time : minutes

Table 3　Travel Time Ratio Assignment by Iteration Method and Division Method, Traffic Demand on Road Section $(n=6)$.

		Road Section	X_{12}	X_{23}	X_{34}	X_{45}	X_{56}	X_{16}	X_{36}
Iteration Method		Traffic Demand	7 060	7 168	11 568	10 335	5 437	5 832	4 605
		Travel Time	15.9	10.2	27.4	20.9	24.5	22.4	31.2
Division Method	$m=5$	Traffic Demand	6 963	7 210	11 657	10 484	5 343	5 790	4 553
		Travel Time	15.7	10.2	27.5	21.1	24.2	22.3	30.9
	$m=10$	Traffic Demand	6 966	7 210	11 658	10 482	5 342	5 790	4 553
		Travel Time	15.7	10.2	27.5	21.1	24.2	22.3	30.9
	$m=100$	Traffic Demand	6 972	7 210	11 658	10 475	5 342	5 790	4 553
		Travel Time	15.7	10.2	27.5	21.1	24.2	22.3	30.9
	$m=1\,000$	Traffic Demand	6 973	7 210	11 658	10 474	5 342	5 790	4 553
		Travel Time	15.7	10.2	27.5	21.1	24.2	22.3	30.9

Traffic Demand : cars,　Travel Time : minutes

iteration method in Table 2 and Table 3.

As regard the division method, in order to observe the difference among solutions caused by the number of the division m, four cases of the calculation are performed; m is 5, 10, 100 and 1 000. The results are shown in the section of division method in Table 2 and Table 3. The interesting here is that there is little difference among those solutions even if the number of the division m is so different.

Now, let us compare the results of the iteration method and the division method. A little difference can be seen in some places, but they may be regarded as errors caused by the difference of the calculation method and it may be concluded that those solutions are the same.

In this example we consider a simple case that the routes of each OD do not overlap each other. But in practice it is more complicated. In such a case, the iteration method must need more complicated computational procedure. Against this the division method can be performed similarly as in this example. From this it can be said that the division method is of practical use. Further, this reduces that the set of solutions is unique even in general case.

4. RELATIONSHIP BETWEEN THE PRINCIPLE OF TRAVEL TIME RATIO AND THAT OF EQUAL TRAVEL TIMES

It can be said that the coefficient of travel time ratio n is the parameter which denotes to what extent travellers get the information to choose their route as a whole (in this paper, only travel time is considered as the information).

Let show actually this by changing the value of n. When $n=1$ and $n=30$, the assignment are performed on the same example. The results are shown in Table 4. Here the division method is used in the calculation and the number of the division is 100. From Table 4 and Table 2 $(n=6)$, it is found that the larger the value of n is, the less the difference among travel times of the routes between each OD becomes. This can be understood easily from eq. (1). That is, the effect of travel time on choosing route goes more conspicuous as n becames larger. And if n is infinity, traffic demands of any OD concentrate into some routes between the OD and travel times of them becomes equal. Namely this result agrees with that of the principle of equal travel times. Conversely, if n is zero, this denotes that travel time is of no value as information for the choice of route. And so, routes are chosen in equal probability. Accordingly, this assignment agrees with the equality assignment. Actually, travel time is uncertain because of variation and irregularity of traffic demand, so each driver chooses his route under the vague information. Therefore, n takes a certain constant between zero and infinity.

Now, in case of the presented division method, each OD traffic demand of any divided layer is respectively assigned to its shortest route under the adjusted travel time. This is equivalent to the case

Table 4　Difference of Solutions Caused by Travel Time Ratio n.

	OD Traffic Demand	$Q_1=2\,000$		$Q_2=6\,000$		$Q_3=5\,000$		$Q_4=9\,000$	
	Path Flow	$X_1{}^1$	$X_2{}^1$	$X_1{}^2$	$X_2{}^2$	$X_1{}^3$	$X_2{}^3$	$X_1{}^4$	$X_2{}^4$
$n=1$	Traffic Demand	824	1 176	2 638	3 362	2 708	2 292	3 576	5 424
	Travel Time	72.7	46.3	67.9	54.1	38.2	46.9	66.7	44.2
$n=30$	Traffic Demand	19	1 981	2 292	3 708	3 291	1 709	2 395	6 605
	Travel Time	64.9	55.7	60.6	60.0	38.4	39.3	51.1	49.6

Traffic Demand : cars,　Travel Time : minutes

Table 5 Path Flows by the Precedent Division Method.

	OD Traffic Demand	$Q_1=2\,000$		$Q_2=6\,000$			$Q_3=5\,000$		$Q_4=9\,000$	
	Path Flow	$X_1{}^1$	$X_2{}^1$	$X_1{}^2$	$X_2{}^2$	$X_3{}^2$	$X_1{}^3$	$X_2{}^3$	$X^{.4}$	$X_2{}^4$
$m=10$	Traffic Demand	—	2 000	1 800	3 600	600	3 500	1 500	1 800	7 200
	Travel Time	(63.2)	56.5	57.8	61.9	59.1	37.3	38.6	48.5	51.2
$m=100$	Traffic Demand	—	2 000	1 800	3 660	540	3 750	1 250	2 250	6 750
	Travel Time	(64.5)	56.0	60.1	60.4	60.8	38.3	39.0	50.4	50.0
$m=1\,000$	Traffic Demand	—	2 000	1 872	3 558	570	3 745	1 255	2 142	6 858
	Travel Time	(64.6)	56.0	60.3	60.3	60.4	38.6	38.6	50.0	50.0

$X_3{}^2$ is path flow 2365

Table 6 Assignment by the Precedent Division Method and That Based on the Principle of Equal Travel Times, Traffic Demand on Road Section.

		Road Section	X_{12}	X_{23}	X_{34}	X_{45}	X_{56}	X_{16}	X_{36}
Principle of E.T.T.		Traffic Demand	7 619	7 381	12 417	10 417	4 583	5 619	3 964
		Travel Time	16.7	10.4	28.9	21.0	21.8	21.8	28.3
Division Method	$m=10$	Traffic Demand	7 300	7 700	12 800	10 800	4 200	5 300	3 900
		Travel Time	16.2	10.6	29.7	21.6	20.5	21.1	27.9
	$m=100$	Traffic Demand	7 550	7 450	12 410	12 410	4 590	5 550	4 040
		Travel Time	16.6	10.4	28.9	21.0	21.8	21.7	28.6
	$m=1\,000$	Traffic Demand	7 617	7 383	12 416	10 416	4 584	5 617	3 967
		Travel Time	16.7	10.4	28.9	21.0	21.8	21.8	28.3

that n is infinity. In other words, it can be concluded that the precedent division method had been performed so that the principle of equal travel times may be realized.

Let us compare the assignment result by the precedent division method with that based on the principle of equal travel times. They are shown in Table 5 and 6. Table 5 shows traffic demands over routes and Table 6 expresses those on road sections. In this case, as compared with the result in Table 2, which is based on the principle of travel time ratio, path flow $X_1{}^1$ of OD traffic 1 vanishes and path flow $X_3{}^2$ of OD traffic 2 appears newly. This is because traffic assignment by the principle of travel time ratio has a unique set of solutions peculiar to how to give the routes. That is, routes shown in Fig. 3 are only designated for the convenience of the calculation. Therefore, we canot but say that the result of this assignment is lack in objectivity. As against this, the routes based on the principle of equal travel times are absolutely determined uniquely and they are shown in Table 5. At this time the degree of the influence of the number of the division m to the solution is pretty larger than the case of the travel time ratio assignment. In these numerical examples, it may be considered that the solution by the division method agrees with that of the assignment based on the principle of equal travel times, when the number of the division is 1 000. Observing the case that the number of the division m is 5, 10 and 100 it is interesting that the choice rate of a route of which

travel time is shorter is not always higher. Traffic demands on the road sections are tabulated in Table 6. They are not so much different from those in Table 3.

Now, a number of assignment methods have been proposed up to date and their fitness with the actual traffic phenomena has been respectively held. As shown in the numerical examples, however, if there are sufficient traffic demands little difference will be sometimes found among them, even if any assignment principle is used. But this is the case that results are viewed from the traffic demand on road sections. In this paper, the assnigment calculation based on the principle of minimizing total travel time is not performed, but the solution will come close to that on the basis of the principle of equal travel times as traffic demand increases. Therefore, it can be found that there was no theoretical ground for the contention that the method which agrees exceedingly with the actual traffic demand is the best. The auther considers about this problem that it is more important whether the mathematical model can explain the law of traffic phenomena or not rather than whether it agrees.

In Table 5 the traffic demands on the routes based on the principle of equal travel times are not tabulated, because they are not determined uniquely[8] And traffic demands on road sections are calculated by cut Method[7]. The cut equations and the equal travel times equations are shown graphically in Fig. 2.

Meanwhile, it is assumed in the precedent division method that existing routes never disappear even if
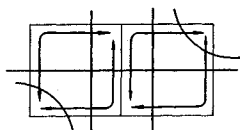
Fig. 2 Pattern of Equal Travel Times and the Cut.

traffic demands increase under the fixed OD pattern. But increasing the traffic demands under the restriction that the OD pattern keeps constant, and following up the change of the assigned flow pattern, the existing routes sometimes, not so often, disappear. In such a case, therefore, the precedent division method does not satisfy the principle of equal travel times. Nevertheless, the solution will not be so much different from the exact solution if traffic demands are sufficient, seeing from the whole aspect of traffic demands on road sections.

Consequently, it can be said that the solution satisfying the principle of equal travel times cannot be always obtained by the precedent division method. And from this the assignment calculation by the precedent division method should be recognized as the method of an approximate calculation for the principle of equal travel times.

5. RELATIONSHIP BETWEEN THE PRINCIPLE OF EQUAL TRAVEL TIMES AND THAT OF MINIMIZING TOTAL TRAVEL TIME

H. Inoue explained the relationship between the principle of equal travel times and that of minimizing total travel time as below[4]. Notation is the same as used in **3**. (1). The total travel time TT is presented as

$$TT = \sum_{\rho}^{r} \phi(X_\rho) X_\rho. \quad\cdots\cdots(16)$$

The condition which X_ρ should satisfy are

$$X_\rho \geqq 0, \quad (\rho=1, 2, \cdots, r) \quad\cdots\cdots(17)$$

$$\sum_{\rho}^{r} X_\rho = Q. \quad\cdots\cdots(18)$$

After all, the problem is to get the set of X_ρ which makes eq. (16) minimum under the constraints of eq. (17) and (18). Then the following Lagrange's function is made by using eq. (17) and (18).

$$Z = \sum_{\rho}^{r} \phi(X_\rho) X_\rho + \lambda(Q - \sum_{\rho}^{r} X_\rho) \quad\cdots\cdots(19)$$

From the theorem of Kuhn-Tucker, the necessary and sufficient conditions are as follows.

for $X_\rho \geqq 0$ $\quad \frac{\partial Z}{\partial X_\rho} = 0$, or $\lambda = \frac{d}{dX_\rho}\{\phi(X_\rho)X_\rho\}$
$$\cdots\cdots(20)$$

at $X_\rho = 0$ $\quad \frac{\partial Z}{\partial X_\rho} \leqq 0$, or $\lambda \leqq \frac{d}{dX_\rho}\{\phi(X_\rho)X_\rho\}$
$$\cdots\cdots(21)$$

further, $\quad \frac{\partial Z}{\partial \lambda} = 0$, or $Q = \sum_{\rho}^{r} X_\rho \quad\cdots\cdots(22)$

Now, let us suppose that the path flows from the first to the ν-th satisfies $X_\rho \geqq 0$. And then eq. (20) is reduced as

$$\frac{d}{dX_1}\{\phi(X_1)X_1\} = \frac{d}{dX_2}\{\phi(X_2)X_2\} = \cdots = \frac{d}{dX_\nu}$$
$$\{\phi(X_D)X_D\} \cdots\cdots(20')$$

If each term of eq. (20′) is regarded as an assumed travel time, this equation is equivalent to equal travel times equation.

Accordingly, the assignment based on the principle of minimizing total travel time can be carried out in the same way as that based on the principle of equal travel times. In short, it can be concluded that the assignment based on the principle of minimizing total travel time in which the capacity function is $T = \phi(X)$ is equivalent to that based on the principle of equal travel times if only the capacity function is converted to $T = d\{\phi(X)X\}dX$. Conversely, the latter assignment is equivalent to the former assignment if only the capacity function is changed to $T = \left\{\int_0^x \phi(X)dX\right\}/X$.

Therefore, even the assignment on the basis of the principle of minimizing total travel time can be performed by the division method by converting the capacity function. But the solution by this method does not always agree with the exact solution is similar to the former case. The author has held that the assigned flow pattern (a normalized ratio of traffic demands on road sections) will change to make the total travel time, which is viewed from a aspect of the pattern, smaller as traffic demands under the fixed OD pattern increase. These are evident from the above discussion. Further it can be recognized that the assignment pattern of equal travel times approaches gradually to that of minimizing total travel time along with the growth of traffic demands, because constants in the capacity function of eq. (15) goes negligible.

6. CONCLUSIONS

In this paper the relationships among the principle of travel time ratio, that of equal travel times and that of minimizing total travel time in the assignment are discussed. Besides, it is clarified the assignment based on any principle can be performed by the division method. But their exact solutions can not be always gained by the division method even if the divided traffic demand of the OD table is very small. After all, the division method in the assignment based on these principle should be admitted as a method of approximation calculation. And this method is very convenient and of

practical use.

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