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DYNAMICS AND EIGENVALUE ANALYSIS OF A RECTANGULAR PLATE WITH STOCHASTIC PROPERTIES

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ABSTRACT

Determination of the mean and variance of eigenvalues of simply supported rectangular plates with stochastic properties is presented by applying the random perturbation techniques of Boyce. Then, determination of the mean and variance of the displacement of a simply supported uniform rectangular plate with uniformly distributed viscous damping having a random value to a distributed random excitation is studied. The technique of Boyce is extended to the inhomogeneous equation for the plate in the latter case. It was found that for free vibration, the distribution of the variance of natural frequency is obtained in exponential form for the given exponential correlation functions of the material properties.

I. INTRODUCTION

In this paper, vibration of a simply supported rectangular plate is investigated when the material properties of the plate are random functions of space co-ordinates and the excitation force is a random process. Although considerable work has been done in dynamic analysis of deterministic structures to random loading (e.g., ref. 1), the study of response of stochastic or probabilistic structural systems to stochastic excitation has been limited to few papers. For example, stochastic equations associated with random free vibration problems of beams and beam-columns have been investigated by Boyce[2)] and Hoshiya[3)], respectively.

The present paper is an extension to the two-dimensional random eigenvalue problem of the techniques used by Boyce[2)]. Moreover, both free and forced vibration problems are discussed. The random perturbation technique of Bellman[4)] is used for both the cases. First, the random eigenvalue problem is solved and the mean and the variances of the $(n\times m)$ eigenvalues are determined. For this case, it is assumed that the plate flexural rigidity and the mass density of the plate are stochastic functions of the space co-ordinates. Second, the forced vibration problem of a uniform rectangular plate is discussed. For this case, the lateral excitation is a random function of space and time. The viscous damping coefficient is considered as a random variable having a constant value over the area of the plate.

Again, it should be emphasized that the problem solved in this paper is different than the classical random vibration problem, where the structure is deterministic and the excitation is random. In our case, either (a) the structural properties or (b) the damping and the excitations are random.

II. GENERAL FORMULATION OF VIBRATION PROBLEM

Let us consider the small lateral vibration of a rectangular plate given by Fig. 1.

With the general assumptions customarily made for the classical vibration problems[5)], we have the following governing equation and the corresponding boundary conditions for a simply supported plate.

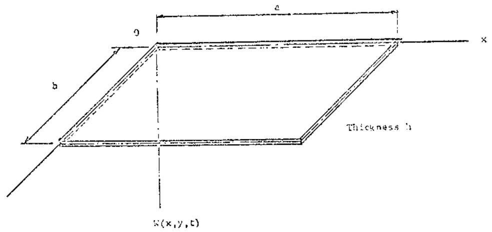

Fig. 1 Simply supported rectangular plate.

$$\frac{\partial^2}{\partial\zeta^2}\left\{D\left(\frac{\partial^2 w}{\partial\zeta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)\right\}+\frac{\partial^2}{\partial\eta^2}\left\{D\left(\frac{a^4}{b^4}\frac{\partial^2 w}{\partial\eta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w}{\partial\zeta^2}\right)\right\}+2\frac{a^2}{b^2}\frac{\partial^2}{\partial\zeta\partial\eta}\left\{D(1-\nu)\frac{\partial^2 w}{\partial\zeta\partial\eta}\right\}+a^4\rho h\frac{\partial^2 w}{\partial t^2}+a^4 C\frac{\partial w}{\partial t}=a^4 Q \quad\cdots\cdots\cdots\cdots(1)$$

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and

$$w(0,\ \eta,\ t)=0\ \frac{\partial^2 w}{\partial \zeta^2}\ (0,\ \eta,\ t)=0 \cdots\cdots\cdots\cdots (2)$$

$$w(1,\ \eta,\ t)=0\ \frac{\partial^2 w}{\partial \zeta^2}\ (1,\ \eta,\ t)=0 \cdots\cdots\cdots\cdots (3)$$

$$w(\zeta,\ 0,\ t)=0\ \frac{\partial^2 w}{\partial \eta^2}\ (\zeta,\ 0,\ t)=0 \cdots\cdots\cdots\cdots (4)$$

$$w(\zeta,\ 1,\ t)=0\ \frac{\partial^2 w}{\partial \eta^2}\ (\zeta,\ 1,\ t)=0 \cdots\cdots\cdots\cdots (5)$$

where w=lateral displacement as a function of spatial co-ordinates ζ and η and of time t.

D=plate flexural rigidity as a function of ζ and η. This is equal to $\frac{Eh^3}{12(1-\nu)}$

E=Young's modulus

ν=Poisson's ratio

h=thickness of the plate

C=coefficient of viscous damping (assumed to be independent of ζ, η, and t).

a=width of the plate

b=length of the plate

ρ=mass density of the plate as a function of ζ and η.

Q=lateral excitation as a function of ζ and η and time t.

Non-dimensional co-ordinates ζ and η are used by replacing x and y by $\zeta=\frac{x}{a}$ and $\eta=\frac{y}{b}$.

Assumptions made in the discussions that follow are :

(1) D and ρ are random stationary functions of spatial co-ordinates ζ and η and can be put in the forms

$$D(\zeta,\eta)=D_0\{1+d(\zeta,\ \eta)\} \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

$$\rho(\zeta,\ \eta)=\rho_0\{1+e(\zeta,\ \eta)\} \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

where D_0 and ρ_0 are constant. $d(\zeta,\ \eta)$ and $e(\zeta,\ \eta)$ are stationary random functions with zero mean and small variance. This assumptions (6) and (7) imply that D and ρ are almost constant over the plate. The assumption of stationarity implies that the small random perturbations of D and ρ are regulated by probability density functions, independent of ζ and η. Thus, we have

$$R_{dd}(\zeta_1-\zeta_2,\ \eta_1-\eta_2)=E[d(\zeta_1,\ \eta_1)d(\zeta_2,\ \eta_2)] \cdots\cdots\cdots\cdots (8)$$

$$R_{ee}(\zeta_1-\zeta_2,\ \eta_1-\eta_2)=E[e(\zeta_1,\ \eta_1)e(\zeta_2,\ \eta_2)] \cdots\cdots\cdots\cdots (9)$$

$$R_{de}(\zeta_1-\zeta_2,\ \eta_1-\eta_2)=E[d(\zeta_1,\ \eta_1)e(\zeta_2,\ \eta_2)] \cdots\cdots\cdots\cdots (10)$$

where R_{dd} and R_{ee} denote the autocorrelation function and R_{de} denotes the corss correlation function of d and e. $E[\ ]$ denotes the expected values.

(2) C is a random variable and is put in the form :

$$C=C_0(1+f) \cdots\cdots\cdots\cdots\cdots\cdots (11)$$

where C_0 is constant and f is a random variable with zero mean and small variance.

(3) Q is a random stationary function with respect to space and time. Q is put in the following form

$$Q(\zeta,\ \eta,\ t)=Q_0(t)[1+q(\zeta,\ \eta)] \cdots\cdots\cdots\cdots\cdots\cdots (12)$$

where $Q_0(t)$ is a stationary random process of t and $q(\zeta,\ \eta)$ is also a stationary function of ζ and η with zero means. $q(\zeta,\ \eta)$ is not necessarily small. This means that $Q(\zeta,\ \eta,\ t)$ has the uniform spatial average intensity $Q_0(t)$ over the plate and has the random variation $q(\zeta,\ \eta)$ at each location of the plate.

(4) f, $Q_0(t)$ and $q(\zeta,\ \eta)$ are statistically independent of each other and of $d(\zeta,\ \eta)$ and $e(\zeta,\ \eta)$.

III. FREE VIBRATION

First of all, let us consider the undamped free vibration of a simply supported plate.

We have from equation (1), by dropping out the damping term and external force

$$\frac{\partial^2}{\partial \zeta^2}\left\{D\left(\frac{\partial^2 w}{\partial \zeta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w}{\partial \eta^2}\right)\right\} +\frac{\partial^2}{\partial \eta^2}\left\{D\left(\frac{a^4}{b^4}\frac{\partial^2 w}{\partial \eta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w}{\partial \zeta^2}\right)\right\} +2\frac{a^2}{b^2}\frac{\partial^2}{\partial \zeta \partial \eta}\left\{D(1-\nu)\frac{\partial^2 w}{\partial \zeta \partial \eta}\right\}+a^4\rho h\frac{\partial^2 w}{\partial t^2}=0 \cdots\cdots\cdots\cdots (13)$$

Assuming the solution of the above equation in the form

$$w(\zeta,\ \eta,\ t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}(\zeta,\ \eta)T_{mn}(t) \cdots\cdots (14)$$

and introducing this into equation (13) leads to

$$\frac{1}{a^4\rho h w_{mn}}\left[\frac{\partial^2}{\partial \zeta^2}\left\{D\left(\frac{\partial^2 w_{mn}}{\partial \zeta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial \zeta^2}\right)\right\} +\frac{\partial^2}{\partial \eta^2}\left\{D\left(\frac{a^4}{b^4}\frac{\partial^2 w_{mn}}{\partial \eta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial \zeta^2}\right)\right\} +2\frac{a^2}{b^2}\frac{\partial^2}{\partial \zeta \partial \eta}\left\{D(1-\nu)\frac{\partial^2 w_{mn}}{\partial \zeta \partial \eta}\right\}\right]=-\frac{\ddot{T}_{mn}}{T_{mn}} \cdots\cdots\cdots\cdots (15)$$

where 'dot' indicates the derivatives with respect to time. In the above equation, the left hand side is a function of ζ and η whereas the right hand side is a function of time. Therefore this equation should be constant and for harmonic oscillation, this constant is required to be positive. By replacing this constant by ω_{mn}^2, we have

$$\frac{\partial^2}{\partial \zeta^2}\left\{D\left(\frac{\partial^2 w_{mn}}{\partial \zeta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial \eta^2}\right)\right\} +\frac{\partial^2}{\partial \eta^2}\left\{D\left(\frac{a^4}{b^4}\frac{\partial^2 w_{mn}}{\partial \eta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial \eta^2}\right)\right\} +2\frac{a^2}{b^2}\frac{\partial^2}{\partial \zeta \partial \eta}\left\{D(1-\nu)\frac{\partial^2 w_{mn}}{\partial \zeta \partial \eta}\right\} -a^4\rho h\,\omega^2_{mn}w_{mn}=0 \cdots\cdots\cdots\cdots (16)$$

where ω_{mn} is natural frequency.

Introducing equations (6) and (7) into the above equation, we have

$$\frac{\partial^2}{\partial\zeta^2}\left\{(1+\alpha\, d(\zeta,\eta))\left(\frac{\partial^2 w_{mn}}{\partial\zeta^2}+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial\eta^2}\right)\right\}$$
$$+\frac{\partial^2}{\partial\eta^2}\left\{(1+\alpha\, d(\zeta,\eta))\left(\frac{a^4}{b^4}\frac{\partial^2 w_{mn}}{\partial\eta^2}\right.\right.$$
$$\left.\left.+\nu\frac{a^2}{b^2}\frac{\partial^2 w_{mn}}{\partial\eta^2}\right)\right\}+2(1-\nu)\frac{a^2}{b^2}\frac{\partial^2}{\partial\zeta\partial\eta}$$
$$\times\left\{(1+\alpha\, d(\zeta,\eta))\frac{\partial^2 w_{mn}}{\partial\zeta\partial\eta}\right\}$$
$$-\lambda_{mn}(1+\beta\, e(\zeta,\eta))w_{mn}=0 \qquad (17)$$

where α and β in general are expansion parameters for perturbation expansions. However, for our case, they will be equated to unity at the end of the analysis.

λ_{mn} is a non-dimensionalized natural frequency and is given by

$$\lambda_{mn}=\frac{a^4\rho_0 h\,\omega^2{}_{mn}}{D_0} \qquad (18)$$

The boundary conditions are satisfied by substituting eq. (14) into equations (2) to (5) to obtain:

$$w_{mn}(0,\eta)=0,\ \frac{\partial^2 w_{mn}(0,\eta)}{\partial\zeta^2}=0 \qquad (19)$$
$$w_{mn}(1,\eta)=0,\ \frac{\partial^2 w_{mn}(1,\eta)}{\partial\zeta^2}=0 \qquad (20)$$
$$w_{mn}(\zeta,0)=0,\ \frac{\partial^2 w_{mn}(\zeta,0)}{\partial\eta^2}=0 \qquad (21)$$
$$w_{mn}(\zeta,1)=0,\ \frac{\partial^2 w_{mn}(\zeta,1)}{\partial\eta^2}=0 \qquad (22)$$

Let us assume the solutions of equation (17) in the following forms

$$\lambda_{mn}=\sum_{i,j=0}^{\infty}\lambda_{ij}\alpha^i\beta^j=\lambda_{00}+\lambda_{10}\alpha+\lambda_{01}\beta+\cdots \qquad (23)$$
$$w_{mn}(\zeta,\eta)=\sum_{i,j=0}^{\infty}w_{ij}(\zeta,\eta)\alpha^i\beta^j=w_{00}(\zeta,\eta)$$
$$+w_{10}(\zeta,\eta)\alpha+w_{01}(\zeta,\eta)\beta+\cdots \qquad (24)$$

Introducing equations (23) and (24) into equation (17) and equating coefficients of each power of α and β leads to the following sequence of differential equations for w_{ij}.

$$\frac{\partial^4 w_{00}}{\partial\zeta^4}+2\frac{a^2}{b^2}\frac{\partial^4 w_{00}}{\partial\zeta^2\partial\eta^2}+\frac{a^4}{b^4}\frac{\partial^4 w_{00}}{\partial\eta^4}-\lambda_{00}w_{00}=0 \qquad (25)$$

$$\frac{\partial^4 w_{10}}{\partial\zeta^4}+2\frac{a^2}{b^2}\frac{\partial^4 w_{10}}{\partial\zeta^2\partial\eta^2}+\frac{a^4}{b^4}\frac{\partial^4 w_{10}}{\partial\eta^4}$$
$$+\frac{\partial^2}{\partial\zeta^2}\left(d\frac{\partial^2 w_{00}}{\partial\zeta^2}\right)+\nu\frac{a^2}{b^2}\frac{\partial^2}{\partial\zeta^2}\left(d\frac{\partial^2 w_{00}}{\partial\eta^2}\right)$$
$$+\frac{a^4}{b^4}\frac{\partial^2}{\partial\eta^2}\left(d\frac{\partial^2 w_{00}}{\partial\eta^2}\right)+\nu\frac{a^2}{b^2}\frac{\partial^2}{\partial\eta^2}\left(d\frac{\partial^2 w_{00}}{\partial\zeta^2}\right)$$
$$+2(1-\nu)\frac{a^2}{b^2}\frac{\partial^2}{\partial\zeta\partial\eta}\left(d\frac{\partial^2 w_{00}}{\partial\zeta\partial\eta}\right)=\lambda_{10}w_{00}+\lambda_{00}w_{10} \qquad (26)$$

$$\frac{\partial^4 w_{01}}{\partial\zeta^4}+2\frac{a^2}{b^2}\frac{\partial^4 w_{01}}{\partial\zeta^2\partial\eta^2}+\frac{a^4}{b^4}\frac{\partial^4 w_{01}}{\partial\eta^4}$$
$$=\lambda_{01}w_{00}+e\,\lambda_{00}w_{00}+\lambda_{00}w_{01} \qquad (27)$$

where the higher terms of α and β beyond first order are neglected. Similarly we obtain the boundary conditions

$$w_{ij}(0,\eta)=0,\ \frac{\partial^2 w_{ij}(0,\eta)}{\partial\zeta^2}=0 \qquad (28)$$
$$w_{ij}(1,\eta)=0,\ \frac{\partial^2 w_{ij}(1,\eta)}{\partial\zeta^2}=0 \qquad (29)$$
$$w_{ij}(\zeta,0)=0,\ \frac{\partial^2 w_{ij}(\zeta,0)}{\partial\eta^2}=0 \qquad (30)$$
$$w_{ij}(\zeta,1)=0,\ \frac{\partial^2 w_{ij}(\zeta,1)}{\partial\eta^2}=0 \qquad (31)$$
$$i,j=0,1$$

Supposing that w_{00} and λ_{00} are obtained from eq. (25) and boundary conditions (28) to (31), then we can express the other solutions λ_{10} and λ_{01} in terms of w_{00} and λ_{00} by solving (26) and (27) subject to (28) thru (31). Multiplying eq. (26) by w_{00} and then integrating it from 0 to 1 with respect to ζ and η (Ref. 2) leads to

$$\lambda_{10}=\frac{\begin{aligned}&\int_0^1\!\!\int_0^1 d\left(\frac{\partial^2 w_{00}}{\partial\zeta^2}\right)^2 d\zeta\, d\eta+2\nu\frac{a^2}{b^2}\int_0^1\!\!\int_0^1 d\left(\frac{\partial^2 w_{00}}{\partial\eta^2}\right)\\&\times\left(\frac{\partial^2 w_{00}}{\partial\zeta^2}\right)d\zeta\, d\eta+\frac{a^4}{b^4}\int_0^1\!\!\int_0^1 d\left(\frac{\partial^2 w_{00}}{\partial\eta^2}\right)^2 d\zeta\, d\eta\\&+2(1-\nu)\frac{a^2}{b^2}\int_0^1\!\!\int_0^1 d\left(\frac{\partial^2 w_{00}}{\partial\zeta\partial\eta}\right)^2 d\zeta\, d\eta\end{aligned}}{\int_0^1\!\!\int_0^1 w_{00}{}^2\, d\zeta\, d\eta} \qquad (32)$$

Similarly from eq. (27)

$$\lambda_{01}=-\frac{\lambda_{00}\int_0^1\!\!\int_0^1 e w^2{}_{00}\, d\zeta\, d\eta}{\int_0^1\!\!\int_0^1 w^2{}_{00}\, d\zeta\, d\eta} \qquad (33)$$

Now putting $\alpha=\beta=1$ in eq. (23), we have for natural frequency

$$\lambda_{mn}=\lambda_{00}+\lambda_{10}+\lambda_{01}+\cdots\cdots \qquad (34)$$

where λ_{00} and w_{00} are given by solving the deterministic equation (25) subject to the boundary conditions (28) to (31).

$$\lambda_{00}=\pi^4\left(m^2+\frac{a^2}{b^2}n^2\right)^2 \qquad (35)$$
$$m=1,2,3,\cdots$$
$$n=1,2,3,\cdots$$
$$w_{00}=\sin m\pi\zeta\,\sin n\pi\eta \qquad (36)$$

Thus, introducing equations (32), (33), (35) and (36) into eq. (34), we obtain

$$\lambda_{mn}=\pi^4\left(m^2+\frac{a^2}{b^2}n^2\right)^2$$
$$+G_{mn}\int_0^1\!\!\int_0^1 d(\zeta,\eta)\sin^2 m\pi\zeta\,\sin^2 n\pi\eta\, d\zeta\, d\eta$$
$$+H_{mn}\int_0^1\!\!\int_0^1 d(\zeta,\eta)\cos^2 m\pi\zeta\,\cos^2 n\pi\eta\, d\zeta\, d\eta$$
$$+I_{mn}\int_0^1\!\!\int_0^1 e(\zeta,\eta)\sin^2 m\pi\zeta\,\sin^2 n\pi\eta\, d\zeta\, d\eta \qquad (37)$$

where

$$G_{mn}=4\pi^4\left(m^4+2\nu\frac{a^2}{b^2}m^2n^2+\frac{a^4}{b^4}n^4\right) \qquad (38)$$
$$H_{mn}=8\pi^4(1-\nu)\frac{a^2}{b^2}m^2n^2 \qquad (39)$$

$$I_{mn}=-4\pi^4\left(m^2+\frac{a^2}{b^2}n^2\right)^2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots (40)$$

Taking the expected value of λ_{mn} from eq. (37), considering zero means of $d(\zeta, \eta)$ and $e(\zeta, \eta)$, we obtain the mean of λ_{mn}.

$$E[\lambda_{mn}]=\lambda_{00}=\pi^4\left(m^2+\frac{a^2}{b^2}n^2\right)^2 \cdots\cdots\cdots\cdots (41)$$

We note that as a result of $E[d]=E[e]=0$ the mean of λ_{mn} is identical to the solution of the deterministic problem and independent of the random variation in properties. Self-multiplication of eq. (37) and introducing the expected value gives the mean square value of λ_{mn} as follows.

$$\begin{aligned}
E[\lambda^2_{mn}]=&\lambda^2_{00}+G^2_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 \\
&\times E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]\sin^2 m\pi\zeta_1 \\
&\times \sin^2 n\pi\eta_1 \sin^2 m\pi\zeta_2 \sin^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \\
&+H^2_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)] \\
&\times \cos^2 m\pi\zeta_1 \cos^2 n\pi\eta_1 \cos^2 m\pi\zeta_2 \\
&\times \cos^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \\
&+I^2_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 E[e(\zeta_1, \eta_1)e(\zeta_2, \eta_2)] \\
&\times \sin^2 m\pi\zeta_1 \sin^2 n\pi\eta_1 \sin^2 m\pi\zeta_2 \\
&\times \sin^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \\
&+2G_{mn}H_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)] \\
&\times \sin^2 m\pi\zeta_1 \sin^2 n\pi\eta_1 \cos^2 m\pi\zeta_2 \\
&\times \cos^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \\
&+2G_{mn}I_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)] \\
&\times \sin^2 m\pi\zeta_1 \sin^2 n\pi\eta_1 \sin^2 m\pi\zeta_2 \\
&\times \sin^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \\
&+2H_{mn}I_{mn}\int_0^1\int_0^1\int_0^1\int_0^1 E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)] \\
&\times \cos^2 m\pi\zeta_1 \cos^2 n\pi\eta_1 \sin^2 m\pi\zeta_2 \\
&\times \sin^2 n\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \cdots\cdots\cdots\cdots\cdots\cdots (42)
\end{aligned}$$

By definition the variance of λ_{mn} is then given by

$$\mathrm{Var}[\lambda_{mn}]=E[\lambda_{mn}^2]-\{E[\lambda_{mn}]\}^2 \cdots\cdots\cdots\cdots\cdots\cdots (43)$$

Equations (42) and (43) are general forms for the mean square and the variance of λ_{mn}. We can evaluate these terms for any random properties specified in the form of auto correlation and cross correlation functions.

Let us first consider the special case for which the auto correlation function of $d(\zeta, \eta)$ and of $e(\zeta, \eta)$ and the cross correlation function of $d(\zeta_1, \eta_1)$ and of $e(\zeta_2, \eta_2)$ are uniform and are equal to their variance or product of their standard deviation respectively, i.e.

$$E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]=\sigma_d^2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots (44)$$

$$E[e(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]=\sigma_d^2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots (45)$$

$$E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)]=\sigma_d\sigma_e \cdots\cdots\cdots\cdots\cdots\cdots (46)$$

Thus, the correlation coefficients are

$$\rho_d=\frac{E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]}{\sigma_d(\zeta_1, \eta_1)\sigma_d(\zeta_2, \eta_2)}=\frac{E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]}{\sigma_d^2}=1$$

Similarly,

$$\rho_e=\frac{E[e(\zeta_1, \eta_1)e(\zeta_2, \eta_2)]}{\sigma_e^2}=1$$

$$\rho_{de}=\frac{E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)]}{\sigma_d\sigma_e}=1$$

Thus, equations (44), (45) and (46) indicate complete correlation between $d(\zeta, \eta)$ and $e(\zeta, \eta)$ for all ζ and η over the plate. There is also complete correlation between $d(\zeta_1, \eta_1)$ and $e(\zeta_2, \eta_2)$. Unity correlation indicates that two variables, say, $d(\zeta_1, \eta_1)$ and $e(\zeta_2, \eta_2)$ are related to each other in a fully deterministic sense. That is, only one value of $d(\zeta_1, \eta_1)$ exists for each value of $e(\zeta_2, \eta_2)$.

For this case, we have

$$E[\lambda^2_{mn}]=\lambda^2_{00}[1+\sigma_d^2+\sigma_e^2-2\sigma_d\sigma_e] \cdots\cdots\cdots (47)$$

$$\mathrm{Var}[\lambda_{mn}]=\lambda^2_{00}[\sigma_d^2+\sigma_e^2-2\sigma_d\sigma_e] \cdots\cdots\cdots\cdots (48)$$

It can be seen that for $\sigma_d=\sigma_e$, $\mathrm{Var}(\lambda_{mn})=0$. This can be explained by the fact that the input parameters have many constraints and hence, the output variance diminishes and in this extreme case, it reduces to zero. Table 1 shows the values of variance of λ_{mn} for given values of σ_d^2 and σ_e^2.

Table 1 Variance of λ_{mn} for given σ_d^2 and σ_e^2
Value of $\mathrm{Var}[\lambda_{mn}]/\lambda_{00}^2$

σ_e^2 \ σ_d^2	0.01	0.02	0.03	0.04	0.05
0.01	0	0.002	0.005	0.010	0.015
0.02	0.002	0	0.001	0.004	0.006
0.03	0.005	0.001	0	0.001	0.003
0.04	0.010	0.004	0.001	0	0.001
0.05	0.015	0.006	0.003	0.001	0

If $d(\zeta, \eta)$ and $e(\zeta, \eta)$ are stochastically independent, we have, after substituting

$$E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)]=E[d(\zeta_1, \eta_1)]E[e(\zeta_2, \eta_2)]=0$$

$$\mathrm{Var}(\lambda_{mn})=\lambda^2_{00}[\sigma_d^2+\sigma_e^2] \cdots\cdots\cdots\cdots (49)$$

Equation (49) indicates that the variance of λ_{mn} increases linearly with σ_d^2 and σ_e^2 (Fig. 2).

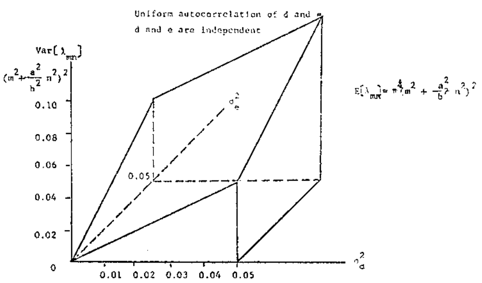

Fig. 2 Variance of λ_{mn} for given σ_d^2 and σ_e^2

As a second special case, consider the auto correlation functions and the cross correlation functions in the exponential form as follows

$$E[d(\zeta_1, \eta_1)d(\zeta_2, \eta_2)]=\sigma_d^2 e^{-k_1\{|\zeta_1-\zeta_2|+|\eta_1-\eta_2|\}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (50)$$

$$E[e(\zeta_1, \eta_1)e(\zeta_2, \eta_2)]=\sigma_e^2 e^{-k_2\{|\zeta_1-\zeta_2|+|\eta_1-\eta_2|\}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (51)$$

$$E[d(\zeta_1, \eta_1)e(\zeta_2, \eta_2)] = \sigma_d\sigma_e e^{-k_3\{|\zeta_1-\zeta_2|+|\eta_1-\eta_2|\}} \qquad (52)$$

where σ_d^2 and σ_e^2 are variances of d and e respectively; k_1, k_2 and k_3 are constants and depend on the correlation length, defined by $\varepsilon=|\zeta_1-\zeta_2|+|\eta_1-\eta_2|$. This definition of the correlation length is reasonable when the distance between two points has one to one correspondence to the sum of its horizontal and vertical components in the homogenous plate. The assumed auto correlation and the cross correlation functions have their maximum value when $\{|\zeta_1-\zeta_2|+|\eta_1-\eta_2|\}$ is zero. κ is selected in such a way that the magnitude of the correlation coefficient becomes five percent of the maximum value at a distance ε from the point where $\{|\zeta_1-\zeta_2|+|\eta_1-\eta_2|\}$ is zero.

$$\sigma_d^2 e^{-k_1\varepsilon_1} = 0.05\,\sigma_d^2$$

$$k_1\varepsilon_1 = 2.9957$$

Similarly,

$$\varepsilon_2 k_2 = \varepsilon_3 k_3 = 2.9957$$

It can be seen that the range of ε_i is between 0 and 2. This is due to the fact that since $0 \leqq \zeta = \frac{x}{a} \leqq 1$ and $0 \leqq \eta = \frac{y}{b} \leqq 1$, we have $0 \leqq |\zeta_1-\zeta_2| \leqq 1$ and $0 \leqq |\eta_1-\eta_2| \leqq 1$ for any points in the plate. Introducing equations (50), (51) and (52) in eq. (42) and using eq. (43), we have,

$$\mathrm{Var}[\lambda_{mn}] = G^2_{mn}\sigma_d^2 A(k_1, m)A(-k_1, m)A(k, n) A(-k_1, n) + H^2_{mn}\sigma_d^2 B(k_1, m)B(-k_1, m) B(k_1, n)B(-k_1, n) + I_{mn}^2\sigma_e^2 A(k_2, m)$$

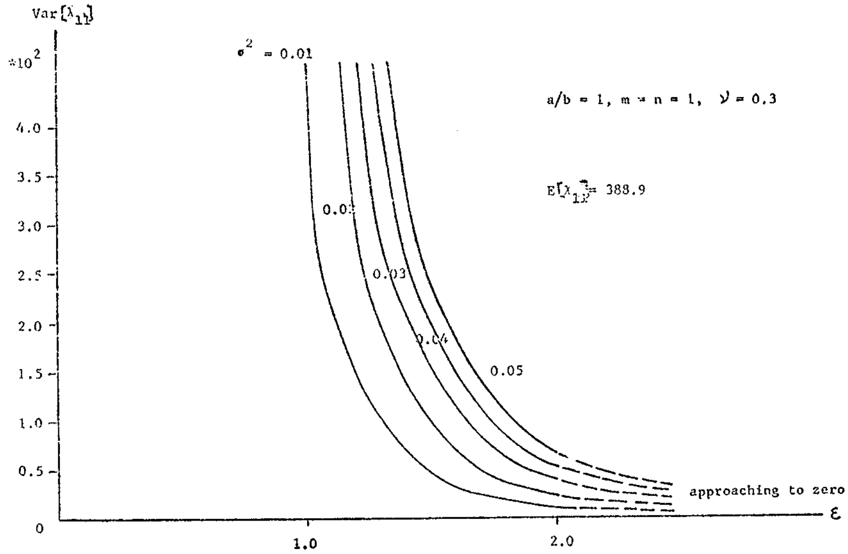

Fig. 3 Variance of λ_{11} for given correlation length and $\sigma_d^2=\sigma_e^2=\sigma^2$

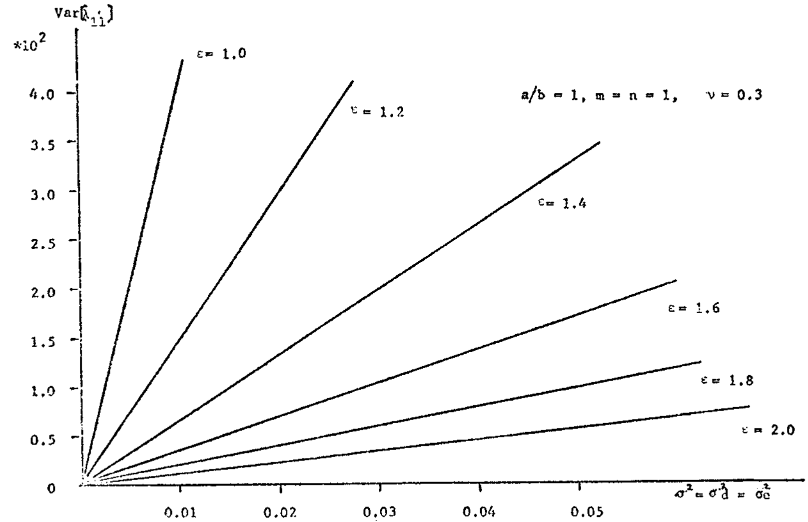

Fig. 4 Variance of λ_{11} for given correlation length and $\sigma^2=\sigma_d^2=\sigma_e^2$

$$A(-k_2, m)A(k_2, n)A(-k_2, n)$$
$$+2\,G_{mn}H_{mn}\sigma_d{}^2A(k_1, m)B(-k_1, m)$$
$$A(k_1, n)B(-k_1, n)+2\,H_{mn}I_{mn}\sigma_d\sigma_e$$
$$B(k_3,m)A(-k_3, m)B(k_3, n)A(-k_3, n)$$
$$+2\,G_{mn}I_{mn}\sigma_d\sigma_eA(k_3, m)A(-k_3, m)$$
$$A(k_3, n)A(-k_3, n) \cdots\cdots (53)$$

where

$$A(k, m)=\frac{2\,m^2\pi^2}{k(k^2+4\,m^2\pi^2)}(1-e^{-k}) \cdots\cdots (54)$$

$$B(k, m)=\frac{k^2+2\,m^2\pi^2}{k(k^2+4\,m^2\pi^2)}(1-e^{-k}) \cdots\cdots (55)$$

Assuming that $\sigma_d{}^2=\sigma_e{}^2=\sigma^2$ and $k_1=k_2=k_3=k$ (thus $\varepsilon_1=\varepsilon_2=\varepsilon_3=\varepsilon$), the variance of λ_{mn} is evaluated for given $a/b=1$ and $m=n=1$. $\nu=0.3$. Fig. 3 indicates how the variance of the natural frequency varies according to the correlation length.

It is interesting to see that the variance of λ_{11} decreases exponentially to zero when the correlation length increases. Although ε does not take more than 2.0, the variance of λ_{11} becomes zero for a given large value of ε and coincides with the results for the uniform correlation obtained previously. If the correlation between random inputs is very weak, and if the randomness between the inputs increase the variance of the output naturally increases to infinity. The variance of λ_{mn} is a linear function of σ^2 as shown in Fig. 4.

IV FORCED VIBRATION

We consider the lateral vibration of the same plate subjected to the lateral random excitation $Q(\zeta, \eta, t)$, which is given by equation (12). We assume that the plate properties are deterministic except for the random damping defined by equation (11).

Hence the governing differential equation (1) is simplified into

$$D\left[\frac{\partial^4 w}{\partial\zeta^4}+2\frac{a^2}{b^2}\frac{\partial^4 w}{\partial\zeta^2\partial\eta^2}+\frac{a^4}{b^4}\frac{\partial^4 w}{\partial\eta^4}\right]$$
$$+a^4\rho h\frac{\partial^2 w}{\partial t^2}+a^4C\frac{\partial w}{\partial t}=a^4Q \cdots\cdots (56)$$

Since the natural frequency and the mode of the deterministic free vibration problem are given by eqs. (35) and (36), we assume the solution of eq. (56) as follows

$$w(\zeta,\eta, t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}(\zeta, \eta)T_{mn}(t) \cdots\cdots (57)$$

where we have

$$w_{mn}(\zeta, \eta)=\sin m\,\pi\zeta\,\sin n\,\pi\eta \cdots\cdots (58)$$

$$\lambda_{mn}=\pi^4\left(m^4+\frac{a^2}{b^2}n^2\right)^2 \cdots\cdots (59)$$

Introducing eqs. (11) and (12) into eq. (56), we have

$$\frac{\partial^4 w}{\partial\zeta^4}+2\frac{a^2}{b^2}\frac{\partial^4 w}{\partial\zeta^2\partial\eta^2}+\frac{a^4}{b^4}\frac{\partial^4 w}{\partial\eta^4}+H_1\frac{\partial^2 w}{\partial t^2}$$
$$+H_2(1+f)\frac{\partial w}{\partial t}=H_3Q_0(t)[1+q(\zeta, \eta)]$$
$$\cdots\cdots (60)$$

where

$$H_1=\frac{a^4\rho h}{D} \cdots\cdots (61)$$

$$H_2=\frac{a^4C_0}{D} \cdots\cdots (62)$$

$$H_3=\frac{a^4}{D} \cdots\cdots (63)$$

Expanding the right hand side of eq. (60) by using the normal mode w_{mn}, we have

$$Q_0(t)[1+q(\zeta, \eta)]=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}(\zeta, \eta)z_{mn}(t)$$
$$\cdots\cdots (64)$$

where

$$z_{mn}(t)=\frac{Q_0(t)\int_0^1\int_0^1[1+q(\zeta, \eta)]w_{mn}(\zeta, \eta)\,d\zeta\,d\eta}{\int_0^1\int_0^1 w^2{}_{mn}(\zeta, \eta)\,d\zeta\,d\eta}$$
$$\cdots\cdots (65)$$

Substituting eq. (58) in eq. (65), we have

$$z_{mn}(t)=4\,Q_0(t)\int_0^1\int_0^1[1+q(\zeta, \eta)]$$
$$\times\sin m\,\pi\zeta\,\sin n\,\pi\eta\,d\zeta\,d\eta \cdots\cdots (66)$$

Replacing the right hand side of eq. (60) by (64) and introducing eq. (58) into eq. (60) leads to the differential equation for T_{mn}.

That is,

$$\ddot{T}_{mn}+\frac{C_0}{\rho h}(1+f)\dot{T}_{mn}+\omega^2{}_{mn}T_{mn}=\frac{1}{\rho h}z_{mn}$$
$$\cdots\cdots (67)$$

Assuming that the plate is initially at rest, we have the following initial conditions

$$w(\zeta, \eta, 0)=\dot{w}(\zeta, \eta, 0)=0 \cdots\cdots (68)$$

Hence for $T_{mn}(t)$, we have

$$T_{mn}(0)=\dot{T}_{mn}(0)=0 \cdots\cdots (69)$$

Now $T_{mn}(t)$ has to be solved by the differential equation (67) and the initial conditions (68). However, since eq. (67) involves the random variable f in the left hand side, the following series expansion for $T_{mn}(t)$ is assumed.

$$T_{mn}(t)=\sum_{i=0}^{\infty}T_i(t)f^i=T_0(t)+T_1(t)f+\cdots (70)$$

Introducing eq. (70) into eq. (67), we have, after some algebraic manipulation, a sequence of differential equations as follows.

$$\ddot{T}_0+\frac{C_0}{\rho h}\dot{T}_0+\omega^2{}_{mn}T_0=\frac{1}{\rho h}z_{mn}(t) \cdots\cdots (71)$$

and

$$\ddot{T}_1+\frac{C_0}{h}(\dot{T}_1+\dot{T}_0)+\omega^2{}_{mn}T_1=0 \cdots\cdots (72)$$

where we neglected the higher terms of f beyond the first order. The initial conditions are from eqs. (69) and (70).

$$T_0(0)=\dot{T}_0(0)=0 \cdots\cdots (73)$$

$$T_1(0)=\dot{T}_1(0)=0 \cdots\cdots (74)$$

Solving eq. (71) with eq. (73) as the initial conditions, we have

$$T_0(t)=\frac{1}{\rho h}\int_0^t h_{mn}(t-u)z_{mn}(u)du \quad \cdots\cdots (75)$$

where

$$h_{mn}(t)=\frac{1}{a_{mn}}e^{\mu t}\sin a_{mn}t \quad \text{for } \omega_{mn}>\frac{C_0}{2\rho h} \quad \cdots\cdots (76)$$

$$=te^{\mu t} \quad \text{for } \omega_{mn}=\frac{C_0}{2\rho h} \quad \cdots\cdots (77)$$

$$=\frac{1}{a_{mn}}e^{\mu t}\sinh a_{mn}t \quad \text{for } \omega_{mn}<\frac{C_0}{2\rho h} \quad \cdots\cdots (78)$$

and

$$a_{mn}=\left|\omega^2{}_{mn}-\frac{C_0^2}{2\rho h}\right|^{1/2} \quad \cdots\cdots (79)$$

$$\mu=-\frac{C_0}{2\rho h} \quad \cdots\cdots (80)$$

Similarly $T_1(t)$ is obtained from solution eq. (72) and (75)

$$T_1(t)=\frac{-C_0}{2\rho h}\int_0^t\int_0^v h_{mn}(t-v) g_{mn}(v-u)z_{mn}(u)dudv \quad \cdots\cdots (81)$$

where

$$g_{mn}(t)=e^{\mu t}\left\{\frac{\mu}{a_{mn}}\sin a_{mn}t+\cos a_{mn}t\right\} \quad \text{for } \omega_{mn}>\frac{C_0}{2\rho h} \quad \cdots\cdots (82)$$

$$=e^{\mu t}(1+\mu t) \quad \text{for } \omega_{mn}=\frac{C_0}{2\rho h} \quad \cdots\cdots (83)$$

$$=e^{\mu t}\left\{\frac{\mu}{a_{mn}}\sinh a_{mn}t+\cosh a_{mn}t\right\} \quad \text{for } \omega_{mn}<\frac{C_0}{2\rho h} \quad \cdots\cdots (84)$$

Introducing eqs. (75) and (81) into eq. (70) and putting the result into eq. (57), together with eq. (58), we obtain the deflection as follows :

$$w(\zeta,\eta,t)=\frac{1}{\rho h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin m\pi\zeta\sin n\pi\eta \times\Bigg[\int_0^t h_{mn}(t-u)z_{mn}(u)du -\frac{C_0 f}{\rho h}\int_0^t\int_0^u h_{mn}(t-u)g_{mn}(u-v)z_{mn}(v)dvdu\Bigg] \quad \cdots\cdots (85)$$

We should note that $w(\zeta, \eta, t)$ includes the random variable f and a random function $z_{mn}(t)$. $z_{mn}(t)$ is a function of $Q_0(t)$ and $q(\zeta, \eta)$ and is given by eq. (66). Taking the mean value of eq. (85) we obtain

$$E[w(\zeta,\eta,t)]=\frac{1}{\rho h}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin m\pi\zeta\sin n\pi\eta \times\Bigg[\int_0^t h_{mn}(t-u)E[z_{mn}(u)]du -\frac{C_0}{\rho h}\int_0^t\int_0^u h_{mn}(t-u)g_{mn}(u-v) \times E[fz_{mn}(v)]dvdu\Bigg] \quad \cdots\cdots (86)$$

Since we have $E[fz_{mn}(v)]=E[f]E[z_{mn}(v)]=0$ and $E[Q_0(u)q(\zeta,\eta)]=E[Q_0(u)]E[q(\zeta,\eta)]=0$, we can write

$$E[z_{mn}(u)]=\frac{4(1-\cos m\pi)(1-\cos n\pi)}{m n\pi^2}E[Q_0(u)]$$

Therefore eq. (86) becomes

$$E[w(\zeta,\eta,t)]=\frac{16}{\rho h\pi^2}\sum_{m=1,3,5\cdots}^{\infty}\sum_{n=1,3,5\cdots}^{\infty} \times\frac{\sin m\pi\zeta\sin n\pi\eta}{mn}\int_0^t h_{mn}(t-u)E[Q_0(u)]du \quad \cdots\cdots (87)$$

If $Q_0(t)$ has zero mean, then the deflection has zero mean. The mean square of the deflectionis evaluated by multiplying eq. (85) by itself and then obtaining the expected value. After some simplification, we will have

$$E[w^2(\zeta,\eta,t)]=\frac{1}{\rho^2h^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \times\sin m\pi\zeta\sin n\pi\eta\sin p\pi\zeta\sin q\pi\eta \times\int_0^t\int_0^t E[z_{mn}(u_1)z_{pq}(u_2)]+\frac{C^2{}_0\sigma_f^2}{\rho^2h^2} \times\int_0^{u_1}\int_0^{u_2} g_{mn}(u_1-v_1)g_{pq}(u_2-v) \times E[z_{mn}(v_1)z_{pq}(v_2)]dv_2dv_1h_{mn}(t-u_1)h_{pq}(t-u_2)du_2du_1 \quad \cdots\cdots (88)$$

where

$$E[z_{mn}(u_1)z_{pq}(u_2)]=16\,E[Q_0(u_1)Q_0(u_2)] \times\int_0^1\int_0^1\int_0^1\int_0^1\{1+E[q(\zeta_1,\eta_1)q(\zeta_2,\eta_2)]\} \times\sin m\pi\zeta_1\sin n\pi\eta_1\sin p\pi\zeta_2 \times\sin q\pi\eta_2 d\zeta_1 d\eta_1 d\zeta_2 d\eta_2 \quad \cdots\cdots (89)$$

and σ_f^2 is the variance of f.

Equation (88) is a general form for the mean square of the deflection and is evaluated for any given σ_f^2 and the auto correlation functions $E[Q_0(u_1)Q_0(u_2)]$ and $E[q(\zeta_1,\eta_1)q(\zeta_2,\eta_2)]$.

Let us consider the case when the external excitation, $Q(\zeta, \eta, t)$ has the following properties

(1) Correlation between $q(\zeta_1, \eta_1)$ and $q(\zeta_2, \eta_2)$ is uniform and is equal to σ_q^2. In other words, they are completely correlated.

$$E[q(\zeta_1,\eta_1)q(\zeta_2,\eta_2)]=R_q(\zeta_1-\zeta_2,\eta_1-\eta_2)=\sigma_q^2 \quad \cdots\cdots (90)$$

(2) $Q_0(t)$ is a white noise process (6) with spectral density function $S(\omega)=S_0$. Hence, auto correlation function of $Q_0(t)$ is given by

$$R_{Q_0}(t_1-t_2)=E[Q_0(t_1)Q_0(t_2)]=2\pi S_0\delta(t_1-t_2) \quad \cdots\cdots (91)$$

If t_1-t_2 is infinity, we may assume that $Q_0(t_1)$ and $Q_0(t_2)$ are almost independent. Therefore we have

$$R_{Q_0}(t_1-t_2)=E[Q_0(t_1)Q_0(t_2)] \cong E[Q_0(t_1)]E[Q_0(t_2)]$$

Since $Q_0(t)$ is a stationary function we have

$$E[Q_0(t_1)]=E[Q_0(t_2)]$$

Hence,

$$E[Q_0(t)]=[R_{Q_0}(t_1-t_2)]^{1/2}=0 \quad \text{for } t_1-t_2=\infty$$

The mean square becomes identical to the variance.

Introducing eqs. (90) and (91) into eq. (88) and (89), we obtain

$$E[w^2(\zeta,\eta,t)]=\frac{256(2\pi S_0)(1+\sigma_q^2)}{\rho^2h^2\pi^4}$$
$$\times\sum_{m,n,p,q=1,3,5\cdots}^{\infty}\sum^{\infty}\sum^{\infty}\sum^{\infty}\frac{\sin m\pi\zeta\sin n\pi\eta\sin p\pi\zeta\sin q\pi\eta}{mnpq}$$
$$\times\left[\Phi_{mnpq}(t)+\frac{C_0^2\sigma_f^2}{\rho^2h^2}\Psi_{mnpq}(t)\right]\cdots\cdots\cdots\cdots(92)$$

where

$$\Phi_{mnpq}(t)=\int_0^t h_{mn}(t-u_1)h_{pq}(t-u_1)du_1\cdots(93)$$
$$\Psi_{mnpq}(t)=\int_0^t\int_0^t\int_0^{u_1}g_{mn}(u_1-v_1)g_{pq}(u_2-v_1)$$
$$\times dv_1h_{mn}(t-u_1)h_{pq}(t-u_2)du_2du_1\cdots\cdots(94)$$

Fig. 5 shows the mean square of the deflection at the center of a square plate when the damping coefficient is deterministic [i.e., $\sigma_f^2=0$] and $\sigma_q^2=0.01$, $\frac{D\pi^4}{a^4\rho h}=0.1$ where the first ten modes are included in the calculation. The curve indicates that the mean square of deflection approaches stationarity after passing through the non-stationary transient stage caused by the given finite initial conditions. (Ref. 7), 8), and 9)). Fig. 6 shows the effect of σ_q^2 to the mean square of deflection. Fig. 7 shows the mean square deflection as a function of σ_q^2 when $D\pi^4/a^4\rho h=0.1$ and $C_0/2\rho h=0.1$. If the damping coefficient is random, we have to evaluate the triple integration of eq. (94). Without doing so, we can examine the mean square of the deflection qualitatively. In the stationary, the expessions (92) with (93) and (94) can be put into the form

$$\frac{\rho^2h^2\pi^4}{256(2\pi S_0)}E[w^2(\zeta,\eta,t)]$$
$$=(1+\sigma_q^2)\left(A+\frac{C_0}{\rho^2h^2}\sigma_f^2B\right)\cdots\cdots\cdots\cdots\cdots(95)$$

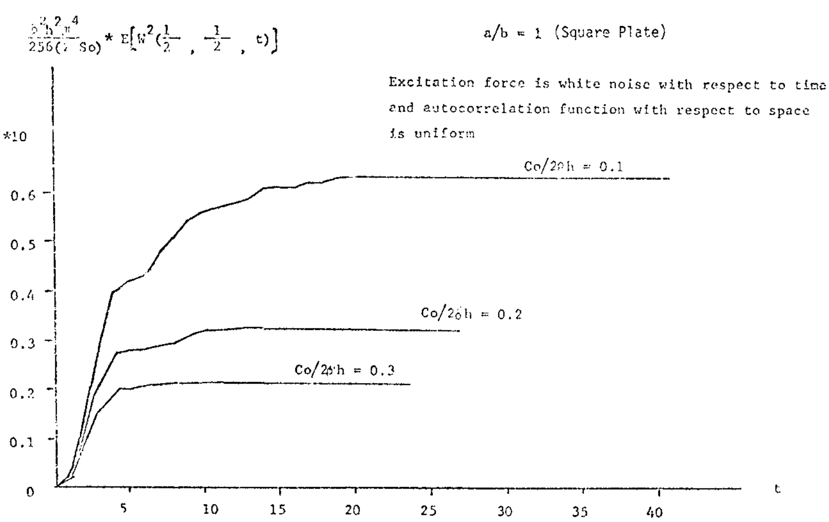

Fig. 5 Mean square of deflection for given $\frac{D\pi^4}{a^4\rho h}=0.1$ and $\sigma_q^2=0.01$

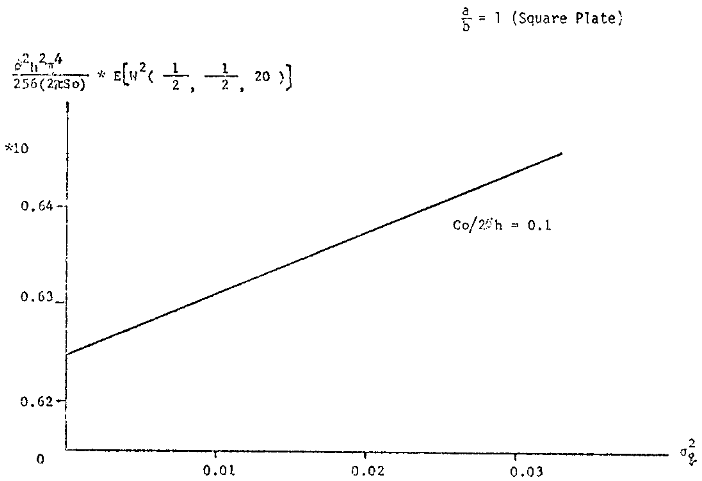

Fig. 6 Mean square of deflection at stationary situation

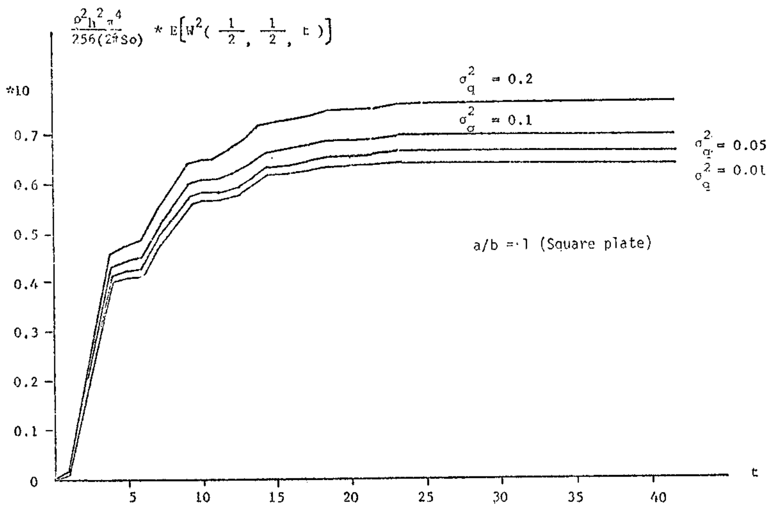

Fig. 7 Mean square of deflection for given $\frac{D\pi^4}{a^4\rho h}=0.1$ and $C_0/2\rho h=0.1$

where A and B are some constants.

We note the mean square is linearly related with σ_f^2 and therefore this value increases as the variance of the damping coefficient increases.

V. CONCLUSIONS

In the free vibration of the plate, we have obtained the general expressions of the mean, mean square and variance of $m\times n^{th}$ natural frequency when the plate flexural rigidity and the mass density of the plate are random functions of space. The effect of the auto correlation the cross correlation of these random inputs to the natural frequency is examined.

By extending perturbation technique to the non-homogeneous stochastic equation, we have evaluated the deflection of a uniform square plate subjected to the random excitation and having uniformly distributed damping having a random value.

We have discussed the case when the excitation is white noise with respect to time and distributed over the plate with uniform correlation. It is very interesting to note the transient stage when the mean square of the deflection fluctuates and reaches steady state after some lapse of time. The duration of the transient stage depends on the amount of damping present in the vibrating plate.

NOMENCLATURE

a=length of a rectangular plate along x axis.
b=length of a rectangular plate along y axis.
C=coefficient of viscouse damping.
C_0=deterministic constant corresponding to C.
$d(\zeta,\eta)$=stationary small random perturbation of $D(\zeta,\eta)$
$D(\zeta,\eta)$=flexural rigidity of a plate
D_0=deterministic constant corresponding to $D(\zeta,\eta)$
$e(\zeta,\eta)$=small random perturbation of $\rho(\zeta,\eta)$
E=Young's modulus of a plate
$E[\ ]$=expected average
f=small random variable corresponding to C
h=thickness of a plate
$q(\zeta,\eta)$=random function of space
$Q(\zeta,\eta,t)$=lateral excitation force
$Q_0(t)$=random function of time
$R_{dd}(\zeta_1,\zeta_2)$=autocorrelation function of $d(\zeta_1)$ and $d(\zeta_2)$
$R_{de}(\zeta_1,\eta_2)$=autocorrelation function of $d(\zeta_1)$ and $e(\zeta_2)$
$R_{ee}(\zeta_1,\zeta_2)$=autocorrelation function of $e(\zeta_1)$ and $e(\zeta_2)$
$R_{Q_0}(t_1-t_2)$=autocorrelation function of $Q_0(t_1)$ and $Q_0(t_2)$
$T_{mn}(t)$=time dependent function
Var[ ]=variance
$W(x,y,t)$=deflection of a plate
$W_{mn}(\zeta,\eta)$=normal mode of a plate
x=coordinate
y=coordinate
α=expansion parameter
β=expansion parameter
ε=correlation length
ζ=nondimensionalized coordinate of x.
η=nondimensionalized coordinate of y.
λ=nondimensionalized natural frequency of a plate.
ν=Poisson ratio
ρ=mass density of a plate
ρ_d, ρ_e, ρ_{de}=corelation coefficient
ρ_0=deterministic constant corresponding to ρ

σ=standard deviation

σ^2=variance

ω_{mn}=natural frequency

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