Proc. of JSCE,
No. 186, Feb. 1971

BASIC STUDY ON STATISTICAL ANALYSIS OF AREAL CONCENTRATION

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1. INTRODUCTION

In recent years, the growing enthusiasm for the use of mathematical models as aids to regional planning derives less from the proven adequacy of such than from the high speed mechanical accuracy with which mathematical models improve the planner's ability to generate sound policy and effective programs.

Nowadays, we know the regional problems to be complex social and economical mechanisms of public facilities, transportation and communication networks, of the rapid concentration of population in the metropolitan area, of the markets for land, housing and even human resources intertwined with the processes of the metropolitan community. Thus, considering this progress, the analysis of areal concentration is the field that is most difficult in research and is unsolved. Then, it is becoming more important that we analyze the descriptive generalizations based on social and economic empirical structure of regional problems.

The builder of a descriptive model has to replicate the relevant features of an existing regional environment or of an already observed process of regional changes. And he must be more explicit in detailing the exact functional forms of his structual relations with the generating empirically relevant output from empirically based input.

We prefer to deal with statistics of mass behavior and properties of regional structures. The macroanalysis consists in large part of descriptive generalizations based on empirical structures.

In this paper, we describe the mathematical standard methods for analysis of areal concentration and apply the methods to analyze the concentration of population and commercial function in the Hokkaido area.

2. MEASUREMENTS OF AREAL CONCENTRATION

In the studies of regional science, various kinds of indices or coefficients have been used so as to measure and describe the extent of areal concentration.

Hoover[1)2)] and Duncan[5)6)] have attempted to extend the graphic technique of the localization curve to compare distributions ordered in accordance with other external criteria. Especially, Hoover's urbanization curve is obtained by the same method as the localization curve except that the order of the graphic cumulation is according the city size.

The present paper relates to such measures as location quotient of concentration, applying to the actual data for the analysis of areal concentration. This direction of research also connects with the basic problem of concentration and a new curve of deriving from Lorenz curve. Prof. Hiroozo Ogawa[7)] referred to the localization curve as the η curve.

We define the notation of the η curve as follows:

$Y=\{y_i/(i=1, \cdots n)\}$

the set of distribution ratios of nonbasic magnitude in the vartical axis.

$X\{x_i/(i=1, \cdots n)\}$

the set of distribution ratios of basic magnitude in the horizontal axis.

n : the number of subareas in the studied area.

Where $\sum_j^n x_j=\sum_j^n y_j=1$

$D=\{D_i/D_i=y_i/x_i(i=1, \cdots, n)\}$

the location quotient.

We rank $\{D_i\}$ in smaller order and designate $\{D_j\}$ with the new number order j.

$Y'=\{y_j'/y_j'=y_{j-1}'+y_j,\ y_1'=y_1\ (j=1, \cdots, n)\}$

$X'=\{x_j'/x_j'=x_{j-1}'+x_j,\ x_1'=x_1\ (j=1, \cdots, n)\}$

In the following, plot the points of $X'\times Y'$ against each j.

Before starting the analysis, we determine whether some curves other than the theoretical power function might fit the data better.

The η curve is founded in formula $Y=X^b$. It becomes necessary to rectify the data by taking logarithms of X and Y before proceeding with the usual least-squares solution. In short, the problem resolved itself to finding the value of the coefficient b in the following linear equation, which is a generalized version $\log Y=b \log X$.

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The value of b designates the coefficient of concentration.

If b is one, then the η curve is identical with the diagonal. The nonbasic magnitude is distributed over the various size groups in the same way as the basic magnitude.

If b increases more than one, then the η curve bows below the diagonal. It indicates that the nonbasic magnitude is more concentrated than the basic magnitude.

Next, the author represents the four new methods to analyze the areal concentration using the η curve.

(1) Areal Concentration Analysis

Areal Concentration Analysis is the method that means to determine the location of a boundary of the concentrated area.

We have to solve the curvature of the η curve to determine the boundary. We can simply solve the curvature of the η curve because the η curve is denoted as X and Y axis.

We put,

$$ds^2=dx^2+dy^2 \qquad dy/dx=\tan\theta$$

The performance equations are as follows :

$$\frac{d}{d\theta}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}(\tan\theta)=\sec^2\theta$$

$$\frac{d}{d\theta}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{d\theta}=\frac{dy^2}{dx^2}\left(\frac{dx}{d\theta}\right)$$

$$=\sec^2\theta=1+\tan^2\theta=1+\left(\frac{dy}{dx}\right)^2$$

Therefore,

$$\frac{dx}{d\theta}=\frac{1}{y''}(1+y'^2)$$

$$\frac{ds}{dx}=\sqrt{1+(dy/dx)^2}=\sqrt{1+y'^2}$$

$$dy/dx=y' \qquad dy^2/dx^2=y''$$

The curvature $d\theta/ds$ is as follows :

$$d\theta/ds=y''/(\sqrt{1+(y')^2})^3$$

We define the breaking point that takes a maximum value of curvature and represent this breaking point as the concentrated point. We define the concentrated area as the area that lies to the right of the concentrated point.

In addition, we can select the homogenious region by this method.

(2) Distribution Analysis

Distribution Analysis is the method that indicates how the distribution percentages of function change as the degrees of the coefficients of concentration increase.

We define the notation of this method as follows:

$$Y=\{y_{ij}/(i=1, \cdots, m)(j=1, \cdots, n)$$

the set of distribution ratios of nonbasic magnitude index j against the coefficients of concentration $\{b_i\}$.

$$F=\{f_{ij}(b_i)/f_{ij}(b_i)=(y_{ij}/\sum_{j=1}^{n} y_{ij})\times 100$$

$$(i=1, \cdots, m)(j=1, \cdots n)\}$$

the set of distribution percentages of nonbasic magnitude index j against the coefficients of concentration $\{b_i\}$.

On the horizontal axis are plotted the coefficients of concentration $\{b_i\}$. And on the vertical axis are plotted the distribution percentages of nonbasic magnitude F.

(3) Time Series Analysis of Areal Concentration

This analysis is the method that indicates how the distributions of areal concentration change as time passes.

$$Y=\{y_i/(i=1, \cdots, n)\}$$

the set of distribution ratios of nonbasic magnitude of next census year.

$$X=\{x_i/(i=1, \cdots, n)\}$$

the set of distribution ratios of basic magnitude of basic time.

n : the number of subareas in the studied area.

$$S=\{S_i/S_i=y_i/x_i \ (i=1, \cdots, n)\}$$

the set of shift ratios of areal concentration between basic year and next census year.

We rank $\{S_i\}$ in smaller number order and designate $\{S_j\}$ with new number order j.

Where $\sum_{j=1}^{n} x_j=\sum_{j=1}^{n} y_j=1$

$$Y'=\{y'_j/y_j'=y_{j-1}'+y_j, \ y_1'=y_1 \ (j=1, \cdots, n)\}$$

$$X=\{x_j'/x_j'=x_{j-1}'+x_j, \ x_1=x_1 \ (j=1, \cdots, n)\}$$

In this case, the ratios of the basic magnitude X are plotted on the horizontal axis and the ratios of the nonbasic magnitude Y on the vertical axis against j. This indicates that the distribution of the curves is concentrated regularly as time passes.

According to a combination of the Distribution Analysis and the Time Series Analysis, we can analyze the changes of the distribution percentages of the economic indexes as time passes. In this case, b_i is the function of time t_i. Where $b_i=g(t_i)$.

In general, the function g is a nonlinear function. This analysis becomes the standard time series analysis if, and only if, the function g is linear.

(4) Interaction Analysis

This analysis is the method calculating the degree of the connection between every pair of the subarea in the studied region. And by this method, we can define the comming area and the living area.

$$I=\{I_{ij}/(i=1, \cdots, m)(j=1, \cdots, n)\}$$

the set of the ratios of the actual trip volumes between a central originating subarea i and any termi-

nating subarea j.

$$E=\{E_{i,j}/E_{i,j}=(P_i\cdot P_j/d_{i,j})/\sum_j^n (P_i\cdot P_j/d_{ij})$$
$$(i=1, \cdots, m)(j=1, \cdots, n)\}$$

the set of ratios of the demographic energies between a central originating subarea i and any terminating subarea j.

Where d_{ij} is the distance which separates i area and j area, and P_i is defined the potential of area i. If a metropolitan traffic study is being conducted, distance of terms much more significant than physical distance.

$$T=\{T_{i_0,j}/T_{i_0,j}=I_{i_0,j}/E_{i_0,j}(j=1, \cdots, n)\}$$

the set of the differential quotients between the actual trip and ideal trip between fix i_0 and any subarea j.

We rank $T_{i_0 j}$ in smaller order and designate $T_{i_0 k}$ with the new number order k.

Where $\sum_j^n T_{i,j}=\sum_j^n I_{i,j}=1 \quad (i=1, \cdots, m)$

$$I'=\{I_{i_0,k}'/I_{i_0,k}'=I_{i_0,k-1}'+I_{i_0,k}I_{i_0,1}'$$
$$=I_{i_0,1}(k=1, \cdots, n)\}$$
$$E'=\{E_{i_0,k}'/E_{i_0,k}'=E_{i_0,k-1}'+E_{i_0,k}E_{i_0,1}$$
$$=E_{i_0,1}(k=1, \cdots, n)\}$$

We plot on a graph on the horizontal axis of $\{E_{i_0,k}\}$ and on the vertical axis of $\{I_{i_0,k}\}$ against each k. We can formulate the η curve $F(E)$ with least squares solution. In addition, we define the coefficients of concentrated are as follows :

$$Q(E)=\int_0^1 F(E)\cdot dE$$

Q indicates the expansion of concentrated area of the central area i.

According to a combination of the four methods mentioned above, we can analyze the function of the nodal region.

When we compare the two percentages distributions of areal concentration, we must have the common units of classification, for example, prefectures, counties, cities, census tracts.

3. AREAL CONCENTRATION OF POPULATION IN HOKKAIDO

One of the most useful sets of data in the regional analysis relates to the population distribution. And the past and current materials on the population distribution are generally available for political administrative functions of regions.

The regional analysis of population begins with the study of areal differences of the population distribution. Since it can provide a fundamental concept of activities of human beings.

In this chapter we shall discuss primarily the empirical laws of the areal concentration of population in the Hokkaido area.

First, for the Time Series Analysis of the areal concentration of population of cities and counties, we plot the cumulative ratios of population in 1956 (the basic time) as the basic magnitude on the horizontal axis and the cumulative ratios of population of every other year from 1958 to 1966 as the nonbasic magnitude on the vertical axis. And the graph of the Time Series Analysis is shown in Fig. 1.

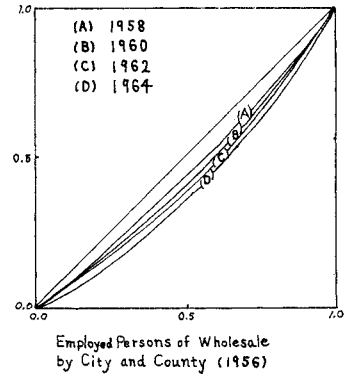


Fig. 1 Time Series Analysis of Population

In addition, we analyze the changes of the distribution percentages of population. Then we plot the distribution percentages of population of oities and counties on the vertical axis and the coefficients of concentration on the horizontal axis against the past census years. And these graphs are shown from Fig. 2 to Fig. 4.

From the graphs described above, the most remarkable feature is the heavy concentration of population around the regional central city Sapporo, central cities Asahikawa, Kushiro, Obihiro, Kitami, Muroran, Tomakomai, Takikawa, Nemuro and satellite cities Chitose, Ebetsu.

This indicates that the concentration of population has created not only a movement of people from rural area to urban area, but also a movement of people from urban area in the coal industrial area to urban area in the regional central city and central cities.

Among these cities Sapporo has the outstanding big amount of net-in-migration. Takikawa is becomming the central city in the Middle Sorachi Shicho with a plan from groups of local authorities under regional basis.

Table 1

Number	Year	Coeficients of Concentration	Correlation Coefficients
(A)	1958	1.0391	0.9999
(B)	1960	1.0738	0.9999
(C)	1962	1.0931	0.9998
(D)	1964	1.1653	0.9996
(E)	1966	1.2408	0.9993

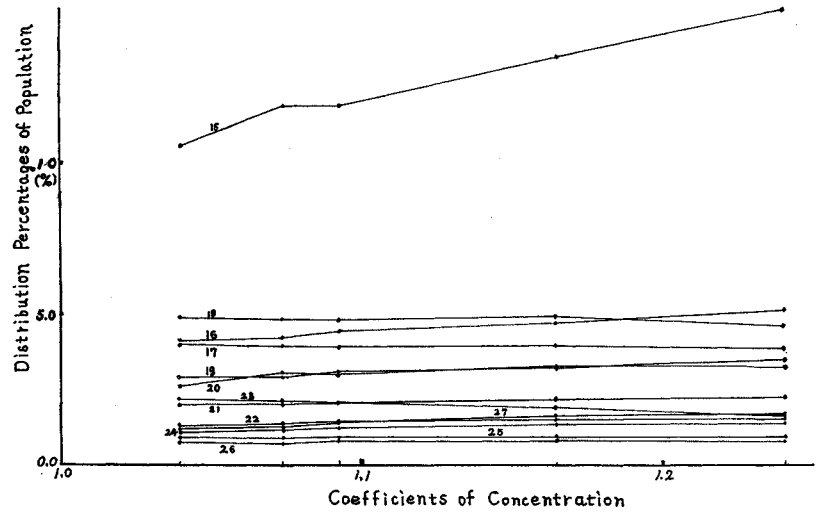


Fig. 2 Distribution Analysis of Population (1)

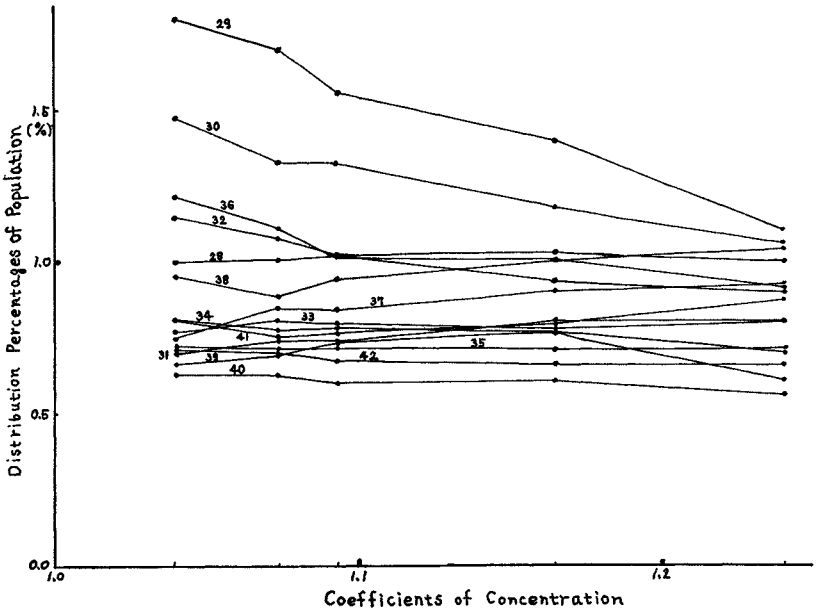


Fig. 3 Distribution Analysis of Population (2)

This discussion has used population as a measure of mass. But if the actural inflow trips is to be studied, it is apparent that the employed persons and persons attending school tend to be more significant indices of mass.

Then we propose to investigate the inflow trips to Sapporo city by the Interaction Analysis the graph of the Interaction Analysis is shown in Fig. 5. This graph indicates the relation between the actual inflow trips and the expected trips to Sapporo city for the employed persons and persons attending school.

We let P_i represent the population i area and especially P_0 the population of Sapporo city. d_{ij} is the time distance from Sapporo to each town. The expansion of concentrated area $Q(E)$ increases from 0.2073 to 0.2848 during five years. It gives the following characteristic points. The areas where the inflow to Sapporo increases from 1960 to 1965 are not only the areas in the neighbourhood of Sapporo city, but also Ebetsu, Chitose and Thobetsu. Then, it has made a remarkable expansion of the comming area of Sapporo. Ebetsu city and Chitose city are situated in 20 km and 40 km at the east and south of Sapporo city as satellite cities.

In addition to the fact that it has made a remarkable development, the characteristic feature is that it is a residential quarter for the professional & clerical workers who have their jobs in Sapporo.

The investigation of the previous residences of immigrants in such cities has relation to the distance

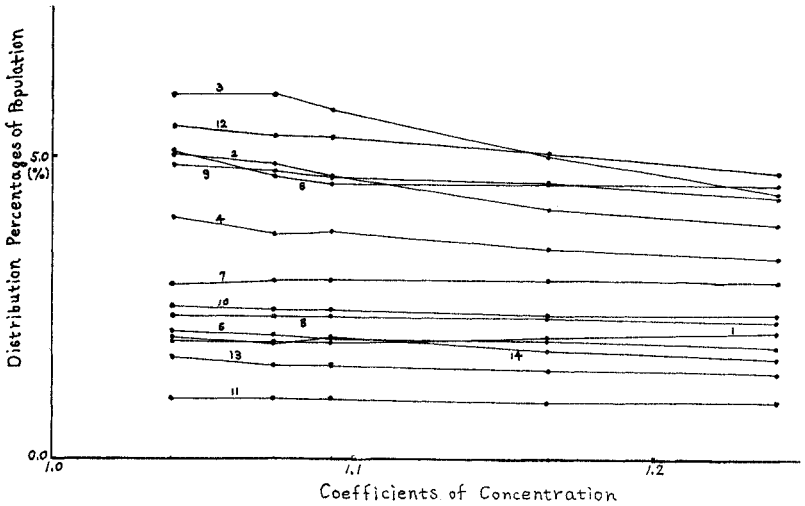


Fig. 4 Distribution Analysis of Population (3)

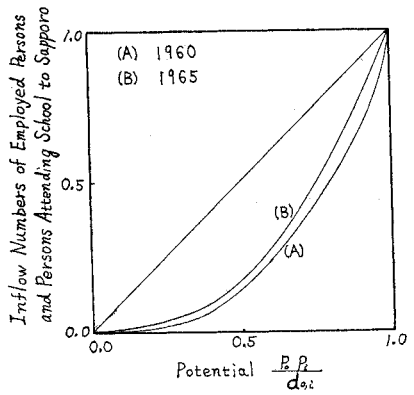


Fig. 5 Interaction Analysis of Inflow to Sapporo

between their previous areas and Sapporo city. In addition, it is indicated as an exceptional aspect that the drift to the city covers the regional-wide area (Gross Sapporo area).

Finally, as far as a comparison of the areal concentration of population of the 46 prefectures and the total cities in Japan, the reader may refer to the author's paper[31].

4. AREAL CONCENTRATION OF COMMERCIAL FUNCTION

In recent years, many new cities have come into existence and central cities have expanded their city areas absorbing their surrounding rural villages. In such a new city area, consumers change their former living condition and consumer habits.

As for commercial activities, many arguments have been presented in as much as many economists are much more likely to think in terms of marketing. They work from a theory of individual choise and have their own version of the market models.

Among these, the microanalytic approach has its problems in that a model based on the theory of rational choice can be implemented only if the chooser's system of relative values can be specified in considerable detail.

In this chapter, the author tries to clarify the correlations between the areal concentration of commercial functions and population from the point of view of regional analysis.

First, we have attempted to contrast the geographic patterns of several commercial trades at a given point of time.

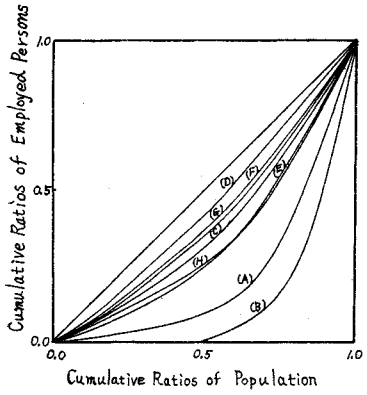
The experimental results of the concentration analysis for persons engaged in Wholesale and Retail trades (1964) by industry are shown in Fig. 6. The cumulative number of population is the basic magnitude on the horizontal axis and the cumulative number of persons engaged is the nonbasic magnitude on the vertical axis.

The total number of persons engaged is concentrated in the regional central city Sapporo and central cities Asahikawa, Otaru, Hakodate, Muroran, Obihiro and Takikawa.

The persons engaged in Wholesale and General merchandise are concentrated in the cities with populations of 100,000 and over. The regional central city Sapporo has the outstanding high concentration of persons engaged in Wholesale (42%) and in General merchandise (47%).

The persons engaged of General merchandise are the persons engaged of Department store.

However, except for Eating house trade which is in satisfactory agreement with the concentration of the total, Food and beverages retail trade which sell household commodities has no areal difference, and other industries don't show higher concentration than that of the total.



(A) Wholesale
(B) General merchandise (Department stores)
(C) Textiles, apparel and accessories
(D) Food and beverages
(E) Eating house
(F) Bicycles and carts
(G) Furniture and house furnishing
(H) Total

Fig. 6 Areal Concentration Analysis of Employed Persons of Commercial Trades (1964)

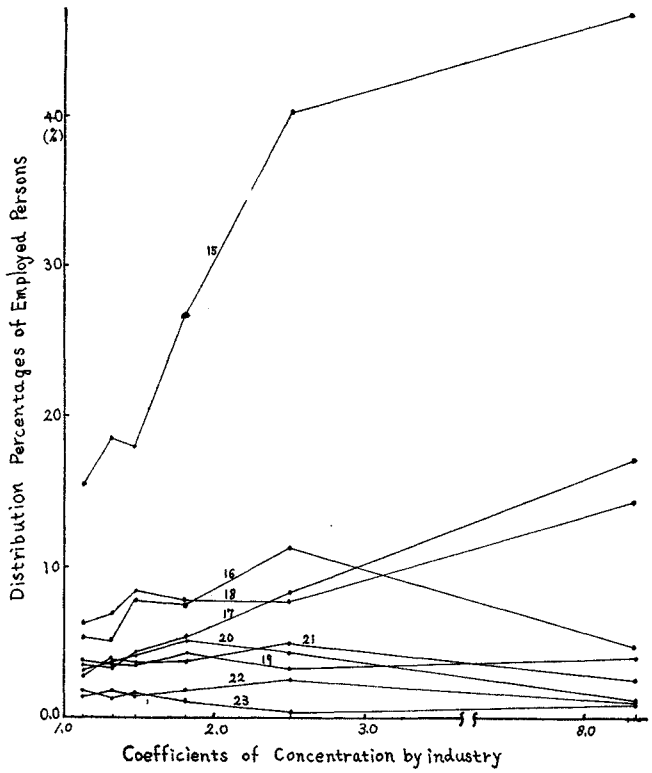


Fig. 7 Distribution Analysis of Industry (1)

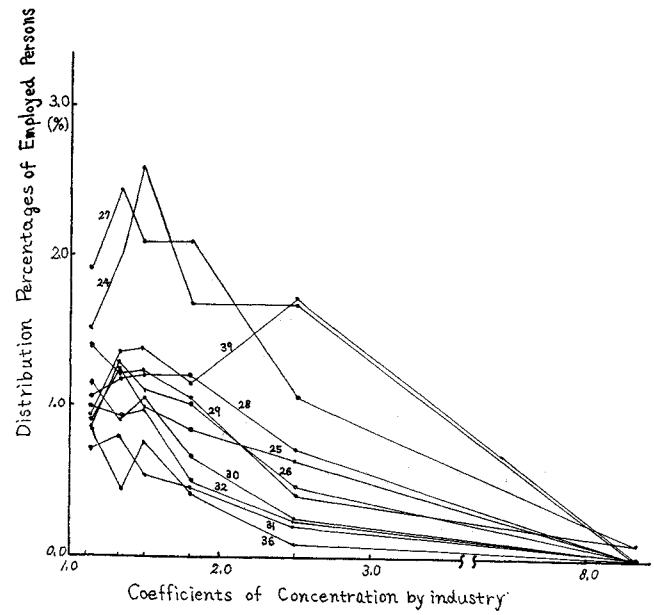


Fig. 8 Distribution Analysis of Industry (2)

Next, we analyze the changes of distribution percentages by city and county as the degrees of the coefficients of industries increase.

The changes of distribution percentages are shown from Fig. 7 to Fig. 10. And the changes of coefficients of the η curves by industries are shown in Table 2.

The set of cities and counties are divided into four groups.

The first group is the regional central city and central cities where the distribution percentages show higher values as the coefficients of concentration increase in values. The names of these cities are Sapporo, Otaru and Hakodate.

The second group is the central cities where the distribution percentages are almost unchange. The names of these cities are Muroran, Kushiro and Kitami.

The third group shows the central cities where the distribution percentages present especially high values in Wholesale concentration. The names of these cities are Asahikawa, Obihiro and Takikawa.

The fourth group is the central cities and counties where the distribution percentages show slightly lower values. This group included all other cities and counties except those mentioned above.

Next, we have attempted to contrast the concentration patterns of several trades in Wholesale trade in 1964. The experimental results of areal concentration are shown in Table 3.

The one group is the trades of Agriculture forestry & fisheries, Lumber & wood, Food &

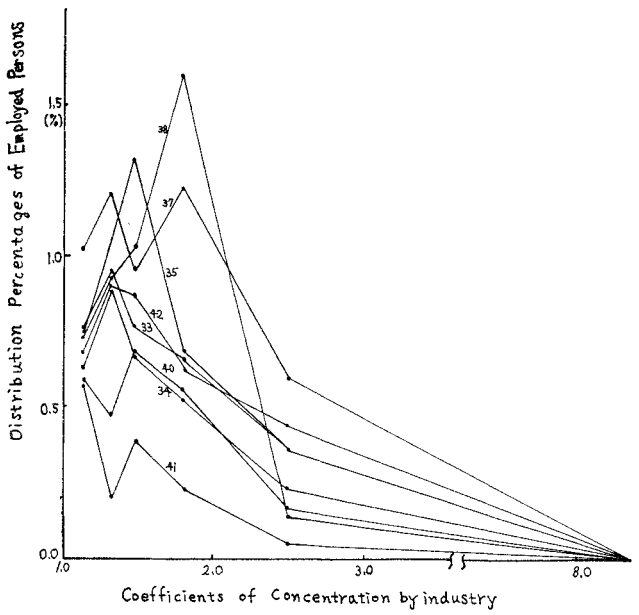


Fig. 9 Distribution Analysis of Industry (3)

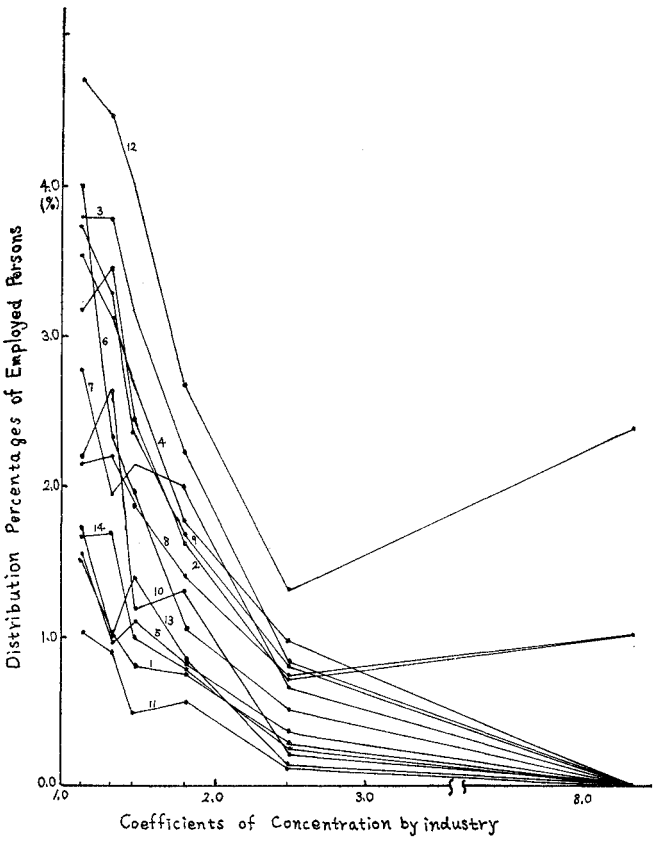


Fig. 10 Distribution Analysis of Industry (4)

beverages and Reclamation materials that are based on household consumer. They show the low values of coefficients of concentration. The other group is the trades that are concentrated in the regional central city and central cities with populations of 50,000 and over. They are Machinery, Furniture & house furnishing, Cloth, Textiles, apparel & accessories, Chemical, Medicine & Miscellaneous. They show the high values of coefficients of concentration.

Next, for the Time Series Analysis of areal concentration of Wholesale, Food & beverages, Furniture & house furnishing and Textiles, apparel & accessories, we plot the cumulative ratios of employed persons in 1956 on the horizontal axis and the cumulative ratios of employed persons of every other year from 1958 to 1966 on the vertical axis. The changes of the curves are shown in Fig. 11. And the changes of coefficients of the η curves are shown in Table 4.

In addition, for the analysis of the differential shift ratio of the areal concentration between the commercial trades mentioned above and population, we plot the coefficients of concentration of industries on the vertical axis and the coefficients of concentration of population on the horizontal axis. This graph is shown in Fig. 12. From the facts of this graph, we may conclude that the shift ratios of areal concentration of Wholesale and Furniture & house furnishing show higher values than that of population and the shift ratio of areal concentration of Food & beverages and Textiles, apparel & accessories show lower values than that of population.

Next, for the detail analysis of the changes of distribution percentages of Wholesale, we use the Distribution Analysis. From the facts of the graph from Fig. 13 to Fig. 15, especially, the distribution percentages of Sapporo, Asahikawa, Tomakomai and Takikawa, show higher values as time passes. But the distribution percentages of Otaru and the cities in the coal industrial region show rapidly values as time passes. This indicates that the changes of the concentration has created not only the decline of the coal industry but also the rapid movements from rural area to central cities, Especially, the decline of Otaru is affected by the outstanding increases of inflow of persons engaged from Otaru to Sapporo.

Table 2

Number	Industry	Coefficients of Concentration	Correlation Coefficients
(A)	Wholesale	2.4978	0.9670
(B)	Department stores	8.3579	0.9754
(C)	Textiles, apparel & accessories	1.4787	0.9956
(D)	Food & beverages	1.1469	0.9996
(E)	Eating house	1.8104	0.9919
(F)	Bicycles and carts	1.4957	0.9970
(G)	Funiture & house furnishing	1.3321	0.9989
(H)	Total	1.7069	0.9950

Table 3

Industry	Coefficients of Concentration	Correlation Coefficients
Agriculture forestry & fisheries	1.8884	0.9956
Reclamation materials	2.1384	0.9670
Lumber & wood	2.4353	0.9470
Food & beverages	2.9834	0.9478
Machinery	3.5286	0.9474
Furniture & house furnishing	6.3481	0.9567
Chemical	7.0791	0.9350
Medicine & Miscellaneous	8.8827	0.8801
Textiles, apparel & accessories	9.1053	0.8686
Cloth	10.6256	0.7537

Table 4

Number	Year	Coefficients of Concentration	Correlation Coefficients
(A)	1958	1.1873	0.9990
(B)	1960	1.2154	0.9997
(C)	1962	1.2762	0.9993
(D)	1964	1.3462	0.9948

Finaly, as far as a comparison of the areal concentration of commercial functions by scales, the reader may refer to the author's paper[46].

5. CONCLUSION

In summary, we have investigated four methods about the areal concentration analysis and applied them to the actual data of population and commercial trades in the Hokkaido area. From the facts described above, we may conclude three points very briefly.

First, according to the Time Series Analysis of the concentration of population, it appears to be

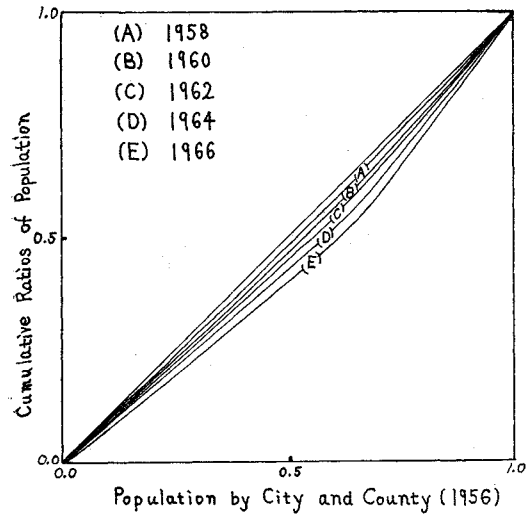


Fig. 11 Time Series Analysis of Wholesale

Table 5

Number	County (Gun)	Number	City
1	Ishikari—Shicho	15	Sapporo
2	Sorachi—Shicho	16	Asahikawa
3	Kamikawa—Shicho	17	Otaru
4	Shiribeshi—Shicho	18	Hakodate
5	Hiyama—Shicho	19	Muroran
6	Oshima—Shich	20	Kushiro
7	Iburi—Shicho	21	Obihiro
8	Hidaka—Shicho	22	Kitami
9	Tokachi—Shicho	23	Yubari
10	Kushiro—Shicho	24	Iwamisawa
11	Nemuro—Shicho	25	Abashiri
12	Abashiri—Shicho	26	Rumoi
13	Soya—Shicho	27	Tomakomai
14	Rumoi—Shicho	28	Wakkanai
		29	Bibai
		30	Ashibetsu
		31	Ebetsu
		32	Akabira
		33	Mombetsu
		34	Shibetsu
		35	Nayoro
		36	Mikasa
		37	Nemuro
		38	Chitose
		39	Takikawa
		40	Sunagawa
		41	Utashinai
		42	Fukagawa

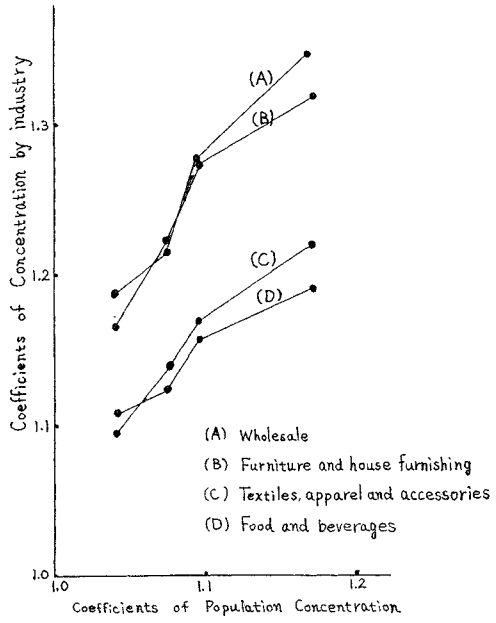


Fig. 12 Relative Growth Chart by Industry

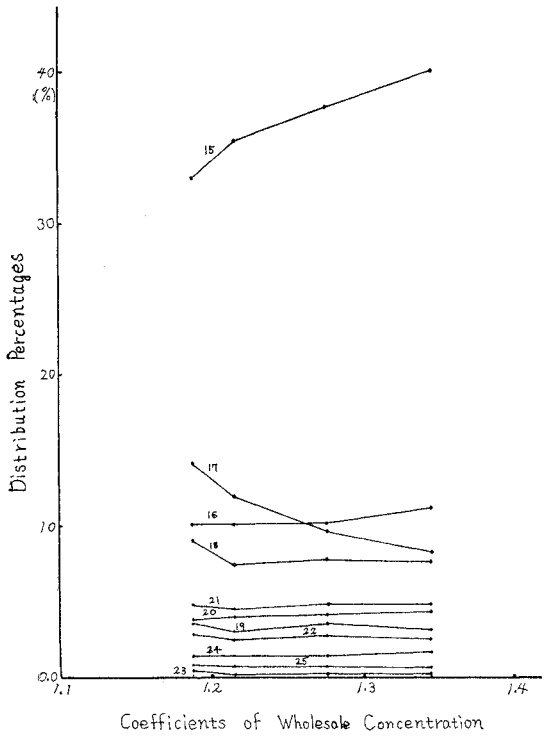


Fig. 13 Distribution Analysis of Wholesale (1)

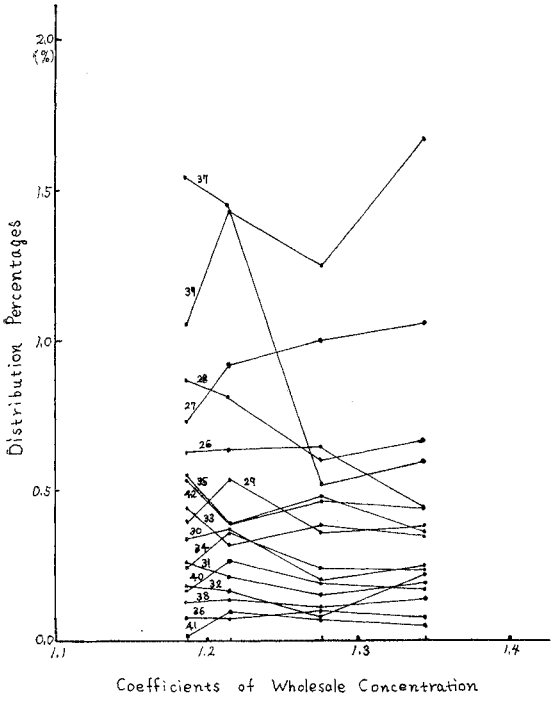


Fig. 14 Distribution Analysis of Wholesale (2)

clear that the concentration of population has created a movement of people from rural area to urban region in the regional central city and central cities.

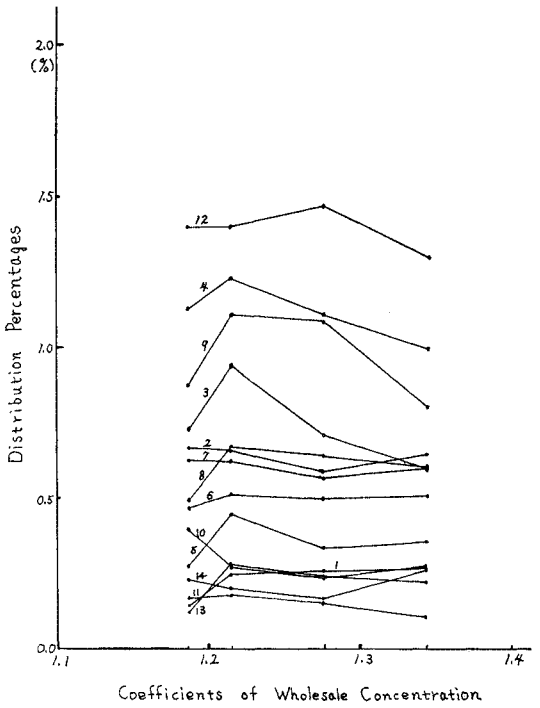


Fig. 15 Distribution Analysis of Wholesale (3)

Second, according to the Concentration Analysis of several commercial trades, the following goods are ranked from low to high : Food & beverages, Furniture & house furnishing, Bicycles & carts, Textiles apparel & accessories, Eating house, Wholesale and General merchandise (Department store).

Third, as for the difference of areal concentration between population and commercial trades, it may appear to be clear that the changes of concentration have created not only the decline of the coal industry but also the rapid expansion of the functions of regional central city and central cities. Especially, the decline of Wholesale trade of Otaru is affected by the outstanding increases of the commercial function of Sapporo.

ACKNOWLEGEMENTS

The author wishes to express his gratitude to Professor H. Ogawa and H. Igarashi of Hokkaido University for suggesting the problem and for their encouragement throughout the work.

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(Recined April 21, 1970)