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# ALTERNATE SCOURS IN STRAIGHT ALLUVIAL CHANNELS

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## I. INTRODUCTION

Two types of meandering patterns in straight alluvial channels with solid banks were indicated by H. W. Shen[1], and H. A. Einstein and H. W. Shen[2]. The first type, a special case of the diagonal dune pattern, resulted from surface waves on the water, and the second type resulted from the difference between the shear stresses at the bed and walls and had deep alternate scour holes. The second type of meandering pattern was caused by secondary flows created by the variation of the Reynolds stresses. Experimental studies on meanderings in straight channels with unerodible walls or banks were conducted by several researchers[3],[4],[5]. By considering a hypothesis that helicoidal currents cause the meandering patterns, W. F. Tanner[6] studied meandering patterns of flows which were suspended underneath nearly horizontal glass plates.

For alluvial channels with erodible banks, a comprehensive experimental study was made by J. F. Friedkin[7]. In 1966, W. B. Langbein and L. B. Leopold[8] studied the river meander by using the theory of minimum variance. Very interesting laboratory studies were conducted by N. A. Rzhanitsyn[9], and G. H. Toebes and A. A. Sooky[10] on the velocity distributions in a rigid boundary meander-flood plain model.

Alternate scour holes and bars can be observed in existing rivers with straight solid banks after a flood. One example of aerial view of alternating bars and scours in the Rio Grande River was given in a paper by the Task Committee on Sedimentation, ASCE[11]. In 1911, R. Jasmund[12] observed rather regular alternate scours in a straight reach with unerodible banks in the Rhein River. In the Rio Grande River near Vinton, Texas, R. K. Fahnestock and T. Maddock, Jr.[13] observed alternate scour holes and bars in reaches having rock revetments. The regions of occurrence of alternate dunes and alternate antidunes on the erodible bed in straight channels were investigated analytically by T. Hayashi[14] as a problem of stability of the erodible bed. In a previous paper, meandering tendencies in straight alluvial channels with solid banks were reported by H. W. Shen and S. Komura[15]. The distance between alternate scour holes in the flow direction (meander length) and the depth of alternate scour holes are analyzed in this paper.

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## II. THEORETICAL CONSIDERATIONS

### (1) Meander Length

The equations of motion and continuity are as follows:

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}=gi-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\nu \nabla^2 \bar{u}-\left[\frac{\partial}{\partial x}(\overline{u'u'})+\frac{\partial}{\partial y}(\overline{u'v'})+\frac{\partial}{\partial z}(\overline{u'w'})\right] \quad (1)$$

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \quad (2)$$

in which $x$ is the coordinate in the flow direction of the channel, and $y$ and $z$ are orientated as in Fig. 1. $\bar{u}$, $\bar{v}$ and $\bar{w}$=time average velocity components in the $x$, $y$ and $z$ directions, respectively; $u'$, $v'$ and $w'$=fluctuating components in the $x$, $y$ and $z$ directions, respectively; $\rho$=fluid density; $p$=pressure; $\nu$=kinematic viscosity; $t$=time; $\nabla^2$=Laplacian operator; $g$=acceleration of gravity; and $i$= bed slope.

If the velocity profiles for walls and bed are assumed to have the form of a power law, then

$$\bar{u}=\bar{u}_{c\,\max}\left(\frac{2y_2}{B}\right)^m\left(\frac{z}{H}\right)^n \quad (3)$$

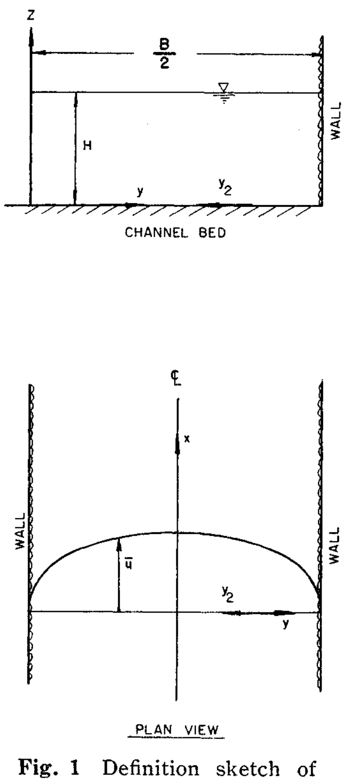


**Fig. 1** Definition sketch of channel.

in which $\bar{u}_{c\max}$=maximum velocity at the middle of the channel; $y_2$=distance from the wall as shown in Fig. 1; $B$=width of the channel; $H$= depth of flow; and $m$ and $n$=dimensionless exponents for velocity distributions. For the validity of this assumption, several evidences can be found in references. For example, W. Nunner[16], H. Schlichting[17], and J. A. Liggett et al.[18] show that the assumption of a simple power law for walls or beds agrees well with experimental data when a suitable choice for the value of power has been made.

Mean velocity for the whole cross-sectional area of the channel, $U$, can be obtained by integrating Eq. (3) from zero to $H$ in $z$ direction and zero to $B/2$ in $y$ direction:

$$U=\frac{\bar{u}_{c\,\max}}{(1+m)(1+n)}\,. \quad (4)$$

Mean velocity at the middle of the channel can be expressed as

$$U_{ce}=\frac{\bar{u}_{c\,\max}}{(1+n)}\,. \quad (5)$$

For the value of $n$, W. Nunner[16] obtained the following equation from Nikuradse's and his own measurements on the velocity distribution:

$$n=\sqrt{\lambda_1} \tag{6}$$

in which $\lambda_1$=friction factor for bed. The coefficient, $\lambda_1$, can be expressed as

$$\lambda_1=8\left(\frac{u_{*1}}{U_{ce}}\right)^2 \tag{7}$$

in which $u_{*1}$=friction velocity for bed. Eliminating $\lambda_1$ and $U_{ce}$ from Eqs. (5), (6) and (7),

$$n=\frac{1}{\left(\dfrac{\bar{u}_{c\,\max}}{\sqrt{8}\,u_{*1}}-1\right)}. \tag{8a}$$

If the maximum velocity at the middle of the channel can be given by Eq. (9), the expression for $n$ becomes Eq. (8b):

$$n=\frac{1}{\left[\dfrac{\ln(M_1H)}{\kappa\sqrt{8}}-1\right]} \tag{8b}$$

$$\bar{u}_{c\,\max}=\frac{u_{*1}}{\kappa}\ln(M_1H) \tag{9}$$

in which $\kappa$=von Kármán's constant, and $M_1=30/k_{s1}$ where $k_{s1}$=representative grain size roughness for bed.

Similar derivation for $m$ can be made by using Eq. (4). The expression for $m$ is

$$m=\frac{1}{\left[\dfrac{\alpha\ln(M_1H)}{\kappa\sqrt{8}(1+n)}-1\right]} \tag{10}$$

in which $\alpha=u_{*1}/u_{*2}$. The value of $\alpha$ can be obtained by trial solution from the following equations as obtained by S. Adachi[19].

The case in which both the bed and walls are hydraulically rough:

for $\alpha\geqq\dfrac{2H}{B}$;

$$\left(\frac{H}{m_rk_{s2}}\right)=\left(\frac{H}{m_rk_{s1}}\right)^{\alpha}\left(\frac{\dfrac{2H}{B}+\alpha^2}{1+\alpha}\right) \tag{11}$$

and for $\alpha\leqq\dfrac{2H}{B}$;

$$\left(\frac{B}{m_rk_{s2}}\right)=2\alpha\left[\left(\frac{H}{m_rk_{s1}}\right)\left(\frac{1+\alpha}{\dfrac{2H}{B}+\alpha^2}\right)\right]^{\alpha} \tag{12}$$

in which $m_r=1/30$, and $k_{s2}$=representative grain size roughness for walls. The subscripts 1 and 2 indicate the bed and the walls, respectively.

The case in which the bed is hydraulically rough and walls are hydraulically smooth:

for $\alpha \geqq \frac{2H}{B}$;

$$\left(\frac{m_s \nu}{H\sqrt{gHS}}\right)=(1+\alpha)\left(\frac{H}{m_r k_{s1}}\right)^{-\alpha}\left(\alpha^2+\frac{2H}{B}\right)^{-3/2} \tag{13}$$

and for $\alpha \leqq \frac{2H}{B}$;

$$\left(\frac{2m_s \nu}{B\sqrt{gHS}}\right)=\left(\alpha^2+\frac{2H}{B}\right)^{\alpha-1/2}\left[\alpha(1+\alpha)\left(\frac{H}{m_r k_{s1}}\right)\right]^{\alpha} \tag{14}$$

in which $m_s=1/9$, and $S$=energy slope of flow.

Assuming that the only one main cell of secondary current exists in the section under consideration, the $\bar{w}$ component at the middle of the channel should be nearly eqaul to zero. Differentiating Eq. (1) with respect to $y$, and since

$$\frac{\partial^2}{\partial x\,\partial y}(\overline{u'u'}) \ll \frac{\partial^2}{\partial y\,\partial z}(\overline{u'w'}),$$

$\frac{\partial^2 \bar{p}}{\partial x\,\partial y}=0$ (after H. Rouse[20]), and $\frac{\partial \bar{v}}{\partial y}=-\frac{\partial \bar{u}}{\partial x}$ from the continuity equation, the expression on $\bar{v}$ at the middle portion of the channel can be expressed as

$$\bar{v}=\frac{-\frac{\partial^2 \bar{u}}{\partial y\,\partial t}-\bar{u}\frac{\partial^2 \bar{u}}{\partial x\,\partial y}+\nu\frac{\partial}{\partial y}(\nabla^2 \bar{u})-\left[\frac{\partial^2(\overline{u'v'})}{\partial y^2}+\frac{\partial^2(\overline{u'w'})}{\partial y\,\partial z}\right]}{\frac{\partial^2 \bar{u}}{\partial y^2}} \tag{15}$$

Von Kármán's equation was used to evaluate the shears:

$$\rho(\overline{u'v'})=\rho\kappa^2\frac{\left(\frac{d\bar{u}}{dy_2}\right)^4}{\left(\frac{d^2\bar{u}}{dy_2{}^2}\right)^2} \tag{16}$$

and

$$\rho(\overline{u'w'})=\rho\kappa^2\frac{\left(\frac{d\bar{u}}{dz}\right)^4}{\left(\frac{d^2\bar{u}}{dz^2}\right)^2} \tag{17}$$

Using Eqs. (3), (16) and (17), and since $y=B/2-y_2$, and $\frac{\partial^2 \bar{u}}{\partial x^2} \ll \frac{\partial^2 \bar{u}}{\partial z^2}$, Eq. (15) becomes

$$\begin{aligned}\bar{v}=&-\frac{y_2}{(m-1)\bar{u}_{c\max}}\left(\frac{\partial \bar{u}_{c\max}}{\partial t}-\frac{n\bar{u}_{c\max}}{H}\frac{\partial H}{\partial t}\right)\\&-\frac{y_2}{(m-1)}\left(\frac{2y_2}{B}\right)^m\left(\frac{z}{H}\right)^n\left(\frac{\partial \bar{u}_{c\max}}{\partial x}-\frac{n\bar{u}_{c\max}}{H}\frac{\partial H}{\partial x}\right)\\&+\nu\left[\frac{-(m-2)}{y_2}+\frac{n(n-1)}{(m-1)}\frac{y_2}{z^2}\right]\end{aligned}$$

$$-2\kappa^2\bar{u}_{c\,\max}\left(\frac{2y_2}{B}\right)\left(\frac{z}{H}\right)^n\left[-\frac{m^2(2m-1)}{(m-1)^3}+\frac{2n^3}{(m-1)(n-1)^2}\frac{y_2}{z}\right]. \tag{18}$$

From the geometry shown in Fig. 2, the meander length, $L$, can be expressed as

$$\frac{B}{L}\propto\frac{|\bar{v}_{c\,\max}|}{\bar{u}_{c\,\max}} \tag{19}$$

in which $\bar{v}_{c\,\max}$=maximum transverse velocity at the middle of the channel. Using Eqs. (18) and (19), and since $m<1$ and $n<1$, the meander length can be expressed as

$$\begin{aligned}\frac{B}{L}=\eta\Bigg\{&\frac{B}{2(1-m)(\bar{u}_{c\,\max})^2}\left[\frac{\partial\bar{u}_{c\,\max}}{\partial t}-\frac{n\bar{u}_{c\,\max}}{H}\frac{\partial H}{\partial t}\right]\\&+\frac{B}{2(1-m)\bar{u}_{c\,\max}}\left(\frac{z_*}{H}\right)^n\left[\frac{\partial\bar{u}_{c\,\max}}{\partial x}-\frac{n\bar{u}_{c\,\max}}{H}\frac{\partial H}{\partial x}\right]\\&+\frac{\nu}{\bar{u}_{c\,\max}}\left[\frac{2(2-m)}{B}+\frac{n(1-n)}{2(1-m)}\frac{B}{z_*^2}\right]\\&+2\kappa^2\left(\frac{z_*}{H}\right)^n\left[\frac{m^2(1-2m)}{(1-m)^3}+\frac{n^3}{(1-m)(1-n)^2}\frac{B}{z_*}\right]\Bigg\}\end{aligned} \tag{20}$$

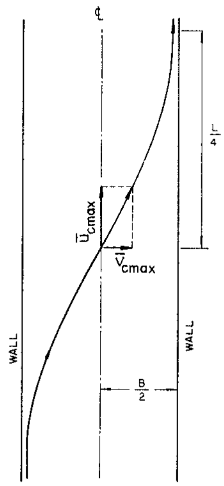


**Fig. 2** Definition sketch on the meander length (distance between alternate scour holes).

in which $\eta$=a constant, and $z_*$=distance from the bed to a point, where a maximum transverse velocity occurs. For practical application, $z_*$ can be replaced by $H$, and since the second and third terms in Eq. (20) are very small, compared with the last term. If the flow is assumed to be steady, Eq. (20) becomes

$$\frac{B}{L}=\eta_a\left[\frac{m^2(1-2m)}{(1-m)^3}+\frac{n^3}{(1-m)(1-n)^2}\left(\frac{B}{H}\right)\right]. \tag{21}$$

As will be shown later, the value of $\eta_a$ was found to be 0.8 from the experimental and field data. The value of $m$ for smooth walls is smaller than that for rough walls. Therefore the value of $B/L$ for smooth walls is small, that is, the value of $L$ for smooth walls is greater than that for rough walls. The tendency to form alternate scours and bars in rough wall channels is higher than that in smooth wall channels.

### (2) Depth of Alternate Scour Holes

The relative scour depth, $H_*$, is defined as

$$H_*=\frac{H_{\max}-H}{H} \tag{22}$$

in which $H_{\max}$=maximum depth of flow in a scour hole. The existence of secondary currents in straight noncircular conduits has been studied rather extensively in the past several years. As explained in the previous paper[15], H. A. Einstein and H. Li[21], J. W. Delleur and D. S. McManus[22], L. C. Hoagland[23], H. J. Tracy[24], and J. A. Liggett et al.[18] all began their analyses from the Reynolds equation of motion with slightly different sets of assumptions. They concluded that the generation of secondary currents $\partial\xi/\partial t$ depended on the variations of Reynolds stresses in the form given by

$$\frac{\partial\xi}{\partial t}=\frac{\partial^2}{\partial y\,\partial z}(\overline{v'^2}-\overline{w'^2})-\frac{\partial^2}{\partial y^2}(\overline{v'w'})+\frac{\partial^2}{\partial z^2}(\overline{v'w'}) \tag{23}$$

in which $\xi=\partial\bar{w}/\partial y-\partial\bar{v}/\partial z$, a measure of the rotation of a fluid particle about an axis normal to the $y$-$z$ plane. H. A. Einstein and H. Li[21] postulated that the sum of the above three terms may be different from zero at corners of a non-circular conduit. Since an analytical solution of Eq. (23) is not possible, H. J. Tracy[24] measured the turbulent structure of the flow in a corner region formed by the junction of two plane boundaries, and obtained information relative to the role of the turbulence with respect to the secondary motions. Then, he examined experimental results with respect to the variation of the normal stress term $\overline{v'^2}$ and $\overline{w'^2}$.

The summation of the three terms on the right side of Eq. (23) should be even greater at a corner between a smooth boundary and a rough boundary than at a corner between two smooth boundaries. This conclusion is obtained by comparing the results of two separate studies made by P. S. Klebanoff[25] and S. Corrsin and A. L. Kistler[26]. H. W. Shen[1] indicated that the formation of alternate scours can be related to a circulation. Therefore it can be considered that the relative scour depth is proportional to a circulation. As indicated by H. J. Tracy[24], the circulation is closely linked to the rotation $\xi$. From the above considerations, it can be assumed that the relative scour depth is given by

$$H_*=C(\xi_s)^{\varepsilon} \tag{24a}$$

in which $C$ and $\varepsilon$ are constants. $\xi_s$ is given by

$$\xi_s=\frac{1}{A}\int_{1/M_1}^{H/2}\int_{1/M_2}^{B/2}\xi\,dy_2\,dz\simeq-\frac{4}{BH}\int_{1/M_1}^{H/2}\int_{0}^{B/2}\left(\frac{\partial\bar{v}}{\partial z}\right)dy_2\,dz \tag{25}$$

in which $A$=cross-sectional area of the channel. From Eqs. (18), (24a) and (25)

$$H_*=C_a\left\{\left(\frac{\bar{u}_{c\max}}{H}\right)\left[\frac{n^3(M_1H)^{(1-n)}}{(1-m)(2+m)(1-n)^2}\left(\frac{B}{H}\right)-\frac{m^2(1-2m)}{(1-m)^3(1+m)}\left(\frac{1}{2}\right)^n\right]\right\}^{\varepsilon}. \tag{24b}$$

Since the second term in Eq. (24b) is very small compared with the first term, the relative scour depth can be expressed as

$$H_*=C_b\left[\frac{n^3(M_1H)^{(1-n)}}{(2+m)(1-m)(1-n)^2}\left(\frac{\bar{u}_{c\max}}{H}\right)\left(\frac{B}{H}\right)\right]^{\varepsilon}. \tag{24c}$$

As will be shown later, the values of $C_b$ and $\varepsilon$ were found to be 0.054 and 0.4, respectively, from the experimental and field data. The value of $m$ for smooth walls is smaller than that for rough walls, therefore the value of $H_*$ by Eq. (24c) gives a small value. From this point of view, the maximum depth of flow for smooth walls is approximately equal to the mean depth of flow, or smaller than that for rough walls. Also, the value of $n$ for smooth bed is smaller than that for hydraulically rough bed.

## III. EVALUATIONS OF EXPERIMENTAL DATA AND OBSERVED DATA IN THE RIO GRANDE RIVER

### (1) Experimental Data

Experiments on meander lengths and alternate scour depths were conducted by the writers in 1966. The experimental flume was 0.38 m deep and 20.73 m long trapezoidal flume with 17.07 m long test section. The bed material used was styrene plastic "Lustrex," a product of Monsanto Chemical Company. The specific gravity of the plastic was 1.054 and the median diameter was 3.4 mm. The experimental results on meander lengths and alternate scour depths for rough wall flume and smooth wall flume were tabulated in a previous paper[15] in 1968. Data on the rough wall flume are tabulated in Table 1. Fig. 3 shows the relationship between the relative scour depth and time, and it clearly indicates that the relative scour depths in the rough wall flume are greater than those in the smooth wall flume. In Fig. 3, $Q$ is the discharge in the test section in the flume, and $Q_d$ refers to the discharge drained from the tailbox. Scour patterns in the rough wall flume were regular, whereas almost all scour patterns in the smooth wall flume were irregular. An examaple of regular scour patterns is shown in Fig. 4.

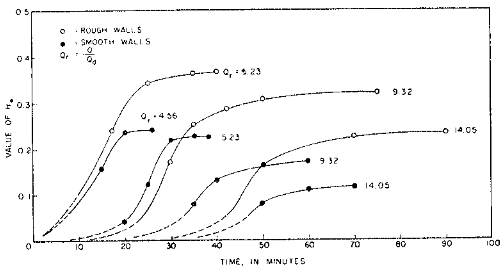


**Fig. 3** Relationship between the relative scour depth and time with $Q/Q_d$ for rough and smooth walls.

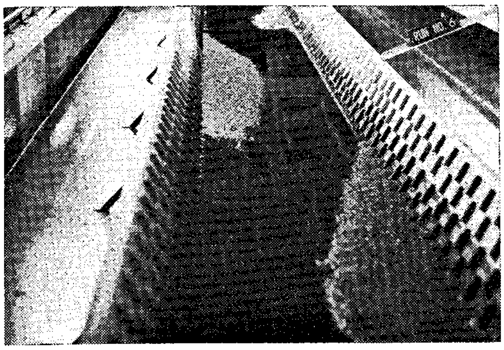

**Fig. 4** Alternate scours in a straight flume with rough walls (looking toward upstream).

### (2) Observed Data in the Rio Grande River

Observations on alternate scours and bars were conducted by R. K. Fahnestock and T. Maddock, Jr.[13] in the Rio Grande River, near Vinton, about 32 km from El Paso, Texas, in 1962. A report on these observations was also published by J. C. Harms and R. K. Fahnestock[27]. Selected sites for observations were Vinton-200 reach, about 805 m upstream from the Vinton bridge, and Vinton-100 reach, 805 m below the Vinton bridge. Each reach is 488 m long, straight, and has a long straight approach. In the Vinton-200 reach, the width is uniform and both banks are covered by

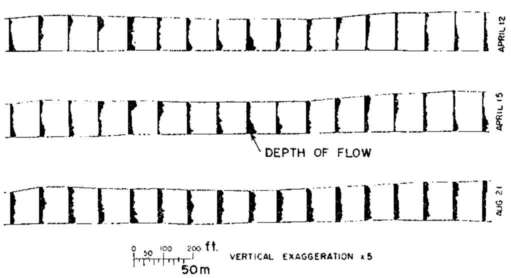


**Fig. 5** Alternate scour patterns in the Rio Grande, Vinton-100 reach.

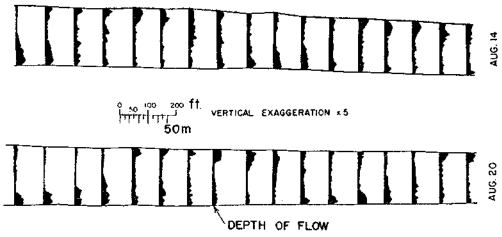


**Fig. 6** Alternate scour patterns in the Rio Grande, Vinton-200 reach.

crushed granite ripraps. In the Vinton-100 reach, the width is constant and its right bank has been stabilized with rock revetments, whereas its left bank is covered with dense brush and small trees. Alternate scour patterns in the Vinton-100 and -200 reaches are shown in Figs. 5 and 6, which were obtained, by courtesy of T. Maddock, Jr., from his unpublished data. Hydraulic data which were used in this investigation are also tabulated in Table 1.

### (3) Empirical Equations on Meander Length

C. C. Inglis[28] presented the following equation by analyzing data collected

**Table 1** Hydraulic data and computations on meander lengths and relative scour depths

| Channel (1) | Run No. or date (2) | Depth, $H$, in meters (3) | Width, $B$, in meters (4) | Slope $\times 10^6$ (5) | Observed mean velocity, $U_{obs}$ in meters per second (6) | Maximum depth of flow, $H_{max}$, in meters (7) | Friction velocity, $u_{*1}$, in meters per second (8) |
|---|---|---|---|---|---|---|---|
| Rough walls | C-2* | 0.0835 | 0.762 | 1220 | 0.3036 | 0.1128 | 0.0305 |
| | C-7* | 0.0829 | 0.762 | 1380 | 0.2804 | 0.1067 | 0.0335 |
| | C-12* | 0.0789 | 0.762 | 960 | 0.2905 | 0.0975 | 0.0271 |
| Vinton-100 | April 12 | 0.5852 | 31.39 | 593 | 0.6645 | 1.2802 | 0.0585 |
| | April 15 | 0.6523 | 31.39 | 533 | 0.6584 | 1.4326 | 0.0582 |
| | Aug. 21 | 1.1156 | 31.39 | 542 | 0.7711 | 1.6154 | 0.0768 |
| Vinton-200 | Aug. 14 | 0.7163 | 60.96 | 680 | 0.7833 | 1.6764 | 0.0692 |
| | Aug. 20 | 0.6157 | 60.66 | 640 | 0.6370 | 1.7069 | 0.0622 |

| $k_{s1}$, in meters (9) | $k_{s2}$, in meters (10) | $\alpha$ (11) | $n$ (12) | $m$ (13) | $\left(\frac{L}{B}\right)_{cal}$ (14) | $\left(\frac{L}{B}\right)_{obs}$ (15) | $H_{*cal}$ (16) | $H_{*obs}$ (17) |
|---|---|---|---|---|---|---|---|---|
| 0.00360 | 0.0191 | 0.84 | 1/6 | 1/3 | 5.78 | 6.2 | 0.268 | 0.348 |
| 0.00360 | 0.0191 | 0.84 | 1/6 | 1/3 | 5.76 | 6.4 | 0.278 | 0.286 |
| 0.00360 | 0.0191 | 0.84 | 1/6 | 1/3 | 5.64 | 6.8 | 0.262 | 0.230 |
| 0.000251 | 0.0732 | 0.61 | 1/10 | 1/4 | 7.70 | 8.0 | 1.24 | 1.19 |
| 0.000251 | 0.0732 | 0.61 | 1/10 | 1/4 | 8.16 | 8.0 | 1.13 | 1.20 |
| 0.000251 | 0.0732 | 0.61 | 1/10 | 1/4 | 10.38 | 9.0 | 0.46 | 0.45 |
| 0.000251 | 0.1524 | 0.57 | 1/10 | 1/4 | 5.83 | 5.5 | 1.59 | 1.34 |
| 0.000251 | 0.1524 | 0.57 | 1/10 | 1/4 | 5.29 | 5.3 | 1.62 | 1.78 |

* These data were obtained from Table 3 in the reference 15.

from alluvial channels:

$$L_m = 6.6B^{0.99} \tag{26}$$

in which $L_m$=meander length in sinuous alluvial channels with erodible banks. Similarly, L. B. Leopold and M. G. Wolman[29] proposed the following equation for rockbound channels:

$$L_m = 10.9B^{1.01}\,. \tag{27}$$

In Eqs. (26) and (27), the exponents on $B$ can be considered as unity, namely, $L_m/B=6.6$ and $L_m/B=10.9$. Field data scattered considerably about these two relationships.

(4) **Determination of $\eta_a$**

The value of $\eta_a$ was found to be 0.8 from the data on rough wall flume and on the Rio Grande River except on August 21 in Vinton-100. Observed values and computations are summarized in Table 1. Fig. 7 shows the relationship between the calculated results and observed values on meander lengths.

(5) **Determination of $C_b$ and $\varepsilon$**

The values of $C_b$ and $\varepsilon$ were found to be 0.054 and 0.4, respectively, from the data on rough wall flume and field data in the Rio Grande River. Computations are tabulated in Table 1. Fig. 8 shows the relationship between the relative scour depth and $\xi_s$.

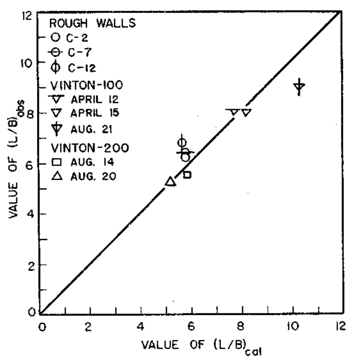


Fig. 7 $(L/B)_{obs}$ versus $(L/B)_{cal}$.

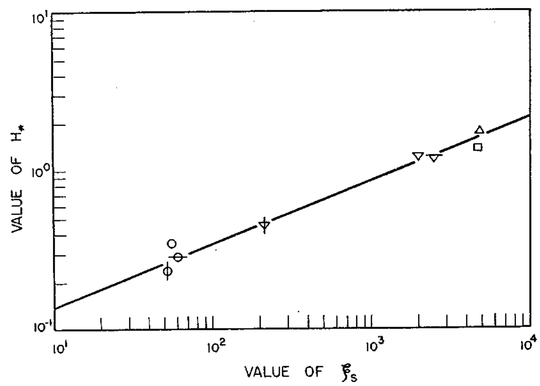


Fig. 8 Relationship between the relative scour depth and $\xi_s$.

## IV. NUMERICAL EXAMPLE

Estimations of meander length and alternate scour depth are illustrated by using data on August 14, 1962, in the Rio Grande River, Vinton-200 reach.

(1) **Estimation of $\alpha$, $m$ and $n$**

From a size distribution curve of the bed material, the value of $k_{s1}$ is $k_{s1}=d_{65}=0.000251$ m. For the wall roughness, $k_{s2}=0.1524$ m was assumed. Other requisite data on the estimations of $n$ and $\alpha$ were as follows: $B=60.96$ m, $\kappa=0.4$,

$H=0.7163$ m and $M_1=119522\ m^{-1}$. By the trial and error method, $\alpha=0.57$ was obtained from Eq. (11). From Eq. (8b), $n=1/10$ was obtained; then the value of $m=1/4$ was obtained by using Eq. (10).

### (2) Meander length

Requiste values for the computation of $B/L$ are as follows: $m=1/4$, $n=1/10$, $H=0.7163$ m and $B=60.96$ m. Eq. (21) gives $B/L=1/5.83$ for these values. From which the meander length is $L=355.40$ m.

### (3) Depth of Alternate Scour Holes

Requisite values for the computation of $H_*$ are as follows: $m=1/4$, $n=1/10$, $M_1=119522\ m^{-1}$, $H=0.7163$ m, $B=60.96$ m and $\bar{u}_{c\,max}/H=2.74\ sec^{-1}$. Eq. (24c) gives $H_*=1.59$, so the maximum depth of flow is $H_{max}=H(1+H_*)=1.86$ m.

## V. CONCLUSIONS

The conclusions obtained from the results of this investigation are as follows:

(1) The difference between the bed roughness and wall roughness is a very important factor for the formation of alternate scour holes and bars.

(2) The tendency to form alternate scours in rough wall channels is higher than that in smooth wall channels. The relative scour depths in rough wall channels are greater than those in smooth wall channels, whereas the meander lengths in rough wall channels are smaller than those in smooth wall channels.

(3) Estimations of meander lengths and relative scour depths can be made by using Eqs. (21) and (24c) in which $\eta_a=0.8$, $C_b=0.054$ and $\varepsilon=0.4$.

(4) In order to prevent the formation of meandering in straight alluvial channels, hydraulically excessive rough-bank protections should be avoided because the formation of alternate scours depends on wall roughnesses.

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## APPENDIX—*Notation*

The following symbols are used in this paper:

$A$=cross-sectional area of the channel;
$B$=width of channel;
$C, C_a, C_b$=constants, ($C_b$=0.054);
$d_{65}$=particle size of bed material for which 65% by weight is finer;
$g$=acceleration of gravity;
$H$=depth of flow;
$H_{\max}$=maximum depth of flow;
$H_*$=relative scour depth, $[=(H_{\max}-H)/H]$;
$i$=bed slope;
$k_s$=representative grain roughness;
ln=natural logarithm;
$L$=meander length;
$L_m$=meander length in sinuous alluvial channels with erodible banks;
$m, n$=dimensionless exponents for the velocity distributions in the lateral and vertical directions, respectively;
$m_r$=constant, (=1/30);
$m_s$=constant, (=1/9);
$M_1=30/k_{s1}$ for hydraulically rough bed;
$M_2=30/k_{s2}$ for hydraulically rough walls;
$p$=pressure;
$Q$=discharge;
$Q_d$=drain discharge in the experimental system;
$S$=energy slope of flow;

$t$=time;
$\bar{u}, \bar{v}, \bar{w}$=time average velocity components in the $x$, $y$ and $z$ directions, respectively;
$u', v', w'$=instantaneous turbulent velocity fluctuations in the $x$, $y$ and $z$ directions, respectively;
$U$=mean velocity for whole cross-sectional area;
$U_{ce}$=mean velocity at the middle of the channel;
$u_*$=friction velocity;
$x, y, z$=coordinate in the flow, lateral and vertical directions, respectively;
$y_2$=distance from the wall;
$z_*$=distance from the bed to a point of maximum transverse velocity;
$\alpha$=dimensionless parameter, $(=u_{*1}/u_{*2})$;
$\xi$=rotation;
$\xi_s$=a function of rotation;
$\eta, \eta_a$=constants, $(\eta_a=0.8)$;
$\varepsilon$=dimensionless exponent, $(\varepsilon=0.4)$;
$\kappa$=von Kármán's constant;
$\lambda$=friction factor;
$\rho$=fluid density;
$\nu$=kinematic viscosity; and
$\nabla^2$=Laplacian operator.

Subscripts:

1 and 2=channel bed and wall, respectively;
cmax=maximum value at the middle of the channel;
obs=observed value; and
cal=calculated value.

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