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RELIABILITY ANALYSIS OF A TAINTER GATE

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I. INTRODUCTION

A tainter gate of a hydroelectric power station in Kyoto, Japan, collapsed in July, 1967. The report[1] of the investigation by the Disaster Prevention Research Institute, Kyoto University, summarized the main causes of failure as follows:

1) coplanar elastic buckling of the foot columns
2) crude stability analysis in the design process
3) primitive assumption of external load pattern

The theoretical investigation and the proposals for the design problems of this type of structure were reported[2], and they verified the causes described above. Only the hydrostatic pressure was considered at the external load on the structure, and the effect of the hydrodynamic pressure was not included. It was found that Euler's buckling load is much more critical for the coplanar instability than for the non-coplanar instability.

This paper considers the tainter gate from the reliability viewpoint, assuming that the external load on the columns is really of two parts. These are the deterministic hydrostatic pressure and the random hydrodynamic pressure which follows the surface disturbance of a large storm or an earthquake. Except for the random nature of the hydrodynamic pressure, all the parameters in this analysis are considered as deterministic quantities, since only the distribution with the greatest dispersion needs to be considered.

A simple structural model of the tainter gate is analyzed. The procedure consists of first constructing a probabilistic model of the random hydrodynamic load. Then the reliability of the simple structure is determined. The criterion for failure is the coplanar buckling of at least one of the foot columns. The reliability obtained serves to logically predict the structural safety, while the concept of the conventional safety factor gives no information about the structure after N repeated loads.

Conversely, given the reliability level associated with the importance of the structure and the other technical and economical reasonings, the size of the structural parameters can be determined.

A Monte Carlo simulation technique is used to evaluate the reliability of the structure. As a verification of the accuracy of this method, a comparison is made in the appendix of the results of both the theoretical and Monte Carlo determinations of the reliability of a simple column with elastic supports.

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II. STRUCTURAL MODEL AND LOAD ENVIRONMENT

A schematic diagram of the tainter gate which actually collapsed in Japan is shown in Fig. 1. As might be expected, the cross members labeled a and b make analyzing the structure very difficult. Another tainter gate, shown in Fig. 2, can be obtained by removing members a and b. This structure is certainly les reliable than the actual gate under the same beam and load parameters.

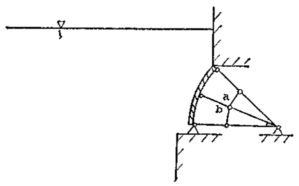

Fig. 1 Actual Tainter Gate

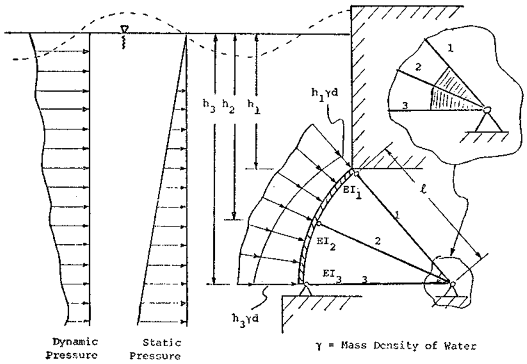

Fig. 2 Model Tainter Gate

However, this modified model is considerably less complicated to analyze. For these reasons, we will use the tainter gate model in Fig. 2.

Our model is composed of two parts. First, the skin plate is a section of a cylindrical shell with a horizontal width d. Second, supporting the plate are three columns of length l. The foot columns are individually attached by pin joints to the plate at one end. The other ends of all three columns are rigidly fastened together and concurrently pin connected to a support shoe (see detail in Fig. 2). The hydrostatic and hydrodynamic loads on the plate are transfered to the foot columns as axial loads.

The axial load, X_i^s, on the ith column due to the hydrostatic pressure is calculated by considering that the hydrostatic pressure increases linearly with the depth of the water. For column 1, this axial load is given by

$$X_1^s \cong \int_{h_1}^{h_1+(h_2-h_1)/2} (\gamma d)x\,dx = \frac{1}{8}\left(\frac{h_2}{h_1}-1\right)\left(3+\frac{h_2}{h_1}\right)\gamma d h_1^2 = \alpha_1^s P \tag{1}$$

where, h_1 and h_2 are depths as shown in Fig. 2

γ = mass density of water

d = width of plate

$\alpha_1^s = \frac{1}{8}\left(\frac{h_2}{h_1}-1\right)\left(3+\frac{h_2}{h_1}\right)$

$P = \gamma d h_1^2$

Similarly, for columns 2 and 3 we have respectively

$$X_2^s \cong \int_{h_1+(h_2-h_1)/2}^{h_2+(h_3-h_2)/2} (\gamma d)x\,dx = \frac{1}{8}\left(1+2\frac{h_2}{h_1}+\frac{h_3}{h_1}\right)\left(\frac{h_3}{h_1}-1\right)\gamma d h_1^2 = \alpha_2^s P \tag{2}$$

$$X_3^s \cong \int_{h_2+(h_3-h_2)/2}^{h_3} (\gamma d)x\,dx = \frac{1}{8}\left(3\frac{h_3}{h_1}+\frac{h_2}{h_1}\right)\left(\frac{h_3}{h_1}-\frac{h_2}{h_1}\right)\gamma d h_1^2 = \alpha_3^s P \tag{3}$$

where,

$$\alpha_2^s = \frac{1}{8}\left(1+2\frac{h_2}{h_1}+\frac{h_3}{h_1}\right)\left(\frac{h_3}{h_1}-1\right) \tag{4}$$

$$\alpha_3^s = \frac{1}{8}\left(3\frac{h_3}{h_1}+\frac{h_2}{h_1}\right)\left(\frac{h_3}{h_1}-\frac{h_2}{h_1}\right) \tag{5}$$

The axial load, X_i^d, on the ith column due to the hydrodynamic pressure is assumed to be of the form

$$X_i^d = \alpha_i^d P \tag{6}$$

where the dynamic coefficient, α_i^d, is a random variable with a joint Gaussian density function with mean, μ_i, and variance, σ_i^2, as follows:

$$f_{\alpha_1^d\alpha_2^d\alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d) = \frac{1}{(2\pi)^{3/2}|S|^{1/2}} \exp\left[\frac{-1}{2|S|}\sum_{j=1}^{3}\sum_{k=1}^{3}|S|_{jk}(\alpha_j^d-\mu_j)(\alpha_k^d-\mu_k)\right] \tag{7}$$

where S is a covariance matrix given by

$$S = \begin{pmatrix} \sigma_1^2 & K_{12} & K_{13} \\ K_{21} & \sigma_2^2 & K_{23} \\ K_{31} & K_{32} & \sigma_3^2 \end{pmatrix} \tag{8}$$

and K_{ij} $(i, j=1, 2, 3)$ is a covariance of α_i^d and α_j^d

and $|S|_{jk}$ is the cofactor of the element in the jth row and kth column of the matrix S.

The variance of σ_i^2 $(i=1, 2, 3)$ of α_i^d is assumed to have the following linear variation with depth (Fig. 3):

$$\sigma_3^2 = B\sigma_1^2 \quad (0 \leq B \leq 1) \tag{9}$$

and

$$\sigma_2^2 = \sigma_1^2 + \frac{h_2-h_1}{h_3-h_1}(\sigma_3^2-\sigma_1^2). \tag{10}$$

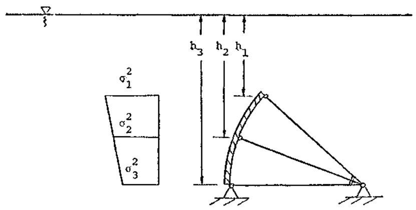

Fig. 3 Variance of α_i^d

In the absence of actual measured data for the depth profile of variance, it is felt that the assumed variance pattern corresponds to the maximum type of turbulence that follows the surface disturbance of a large storm. It seems natural that the correlation coefficient, ρ_{ij}, between α_i^d and α_j^d decreases with an increment in the distance between point i and j. Thus, if we assume a linear relation, ρ_{ij} is defined as follows:

$$\rho_{ij} = 1 - D|h_i - h_j| \tag{11}$$

where

$$0 \leq D \leq \frac{1}{h_3-h_1}.$$

The covariance, K_{ij}, is now expressed as

$$K_{ij}=\sigma_i\sigma_j\rho_{ij}=\sigma_i\sigma_j[1-D|h_i-h_j|] \tag{12}$$

Since $\rho_{ij}=\rho_{ji}$, we have

$$K_{12}=K_{21}=\sigma_1\sigma_2[1-D(h_2-h_1)] \tag{13}$$

$$K_{13}=K_{31}=\sigma_1\sigma_3[1-D(h_3-h_1)] \tag{14}$$

and

$$K_{23}=K_{32}=\sigma_2\sigma_3[1-D(h_3-h_2)]\,. \tag{15}$$

There are two basic reasons for assuming a Gaussian density function for the dynamic coefficient. First, since α_i^d is dependent on several factors, the Central Limit Theorem suggests a normal function. Second, we have no data to to support any particular density function.

In the analysis which follows, the axial loads for the columns are

$$X_1=X_1^s+X_1^d=(\alpha_1^s+\alpha_1^d)P \tag{16}$$

$$X_2=X_2^s+X_2^d=(\alpha_2^s+\alpha_2^d)P \tag{17}$$

and

$$X_3=X_3^s+X_3^d=(\alpha_3^s+\alpha_3^d)P\,. \tag{18}$$

III. STABILITY ANALYSIS

The stability analysis for elastic buckling of this model tainter gate is given in Reference[2]. Only a brief summary is presented here.

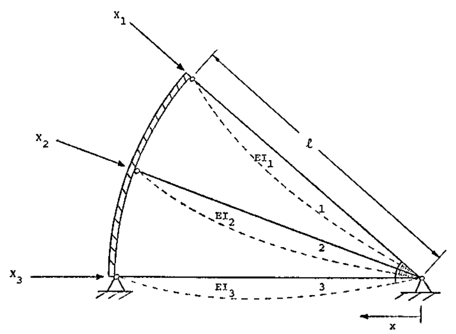

Fig. 4 Stability Analysis

The governing equation is given by (see Fig. 4)

$$EI_i(y_i)''''+X_iy_i''=0 \qquad (i=1, 2, 3) \tag{19}$$

where, E=Young's modulus
I_i=moment of inertia of the ith column. Letting

$$k_i=\sqrt{\frac{X_i}{EI_i}} \tag{20}$$

The general solution is

$$y_i(x)=A_i\sin(k_ix)+B_i\cos(k_ix)+C_ix+D_i\,. \tag{21}$$

The boundary conditions are as follows:

at $x=l$,

$$y_i(l)=y_i''(l)=0; \qquad (i=1, 2, 3) \tag{22}$$

at $x=0$,

$$EI_1y_1''(0)+EI_2y_2''(0)+EI_3y_3''(0)=0 \tag{23}$$

$$y_i'(0)=y_{i+1}'(0)=0; \qquad (i=1, 2) \tag{24}$$

$$y_i(0)=0; \qquad (i=1, 2, 3)\,. \tag{25}$$

Applying the boundary conditions (22) to (25) to Eq. (21), we have for the characteristic equation of the buckling

$$\frac{\alpha_1^s+\alpha_1^d}{k_1 l \cot(k_1 l)-1}+\frac{\alpha_2^s+\alpha_2^d}{k_2 l \cot(k_2 l)-1}+\frac{\alpha_3^s+\alpha_3^d}{k_3 l \cot(k_3 l)-1}=0. \tag{26}$$

Since the condition of buckling for a simple column is $k_i l=\pi$, we may apploximate our solution in the form

$$k_i l \cong \pi+\xi_i \tag{27}$$

where ξ_i is small.

From Eq. (26) and (27), we get

$$P_{cr}=\frac{\pi^2 E I_1}{l^2}\left\{\frac{(\alpha_1^s+\alpha_2^s+\alpha_3^s)+(\alpha_1^d+\alpha_2^d+\alpha_3^d)}{(\alpha_1^s+\alpha_1^d)^{1.5}+\sqrt{\frac{I_1}{I_2}}(\alpha_2^s+\alpha_2^d)^{1.5}+\sqrt{\frac{I_1}{I_3}}(\alpha_3^s+\alpha_3^d)^{1.5}}\right\}^2. \tag{28}$$

The degree of accuracy of P_{cr} is proven satisfactory[2].

We note that this P_{cr} is a random variable as a function of α_i^d $(i=1, 2, 3)$. It is also assumed that $(\alpha_i^s+\alpha_i^d)$ is never negative (i.e. the columns are always in compression).

IV. RELIABILITY OF A TAINTER GATE

The probability of failure of the tainter gate under one load application is given by the following integrations:

$$P_f=P(P>P_{cr})=\iiint_D f_{\alpha_1^d\alpha_2^d\alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d)\,d\alpha_1^d\, d\alpha_2^d\, d\alpha_3^d \tag{29}$$

where, $P(\ )$ reads "probability that"
$P=\gamma d h_1^2$
$P_{cr}=P_{cr}$ of equation (28)
D =domain defined by $P>P_{cr}$

and $f_{\alpha_1^d\alpha_2^d\alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d)$ is given by Eq. (7). The evaluation of Eq. (29) is very difficult since the given domain is specified by a three dimensional space $\{\alpha_1^d, \alpha_2^d, \alpha_3^d\}$. A method of Monte Carlo simulation is developed to solve this equation and is given in the next section.

If we are interested in the probability of failure of the tainter gate under several load applications, further calculation and assumptions are required. If the same distribution of the magnitude of surface disturbances repeats N times, and if the probability of failure due to a single load (Eq. (29)) is small, we can approximate the probability of failure as

$$P_f(N)=NP_f \tag{30}$$

where $P_f(N)$ is the total probability of failure in N load applications and $P_f=$ probability of failure under one load application. If the occurance of these disturbances follows the Poisson probability law, then the probability of finding N events in time, t, with the mean rate of occurence, μ, is given as[3]

$$P(N/t)=\frac{e^{-\mu t}(\mu t)^N}{N!}. \tag{31}$$

For example, if the historical record shows 25 storms or equivalent disturbances, then by engineering judgement, for the last ten years,

$$\mu=\frac{25}{10}=2.5 \text{ disturbances per year.}$$

By applying Eq. (31) to Eq. (30), with the relation,

$$P_s(N)=1-P_f(N)$$

we obtain for the mean of the reliability for a given time, t, as

$$E[P_s(N/t)]=\sum_{N=0}^{\infty} P(N)\, P(N/t)\,. \tag{32}$$

The variance of $P_s(N/t)$ is

$$\mathrm{Var}\,[P_s(N/t)]=\sum_{N=0}^{\infty} P_s^2(N)\, P(N/t)-E[P_s(N/t)]^2\,. \tag{33}$$

V. MONTE CARLO SIMULATION

In this section a Monte Carlo Simulation is developed for the evaluation of Eq. (29). Five steps are required as follows:

(**Step 1**) A set of random variables is generated from a uniform distribution function such that

$$0\leq d_i\leq 1 \tag{34}$$

$$0\leq d_{i+1}\leq 1 \tag{35}$$

$$0\leq d_{i+2}\leq 0 \tag{36}$$

(**Step 2**) A marginal density function of α_1^d is calculated from Eq. (7) such that

$$f_{\alpha_1^d}(\alpha_1^d)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\alpha_1^d\alpha_2^d\alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d)d\alpha_2^d\, d\alpha_3^d=\frac{1}{\sqrt{2\pi}\,\sigma_1}\exp\left[-\frac{1}{2\sigma_1^d}(\alpha_1^d-\mu_1)^2\right] \tag{37}$$

α_1^d is a normal random variable with mean, μ_1, and standard deviation, σ_1. The distribution function of α_1^d is

$$F_{\alpha_1^d}(\alpha_1^d)=\int_{-\infty}^{\alpha_1^d} f_{\alpha_1^d}(\alpha_1^d)d\alpha_1^d\,. \tag{38}$$

Letting $d_i=F_{\alpha_1^d}(\alpha_1^d)$, and solving for α_1^d,

$$\alpha_1^d=F_{\alpha_1^d}^{-1}(d_i)\,. \tag{39}$$

(**Step 3**) A conditional joint density function of α_2^d and α_3^d is now

$$f_{\alpha_2^d\alpha_3^d/\alpha_1^d}(\alpha_2^d, \alpha_3^d)=\frac{f_{\alpha_1^d\alpha_2^d\alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d)}{f_{\alpha_1^d}(\alpha_1^d)}$$

$$=\frac{1}{2\pi S_2 S_3\sqrt{1-\rho_{23}^2}}\exp\left\{\frac{-1}{2(1-\rho_{23}^2)}\left[\left(\frac{\alpha_2{}^d-m_2}{S_2}\right)^2+\left(\frac{\alpha_3{}^d-m_3}{S_3}\right)^2 -2\rho_{23}\left(\frac{\alpha_2{}^d-m_2}{S_2}\right)\left(\frac{\alpha_3{}^d-m_3}{S_3}\right)\right]\right\} \tag{40}$$

where

$$m_2=\mu_2+\rho_{12}\frac{\sigma_2}{\sigma_1}(\alpha_1{}^d-\mu_1) \tag{41}$$

$$S_2=\sigma_2\sqrt{1-\rho_{12}^2} \tag{42}$$

$$m_3=\mu_3+\rho_{13}\frac{\sigma_3}{\sigma_1}(\alpha_1{}^d-\mu_1) \tag{43}$$

$$S_3=\sigma_3\sqrt{1-\rho_{13}^2} \tag{44}$$

Integrating Eq. (40) with respect to $\alpha_3{}^d$ from $-\infty$ to ∞, we get

$$f_{\alpha_2{}^d/\alpha_1{}^d}(\alpha_2{}^d)=\int_{-\infty}^{\infty} f_{\alpha_2{}^d\alpha_3{}^d/\alpha_1{}^d}(\alpha_2{}^d,\ \alpha_3{}^d)d\alpha_3{}^d=\frac{1}{\sqrt{2\pi}\,S_2}\exp\left[\frac{-1}{2S_2{}^2}(\alpha_2{}^d-m_2)^2\right] \tag{45}$$

$\alpha_2{}^d$ is normal with mean, m_2, and standard deviation, S_2. The distribution function of $\alpha_2{}^d$ is

$$F_{\alpha_2{}^d/\alpha_1{}^d}(\alpha_2{}^d)=\int_{-\infty}^{\alpha_2{}^d} f_{\alpha_2{}^d/\alpha_1{}^d}(\alpha_2{}^d)d\alpha_2{}^d \tag{46}$$

Letting $d_{i+1}=F_{\alpha_2{}^d/\alpha_1{}^d}$, solving for $\alpha_2{}^d$,

$$\alpha_2{}^d=F^{-1}_{\alpha_2{}^d/\alpha_1{}^d}(d_{i+1})\,. \tag{47}$$

(**Step 4**) The conditional density function of $\alpha_3{}^d$ is now

$$f_{\alpha_3{}^d/\alpha_1{}^d\alpha_2{}^d}(d_3{}^d)=\frac{f_{\alpha_1{}^d\alpha_2{}^d\alpha_3{}^d}(\alpha_1{}^d,\ \alpha_2{}^d,\ \alpha_3{}^d)}{f_{\alpha_1{}^d}(\alpha_1{}^d)f_{\alpha_2{}^d/\alpha_1{}^d}(\alpha_2{}^d)}$$

$$=\frac{1}{\sqrt{2\pi}\,S_3\sqrt{1-\rho_{23}^2}}\exp\left[-\frac{1}{2}\ \frac{\alpha_3{}^d-m_3-\left(\frac{S_3}{S_2}\right)\rho_{23}(\alpha_2{}^d-m_2)}{S_3\sqrt{1-\rho_{23}^2}}\right] \tag{48}$$

$\alpha_3{}^d$ is normal with

$$\text{mean}=m_3+\left(\frac{S_3}{S_2}\right)\rho_{23}(\alpha_2{}^d-m_2) \tag{49}$$

$$\text{standard deviation}=S_3\sqrt{1-\rho_{23}^2} \tag{50}$$

The distribution function of $\alpha_3{}^d$ is

$$F_{\alpha_3{}^d/\alpha_1{}^d\alpha_2{}^d}(\alpha_3{}^d)=\int_{-\infty}^{\alpha_3{}^d} f_{\alpha_3{}^d/\alpha_1{}^d\alpha_2{}^d}(\alpha_3{}^d)d\alpha_3{}^d \tag{51}$$

Letting $d_{i+2}=F_{\alpha_3{}^d/\alpha_1{}^d\alpha_2{}^d}(\alpha_3{}^d)$ and solving for $\alpha_3{}^d$,

$$\alpha_3{}^d = F^{-1}_{\alpha_3{}^d/\alpha_1{}^d\alpha_2{}^d}(d_{i+2})\,. \tag{52}$$

(**Step 5**) The data set $\{\alpha_1{}^d, \alpha_2{}^d, \alpha_3{}^d\}$ given from Eq. (39), (47) and (52) is substituted into Eq. (28). A sample value of P_{cr} is obtained.
By repeating this procedure and keeping a record of the number of trials and the number of times $P=\gamma dh_1{}^2$ is greater than P_{cr}, we obtain an estimation of the probability of failure, P_f, as

$$P_f = P(P>P_{cr}) = \frac{\text{Number of data sets where } P>P_{cr}}{\text{total number of data sets}} \tag{53}$$

The reliability is therefore

$$P_s = 1 - P_f \tag{54}$$

VI. NUMERICAL EXAMPLE

In order to demonstrate the Monte Carlo solution derived in the last section, a simulation is performed on a specific model with specific parameters by use of a digital computer. Since we have no actual data on the size of the structure which failured in Japan or on the dynamic effect parameters, we estimate sizes and values which seem reasonable as follows:

mean of $\alpha_i{}^d=0$ $(i=1, 2, 3)$
standard deviation of $\alpha_1{}^d=0.01$ (ft.)

$I_1=6\times10^{-2}$ (ft.)^4	$E=2.5\times10^6$ (psf)
$I_2=13\times10^{-2}$ (ft.)^4	$l=20$ (ft.)
$I_3=7\times10^{-2}$ (ft.)^4	$\gamma=62.4$ (#/ft.^{3})
$h_1=20$ (ft.)	$d=1$ (ft.)
$h_2=25$ (ft.)	$B=0.1$, 0.5, 0.7
$h_3=30$ (ft.)	$D=0$-01, 0.05, 0.09

Since we have no accurate values or estimates for B and D, we choose a range of values which will allow us to find out how sensitive the overall reliability is to changes in these parameters. The results of this demonstration need not yield reliability levels high enough for an actual tainter gate since this is

Table 1 Results of Monte Carlo Simulation

D \ B	.1	.5	.7
.01	.8300	.7680	.7580
.05	.8520	.8100	.8080
.09	.8780	.8320	.8180

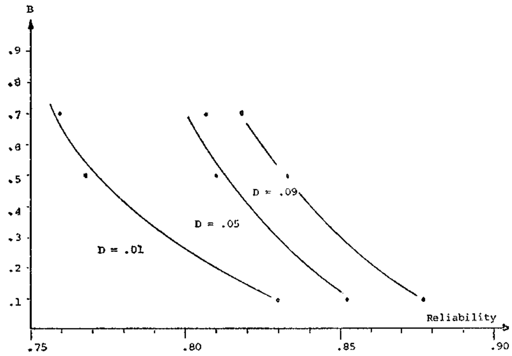

Fig. 5 B vs. Reliability

only a demonstration of a method.

The results of the simulation are tabulated in Table 1. As would be expected, as increase in D increases the reliability while an increase in B decreases the reliability. These effects can be more easily seen in Figs. 5 and 6. Because of the small number of trials (500), no high degree of accuracy is expected in these values. The general trends are, however, well defined by this demonstration.

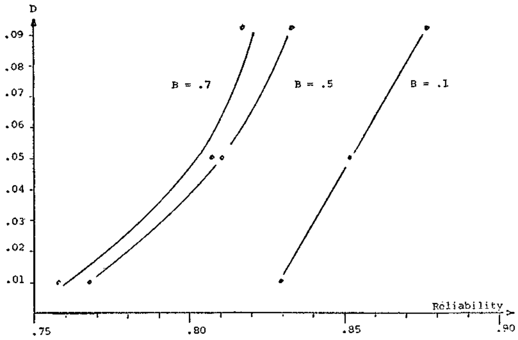

Fig. 6 D vs. Reliability

VII. CONCLUSION AND ACKNOWLEDGEMENT

The reliability of a tainter gate is investigated when it is subjected to a combination of hydrostatic and hydrodynamic loads. The results of a Monte Carlo simulation, derived and demonstrated as a numerical solution to the reliability of the structure, indicate a rational evaluation of the safety of a tainter gate.

This work was done as a part of the senior program in the Department of Engineering Mechanics at Virginia Polytechnic Institute and State University.

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APPENDIX

RELIABILITY OF A SIMPLE COLUMN WITH ELASTIC SUPPORTS

There is a feature of the Monte Carlo simulation used in the main paper which needs to be demonstrated. The accuracy of the simulation as compared to an analytical solution of a simple structure is shown in the following example.

The computer oriented algorithm used in this example is as follows:

1. Solve the structure for deterministic loads and resistances.
2. Generate a random load set and a random resistance set from predetermined

distribution functions. (There are several programs available for this purpose. For example, GAUSS[4] generates Gaussian variables.)
3. Determine if the structure would fail under the random situation.
4. Repeat Steps 2) and 3) many times keeping a tally of the total number of trials and failures.
5. Divide the number of failures by the total number of trials for a very close approximation of the probability of failure.

The structure used is a single flexible column with three spring supports (Fig. A1). The axial load is assumed to be a stochastic variable and the spring constants are assumed to be identical but stochastic variables. The other parameters including length, modulus of elasticity and moment of inertia are assumed to be deterministic.

Since a previous paper[5] in this study included a relatively complete analytical solution to reliability of this structure, only the pertienent results will be stated here. There are three modes of failure for this structure as shown in Fig. A2. These modes are represented along with appropriate limiting equations

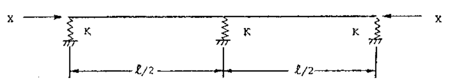


Fig. A1 Column

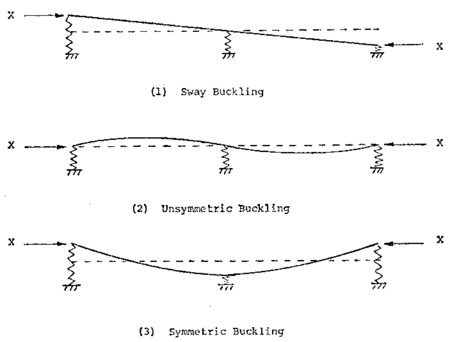
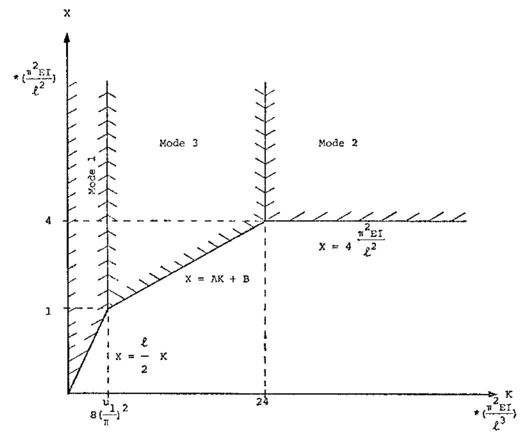


Fig. A2 Modes of Failure



Note: a) u_1 is the minimum value of u in the equation $\tan u + 2u = 0$.

b) $A = \dfrac{l(\pi^2 - u_1^2)}{6\pi^2 - 2u_1^2}$

c) $B = \dfrac{8\pi^2 EIu_1^2}{(3\pi^2 - u_1^2)l^2}$

Fig. A3 X-K Plane

in Fig. A3. An analytical method was derived which approximated the reliability of the column under normal, log-normal, and deterministic distributions of load and resistance. Some of these results are shown in Tables A1 and A2.

The Monte Carlo simulation described above was applied to this structure with proper consideration of Fig. A3. The normal and log-normal distribution functions were chosen with appropriate parameters such that the central safety factor was two. Load and spring constants were randomly selected from their respective distribution functions. A maximum allowable value of the load, as dictated by the spring constants, was calculated by considering the proper limiting equation suggested by Fig. A3. If the randomly selected value of the load

Table A1 Column Reliability with Random X and K

Load (#)†		Normal		Log-Normal	
		Analytical	Monte Carlo*	Analytical	Monte Carlo*
42,840	model 1	0.9745	0.9767	0.9926	0.9913
	model 2	0.0000	0.0000	0.0000	0.0000
	model 3	0.0000	0.0000	0.0002	0.0000
	total	0.9145	0.9767	0.9928	0.9913
165,630	model 1	0.0000	0.0000	0.0000	0.0000
	model 2	0.0000	0.0000	0.0019	0.0020
	model 3	0.9992	0.9986	0.9968	0.9953
	total	0.9992	0.9986	0.9987	0.9973
245,410	model 1	0.0000	0.0000	0.0000	0.0000
	model 2	0.8413	0.8433	0.8640	0.8620
	model 3	0.1581	0.1553	0.1355	0.1373
	total	0.9994	0.9986	0.9995	0.9993

† Load is mean for normal and median for log-normal and the central safety factor is 2. Standard deviations are 20%.
* 1500 trials for each total.

Table A2 Column Reliability with Deterministic X and K

X (kips)	Analytical	Monte Carlo*
25	1.000	1.000
50	0.999	⋮
75	⋮	⋮
100	⋮	⋮
⋮	⋮	⋮
250	⋮	1.000
275	⋮	0.999
300	0.999	0.999
325	0.997	0.998
350	0.994	0.993
375	0.989	0.985
400	0.977	0.975
425	0.959	0.956
450	0.926	0.914
475	0.874	0.857
500	0.000	0.000

Mean of K=10,950 (#/in) Std. Dev.=20%
* 1000 trials

Table A3 Variation of Approximation with Number of Trials

Number of Trials	Reliability
200	1.0000
400	1.0000
600	0.9999
800	0.9999
1000	0.9989
1200	0.9991
1400	0.9985
1600	0.9987
1800	0.9988
2000	0.9989

$\bar{X}$=245,410 (#)
Std. Dev.=20%
Central Safety Factor=2

was less than this calculated critical value, the column was assumed to have survived the test. Fifteen hundred random sets of load and spring constants were evaluated for each of six distribution function sets. By keeping a tally of the number of trials and survivals, an approximation of the reliability of the column was made for each set of distributions. The results of "experiments" are shown in Table A1 and A2.

It can be seen that the analytical and Monte Carlo solutions of reliability agree very closely for this structure. One advantage to the Monte Carlo determination of reliability is that the number of trials is the only major influence on the accuracy of approximation. An example of this influence is shown in Table A3. As can be easily seen, there is no real guarantee that any particular number of trials will yield an exact answer. The Monte Carlo solution is an approximation.

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