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RELIABILITY OF A SINGLE FLEXIBLE COLUMN WITH THREE SPRING SUPPORTS

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I. INTRODUCTION AND COLUMN RELIABILITY

A structure designed with the conventional safety factor may fail despite the fact that the safety factor, defined by the ratio of minimum resistance to maximum load being greater than unity, is supposed to guarantee one hundred percent safety of the structure. One of the main causes of failure is that statistical data relating loads and resistances are never predictable; and hence, loads and resistances are more rationally considered as random variables. Therefore this one hundred percent safety is nothing more than convenient fiction. A probabilistic analysis is, then, inevitable to give a rational design method [1].

With this in mind, we considered the probability of buckling modes of a single flexible column with three spring supports, one at each end and one at the middle point of the column, when the column is subjected to a single random compressive load which is time independent. The springs are assumed to be identical, but random variables. The assumption of the load and springs being random variables is due to the fact that load is not often known in advance and the true strength of the springs cannot be ascertained without individually testing the property. We also assume that the slenderness ratio of the column is large enough to guarantee the impossibility of inelastic buckling.

This is one of the beam-column problems on elastic foundation and might be considered as a simplified model of a tower swayed by cables or as a pile supporting a structural foundation.

Two problems are discussed:

(1) Probabilities of possible failure modes were obtained for a given distribution of the springs and a single load. Then the overall stability of the column was determined in accordance with these probabilities of failure modes.

(2) An allowable maximum load with the prescribed reliability level was obtained for a given distribution of the springs.

Let R be the overall resistance of the column and x be a single load applied on the column. The probability of the event of overall failure, $P(E_f)$, is then given by

$$P(E_f)=P(R<X), \tag{1}$$

where $P(\;)$ reads "probability that."

Then the reliability of the column is given by

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$$P(E_s)=1-P(E_f)\,. \tag{2}$$

Although a single load condition is discussed in this paper, we can determine the overall reliability of the column to repeated N load applications, $X_1, X_2, \cdots, X_N$.

Given a joint density function of N loads and overall resistance, $f_{X_1,X_2,X_3,\cdots,X_N,R}(x_1, x_2, x_3, \cdots, x_N, r)$, the reliability for N load applications is calculated by

$$P_s(N)=\int_0^\infty \underbrace{\int_0^r\int_0^r \cdots \int_0^r}_{N \text{ integrations}} f_{X_1,X_2,\cdots,X_N,R}(x_1, x_2, \cdots, x_N, r)dx_1, dx_2, \cdots, dx_N, dr\,. \tag{3}$$

If the loads are independent of the resistance, Eq. (3) becomes

$$P_s(N)=\int_0^\infty P \text{ (all loads are less than } r) \, f_R(r)dr \tag{4}$$

$$=\int_0^\infty P(X_1<r \cap X_2<r \cap \cdots \cap X_N<r) f_R(r)dr\,, \tag{5}$$

where the symbol $\cap$ indicates intersection and $f_R(\ )$ is a density function of resistance. Moreover, if the loads are independent and identically distributed, then

$$P_s(N)=\int_0^\infty P(X<r)^N f_R(r)dr=\int_0^\infty F_X{}^N(r) f_R(r)dr\,, \tag{6}$$

where $F_X(\ )$ is a distribution function of X (a single load). If $P(X>r)$ is significantly small, then an approximation of Eq. (6) is possible; that is,

$$P_s(N)=\int_0^\infty \{1-P(X>r)\}^N f_R(r)dr \cong \int_0^\infty \{1-NP(X>r)\} f_R(f)dr\,,$$

or

$$P_s(N) \cong 1-N\int_0^\infty P(X>r) f_R(r)dr\,.$$

The second term of the right hand side of the above equation is the product of N and the probability of failure of the column due to a single load application, $P(E_f)$. Hence we have

$$P_s(N) \cong 1-NP(E_f)\,. \tag{7}$$

We note that the reliability of the column decreases as the number of the repeated loads increases; and for the small $P(E_f)$, the decrease is linear with N. Since given $P(E_f)$, we can obtain the reliability of the column to N repeated load applications from Eq. (7), the discussion of a single load application is very meaningful. According to Cornell [2], we have for imperfect dependence of loads,

$$1-P(E_f) \geq P_s(N) \geq 1-NP(E_f)\,.$$

Regarding reliability and safety analysis of structures, recent papers by Heller [3] or Ang [4] are available in which they critically describe the related problems.

II. INSTABILITY OF COLUMN

Consider the column illustrated in Fig. 1. If the applied load X is beyond the critical value (which is actually dependent upon some functional relationship between X, spring modulus K, and the column properties) an instability of the column occurs. In reality, three modes of failure can be considered, i.e., sway buckling, unsymmetric buckling, and symmetric buckling (Fig. 2). The mode of sway buckling occurs when $x=\frac{Kl}{2}$. Unsymmetric buckling starts when $x=\frac{4\pi^2 EI}{l^2}$. Symmetric buckling can be determined from the following consideration. Assume the mode of symmetric buckling as indicated in Fig. 3. The end springs are expanded by δ_1, while the middle one is contracted by δ_2. The reactions induced in the springs at both ends and at the middle point are consequently $K\delta_1$ and $K\delta_2$, respectively. From the vertical equilibrium condition, we have

$$\frac{\delta_2}{\delta_1}=2. \tag{9}$$

The expression for bending moment at x is given by

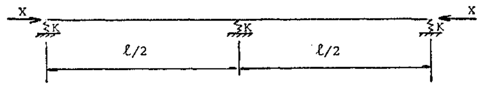
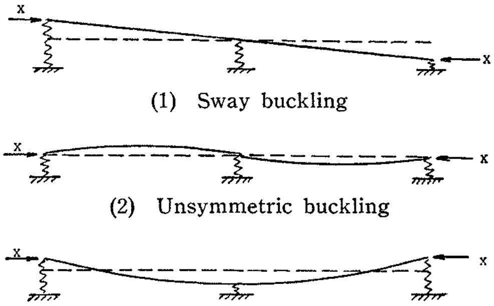


Fig. 1 Column



(1) Sway buckling

(2) Unsymmetric buckling

(3) Symmetric buckling

Fig. 2 Modes of Failures

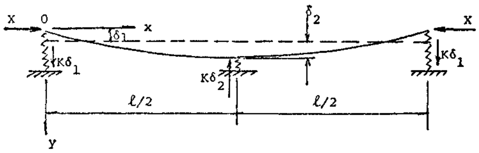


Fig. 3 Symmetric buckling

$$M(x)=Xy-K\delta_1 x=-EIy'' \quad \text{for} \quad 0\leq x\leq\frac{l}{2}, \tag{10}$$

where the coordinate system (x, y) is given in Fig. 3. E and I are Young's modulus and moment of inertia of the column, respectively. Solving Eq. (10) for y and applying the boundary conditions $y(0)=0$, $y\left(\frac{l}{2}\right)=\delta_1+\delta_2$ and $\frac{dy(l/2)}{dx}=0$, together with Eq. (9), we have the critical equation for the buckling as follows:

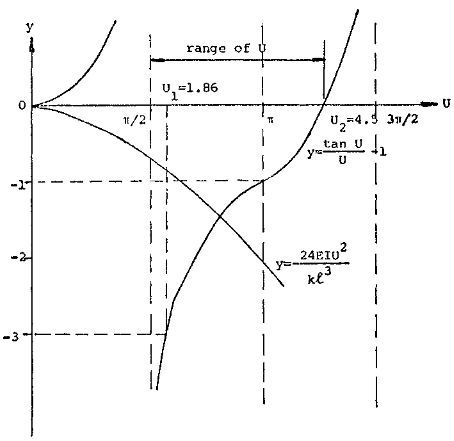


Fig. 4 Range of U value

$$\frac{\tan U}{U}-1=-24\frac{EIU^2}{Kl^3} \tag{11}$$

where U is an unknown variable given by the relationship $U^2=\frac{X}{EI}\frac{l^2}{4}$. Solving U from Eq. (11), we obtain the critical load which is

$$X=\frac{4EIU^2}{l^2} \tag{12}$$

Note that the value U varies in the range $\frac{\pi}{2}<U<U_2$, depending on the value K

(Fig. 4). Consequently the determination of the buckling mode and the corresponding critical value X are dependent upon the relationship between X and K, even if we assume the column properties to be given constants.

The detail of derivation of Eq. (11) is given in Appendix A.

III. PROBABILITY OF FAILURE OF COLUMN

In the following discussions, assume that axial load x and spring modulus K are random variables.

Let $$R_1=\frac{Kl}{2} \quad \text{(critical value of sway buckling),} \tag{13}$$

$$S=\frac{4\pi^2 EI}{l^2} \quad \text{(critical value of unsymmetric buckling which is deterministic),} \tag{14}$$

and $$R_3=\frac{4EIU^2}{l^2} \quad \text{(critical value of symmetric buckling).} \tag{15}$$

Let E_{f_1} represent the event of sway buckling of the column when it is subjected to a single load X. Then, this event can be described as

$$E_{f_1}=X>R_1\cap R_1<S\cap R_1<R_3\,, \tag{16}$$

where event E_{f_1} occurs when X exceeds R_1, the smallest of R_1, S and R_3. Similarly, we can write respectively for events of unsymmetric and symmetric buckling

$$E_{f_2}=X>S\cap S<R_1\cap S<R_3 \tag{17}$$

and

$$E_{f_3}=X>R_3\cap R_3<R_1\cap R_3<S\,. \tag{18}$$

Therefore, the event of the column failure by any mode, E_f, can be written by

$$E_f=E_{f_1}\cup E_{f_2}\cup E_{f_3}\,, \tag{19}$$

where symbol $\cup$ means union.

The domains for E_{f_1}, E_{f_2} and E_{f_3} shown in R_1-R_3 plane (Fig. 5) obviously indicate that events E_{f_1}, E_{f_2} and E_{f_3} are mutually exclusive. Thus, the probability of failure of the column due to a single load application, $P(E_f)$ is given from Eq. (19),

$$P(E_f)=P(E_{f_1}\cup E_{f_2}\cup E_{f_3})=P(E_{f_1})+P(E_{f_2})+P(E_{f_3})\,. \tag{20}$$

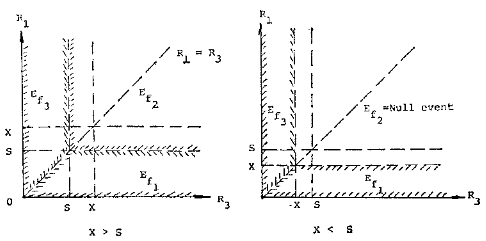


Fig. 5 Domains of E_{f_1}, E_{f_2} and E_{f_3}

The reliability of the column under a single load application is, from Eqs. (2) and (20),

$$P(E_s)=1-P(E_{f_1})-P(E_{f_2})-P(E_{f_3})\,, \tag{21}$$

where $P(E_{f_1})$, $P(E_{f_2})$, and $P(E_{f_3})$ can be expressed by the following conditional probabilities (refer to Fig. 5)

$$P(E_{f_1})=P(R_1<R_3\cap R_1<S|X>S)+P(R_1<R_3\cap R_1<X|X<S), \tag{22}$$

$$P(E_{f_2})=P(S<R_3\cap S<R_1|X>S), \tag{23}$$

and

$$P(E_{f_3})=P(R_1>R_3\cap R_3<S|X>S)+P(R_1>R_3\cap R_3<X|X<S). \tag{24}$$

Difficulties in the evaluations of Eqs. (22), (23), and (24) may arise because of the fact that R_1 and R_3 are random functions of K. An alternate approach is the transformation of domains E_{f_1}, E_{f_2}, and E_{f_3} in the R_1-R_3 plane into those in the X-K plane, (Fig. 6). The transformation procedures are as follows. First of all, we draw lines $X=R_1=\frac{Kl}{2}$ and $X=S=\frac{4\pi^2EI}{l^2}$. The curve, $X=\frac{4EIU^2}{l^2}$ = function of K, can be obtained with the help of Fig. 4. Next, determine the critical values of K by letting $R_1=S$, $R_1=R_3$, and $S=R_3$ from Eqs. (13), (14), and (15). Event E_{f_1} on the X-K plane is now determined by the intersection of $X>R_1$, $R_1<S$, and $R_1<R_3$. Similarly, we can obtain the domains of E_{f_2} and E_{f_3} on the X-K plane. In Fig. 6, U_1 is solved from the relationship $\frac{\tan U_1}{U_1}-1=-3$. U_2 is given by $\frac{\tan U_2}{U_2}-1=0$. Fig. 6 indicates that we will most likely have sway buckling for small values of K; and that when K increases, we will have symmetric buckling; and for the larger values of K, unsymmetric buckling will govern the stability. We may replace the corresponding curve with the line segment AB for a conservative estimation of the probability of event E_{f_3}.

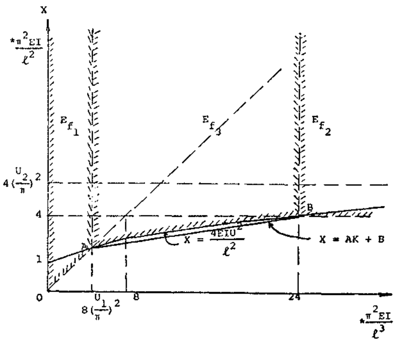


Fig. 6 Domains of E_{f_1}, E_{f_2} and E_{f_3} in X-K plane.

By making use of the domains of E_{f_1}, E_{f_2}, and E_{f_3} in the X-K plane, we can express

$$P(E_{f_1})=\int_0^a\int_{bK}^{\infty}f_{XK}(x, k)dx\,dk, \tag{25}$$

$$P(E_{f_2})=\int_c^{\infty}\int_d^{\infty}f_{XK}(x, k)dx\,dk, \tag{26}$$

and

$$P(E_{f_3})=\int_a^c\int_{AK+B}^{\infty}f_{XK}(x, k)dx\,dk, \tag{27}$$

where

$$a=\frac{8EIU_1^2}{l^3} \tag{28}$$

$$b=\frac{l}{2} \tag{29}$$

$$c=\frac{24\pi^2EI}{l^3} \tag{30}$$

$$d=\frac{4\pi^2EI}{l^2} \tag{31}$$

$$A=\frac{l(\pi^2-U_1^2)}{6\pi^2-2U_1^2} \tag{32}$$

$$B=\frac{8\pi^2 EIU_1^2}{(3\pi^2-U_1^2)l^2}\,, \tag{33}$$

and $f_{XK}(x, k)$ is a joint probability density function of X and K. In actual problems, the determination of $f_{XK}(x, k)$ is very difficult unless sufficient data are available. The existing uncertainty may be overwhelmed by information theory or by subjectively predicted probability models of X and K [5] [6]. If X and K are statistically independent, we have $f_{XK}(x, k)=f_X(x)f_K(k)$, where $f_X(x)$ and $f_K(k)$ are density functions of X and K, respectively. Then Eqs. (25), (26), and (27) are simplified into

$$P(E_{f_1})=\int_0^a [1-F_X(bk)]f_K(k)dk\,, \tag{34}$$

$$P(E_{f_2})=[1-F_X(d)][1-F_K(c)]\,, \tag{35}$$

and

$$P(E_{f_3})=\int_a^c [1-F_X(Ak+B)]f_K(k)dk\,. \tag{36}$$

$F_X(\ \ )$ and $F_K(\ \ )$ are distribution functions of X and K. Finally, we can evaluate the probability of each mode by Eqs. (25), (26), and (27), or by Eqs. (34), (35), and (36) for the independence of X and K. Then the overall reliability is obtained by Eq. (21).

IV. NUMERICAL ANALYSIS

In the following discussion we will derive and demonstrate a numerical method for evaluating the reliability of our system under random distributions of load, X, and resistance, K. The examples will include deterministic and normally and lognormally distributed values of load and resistance and certain combinations of these distributions.

From Eq. (34) it is obvious that

$$P(E_{f_1})=\int_0^a\int_0^\infty f_{XK}(x, k)dx\,dk-\int_0^a\int_0^{bk} f_{XK}(x, k)dx\,dk\,, \tag{37}$$

and assuming statistical independence of $f_X(x)$ and $f_K(k)$,

$$P(E_{f_1})=F_K(a)-\int_0^a\int_0^{bk} f_X(x)f_K(k)dx\,dk=F_K(a)-P_{s_1}\,. \tag{38}$$

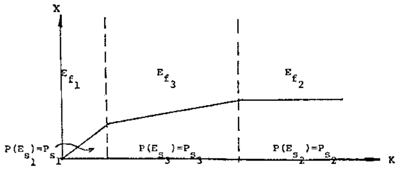


Fig. 7 Definitions of P_{s_1}, P_{s_2} and P_{s_3}.

The double integral determines the probability of success on the interval $K<a$ and $0<X<bK$. Therefore, from here on, the integral will be referred to as P_{s_1}, with the subscript "1" to distinguish it from P_{s_2} and P_{s_3}, which will be used to represent the probabilities of survival corresponding to $P(E_{f_2})$ and $P(E_{f_3})$, respectively (see Fig. 7).

Similarly, we can write

$$P(E_{f_2})=1-F_K(c)-\int_0^{\infty}\int_0^{d} f_X(x)f_K(k)dx\,dk=1-F_K(c)-P_{s_2} \tag{39}$$

and

$$P(E_{f_3})=F_K(c)-F_K(a)-\int_a^{c}\int_0^{Ak+B} f_X(x)f_K(k)dx\,dk$$
$$=F_K(c)-F_K(a)-P_{s_3}\,. \tag{40}$$

The total unreliability is the sum of the individual unreliabilities,

$$P(E_f)=P(E_{f_1})+P(E_{f_2})+P(E_{f_3})$$
$$=1-(P_{s_1}+P_{s_2}+P_{s_3})\,, \tag{41}$$

and the total reliability is, then,

$$R=1-P(E_f)=P_{s_1}+P_{s_2}+P_{s_3}\,. \tag{42}$$

The defining integrals of P_{s_1}, P_{s_2}, and P_{s_3} are all very similar. With the proper choice of Q, R, S, and T (see Table 1 and Fig. 8) we have,

$$P_{s_j}=\int_S^{T}\int_0^{Qk+R} f_X(x)f_K(k)dx\,dk; \qquad j=1, 2, 3$$

or

$$P_{s_j}=\int_S^{T} F_X(kQ+R)f_K(k)dk; \qquad j=1, 2, 3\,. \tag{43}$$

Table 1 Definition of constants Q, R, S and T

	Q	R	S	T
P_{s_1}	b	0	0	a
P_{s_2}	0	d	c	∞
P_{s_3}	A	B	a	c

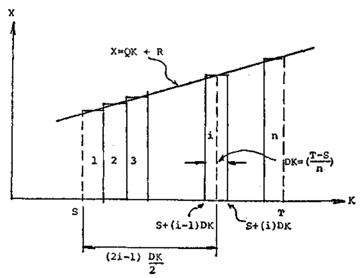


Fig. 8 Evaluation of P_{s_j}

The evaluation of P_{s_j} can be numerically approximated by the following formula:

$$P_{s_j}\approx\sum_{i=1}^{n}\left[F_X\left\{Q\left(S+(2i-1)\frac{DK}{2}\right)+R\right\}\{F_K(S+i-DK)-F_K[S+(i-1)DK]\}\right]; \tag{44}$$

where

$$n=(T-S)/DK\,.$$

Obviously, as n approaches infinity, theoretically this summation approaches the true integral. For computer applications, however, n should be chosen such that DK stays within the accuracy of the function generator. The total reliability can be obtained by evaluating P_{s_j} for $j=1, 2, 3$ and adding the results.

Thus far, nothing has been said about the nature of the functions $f_X(x)$ and $f_K(k)$ except that $F_X(0)=F_K(0)=0$ and that, statistically, they are independent.

Any distribution function may be used, as will be demonstrated in the following examples. In all the examples, the following beam parameters will be used:

$$I=48.4 \text{ inches}$$

$$L=336 \text{ inches}$$

$$E=29\times10^6 \text{ psi}$$

These values correspond to an American Standard Steel I-Beam that is 24″×7″ and 100 lb/ft. There is no loss of generality by using only one beam, and in fact, it will help make the final comparisons more meaningful.

Example 1:

If we were designing without any regard for randomness, we might choose a safety factor of 2. Since there are three modes of failure, let us look at three different K values, one in each region, and evaluate the corresponding X values. For the I-Beam under consideration, Fig. 6 becomes Fig. 9. Table 2 shows a reasonable set of values to consider. Since the safety factor is 2, we might say we have a 200% reliability.

However, we understand that a 200% reliability is fictitious and does not mean more than 100% reliability.

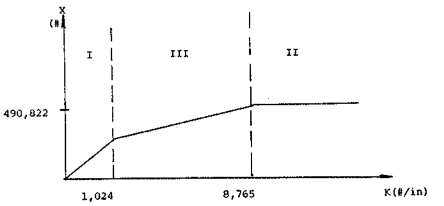


Fig. 9 X-K plane used in examples.

Example 2:

Suppose that we know, deterministically, a value of the load, X, and that the resistance, K, is normally distributed. We can determine a reliability for each given X and mean and standard deviation of K.

Since we are only interested in the probability that K is greater than the critical value, we can modify P_{s_j} as follows:

$$P_{s_j}=P\{K_{0_j}<K<T\}=P\left\{\frac{X-R}{Q}<K<T\right\}.$$

It is obvious that there will be, at most, only one point where X, (a constant), can cross into the region which allows success. We only need to determine the corresponding K and

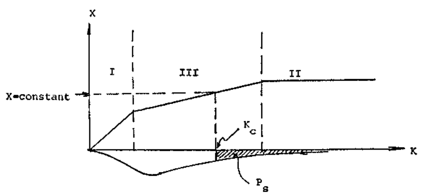


Fig. 10 Example 2: Deterministic X Normal K

Table 2

Example 1: Deterministic K and X with S.F.=2

K (#/in)	X (#)	Reliability
510	42,840	200%
4,890	165,630	200%
10,950	245,410	200%

Example 3: Normal X and K

μK (#/in)	μX (#)	Reliability
510	42,840	97.45%
4,890	165,630	99.92%
10,950	245,410	99.94%

st. dev.=20% of mean C.S.F.=2

Example 4: Lognormal X and K

$\tilde{K}$ (#/in)	$\tilde{X}$ (#)	Reliability
510	42,840	99.28%
4,890	165,630	99.87%
10,950	245,410	99.95%

st. dev.=20% C.S.F.=2

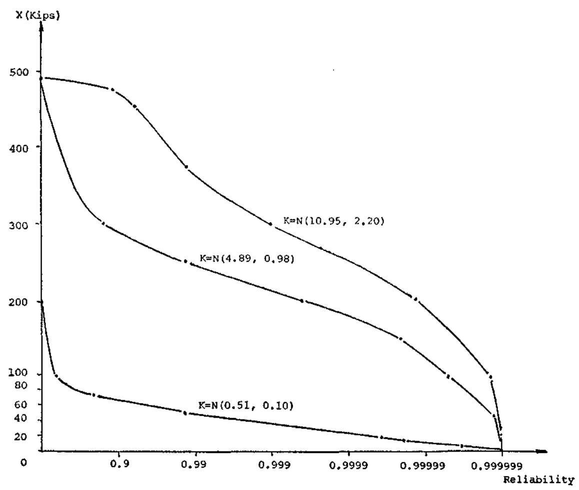


Fig. 11 Reliability vs. Axial load

evaluate $P\{K>K_c\}$ to determine the reliability (see Fig. 10). The results are plotted in Fig. 11 and tabulated in Appendix B. Fig. 11 indicates that we can determine the maximum allowable load X for the given reliability level.

For example, given the reliability, 99.94% and normal distribution of K with mean, $\mu_K=4.89$ and standard deviation, $\sigma_K=0.98$, we have the maximum allowable load, $X=200$ kips.

Example 3:

If both X and K are normally distributed, with known means and standard deviations, the reliability can be determined. The original equation for P_{s_j} is used here. Since a safety factor of 2 was chosen in Example 1, a central safety factor of 2 will be used here and in the next example so that later comparisons of the results can be made. Also, a standard deviation of 20% of the mean will be used.

The results are shown Table 2, where st. dev. = standard deviation of both K and X, C.S.F. = central safety factor, μ_K=mean of K and μ_X=mean of X. For example, if $\mu_K=510$ #/in, and $\mu_X=42{,}840$ #, we have 97.45% reliability of the column.

Example 4:

If both X and K are lognormally distributed, with known mean and a 20% deviation, the reliability can again be determined. The results are in Table 2, where $\tilde{K}$ and $\tilde{X}$ are medians of K and X respectively. The similar comment of Example 3 can be made.

V. CONCLUSION AND ACKNOWLEDGEMENT

Reliability of a column with spring supports is investigated when it is subjected to a single load application. Comparisons made for deterministic and probabilistic loads and resistances indicate that the probabilistic approach gives a rational evaluation of safety of a structure, compared to the deterministic approach. It is obvious that the fictitious 200% reliability in Example 1 reduces to less than 100% reliability in Examples 3 and 4 when the probability distribution of X and K are considered. We may interpret 99.28% reliability in Example 4 as the possibility of failure of 72 columns out of 10,000 norminally designed columns. Similar comments can be made on the other calculated reliabilities.

This work was done for a part of senior program conducted in Engineering Mechanics Department, Virginia Polytechnic Institute and State University.

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APPENDIX A:

Solution of Case 3 differential equation

$$M(X)=Xy-K\delta_1 x=-EIy'' \qquad \text{for} \quad 0\leq X\leq l/2\,.$$

The solution is of the form,

$$y(x)=A\sin gx+B\cos gx+\left(\frac{K\delta_1}{g^2EI}\right)x; \qquad \text{where} \quad g^2=\frac{X}{EI}\,.$$

Boundary conditions yield

$$y(0)=0\,, \quad y\left(\frac{l}{2}\right)=\delta_1+\delta_2\,, \quad \text{and} \quad y'\left(\frac{l}{2}\right)=0\,,$$

and applying these we get,

$$y(0)=0=B \quad \longrightarrow \quad B=0\,,$$

$$y\left(\frac{l}{2}\right)=\delta_1+\delta_2=A\sin\left(\frac{gl}{2}\right)+\frac{K\delta_1 l_1}{2g^2EI} \tag{1}$$

or
$$A=\delta_1 \csc\left(\frac{gl}{2}\right)\left(3-\frac{Kl}{2g^2EI}\right),$$

and
$$y'\left(\frac{l}{2}\right)=0 \quad\longrightarrow\quad Ag\cos\left(\frac{gl}{2}\right)+\frac{K\delta_1}{g^2EI}=0. \qquad (2)$$

Combining (1) and (2) we get

$$\cot\left(\frac{gl}{2}\right)\left(3-\frac{Kl}{2g^2EI}\right)=-\frac{K}{g^3EI}. \qquad (3)$$

Let
$$U=\frac{gl}{2}, \quad \text{then} \quad U^2=\frac{g^2l^2}{4}=\frac{X}{EI}\,\frac{l^2}{4}$$

and
$$X=\frac{4EIU^2}{l^2}.$$

Substituting into (3) for U and reducing, we get

$$\frac{\tan U}{U}-1=-\frac{24EI}{kl^3}U^2.$$

APPENDIX B:

Results of Example 2

X (kips)	Reliability in percent		
	μ_k=510 (#/in)	μ_k=4890 (#/in)	μ_k=10,950 (#/in)
5	99.99	99.99	99.99
10	99.99	|	|
15	99.99	|	|
20	99.99	↓	|
25	99.87	99.99	|
50	95.99	99.99	|
100	24.20	99.99	|
150	0.07	99.99	|
200	0.00	99.94	↓
250	|	97.73	99.99
300	|	78.81	99.90
350	|	32.64	99.46
400	|	4.46	97.73
450	|	0.16	92.65
500	↓	0.00	0.00

$\sigma_k=0.2(\mu_k)$

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