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PRODUCING OF ORTHOTROPIC PLATE FOR PHOTO-ELASTIC EXPERIMENT AND ITS APPLICATION TO EXPERIMENT

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I. INTRODUCTION

There are many technical problems which are not precisely analyzed without considering the orthotropy of materials. Some examples of these problems are found in a plate of isotropic material with stiffners uniformly stretched in one direction, corrugated plate and stratiform ground etc.. As the theoretical analysis of stress concentration in an orthotropic plate is generally difficult, the experimental analysis by photo-elastic experiment is often used. But in this field there has been so far difficulty in getting a proper photo-elastic material with the expected orthotropic characteristics. Photo-elastic experiments of orthotropic plates have been reported by Takeshi Hayashi, Yoshitsugu Niwa, Toshikazu Kawamoto and C. Sonntag.

T. Hayashi reported on the two-dimensional photo-elastic experiment with an orthotropic plate, composed of glass fiber cloth and Rigolac polyester resin, where the former gives orthotropy to isotropic Rigolac polyester resin and Rigolac polyester resin alone has photo-elastic effect. Therefore, its photo-elastic strain sensitivity, as in the case of isotropic material, is independent of both position and direction. It seems that the method of T. Hayashi is adequate for photo-elastic experiment, but in its application there are some points to be improved. Namely, (1) the available range for ratio of principal elastic moduli is considerably limited, (2) because of low photo-elastic sensitivity of Rigolac polyester resin, the comparatively large load is necessary to obtain sufficient number of isochromatic fringe order to analyze stress distribution, (3) the clear cut of isochromatic fringe pattern seems not to be desired, (4) an uniform and homogeneous orthotropic plate seems to be difficult to be produced.

Y. Niwa and T. Kawamoto adopted the photo-elastic coating method in their experiments carried out with orthotropic plate, composed of Epoxy resin and metallic wires. The coating material in the experiments was Epoxy rubber. However, the photo-elastic coating method itself is still on the way of developing and establishment of the technics for quantitative study is to be seeked.

Authors have succeeded in producing an orthotropic plate suitable for photo-elastic experiment and carried out some experiments on stress concentration with this plate, to examine its characteristics. They call the method of experiment "Stress Sandwitch Method" and the plate "Sandwitch Plate".

Its basis is on the stress freezing method by the sandwitch plate which con-

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sists of one middle layer of Epoxy resin with high photo-elastic sensitivity and of two outside layers reinforced by nylon fiber to give the plate orthotropy. In the stress freezing process of the sandwitch plate, orthotropy is given to the plate by utilizing the difference between the second transition points of Epoxy resin and nylon fiber. The strain condition of the orthotropic plate are freezed in the fiberless middle layer between the fiber reinforced outside layers. After having finished a cycle of the stress freezing, the fiber reinforced outside layers are sliced off and two-dimensional photo-elastic experiment is carried out with the fiberless middle layer.

According to their method, it is easy to produce an uniform and homogeneous orthotropic plate with desired conditions on principal elastic moduli over a wide range. Moreover, the clear isochromatic fringe pattern is obtained, though there are some usual difficulties caused by the stress freezing method. Therefore, this method is also effective for a supplement of the experiment means developed by T. Hayashi, Y. Niwa and T. Kawamoto.

Some photo-elastic experiments have been carried out with the above-mentioned sandwitch plates as the examples on the problem of a circular hole subjected to uniform tension and the experimental results have been compared with theoretical calculation.

II. FUNDAMENTALS OF STRESS FREEZING METHOD

1. Law of Elasticity of Orthotropic Plate

In Fig. 1, the axes 1, 2 are the elastic principal axes of an orthotropic plate and the axes x, y are the axes rotated counterclockwise by θ from the principal axes. The law of elasticity related to the principal axes 1, 2 in the case of plane stress is

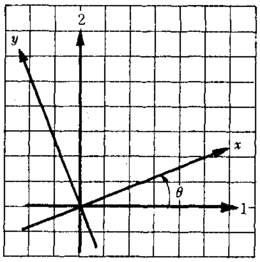


Fig. 1

$$\left.\begin{aligned} \varepsilon_1 &= \frac{1}{E_1}\sigma_1 - \frac{\nu_{21}}{E_2}\sigma_2 = a_{11}\sigma_1 + a_{12}\sigma_2 \\ \varepsilon_2 &= -\frac{\nu_{12}}{E_1}\sigma_1 + \frac{1}{E_2}\sigma_1 = a_{12}\sigma_1 + a_{22}\sigma_2 \\ \gamma_{12} &= \frac{1}{G_{12}}\tau_{12} = a_{66} \cdots \tau_{12} \end{aligned}\right\} \quad (1)$$

where

$\varepsilon_1, \varepsilon_2, \gamma_{12}$: normal strains and shearing strain related to 1, 2 axes

$E_1, E_2, \nu_{12}, \nu_{21}$: Young's moduli and Poisson's ratios related to 1, 2 axes

G_{12}: shear modulus related to 1, 2 axes.

Then the law of elasticity related to the axes x, y which have rotating angle θ from the principal axes 1, 2, is given in the following form by the coefficients in the equation (1).

$$\left.\begin{aligned} \varepsilon_x &= c_{11}\sigma_x + c_{12}\sigma_y + c_{16}\tau_{xy} \\ \varepsilon_y &= c_{12}\sigma_x + c_{22}\sigma_y + c_{26}\tau_{xy} \\ \gamma_{xy} &= c_{16}\sigma_x + c_{26}\sigma_y + c_{66}\tau_{xy} \end{aligned}\right\} \quad (2)$$

where

$$\left.\begin{aligned}
c_{11}&=a_{11}\cos^4\theta+a_{22}\sin^4\theta+(2a_{12}+a_{66})\sin^2\theta\cos^2\theta\\
c_{22}&=a_{11}\sin^4\theta+a_{22}\cos^4\theta+(2a_{12}+a_{66})\sin^2\theta\cos^2\theta\\
c_{66}&=a_{66}+4(a_{11}+a_{22}-2a_{12}-a_{66})\sin^2\theta\cos^2\theta\\
c_{12}&=a_{12}+(a_{11}+a_{22}-2a_{12}-a_{66})\sin^2\theta\cos^2\theta\\
c_{16}&=-2a_{11}\cos^3\theta\sin\theta+2a_{22}\cos\theta\sin^3\theta+(2a_{12}+a_{66})\sin\theta\cos\theta\,(\cos^2\theta-\sin^2\theta)\\
c_{26}&=-2a_{11}\cos\theta\sin^3\theta+2a_{22}\cos^3\theta\sin\theta-(2a_{12}+a_{66})\sin\theta\cos\theta\,(\cos^2\theta-\sin^2\theta)
\end{aligned}\right\}\quad(3)$$

2. Stress Freezing Sandwitch Method

In Fig. 2, a section of sandwitch plate is shown. The middle layer of plain Epoxy resin of high photo-elastic sensitivity is sandwitched by the two outside layers reinforced by nylon fibers. Such sandwitch plates as this have been used in all the experiments of this paper.

The middle layer of the sandwitch plate is isotropic, but the deformation is subjected to the orthotropy of the plate at high temperature. And the orthotropic strain distribution of the plate is fixed in the isotropic fiberless middle layer in the process of stress-freezing. Then the strain distribution fixed in that layer is analyzed by the same way as a two-dimensional photo-elastic experiment of isotropic body, after the outside layers were sliced off.

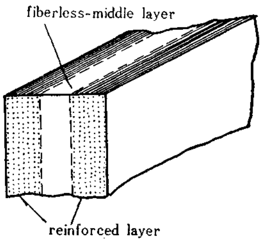


Fig. 2

In the photo-elastic experiment of the fiberless middle layer, an isochromatic line is given as the locus of points where the difference between principal strains is constant and an isoclinic line as the locus of points where the direction angle of principal strain is constant. And the law of photo-elasticity of that layer is also expressed in the form

$$N_A=d\beta_h|\varepsilon_p-\varepsilon_q|=\frac{E_e}{1+\nu_e}\alpha_h d|\varepsilon_p-\varepsilon_q| \qquad (4)$$

where

N_A :	fringe order at a point A
α_h, β_h :	photo-elastic stress sensitivity and strain sensitivity in rubber-like elastic region of Epoxy resin, respectively,
ν_e, E_e :	Poisson's ratio and Young's modulus in rubber-like elastic region of Epoxy resin, respectively,
ε_p, ε_q :	principal strains at a point A.

3. Analysis of Circumferential Stress Along Free Boundary

The difference between the principal strains related to x, y axes is expressed by

$$\begin{aligned}
|\varepsilon_p-\varepsilon_q|&=\left|\sqrt{(\varepsilon_x-\varepsilon_y)^2+\gamma_{xy}^2}\right|\\
&=\left|\sqrt{\{(c_{11}-c_{12})\sigma_x+(c_{12}-c_{22})\sigma_y+(c_{16}-c_{26})\tau_{xy}\}^2+(c_{16}\sigma_x+c_{26}\sigma_y+c_{66}\tau_{xy})^2}\right| \qquad (5)
\end{aligned}$$

If x, y axes determined to coincide to the direction normal and tangential to a free boundary, respectively, as shown in Fig. 3, the stress conditions along the

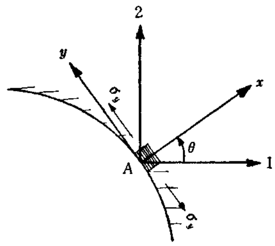


Fig. 3

free boundary satisfy

$$\sigma_x = \tau_{xy} = 0 \tag{6}$$

Substituting these into (5) and applying it to Eqs. (4), we have

$$|\sigma_y| = \frac{N_A}{d\beta_h \sqrt{(c_{12}-c_{22})^2 + c_{26}^2}} \tag{7}$$

This is the equation for determining circumferential stresses along a free boundary.

III. METHOD OF PREPARING SANDWITCH PLATE

Before making the sandwitch plate of Epoxy resin and nylon fiber, authors tried to make a sandwitch plate of Epoxy resin and glass fiber cloth. But by the difference of linear expansion coefficients between Epoxy resin and glass fiber, large thermal stress was induced. To overcome this trouble, combination of Epoxy resin and nylon fiber was chosen for the sandwitch plate. Because the coefficients of linear expansion of Epoxy resin and nylon fiber are rather similar in the rubber-like elastic region of Epoxy resin.

1. Properties of Nylon Fiber at a Temperature about 130°C

Usually high initial stresses of tension are fixed in nylon fiber on the market. If such nylon fiber is kept at a temperature about 130°C for several hours, the tensile strains fixed in the nylon fiber are released and it shrinks about 10% in length. Thus the use of an untreated nylon fiber in production of the sandwitch plate is not favorable, because the large shrinkage of the fiber introduces high initial stresses in heat-treating during production of the plate. For this reason, nylon fiber is annealed at a temperature about 150°C for 5 hours to remove the shrinkage, before it is used for reinforcement of sandwitch plate.

The nylon fiber used for making the sandwitch plate is "840 Denears" in thickness. The relation between the annealing-hours and the amount of shrinkage of the nylon fiber is shown in Fig. 4. The testing setup for measurement of Young's modulus of the nylon fiber after the annealing is shown in Photo. 1.

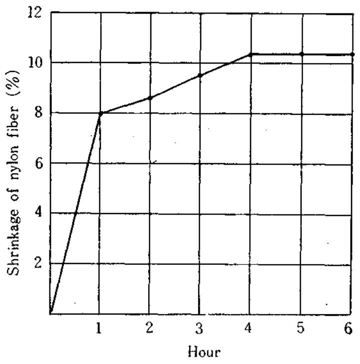


Fig. 4

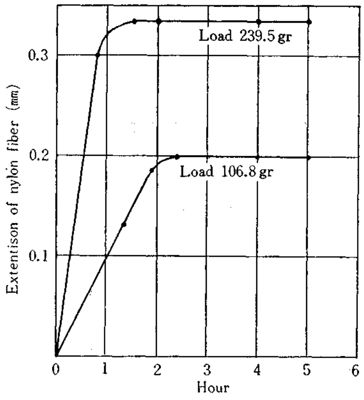


Fig. 5

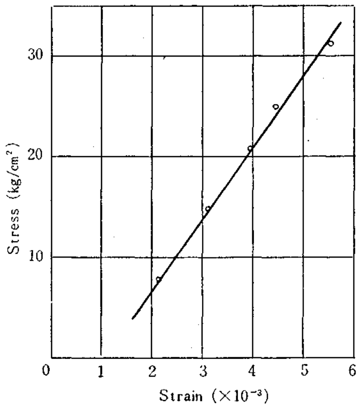


Fig. 6

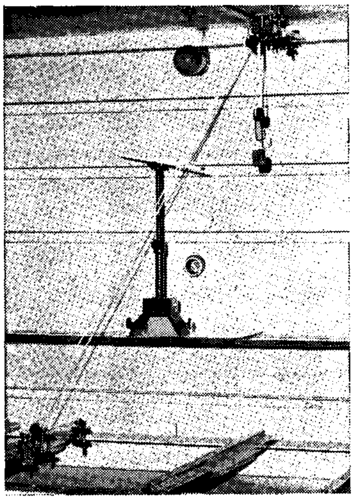


Photo. 1

Fig. 5 and Fig. 6 are also some examples of the creep diagrams and the stress-strain diagram of the nylon fiber. Properties of the nylon fiber are as follows.

Young's modulus	7.9×10^3 kg/cm^2
coefficient of linear expansion at about 130°C	0.8×10^{-4}/C°
melting point	215°C

2. Properties of Epoxy Resin in Rubber-Like Elastic Region

Epoxy resin has been used as a good photo-elastic material, for it gives a clear fringe pattern by its high photo-elastic sensitivity and transparency, etc.. Epoxy resin used in this experiments is the mixture of Araldite B and hardener "HT-901" in the ratio of 100:30 by weigth at 130°C. It is cured for about 18 hours at the same temperature after its casting.

Properties of Epoxy resin in rubber-like elastic region are as follows.

Young's modulus	1.8×10^2 kg/cm^2
Poisson's ratio	0.5
coefficient of linear expansion at about 130°C	1.7×10^{-4}/C°
photo-elastic strain sensitivity	4.8×10^2 cm

3. Casting Mold of Epoxy Resin and Its Casting

After the above-mentioned annealing, the nylon fiber is wound on a flat reel giving a proper tensile stress to it. If proper tensile stress is not given to the nylon fiber, the initial stress would be caused by the reason that the free shrinkage caused by the hardening of Epoxy resin in producing is restrained by the nylon fiber. The proper value of the tensile stress can be experimentally determined by the measurement of residual stress of prepared sandwitch plate.

Photo. 3 is the equipment for winding the nylon fiber on the reel. To wind the nylon fiber uniformly on the reel, a guide is provided which moves parallel to the axis of the reel in proportion to its rotating speed. As the nylon fiber is wound by two layers on the reel at a time, fourteen layers are obtained after repeating the same operation seven times. Spacers are inserted between the

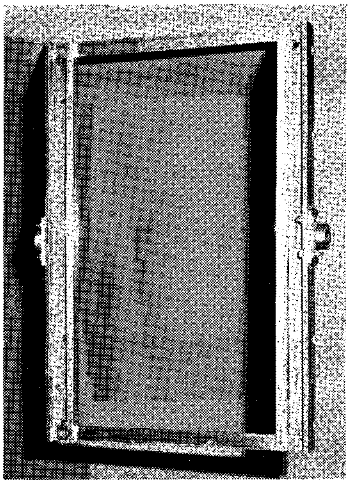


Photo. 2 The detail of the reel.

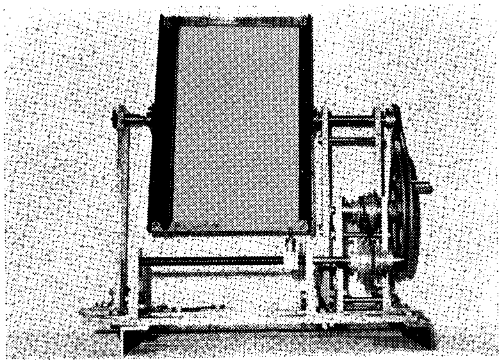


Photo. 3 The front view of the fiber winding equipment.

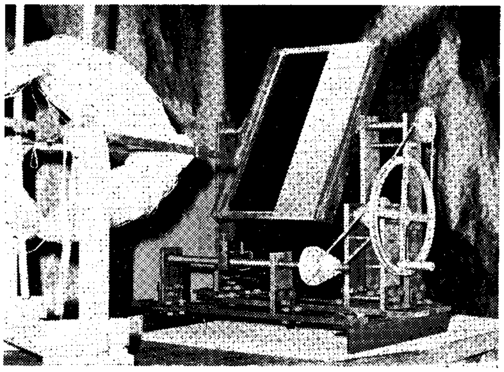


Photo. 4 The winding operation of nylon fiber on the reel.

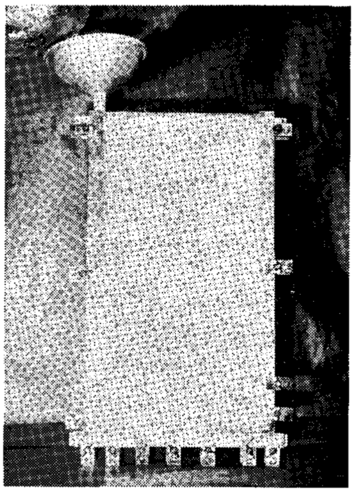


Photo. 5 The pouring of Epoxy resin into

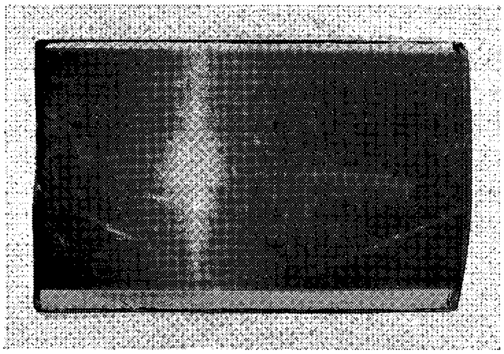


Photo. 6 A sandwitch plate after the mold is disjointed.

layers so that the clearance between the layers keep constant.

To form a mold, after the protection against leakage of Epoxy resin around the reel was made, both the side plane of reel wound by the nylon fiber are covered with two sheets of glass coated by cellophane. The melted Epoxy resin mixed up with the hardener at 130°C for 15 minutes is poured into the mold at the same temperature. The mold is set to make the nylon fiber run vertically. This is very effective to avoid suspension of small air bubbles among the nylon fiber during casting of the Epoxy resin. In practice, the amount of air bubbles contained in the casted plate differs greatly in the direction of pouring of Epoxy resin to the nylon fiber. With hardening of Epoxy resin to some extent, the mold is disjointed to take off the two sheets of glass. And the nylon fibers are cut from the reel to permit Epoxy resin to shrink freely in the process of its hardening. After cured at about 130°C for about 18 hours, the casted sandwitch plate is gradually cooled down till room temperature. And, stripping the cellophane adhering to both surface of the plate and arranging its shape, the sandwitch plate is ready for a photo-elastic experiment of an orthotropic plate.

The making process of the sandwitch plate are shown in Photo. 2~6 together with its equipments.

IV. ELASTIC CHARACTERISTICS OF SANDWITCH PLATE IN RUBBER-LIKE ELASTIC REGION OF EPOXY RESIN

In the analyses of the elastic characteristics of the sandwitch plate at a high temperature by stress freezing method, some assumptions are made; (1) The sandwitch plate behaves to a homogeneous and orthotropic body as a whole. (2) Any relative displacement between nylon fiber and Epoxy resin does not occur, when the sandwitch plate deforms. (3) Because adhesion of Epoxy resin and nylon fiber is perfect, the characteristics of sandwitch plate in tension and compression are completely the same. (4) Nylon fiber has no rigidity for bending and nylon fiber gives additional stresses to the plate only in the direction parallel to it. (5) In the calculation shear deformation of nylon fiber is neglected as being very small. For nylon fiber has shear modulus extremely larger than that of Epoxy resin at the high temperature and does not occupy a large position of the sandwitch plate.

By these assumptions, the characteristics of the sandwitch plate are determined as follows.

1. Elastic Moduli in Direction of Nylon Fiber

Consider a small rectangular element ABCD cut out from the sandwitch plate, as shown in Fig. 7. In this element sideplanes AB, CD are perpendicular to the direction of nylon fiber on which normal stress σ_1 is acting. The axes 1, 2 are those of elastic symmetry, one of which is parallel to nylon fiber and the other perpendicular to it. When external load is applyed, the stresses in Epoxy resin and nylon fiber are different because of the difference of the elastic moduli. In this calculation the stress caused by external load is considered to be separated into two portions. One is the normal stress when nylon fiber are replaced by Epoxy resin, then the stress F is added considering the difference of elastic moduli of Epoxy resin and nylon fiber. And denoting by σ_{e1}, σ_{e2} the normal stresses in the side-planes in the direction of elastic axes 1, 2 and by $\varepsilon_1, \varepsilon_2$ the

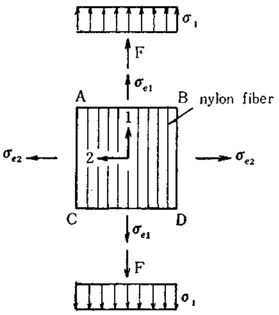


Fig. 7

normal strain components of the sandwitch plate in the same direction, respectively, we have for the relation satisfied in the portion of nylon fiber in the sideplane AB, CD

$$F=n(E_f-E_e)\varepsilon_1 \tag{8}$$

and we have for Hooke's law of the element replaced by Epoxy resin in the state of plane stress

$$\sigma_{e1}=\frac{E_e}{1-\nu_e^2}(\varepsilon_1+\nu_e\varepsilon_2)\,,\qquad \sigma_{e2}=\frac{E_e}{1-\nu_e^2}(\varepsilon_2+\nu_e\varepsilon_1) \tag{9}$$

where

E_f, E_e: Young's moduli of nylon fiber and Epoxy resin at the high temperature, respectively

n: ratio of the cross area of nylon fiber contained in the sideplane AB or CD to its area

ν_e: Poisson's ratio of Epoxy resin at the high temperature.

Then, considering the forces on the sideplane AB, CD of the element in the state of uni-axial stress σ_1, we have

$$\sigma_1=F+\sigma_{e1}=n(E_f-E_e)\varepsilon_1+\frac{E_e}{1-\nu_e^2}(\varepsilon_1+\nu_e\varepsilon_2) \tag{10}$$

There is no normal stress in the direction of axes 2, since nylon fiber has not any resistance to the deformation perpendicular to its direction.

$$\sigma_{e2}=\frac{E_e}{1-\nu_e^2}(\varepsilon_2+\nu_e\varepsilon_1)=0 \tag{11}$$

Now regarding that Young's modulus E_1 and Poisson's ratio ν_1 of the sandwich plate in direction of the elastic symmetry axis 1 are expressed by

$$E_1=\frac{\sigma_1}{\varepsilon_1}\,,\qquad \nu_1=-\frac{\varepsilon_2}{\varepsilon_1} \tag{12}$$

we find from Eqs. (10), (11) and (12).

$$E_1=E_e+n(E_f-E_e)\,,\qquad \nu_1=\nu_e \tag{13}$$

2. Elastic Moduli in Direction Perpendicular to Nylon Fiber

Consider again the element ABCD, as shown in Fig. 8, on the sideplanes AC, BD parallel to the direction of nylon fiber normal stress σ_2 is acting. Then nylon fiber has no influence to the normal stress of the sandwitch plate in direction of axis 2 perpendicular to nylon fiber. Consequently, we have

$$\sigma_{e2}=\sigma_2 \tag{14}$$

From that $\sigma_1=0$ on the sideplanes AB, CD, we have

$$F+\sigma_{e1}=0 \tag{15}$$

Substituting Eqs. (9) into Eqs. (14) and (15), we have

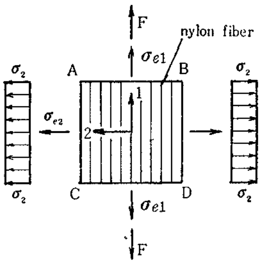


Fig. 8

$$\left.\begin{aligned} &\frac{\nu_e E_e}{1-\nu_e^2}\varepsilon_1+\frac{E_e}{1-\nu_e^2}\varepsilon_2=\sigma_2 \\ &\left\{\frac{E_e}{1-\nu_e^2}+n(E_f-E_e)\right\}\varepsilon_1+\frac{\nu_e E_e}{1-\nu_e^2}\varepsilon_2=0 \end{aligned}\right\} \tag{16}$$

Solving these equations Young's modulus E_2 and Poisson's ratio ν_2 of the sandwitch plate in direction of elastic symmetry axis 2 are obtained in the forms

$$\left.\begin{aligned} E_2&=\frac{\sigma_2}{\varepsilon_2}=\frac{E_1 E_e}{E_1-n(E_f-E_e)\nu_e^2} \\ \nu_e&=-\frac{\varepsilon_1}{\varepsilon_2}=\frac{\nu_e E_e}{E_1-n(E_f-E_e)\nu_e^2} \end{aligned}\right\} \tag{17}$$

From Eqs. (13) and (17), we find

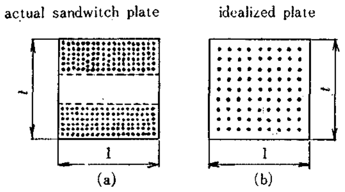
$$\frac{E_1}{E_2}=\frac{\nu_1}{\nu_2}=\frac{E_1-n(E_f-E_e)\nu_e^2}{E_e} \tag{18}$$

This shows that Maxwell's reciprocal theorem is satisfied.

3. Shear Modulus Related to Elastic Symmetry Axes

In calculation of shear deformation related to the elastic symmetry axes of the sandwitch plate, the assumptions have been made that nylon fiber has no shear deformation, for having shear modulus considerably larger than that of Epoxy resin. Thus the shear deformation of the sandwitch plate is influenced by the amount of nylon fibers existing in its cross section. Therefore it is neccessary to give special consideration for calculating the shear deformation.

Actual distribution of nylon fiber in a sandwitch plate is shown in Fig. 9 (a). But in analysis the assumption may be admitted that the sandwitch plate acts as a homogeneous material in which the nylon fiber are uniformly distributed as shown in Fig. 9 (b). In the plane stress condition there is no shear strain in the direction of thickness of this sandwitch plate. However in the stressed plane the shear deformation should be considered separately for Epoxy resin and nylon fiber. Let d be the total width of the nondeforming nylon fibers in the unit width of sandwitch plate, then d is given in the form



(a) (b)

Fig. 9

$$d=\phi\sqrt{\frac{m}{1\times t}} \tag{19}$$

where ϕ and m are the mean diameter and the number of nylon fiber existing in the cross section $1\times t$ of the sandwitch plate of Fig. 9 (a), respectively, and t is its thickness.

When the element ABCD is in the state of simple shear τ_{12}, as shown in Fig. 10, the each fiber of no shear deformation is shifted to one side to constitute the width d. If this element consists of only Epoxy resin, the shear strain is then

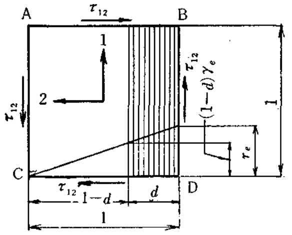


Fig. 10

$$\gamma_{e12}=\frac{\tau_{12}}{Ge} \tag{20}$$

according to Hooke's law, where Ge is the shear modulus of Epoxy resin at the high temperature. But, due to the existence of the part of no shear deformation, the shearing strain of the sandwitch plate decrease to

$$\gamma_{12}=(1-d)\gamma_{e12} \tag{21}$$

as being easily understood from Fig. 10. Substituting Eqs. (20) into Eqs. (21), we have

$$\gamma_{12}=\frac{1-d}{Ge}\tau_{12} \tag{22}$$

This shows a linear relation between shearing stress and strain. Accordingly, the shear modulus of the sandwitch plate related to the elastic symmetry axes 1, 2 is given by

$$G_{12}=\frac{\tau_{12}}{\gamma_{12}}=\frac{Ge}{1-d} \tag{23}$$

4. Comparison of Theories and Experimental Results

The elastic moduli of a sandwitch plate obtained by the proposed theories are compared with their experimental results. Four narrow rectangular cross section test-specimens were prepared, of which axes are inclined by $\theta=0°$, $45°$, $60°$ and $90°$ to the direction of nylon fiber, as shown in Fig. 11. These specimens were tested by tensile load except the specimen (a) which was tested by bending loads. The process of the experiments was similar to that of the stress freezing method.

The experimental results are given in Table 1 with theoretical ones. Since it was rather difficult to obtain shear modulus with tests, its experimental value was determined indirectly from the following relation

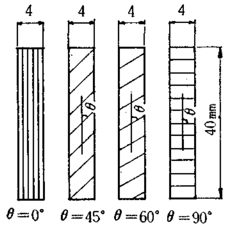


Fig. 11

Table 1

Test Piece	θ	Experimental Value		Theoretical Value	
		Young's modulus	Poisson's ratio	Young's modulus	Poisson's ratio
1	0°	950	0.5	940	0.50
2	45°	270		270	
3	60°	230		230	
4	90°	230	0.1	220	0.11

$$\frac{1}{G_{12}}=\frac{1}{E_{45°}}-\left(\frac{1}{E_{0°}}+\frac{1}{E_{90°}}-\frac{2\nu_{0°}}{E_{0°}}\right) \tag{24}$$

where

E_{0°, E_{45°, E_{90°: Young's moduli of sandwitch plate in the direction $\theta=0^\circ$, 45°, and 90°, determined by test, respectively,

ν_0: Poisson's ratio in the direction $\theta=0^\circ$ also determined by test.

The theoretical values of elastic moduli were taken by substituting the following values into Eqs. (13), (17) and (23)

$$E_f=7.9\times10^3\,\text{kg/cm}^2,\qquad E_e=1.8\times10^2\,\text{kg/cm}^2$$

$$e=0.5,\quad n=0.10,\quad d=0.38$$

It is seen from Table 1 that the experimental results correspond very well to the theoretical ones. Thus these theoretical values were used in the next section as being applicable to the analyses for comparison with the experimental results.

V. PHOTO-ELASTIC EXPERIMENTS BY SANDWITCH PLATE

To examine whether a sandwitch plate was a new orthotropic plate suitable for photo-elastic investigation, some experiments were conducted. The problem of a plate with a circular hole under uniform tensile load was chosen as an example, for this kind of problems have been dealt with by several researchers. In these experiments our investigation was limited to examine the circumferential stresses around a circular hole of a sandwitch plate. The results are also applicable to the state of stresses around a tunnel without lining made in an orthotropic elastic rock ground subjected to graviation only. As shown in Fig. 12, the test specimen was loaded by three pairs of equal force in order to make a state of uniform tensile stress in the region properly apart from the circular hole. To ensure the reliability of the loading, each of these pairs of forces was independently given through the wire laid on the pulleys. The setup of the loading is shown in Photo. 7.

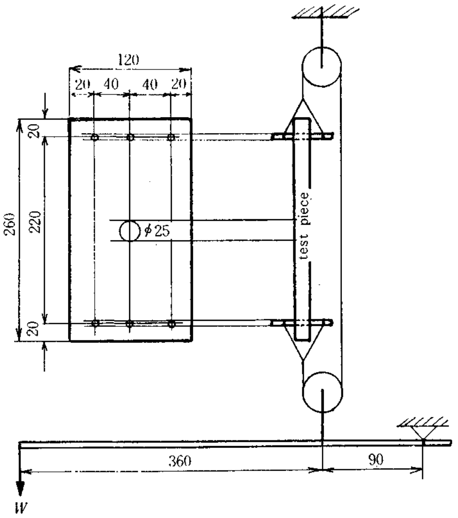


Fig. 12

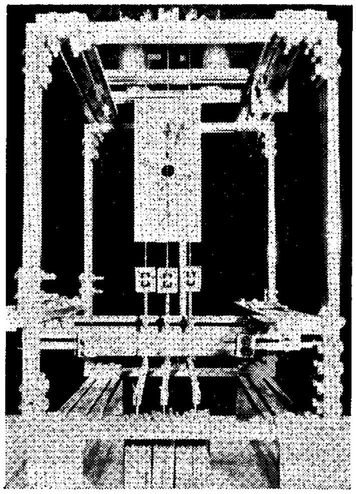
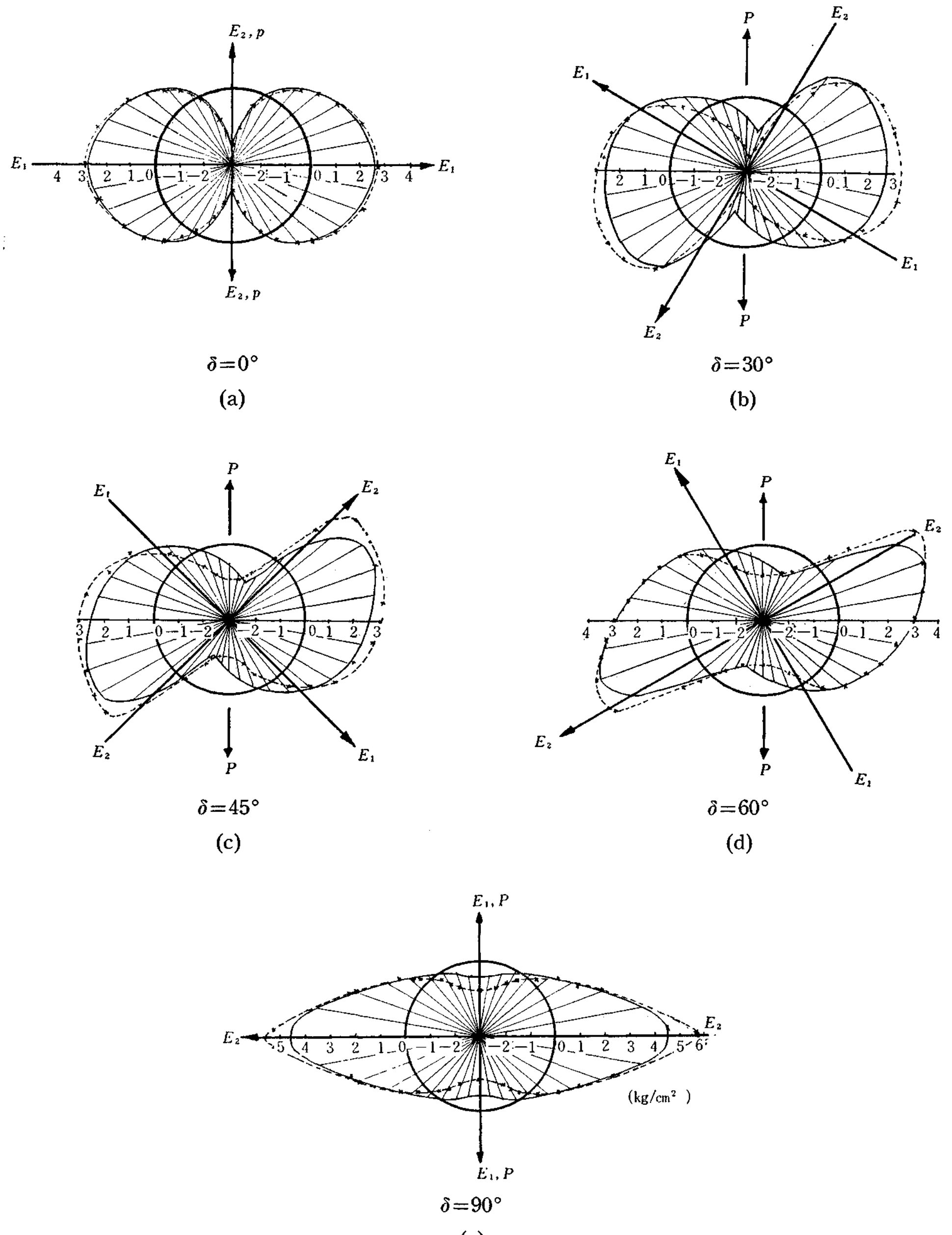


Photo. 7



$\delta=0°$
(a)

$\delta=30°$
(b)

$\delta=45°$
(c)

$\delta=60°$
(d)

$\delta=90°$
(e)

Fig. 13

In Table 2 the condition of the specimens and the loads used in the experiments are given. Photo. 8 shows the fringe patterns taken from photo-elastic experiments of the middle layer sliced out from the test specimens in which the stresses were frozen. All the experimental results as to the circumferential stresses around the circular hole are shown in Fig. 13, together with the calcu-

Table 2 Loading Condition

No.	δ	Cross Section	Load (kg)	p (kg/cm²)
1	0°	18.36 cm²	5.00	2.173
2	30°	19.32	5.44	2.100
3	45°	19.99	4.98	2.000
4	60°	19.27	4.58	2.200
5	90°	19.24	3.55	2.500

lated ones obtained from the theory by G. S. Sonntag for comparison. Referring to Fig. 14, the theoretical stresses around a circular hole made in an orthotropic plate subjected to an uniform tensile stress p are given in the form

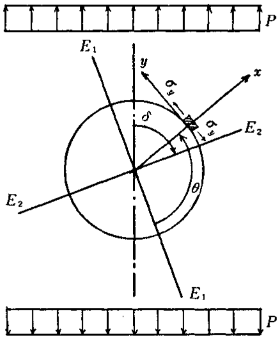


Fig. 14

$$\left.\begin{aligned}
\sigma_y=&\frac{p}{E_2N}[\{(1+n)\cos^2\delta+m\sin^2\delta\}\cos^2\theta\\
&+n(1+n-m)\sin\delta\cos\delta\sin\theta\cos\theta\\
&+\{\cos^2\delta+(m-n)\sin^2\delta\}m\sin^2\theta]\\
m=&-\sqrt{\frac{E_2}{E_1}},\qquad n=\sqrt{2\left(\sqrt{\frac{E_2}{E_1}}-\nu_{21}\right)+\frac{E_2}{G_{12}}}\\
N=&\frac{\sin^4\theta}{E_1}+\left(\frac{1}{G_{12}}-\frac{2\nu_{21}}{E_2}\right)\sin^2\theta\cos^2\theta+\frac{\cos^4\theta}{E_2}
\end{aligned}\right\}\quad(25)$$

where δ is the angle between the tensile axis and principal axis 2 and θ is the angle which is measured anticlockwise from the elastic principal axis 1.

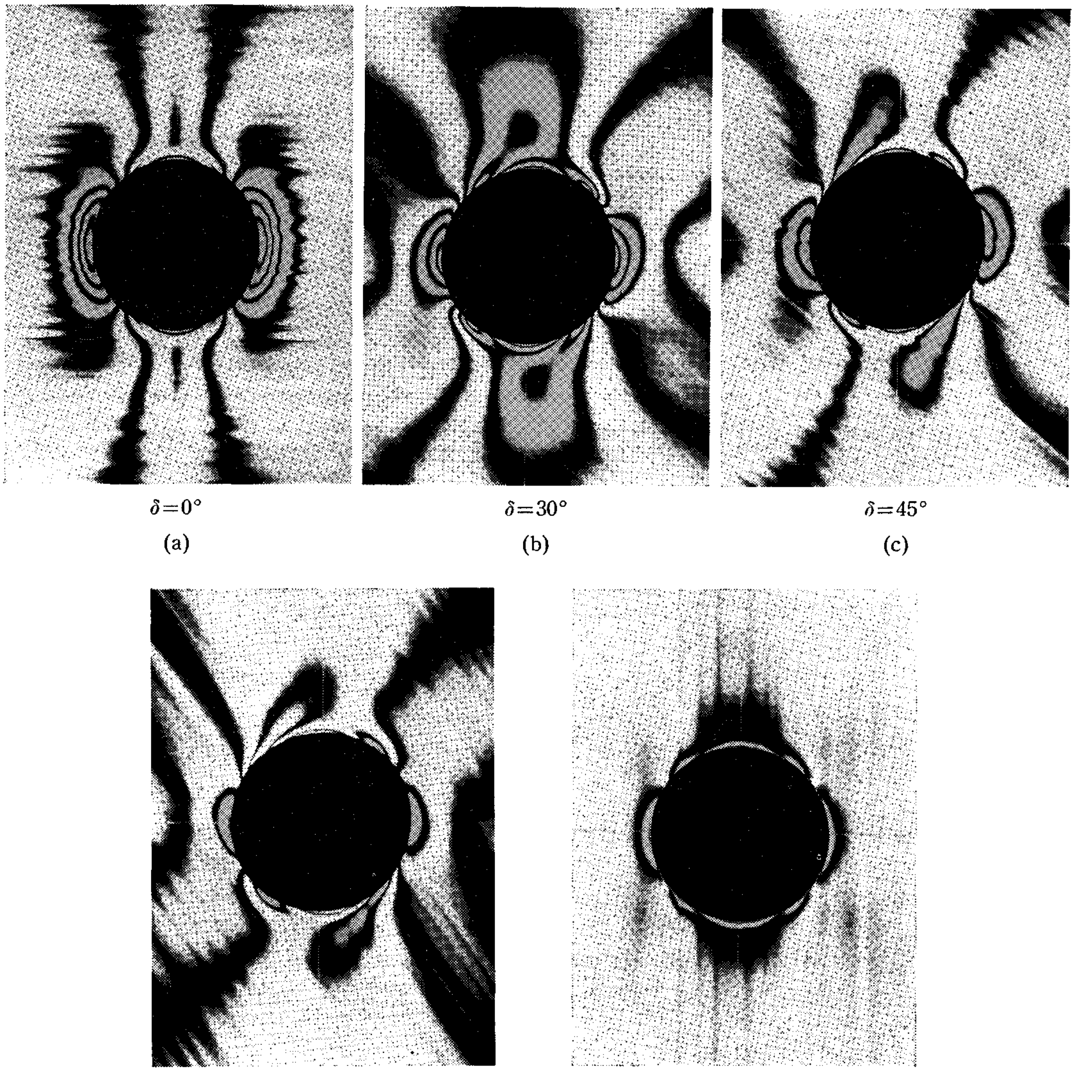
The experimental results in Fig. 13 correspond well to the theoretical ones as a whole and fringe patterns in Photo. 8 are also more clearcut than have been taken in the experiments already reported. Thus it seems to be apparent that the sandwitch plate proposed here is an useful orthotropic plate for photo-elastic experiment.

VI. CONCLUSION

A new orthotropic material named sandwitch plate is proposed and some experiments were carried out for ascertaining its utility to photo-elastic experiment. And the validity of the theory proposed for calculation of its elastic moduli was examined.

The calculated values of the elastic properties correspond to measured results. This fact suggests that the assumption made for calculation of the elastic moduli of the sandwitch plate are effective for the analyses of its actual behavior. Through the results of some photo-elastic experiments by the sandwitch plate, it becomes also apparent that this plate is suitable as an orthotropic plate for photo-elastic experiment.

By adopting the experimental method developed here, some kinds of inconveniency in the photo-elastic experiment for an orthotropic plate can be success-



$\delta=0°$ (a)

$\delta=30°$ (b)

$\delta=45°$ (c)

$\delta=60°$ (d)

$\delta=90°$ (e)

Photo. 8

fully eliminated. For instance, the ratio of Young's moduli of both elastic principal axes can be easily changed within a wide range, a more clear-cut fringe pattern can be taken than has been already seen, suitable fringe order for stress analysis can be obtained by comparatively small load, etc..

Generally it is very difficult to solve the problems on stress concentration in an orthotropic plate analytically. Therefore, the new experimental method proposed will supply an additional means to those already developed in that field. But, inspecting the conditions of stress concentration in the examples of photoelastic experiment given here, a certain tendency can be found that the difference between the values obtained from the experiments and the calculations are rather large along the limited boundary in the part where the direction of nylon fiber is perpendicular to the circular hole. This result seems to be caused by the characteristics of the sandwitch plate itself. The detail of the cause shall be investigated in future.

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