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DETERMINATION OF LOSS MINIMIZING OFFSETS PATTERN THROUGH THE MAXIMUM PRINCIPLE

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1. INTRODUCTION

When the traffic volume increases exceeding some degree over the whole of the network where all intersections have traffic signals, the independent operation of each signal leads to inviting a fall of the capacity of the network. To avoid such inefficiency there is a control procedure which aims to relate all signals in their indication through some method so as to mitigate the traffic congestion. The procedure is a traffic signal control system called area traffic control which has become a staple problem of traffic control in some big cities in the world these days, being supported by the rapid spread of computers. The area traffic control has already been executed in the principal cities of the U.S.A., England, Canada and other countries and it seems that those control systems have been producing effective results.

But as to the development of a control procedure it still remains first step although there have been much technical improvement for an electronic computer; the present is so to speak an experimental stage or a trial and error stage. For this reason an efficient software for the area traffic control procedure is being required to be developed.

When we limit the argument to the fixed-time signal system, there are only several methods which have already been developed so far as the author knows, for example, the Combination Method which was developed by Hillier[1] and Whiting and extended by Allsop[2], the TRANSYT Method[3] or the Method through dynamic programming developed by the author[4],[5].

In the present paper a method for minimizing the loss to traffic in a network controlled by fixed-time signals through the discrete maximum principle subject to assumptions which are similar to those adopted in the Methods mentioned above is described.

When we say about an area traffic control procedure, we mean the method for rational determination of cycle time, split and offset of each signal.

But the former two control parameters can be determined independently at each intersection and so the determination of offset will be the main problem in the substantial sence of the area traffic control whose purpose is to transact the the traffic stream smoothly by some combination of control parameters at all the intersections. Therefore in the present paper we focus the problem on determining the optimum offsets pattern for traffic signals in a network.

2. ASSUMPTIONS

Here we adopt the loss to traffic for estimating the effectiveness of a control

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procedure and discuss on the optimum area traffic control procedure (or offsets pattern) which minimizes the total loss to traffic in a network. Here in the paper the loss to traffic may be anything that can be quantified, for example, the number of stops of vehicles, the running time, the delay or the queue length of vehicles. The important thing is that the criterion is the appropriate measure for estimating the effectiveness of transacting traffic in a network and so if the pattern of the traffic flow is eminently different with time even in the same network, it may be preferable to adopt the most appropriate measure with time. Whence for convenience we use the word "loss" for designating the measure of the estimation of the control effectiveness.

The main assumptions in the paper are as follows;

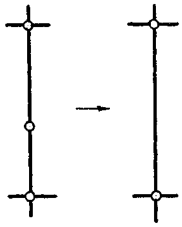
1) The split is given at each intersection in the network. All signals have a common cycle time which may be a standard cycle time at the most congested intersections.
2) The loss generated in a link which has intersections at both ends of it depends only on the flow on it and the offset policy at those intersections and is not affected by any other offset policy; that is, the loss to the flow q_{ab} between the two intersections a, b is represented by the function $g(k_a, k_b, q_{ab})$ where k_a and k_b are the offset policy at the intersections a and b respectively.

The assumption 2) seems to be in conflict with the substance of the area traffic control which aims to control the traffic taking into account the relations among the intersections in the network but it is considered to be not so unreal when the traffic volume increases so that the area traffic control is required though not true in the case that the traffic flow is small; that is, the reasonableness of this assumption may be allowed when the saturation flow incessantly flows during every green time.

3. SIMPLIFICATION OF A NETWORK

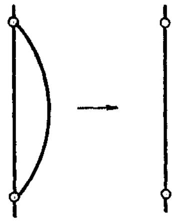
By the assumption 2) it becomes possible to simplify or reduce a network[6] in the following ways.

1) Suppose that there is one or more minor intersections on a link between two major intersections as shown in Fig. 1(a) then by the assumption 2) the optimum offset policy at the minor intersections can be determined depending on a pair of offsets at the major intersections and it is independent of the policy at all the other intersections. Hence it may be permitted to omit those minor intersections from the network. Thus the network shown in Fig. 1(a) becomes as the network in Fig. 1(b).



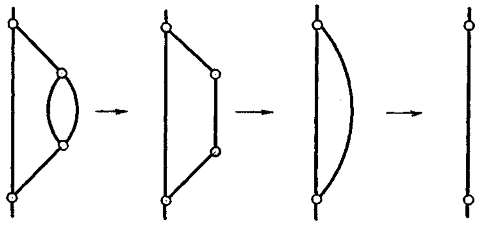
(a) (b)

Fig. 1



(a) (b)

Fig. 2



(a) (b) (c) (d)'

Fig. 3

2) In the case that several links are connected to a pair of intersections in parallel as Fig. 2 (a) those links can be delt with as single link as shown in Fig. 2 (b) because the loss generated in a link is identified according to a pair of policies at the intersections.

If we use the combination of the above two conversions, a graph in Fig. 3 (a) is in the first place changed to the graph in Fig. 3 (b) by the conversion 2), then to the graph in Fig. 3 (c) by the conversion 1) and eventually to the graph in Fig. 3 (d) which is merely a link with intersections at both ends of it by using the conversion 2) again.

3) When a node n is an articulation point as in Fig. 4 (a) and the graph can be partitioned into two subgraphs by detaching the graph at the node (Fig. 4 (b)), there arise no troubles if the determination of the control policy is separately done each subgraph so long as the policy at the node n is considered in common. Therefore in such a case the optimum control policy for a main subgraph is determined at first and after that the control policies for the other subgraphs are determined depending on the policy of the main subgraph in relation to articulation points. Whence the graph in Fig. 4 (a) can be reduced to such a graph as in Fig. 4 (c).

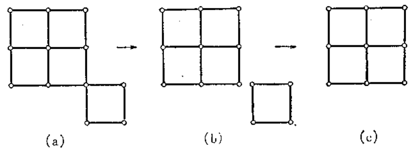


Fig. 4

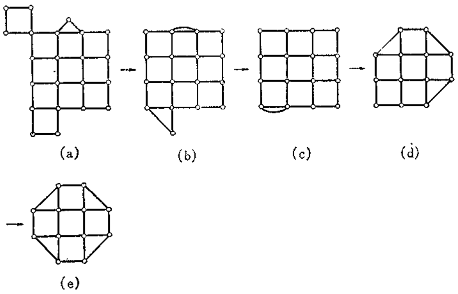


Fig. 5

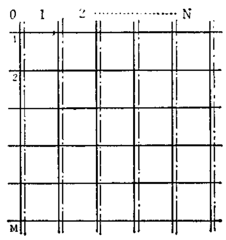


Fig. 6 Grid system network.

By utilizing the conversions 1), 2) and 3) the graph in Fig. 5 (a) eventually becomes as the graph in Fig. 5 (e). When we examine the final graph in Fig. 5 (e) we notice the fact that the degree of every node is no less than three. Generally, the degree of every node in a simplified graph is always no less than three and so it may as well to deal with only such a graph in an analysis.

However, since we can expect no improvement in the process of seeking an optimum control policy even if the shape of a graph for analysis is simplified as can be seen from the process of simplification of a graph mentioned above, a grid system network is delt with here for the convenience of formulation.

4. DETERMINATION OF AN OPTIMUM OFFSETS PATTERN

Here we consider a grid system network which has $(N+1)$ intersections in the horizontal direction and M intersections in the vertical direction as one for analysis and divide it into $(N+1)$ sections $0, 1, 2, \cdots, N$ as shown in Fig. 6.

Let

${}^hq_{i1}^n$; the traffic flow which flows from the intersection in the $(n-1)$-th section to the intersection in the n-th section on the i-th horizontal link.

${}^hq_{i2}^n$; the opposite flow to ${}^hq_{i1}^n$.

${}^vq_{i1}^n$; the flow which flows from the i-th intersection to the $(i+1)$-th intersection on the i-th vertical link in the n-th section.

${}^vq_{i2}^n$; the opposite flow to ${}^vq_{i1}^n$.

k_i^n ; the offset policy at the i-th intersection in the n-th section which is the time length from a standard time to the beginning of the green time in the vertical direction and that to the beginning of the red time in the horizontal direction. Suppose that the common cycle time to all the intersections is C and the optimum offsets pattern is seeked at discrete points of τ seconds interval, then k_i^n is concretely defined as follows;

$k_i^n=0$; the offset is equal to zero; in the vertical direction the offset of the green time is zero and in the horizontal direction the offset of the red time is zero.

$k_i^n=1$; the offset is τ.

⋮

$k_i^n=K$; the offset is $K\cdot\tau$, where $K=C/\tau$ (positive integer)

${}^hg_{i1}^n(k_i^{n-1}, k_i^n, {}^hq_{i1}^n)$; the loss suffered by ${}^hq_{i1}^n$ on the i-th horizontal link in the n-th section when the policy at the i-th intersection in the $(n-1)$-th section is k_i^{n-1} and the policy at the i-th intersection in the n-th section is k_i^n.

${}^hg_{i2}^n(k_i^{n-1}, k_i^n, {}^hq_{i2}^n)$; similarly the loss suffered by ${}^hq_{i2}^n$.

${}^vg_{i1}^n(k_i^n, k_{i+1}^n, {}^vq_{i1}^n)$; the loss suffered by ${}^vq_{i1}^n$ on the i-th vertical link when the policy at the i-th intersection in the n-th section is k_i^n and the policy at the $(i+1)$-th intersection in the same section is k_{i+1}^n.

${}^vg_{i2}^n(k_i^n, k_{i+1}^n, {}^vq_{i2}^n)$; similarly the loss suffered by ${}^vq_{i2}^n$.

$$(n=0, 1, 2, \cdots, N;\quad i=1, 2, \cdots, M)$$

Here

$${}^hg_{i1}^0(k_i^{-1}, k_i^0, {}^hq_{i1}^0)={}^hg_{i2}^0(k_i^{-1}, k_i^0, {}^hq_{i2}^0)=0 \qquad (i=1, 2, \cdots, M)$$

$${}^vg_{M1}^n(k_M^n, k_{M+1}^n, {}^vg_{M1}^n)={}^vg_{M2}^n(k_M^n, k_{M+1}^n, {}^vq_{M2}^n)=0 \qquad (n=0, 1, 2, \cdots, N)$$

The reason why ${}^hg_{i\nu}^0(\ )$ and ${}^vg_{M\nu}^n(\ )$ $(\nu=1, 2)$ are set to be zero is that the loss generated on the exterior links of the network is not related to the control effectiveness of the network and so they may be some constant values instead of zeros.

The purpose of this study of determining the optimum control policy at each intersection that minimizes the total loss generated in the network comes to a mathematical problem of seeking k_i^n $(n=0, 1, 2, \cdots, N;\ i=1, 2, \cdots, M)$ so as to minimize the objective function represented by

$$F=\sum_{n=0}^{N}\left[\sum_{i=1}^{M}\left\{{}^hg_{i1}^n(k_i^{n-1}, k_1^n, {}^hq_{i1}^n)+{}^hg_{i2}^n(k_i^{n-1}, k_i^n, {}^hq_{i2}^u)\right\}+\sum_{i=1}^{M}\left\{{}^vg_{i1}^n(k_i^n, k_{i+1}^n, {}^vq_{i1}^n)+{}^vg_{i2}^n(k_i^n, k_{i+1}^n, {}^vq_{i2}^n)\right\}\right] \tag{1}$$

There are many mathematical techniques which can be used to minimize F represented by equation (1). Here we use the discrete maximum principle developed by L. T. Fan and C. S. Wang which originated from Pontryagin's maximum principle. This discrete maximum principle by Fan and Wang[7] is the generalized method of the discrete maximum principle which was published by S. Katz[8] in 1962.

Now let the sections of the network correspond to the stages in multi-stage decision process. And the total loss generated in the network is minimized in passing through the $(N+1)$ stages which consist of state variable $g(k, k', q)$'s and decision variables k's.

However, it is impossible to apply the discrete maximum principle to the problem shown here as it is, and so it is required to change the problem to a standard type of problem of the discreate maximum principle by adding or modyfying the state variables and the decision variables. For this the following variable is introduced as a state variable in the n-th section.

$$x_1^n = x_1^{n-1} + \sum_{i=1}^{M} \left\{ {}^h g_{i1}^n(k_i^{n-1}, k_i^n, {}^h q_{i1}^n) + {}^h g_{i2}^n(k_i^{n-1}, k_i^n, {}^h q_{i2}^n) \right\} + \sum_{i=1}^{M} \left\{ {}^v g_{i1}^n(k_i^n, k_{i+1}^n, {}^v q_{i1}^n) + {}^v g_{i2}^n(k_i^n, k_{i+1}^n, {}^v q_{i2}^n) \right\} \qquad (n=1, 2, \cdots, N) \tag{2}$$

Where

$$x_1^0 = \sum_{i=1}^{M} \left\{ {}^v g_{i1}^0(k_i^0, k_{i+1}^0, {}^v q_{i1}^0) + {}^v g_{i2}^0(k_i^n, k_{i+1}^n, {}^v q_{i2}^0) \right) \right\} \tag{3}$$

This state variable x_1^n means the accumulative loss from the 0-th section to the n-th section and so the total loss generated over the whole of the network can be described by x_1^N.

Let

$$y_i^n = k_i^n \tag{4}$$

$$\theta_i^n = k_i^n - k_i^{n-1} \qquad (-K \leqq \theta_i^n \leqq K) \tag{5}$$

then the state variable x_1^n given by equation (2) is rewritten as follows

$$x_1^n = x_1^{n-1} + \sum_{i=1}^{M} \left\{ {}^h g_{i1}^n(y_i^{n-1}, \theta_i^n + y_i^{n-1}, {}^h q_{i1}^n) + {}^h g_{i2}^n(y_i^{n-1}, \theta_i^n + y_i^{n-1}, {}^h q_{i2}^n) \right\} + \sum_{i=1}^{M} \left\{ {}^v g_{i1}^n(\theta_i^n + y_i^{n-1}, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i1}^n) + {}^v g_{i2}^n(\theta_i^n + y_i^{n-1}, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i2}^n) \right\} \tag{6}$$

In the above equation we regard y_i^n as a state variable and θ_i^n as a decision variable. From equations (4) and (5) the performance equation associated with y_i^n becomes

$$y_i^n = \theta_i^n + y_i^{n-1} \qquad (n=1, 2, \cdots, N;\ i=1, 2, \cdots, M) \tag{7}$$

Here y_i^0 is the policy k_i^0 $(i=1, 2, \cdots, M)$ at the i-th intersection in the 0-th section. Thus the problem delt with here is converted to the problem where decision variables θ_i^n's in each section are determined so as to minimize x_1^N subject to the performance equations (6) and (7). This is a general problem which can be solved by the discrete maximum principle and the concrete method for solving the problem is as follows.

First of all we introduce the Hamiltonian function corresponding to the n-th section represented by the following equation.

$$H^n = z_1^n x_1^n + \sum_{i=1}^{M} z_{i+1}^n y_i^n \qquad (n=1, 2, \cdots, N) \tag{8}$$

where z_i^n is a covariant variable following the equations

$$z_1^{n-1} = \frac{\partial H^n}{\partial x_1^{n-1}} \qquad (n=1, 2, \cdots, N) \tag{9}$$

$$z_{i+1}^{n-1} = \frac{\partial H^n}{\partial y_i^{n-1}} \qquad (n=1, 2, \cdots, N;\ i=1, 2, \cdots, M) \tag{10}$$

Here the partial differential equations (9) and (10) means the difference equations in fact since the problem in the present paper is a discrete type.

Substituting the equations (6) and (7) in the Hamiltonian function represented by equation (8) gives

$$\begin{aligned} H^n = z_1^n \Bigg[x_1^{n-1} &+ \sum_{i=1}^{M} \left\{ {}^h g_{i1}^n (y_i^{n-1}, \theta_i^n + y_i^{n-1}, {}^h q_{i1}^n) + {}^h g_{i2}^n (y_i^{n-1}, \theta_i^n + y_i^{n-1}, {}^h q_{i2}^n) \right\} \\ &+ \sum_{i=1}^{M} \left\{ {}^v g_{i1}^n (\theta_i^n + y_i^{n-1}, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i1}^n) + {}^v g_{i2}^n (\theta_i^n + y_i^{n-1}, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i2}^n) \right\} \Bigg] \\ &+ \sum_{i=1}^{M} z_{i+1}^n (\theta_i^n + y_i^{n-1}) \end{aligned} \tag{11}$$

The following relations among the covariant variables are obtained from equations (9) and (10) if we consider equation (11).

$$z_1^{n-1} = \frac{\partial H^n}{\partial x_1^{n-1}} = z_1^n \tag{12}$$

$$\begin{aligned} z_{i+1}^{n-1} = \frac{\partial H^n}{\partial y_i^{n-1}} = z_{i+1}^n + z_1^n \Bigg(\sum_{\nu=1}^{2} \Bigg[&\left\{ {}^h g_{i\nu}^n (y_i^{n-1}+1, \theta_i^n + y_i^{n-1}+1, {}^h q_{i\nu}^n) - {}^h g_{i\nu}^n (y_i^{n-1}, \theta_i^n + y_i^{n-1}, {}^h q_{i\nu}^n) \right\} \\ &+ \left\{ {}^v g_{i\nu}^n (\theta_i^n + y_i^{n-1}+1, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i\nu}^n) - {}^v g_{i\nu}^n (\theta_i^n + y_i^{n-1}, \theta_{i+1}^n + y_{i+1}^{n-1}, {}^v q_{i\nu}^n) \right\} \\ &+ \left\{ {}^v g_{i-1,\nu}^n (\theta_{i-1}^n + y_{i-1}^{n-1}, \theta_i^n + y_i^{n-1}+1, {}^v q_{i-1,\nu}^n) - {}^v g_{i-1,\nu}^n (\theta_{i-1}^n + y_{i-1}^{n-1}, \theta_i^n + y_i^{n-1}, {}^v q_{i-1,\nu}^n) \right\} \Bigg] \Bigg) \end{aligned} \tag{13}$$

On the other hand the objective function F is described as follows as previously mentioned.

$$F = 1 \cdot x_1^N + 0 \cdot y_1^N + 0 \cdot y_2^N + \cdots + 0 \cdot y_M^N$$

Hence the final condition of z_i^n becomes

$$z_1^N = 1 \tag{14}$$

$$z_{i+1}^N = 0 \qquad (i=1, 2, \cdots, M) \tag{15}$$

From equations (12) and (14) the covariant variable z_1^n is uniquely determined as

$$z_1^n=1 \qquad (n=1, 2, \cdots, N) \tag{16}$$

Whence equation (13) becomes

$$\begin{aligned} z_{i+1}^{n-1}=z_{i+1}^n+\sum_{\nu=1}^{2}\Bigg[&\left\{{}^h g_{i\nu}^n(y_i^{n-1}+1, \theta_i^n+y_i^{n-1}+1, {}^h q_{i\nu}^n)-{}^h g_{i\nu}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i\nu}^n)\right\} \\ &+\left\{{}^v g_{i\nu}^n(\theta_i^n+y_i^{n-1}+1, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i\nu}^n)-{}^v g_{i\nu}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i\nu}^n)\right\} \\ &+\left\{{}^v g_{i-1,\nu}^n(\theta_{i-1}^n+y_{i-1}^{n-1}, \theta_i^n+y_i^{n-1}+1, {}^v q_{i-1,\nu}^n)-{}^v g_{i-1,\nu}^n(\theta_{i-i}^n+y_{i-1}^{n-1}, \theta_i^n+y_i^{n-1}, {}^v q_{i-1,\nu}^n)\right\}\Bigg] \end{aligned} \tag{17}$$

Therefore the Hamiltonian function eventually becomes

$$\begin{aligned} H^n=x_1^{n-1}+&\sum_{i=1}^{M}\left\{{}^h g_{i1}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i1}^n)+{}^h g_{i2}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i2}^n)\right\} \\ &+\sum_{i=1}^{M}\left\{{}^v g_{i1}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i1}^n)+{}^v g_{i2}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i2}^n)\right\} \\ &+\sum_{i=1}^{M} z_{i+1}^n(\theta_i^n+y_i^{n-1}) \qquad (n=1, 2, \cdots, N) \end{aligned} \tag{18}$$

The optimum decision variables θ_i^n's in each section are obtained by finding out the values which minimize H^n represented by equation (18) in their domain. Here the domain of θ_i^n can be written as $-y_i^{n-1}\leqq\theta_i^n\leqq K-y_i^{n-1}$ since $y_i^n=\theta_i^n+y_i^{n-1}$ from equation (7) and $0\leqq y_i^n\leqq K$.

We solve the problem of minimizing H^n through applying dynamic programming. In the first place $f_j(\theta_{j-1}^n)$ is defined to be the minimum value of

$$\begin{aligned} &\sum_{i=j-1}^{M}\left\{{}^h g_{i1}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i1}^n)+{}^v g_{i2}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i2}^n)\right\} \\ &\quad+\sum_{i=j-1}^{M}\left\{{}^v g_{i1}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i1}^n)+{}^v g_{i2}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i2}^n)\right\} \\ &\quad+\sum_{i=j-1}^{M} z_{i+1}^n(\theta_i^n+y_i^{n-1})\,. \end{aligned}$$

Then the minimization of H^n gets equivalent to obtaining θ_i^n's ($n=1, 2, \cdots, N$; $i=1, 2, \cdots, M$) which correspond to f_1.

Now from the definition first we get

$$\begin{aligned} f_M(\theta_{M-1}^n)=\min_{-y_M^{n-1}\leqq\theta_M^n\leqq K-y_M^{n-1}}\Bigg[&\sum_{i=M-1}^{M}\left\{{}^h g_{i1}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i1}^n)+{}^h g_{i2}^n(y_i^{n-1}, \theta_i^n+y_i^{n-1}, {}^h q_{i2}^n)\right\} \\ &+\sum_{i=M-1}^{M}\left\{{}^v g_{i1}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i1}^n)+{}^v g_{i2}^n(\theta_i^n+y_i^{n-1}, \theta_{i+1}^n+y_{i+1}^{n-1}, {}^v q_{i2}^n)\right\} \\ &+\sum_{i=M-1}^{M} z_{i+1}^n(\theta_i^n+y_i^{n-1})\Bigg] \end{aligned} \tag{19}$$

All θ_M^n's sufficing the above equation are calculated, corresponding to all θ_{M-1}^n's in its domain $-K \leqq \theta_{M-1}^n \leqq K$. The obtained θ_M^n is denoted by $\dot{\theta}_M^n(\theta_{M-1}^n)$ in detail.

Next $f_{M-1}(\theta_{M-1}^n)$ is expressed by the following equation, using the recurrence relation of dynamic programming.

$$f_{M-1}(\theta_{M-2}^n) = \min_{-y_{M-1}^{n-1} \leqq \theta_{M-1}^n \leqq K - y_{M-1}^{n-1}} \Big[\Big\{ {}^h g_{M-2,1}^n (y_{M-2}^{n-1}, \theta_{M-2}^n + y_{M-2}^{n-1}, {}^h q_{M-2,1}^n) + {}^h g_{M-2,2}^n (y_{M-2}^{n-1}, \theta_{M-2}^n + y_{M-2}^{n-1}, {}^h q_{M-2,2}^n) \Big\} + \Big\{ {}^v g_{M-2,1}^n (\theta_{M-2}^n + y_{M-2}^{n-1}, \theta_{M-1}^n + y_{M-1}^{n-1}, {}^v q_{M-2,1}^n) + {}^v g_{M-2,2}^n (\theta_{M-2}^n + y_{M-2}^{n-1}, \theta_{M-1}^n + y_{M-1}^{n-1}, {}^v q_{M-2,2}^n) \Big\} + z_{N-1}^n (\theta_{M-2}^n + y_{M-2}^{n-1}) + f_M(\theta_{M-1}^n) \Big] \tag{20}$$

From this equation $\dot{\theta}_{M-1}^n$ corresponding to given θ_{M-2}^n is obtained.

Likewise the general recurrence relation is represented by the following equation.

$$f_j(\theta_{j-1}^n) = \min_{-y_j^{n-1} \leqq \theta_j^n \leqq K - y_j^{n-1}} \Big[\Big\{ {}^h g_{j-1,1}^n (y_{j-1}^{n-1}, \theta_{j-1}^n + y_{j-1}^{n-1}, {}^h q_{j-1,1}^n) + {}^h g_{j-1,2}^n (y_{j-1}^{n-1}, \theta_{j-1}^n + y_{j-1}^{n-1}, {}^h q_{j-1,2}^n) \Big\} + \Big\{ {}^v g_{j-1,1}^n (\theta_{j-1}^n + y_{j-1}^{n-1}, \theta_j^n + y_j^{n-1}, {}^v q_{j-1,1}^n) + {}^v g_{j-1,2}^n (\theta_{j-1}^n + y_{j-1}^{n-1}, \theta_j^n + y_j^{n-1}, {}^v q_{j-1,2}^n) \Big\} + z_j^n (\theta_{j-1}^n + y_{j-1}^{n-1}) + f_{j+1}(\theta_j^n) \Big] \quad (j = 2, 3, \cdots, M-1) \tag{21}$$

By using this recurrence relation all the conditional optimum values θ_j^n's ($j=2, 3, \cdots, M-1$) are determined.

Eventually the following relation holds between the first intersection and the second intersection.

$$f_1 = \min_{\theta_1^n} \{ f_2(\theta_1^n) \} \tag{22}$$

θ_1^n gotten at this final stage is no more a conditional optimum value but an absolute optimum value from the definition of f_1. Let the value be designated as $\overset{*}{\theta}{}_1^n$ then $\dot{\theta}_2^n$ which corresponds to $f_2(\overset{*}{\theta}{}_1^n)$ of $f_2(\theta_1^n)$ previously obtained become the absolute optimum decision variable $\overset{*}{\theta}{}_2^n$ at the second intersection. The optimum decision variable $\overset{*}{\theta}{}_3^n$ at the third intersection is obtained by choosing $\dot{\theta}_3^n$ which corresponds to $f_3(\overset{*}{\theta}{}_2^n)$. Similarly, by examining back to the first intersection using $f_2, f_3, \cdots, f_M$ calculated beforehand all the absolute optimum decision variables which minimize the Hamiltonian function H^n are determined as $\overset{*}{\theta}{}_j^n$ ($j=1, 2, \cdots, M$).

Now we have seen that the optimum offset policy at each intersection minimizing the total loss over the whole of the network can be determined through the discrete maximum principle. But in the above we show only the main points of the technique for determining the optimum offsets pattern and so the concrete calculation procedure is shown below.

[Calculation Step]

If we regard the problem delt with here as a mathematical problem through the discrete maximum principle, it cannot be solved by using the usual method since the boundary conditions of the state variables, that is, the initial condition $x_1^0, y_1^0, y_2^0, \cdots, y_M^0$ and the final condition $x_1^N, y_1^N, y_2^N, \cdots, y_M^N$ are both free. Therefore here we consider the initial points to be fixed and determine the optimum solutions successively corresponding to any different sets of $x_1^0, y_1^0, y_2^0, \cdots, y_M^0$.

Step 1. Assume a set $y_1^0, y_2^0, \cdots, y_M^0$ and calculate x_1^0 by equation (3).

Step 2. Determine $\overset{*}{\theta}{}_1^N, \overset{*}{\theta}{}_2^N, \cdots, \overset{*}{\theta}{}_M^N$ using equations from (19) to (22), assuming $x_1^N, y_1^N, y_2^N, \cdots, y_M^N$.

Step 3. Then $x_1^{N-1}, y_1^{N-1}, y_2^{N-1}, \cdots, y_M^{N-1}$ are calculated from equations (6) and (7). Calculate z_{i+1}^{N-1} ($i=1, 2, \cdots, M$) by equation (13) using those state variables in the $(N-1)$-th section and determine the optimum decision variables $\overset{*}{\theta}{}_1^{N-1}, \overset{*}{\theta}{}_2^{N-1}, \cdots, \overset{*}{\theta}{}_M^{N-1}$ by equations from (19) to (22).

Step 4. Itelate the same procedure as step 3 until $x_1^1, y_1^1, y_2^1, \cdots, y_M^1$ and $\overset{*}{\theta}{}_1^1, \overset{*}{\theta}{}_2^1, \cdots, \overset{*}{\theta}{}_M^1$ are obtained.

Step 5. Calculate $x_1^0, y_1^0, y_2^0, \cdots, y_M^0$ from equations (6) and (7) using $x_1^1, y_i^1, \overset{*}{\theta}{}_i^1$ obtained at Step 4.

Step 6. If the state variables in the 0-th section calculated at Step 5 are equal to the assumed values at Step 1, go to Step 7. Otherwise return to Step 2.

Step 7. Memorize the obtained values x_1^N and y_i^n ($n=0, 1, 2, \cdots, N$; $i=1, 2, \cdots, M$) and return to Step 2 assuming new initial condition $y_1^0, y_2^0, \cdots, y_M^0$ and calculating x_1^0 by them.

Step 8. After calculating for all different initial sets $y_1^0, y_2^0, \cdots, y_M^0$ and x_1^0, compare x_1^N corresponding to each initial set and find out the minimum value and corresponding y_i^n ($n=0, 1, 2, \cdots, N$; $i=1, 2, \cdots, M$). Then those state variables give the optimum offsets pattern.

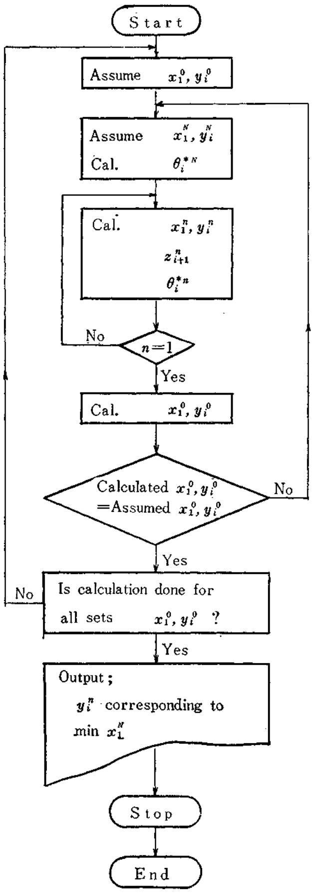


Fig. 7 Flow-Chart.

The above calculation process is summarized in the flow chart shown in Fig. 7.

5. DISCUSSION AND CONCLUSION

In the present paper a fundamental aspect is shown for determining the optimum offsets pattern through the discrete maximum principle.

As it is reported that the effectiveness of reducing the total running time by about twelve percent was obtained in the Glasgow's experiment[9] of the area traffic control over the existing network based on the similar assumptions, it may be possible to reduce the traffic loss by using the present technique for an actual network if the traffic loss function ${}^{h}g_{iv}^{n}(k_i^{n-1}, k_i^n, {}^{h}q_{iv}^{n})$ or ${}^{v}g_{iv}^{n}(k_i^n, k_{i+1}^n, {}^{v}q_{iv}^{n})$ is given through simulation technique or through surveys.

There is a problem what quantity we should adopt as the values of ${}^{h}q_{iv}^{n}$ and ${}^{v}q_{iv}^{n}$ since they fluctuate at all times. Both the value and the degree of the fluctuation of the traffic volume vary depending on the time length during which they are counter. If a control procedure is changed successively after some time interval, the length of it is considered to be a rational time length during which the traffic volume should be counted. That is, since it is possible to change a control procedure (including cycle time and split) at every time interval if we use the technique for seeking the optimum offsets pattern proposed here at such time interval during which the traffic volume and the flow pattern are considered to be nearly constant, for example, if the time interval is thirty minutes we may adopt the mean traffic volume per thirty minutes.

As the minimum interval in which it is possible to change the control procedure we can consider the time length in which the estimation of the traffic volume appearing in the subsequent interval and the calculation for determining the optimum control procedure through the method mentioned here using the estimated traffic volume as input are possible. However, if the time interval is too short, there appear many inconveniences, for example, the fluctuation of the traffic volume becomes violent or it gets difficult to estimate the traffic volume. And so it is important to adopt an appropriate time interval during which the traffic volume is comparatively stable. It may not require that the control procedure adapts itself to every fluctuation of the traffic volume in a short time.

In this paper the technique of determination of the control procedure for a network controlled by fixed-time signals but the same argument may hold good for a vehicle-actuated signals system when the traffic volume increases to some extent because the maximum green time set beforehand is always displayed when the traffic becomes congested and we cannot find any substantial differences between a fixed-time signal and a vehicle-actuated signal.

Here we assume the common cycle time at all the intersections but in fact each intersection has its own optimum cycle time corresponding to the traffic condition there. The proposed method seems no to be general in this point. But if we set an independent cycle time at each intersection it is feared that the effectiveness of offsets which are considered to take the most important role in synchronization is destroyed periodically and as the result it becomes doubtful whether the original purpose of executing the optimum control for two dimensional signals system is achieved or not; that is, it is considered to get difficult to guarantee a significant difference from a randomly selected control policy. Though any particular difficulties do not arise in using the method for a network controlled by signals having unequal cycle times if we do not hate the increment of the calculation, for the above reasons we had better not give an independent cycle time at each intersection but adopt a half or third (or two times or three times) of some standard cycle time or divide the network into some subareas each of which has a roughly homogeneous traffic condition and use a common cycle time in each subarea which may be rather independently controlled.

The volume of the calculation is anticipated to amount huge especially for a large network because the method requires much iterative calculation as we illustrate in the calculation step. It also depends on the interval τ seeking the optimum offsets pattern. The interval should be one such as balanced with the irregular fluctuation of the traffic volume and so forth and so it is meaningless to take an unnecessarily fine interval. About ten seconds or twenty seconds may be sufficient.

We do not refer to the way of assuming x_1^N but there is a way as follows. That is, the lower limit is the sum of the minimum loss generated in each link when it is independently delt with and the upper limit may be the total loss of the traffic in the network which is controlled by random offsets pattern since x_1^N means the minimum total loss generated in a network when a certain set of offsets in the 0-th section is given and it is probably less than the loss corresponding to random offsets pattern. However, speaking in conclusion, we do not have to become nervous about the way of assuming x_1^N but may only add the difference between x_1^0 initially assumed and x_1^0 obtained as a result of the iterative calculation to assumed x_1^N since the important thing is whether the calculated $y_1^0, y_2^0, \cdots, y_M^0$ correspond with those initially assumed or not.

The most important problem left for the future is to examine the adaptiveness of the loss function which is the fundamental premise of the method. We are now doing work associated with the point through simulation technique and we will be very happy if we can obtained the lower limit of the traffic volume for which the assumed form of the loss function is adaptable.

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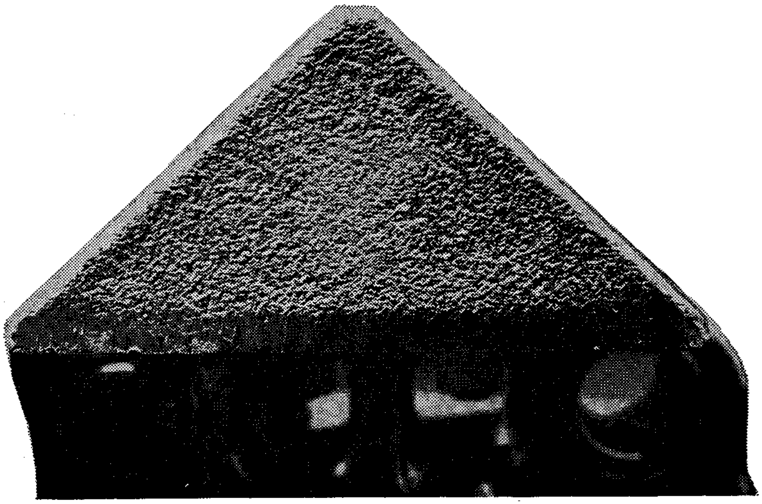
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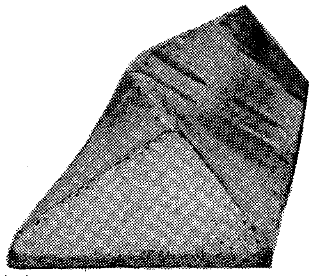
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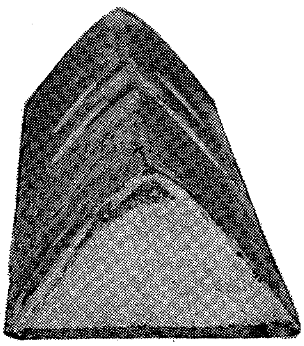
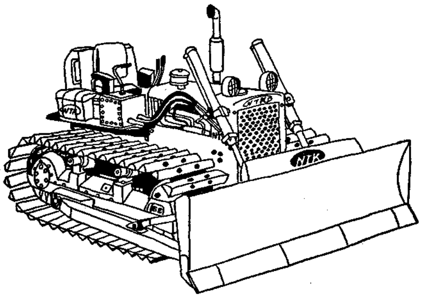

側面に丸味をつけた三角シュー

悪質地盤の処理に抜群の働きを示す湿地ブルドーザは、日本のユーザーにとって欠かせない機動力であることはご承知の通りですが、その普及の目覚ましさにつれて、あるメーカーが「日本No.1の技術が開発した…」といえば、一方は「世界No.1の技術が…」とうたう。しかし、一寸待って下さい。湿地ブルドーザの生命である三角シューは、北海道の泥炭地と取組んだ日特金属が発明したもの。それまでシューは、グローサしかなかったのです。日特の開発した三角シューは、〝土がつかず、土を乱さず、転圧力が大きく、傾斜地にも強い〟という画期的な性能を備えており、日本のユーザーの要望に完全に応えたもの。そして湿地研究から生まれた頑強な足廻り、理想の車体バランス等正統派ＮＴＫの湿地ブルドーザは断然ピカ一です。超々湿地ブルまで種類も豊富。湿地ブルについては専門家のＮＴＫにまずご相談下さい。

豊富な湿地シリーズ	接地圧	定格出力	総重量
NTK－6C 湿地	0.29kg／cm²	120ps	15,000kg
NTK－5 湿地	0.27kg／cm²	76ps	9,200kg
NTK－5 超湿地	0.19kg／cm²	76ps	10,300kg
NTK－5 超々湿地	0.13kg／cm²	76ps	10,000kg
NTK－4 湿地	0.25kg／cm²	61ps	8,100kg



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