Proc. of JSCE,
No. 172, 1969-12

RESIDUAL STRESS AND LOCAL BUCKLING STRENGTH OF STEEL COLUMNS

*Fumio Nishino**
*Lambert Tall***

ABSTRACT

This report presents the results of a study of the local buckling strength of steel columns. The effect of residual stress is given attention. The finite difference method was employed throughout the analysis and proved to be a suitable method for obtaining solutions for this type of problem. Numerical results are presented for plate buckling curves for plates with idealized residual stress distributions of various magnitudes. The boundary conditions of the plates are simply supported at the loading edges, and four combinations of free, simply supported and fixed at the unloaded edges. An illustrative result is also presented for the local buckling of a column cross section.

The theoretical results were correlated with experimental results of eight pilot tests of square welded columns of ASTM A7 and A514 steels.

I. INTRODUCTION

The strength of steel columns has been investigated to a great extend and variety by many investigators[1)~5)], who introduced residual stresses as the main factor influencing the buckling strength of centrally loaded columns. As a rule, most of the cross sections of steel columns consist of plate elements. It is possible, therefore, that even before instability of a column takes place, the component plates may buckle locally so that a premature failure of the entire column will occur characterized by a distortion of the cross section. The main purpose of this paper is the analysis of the local buckling strength of steel columns with residual stress.

The buckling load of plates is different from the ultimate load which the plates can carry, as opposed to the case of a column for which the buckling load has been found to be of a similar magnitude to the ultimate load for practical columns. Plates may be able to sustain the buckled state with ultimate loads considerably exceeding the buckling load. However, the difference between buckling and ultimate loads becomes significant only for very thin plates, which is not the case for plate elements of structural steel columns. Once buckling occurs in plate elements of columns, the stiffness for axial compression of the plates reduces, and this in turn reduces the bending rigidity of the column, possibly leading to overall failure of the column. Hence, the buckling load of plate

* Associate Professor, Structural Testing Laboratory, Engineering Research Inst. Faculty of Eng., Univ. of Tokyo; Formerly Research Instructor, Fritz Eng. Lab., Lehigh Univ.

** Associate Professor of Civil Engineering and Director, Division for Fatigue and Fracture, Fritz Eng. Laboratory, Lehigh University, Bethlehem, Pa., U.S.A.

elements or plate assemblies is more important as a guide for the design of column cross sections than in determining the ultimate load.

The development of the thory of elastic buckling of thin plates is reviewed in References 6 and 7. Attempts to extend the theory of plate stability into the inelastic range were made by many investigators. There are two main current trends in the development of inelastic buckling of plates, one based on Hencky's total strain or deformation theory and the other on Prandtl-Reuss' incremental theory. Bijlaard appears to have been the first to arrive at satisfactory theoretical solutions for inelastic buckling theories of plates[8),9),10)]. His work is the most comprehensive of all available including those which appeared later. He considered both the incremental and the total strain theories.

Although there have been many discussions, no definite conclusions as to the applicability of these theories of plasticity have yet been made[11)~15)]. The incremental theory appears logical since the loading history must play a role, in general. Test results have, however, shown that only the total strain theory gives good agreement[16)~19)]. In this study, both theories are used to determine the stiffness of the yielded portion of the plates.

The theoretical studies and the experiments for inelastic buckling have been advanced mainly for aluminum plates. A plastic buckling theory for steel plates was developed in Ref. 20. In that study, the four independent instantaneous flexure and shear moduli of an orthotropic plate were determined from the test results of the material under consideration.

The effect of residual stress on the elastic buckling strength of steel plates are studied and presented in Ref. 21, where it was shown that the residual stresses could influence the elastic buckling strength of a plate. The analysis was further developed into the inelastic range, where it was shown that the energy principle could be used to solve the inelastic buckling problem[18)]. An analytical solution was obtained in Ref. 18 for simply supported, fixed and elastically restrained plates at the unloaded edges, together with a numerical solution for simply supported plates with a particular distribution of residual stress.

II. ANALYSIS OF PLATE BUCKLING

2.1 Assumptions

In addition to the usual assumptions for the analysis of thin plates[6),7)] and to the assumption that no strain reversal takes place at the instant of buckling, the following specified conditions are implied in the analysis of this study:

Specified Conditions

(1) The thrust is at the two opposite edges of the plate element in the middle plane, where the plate is simply supported.

(2) The boundary conditions at the two edges where no loading is applied, are either free, simply supported or fixed if no plate intersects; the boundary at the intersection of component plates in column cross sections is considered as rigidly connected where the line of intersection remains straight.

(3) The plate thickness and material properties are constant in the same direction as the application of thrust.

(4) The residual stress is present only along the same directions as the

thrust and its magnitude is constant in that direction.

(5) The wave length of buckling is identical on each plate element in a buckled plate assembly, and there is no phase lag between plates.

Further, the following assumptions are made in obtaining numerical solutions:

(1) The stress-strain relationship of uniaxially loaded steel is elastic-perfectly-plastic.

(2) The residual stress distribution is symmetric if any symmetric axis is present.

(3) The residual stress varies linearly inside the mesh cell.

The coordinate systems are as follows: The coordinate x is perpendicular to the middle plane of the plate, y is normal to the thrust in the middle plane and z is the coordinate parallel to the thrust and to the residual stress.

2.2 Stress-Strain Relationship and Basic Differential Equation

Denoting the normal stress components in the z and y direction by σ_z and σ_y and shear stress by τ, for buckling from a state of uniaxial compression ($\sigma_z \neq 0$, $\sigma_y=0$ and $\tau=0$) and with no strain reversal the stress-strain relationship by Bijlaard[8] is given by the following equations

$$\left.\begin{aligned} \delta\sigma_z &= E(k_1\delta\varepsilon_z + k_2\delta\varepsilon_y) \\ \delta\sigma_y &= E(k_2\delta\varepsilon_z + k_3\delta\varepsilon_y) \\ \delta\tau &= Ek_4\delta_r \end{aligned}\right\} \quad (1)$$

where k_1 to k_4 are defined as follows by Poisson's ratio ν, tangent modulus E_t on the stress-strain curve of the compression coupon at the stress intensity of σ_z, and by the secant modulus E_s, which is shown in Fig. 1.

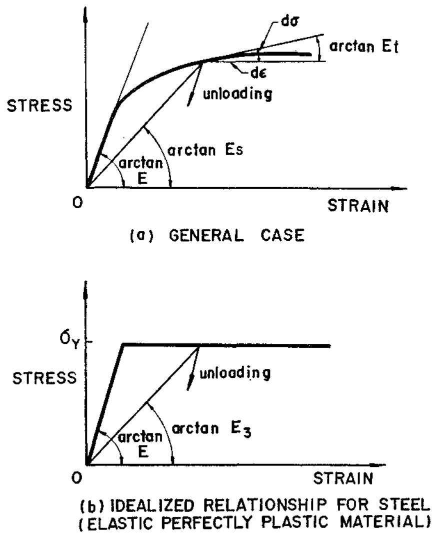


Fig. 1 Stress-strain relationship

$$\left.\begin{aligned} k_1 &= \frac{1+3\left(\dfrac{E_t}{E_s}\right)}{(5-4\nu+3e)-(1-2\nu)^2\left(\dfrac{E_t}{E}\right)}, \quad k_2 = \frac{2-2(1-2\nu)\left(\dfrac{E_t}{E}\right)}{(5-4\nu+3e)-(1-2\nu)^2\left(\dfrac{E_t}{E}\right)}, \\ k_3 &= \frac{4}{(5-4\nu+3e)-(1-2\nu)^2\left(\dfrac{E_t}{E}\right)}, \quad k_4 = \frac{1}{2+2\nu+3e} \\ e &= \frac{E}{E_s} - 1 \end{aligned}\right\} \quad (2)$$

E is the modulus of elasticity, $\delta\sigma_z$, $\delta\sigma_y$, and $\delta\varepsilon_z$, $\delta\varepsilon_y$ are the stresses and strains due to the buckling of the plate in the z- and y-directions, respectively. Substitution of $E_s=E_t=E$ and $e=0$ into Eq. (2) changes the relationship of Eq. (1) to that in the elastic domain.

A loss of strain in the strain history, which is the main assumption of the incremental theory, amounts only to a neglect of the initial plastic deformation at the instant of buckling, which is taken into account by equating e, as defined by Eq. (2) (which is equal to the ratio of plastic strain to elastic strain, $\varepsilon_p/\varepsilon_e$) to zero. Equation (2) then becomes identical with the ones derived by Handelman and Prager[22].

With the stress-strain relationship, Bijlaard derived the following basic differential equation of equilibrium of an element in a slightly bent plate[8),9),10)].

$$E\left[\frac{\partial^2}{\partial z^2}\left(Ik_1\frac{\partial^2 w}{\partial z^2}+Ik_2\frac{\partial^2 w}{\partial y^2}\right)+4\frac{\partial^2}{\partial z\,\partial y}\left(Ik_4\frac{\partial^2 w}{\partial z\,\partial y}\right)+\frac{\partial}{\partial y^2}\left(Ik_2\frac{\partial^2 w}{\partial z^2}+Ik_3\frac{\partial^2 w}{\partial y^2}\right)\right]+t\sigma_z\frac{\partial^2 w}{\partial z^2}=0 \tag{3}$$

The equation is applicable both in the elastic and in the inelastic domain of the plate.

2.3 General Approach

In the above plate buckling equation, Eq. (3), the stress σ_z is a function of the residual strain distribution and the strain distribution due to thrust. By assumption, both of these strains change their intensities in the direction of the width of the plate, and consequently, the stress intensity is a function of the coordinate y. Since k_1 through k_4 are functions of strain intensities, they are also functions of the coordinate y.

The distribution of residual strain varies considerably as presented in Ref. 3, and 23 through 27. Hence, idealization could be made in several ways for analysis, such as a triangular or parabolic distribution or a combination of broken straight lines. It would be evident, therefore, that a rigorous solution of the present problem is a quite difficult task.

Several approximate methods to obtain an eigenvalue have been developed. Among the methods, the finite difference method with the help of a high speed computer affords a powerful tool for the solution of the many problems involving ordinary differential equations.

In the subsequent analysis, the deflected shape of the plate is assumed by the following product function which satisfies the boundary conditions at the loading edges

$$w=Y\sin\frac{\pi}{L}z \tag{4}$$

where Y is a function of the coordinate y alone. This assumed shape is known to be the exact deflected shape for an elastic plate free of residual stress and the shape has been presumed as satisfactory by many investigators[8),16),22)] for the plastic buckling of a plate. Substituting Eq. (4) into Eq. (3), the basic differential equation can be shown finally in the following form, where the equation is divided by a constant I_0

$$\frac{d^2}{dy^2}\left(\frac{I}{I_0}k_3\frac{d^2Y}{dy^2}-\frac{\pi^2}{L^2}\frac{I}{I_0}k^2Y\right)-4\frac{\pi^2}{L^2}\frac{d}{dy}\left(\frac{I}{I_0}k_4\frac{dY}{dy}\right)-\frac{\pi^2}{L^2}\frac{I}{I_0}k_2\frac{d^2Y}{dy^2}+\frac{\pi^2}{L^2}\left(\frac{\pi^2}{L^2}\frac{I}{I_0}k_1-\frac{t\sigma_z}{EI_0}\right)Y=0 \tag{5}$$

2.4 Differential Equation and Boundary Conditions in Finite Differences

The basic difference equation for an evenly spaced mesh is derived as follows by replacing the derivatives of Eq. (5) by the corresponding central difference quotients[28].

$$C_{1,i}Y_{i+2}+C_{2,i}Y_{i+1}+(C_{3,i}-\lambda^2 C_i)Y_i+C_{4,i}Y_{i-1}+C_{5,i}Y_{i-2}=0 \qquad (6)$$

where subscript i of the coefficients C shows that it is for the equation at mesh point i and subscript to the function Y denotes the deflections at the points. λ is the non-dimentionalized width-thickness ratio defined as:

$$\lambda=\frac{b}{t}\sqrt{\frac{\sigma_Y}{E}} \qquad (7)$$

The coefficients are defined as:

$$\left.\begin{aligned}
C_{1,i}&=\frac{I_{i+1}}{I_0}k_{3,i+1}\\
C_{2,i}&=-2\left(\frac{I_{i+1}}{I_0}k_{3,i+1}+\frac{I_i}{I_0}k_{3,i}\right)-\frac{\pi^2r^2}{L^2}\left(\frac{I_{i+1}}{I_0}k_{2,i+1}+\frac{I_i}{I_0}k_{2,i}+4\frac{I_{i+1/2}}{I_0}k_{4,i+1/2}\right)\\
C_{3,i}&=\frac{I_{i+1}}{I_0}k_{3,i+1}+4\frac{I_i}{I_0}k_{3,i}+\frac{I_{i-1}}{I_0}k_{3,i-1}\\
&\quad+\frac{\pi^2r^2}{L^2}\left(4\frac{I_i}{I_0}k_{2,i}+4\frac{I_{i+1/2}}{I_0}k_{4,i+1/2}+4\frac{I_{i-1/2}}{I_0}k_{4,i-1/2}+\frac{\pi^2r^2}{L^2}k_{1,i}\right)\\
C_{4,i}&=-2\left(\frac{I_{i-1}}{I_0}k_{3,i-1}+\frac{I_i}{I_0}k_{3,i}\right)-\frac{\pi^2r^2}{L^2}\left(\frac{I_{i-1}}{I_0}k_{2,i-1}+\frac{I_i}{I_0}k_{2,i}+4\frac{I_{i-1/2}}{I_0}k_{4,i-1/2}\right)\\
C_{5,i}&=\frac{I_{i-1}}{I_0}k_{3,i-1}\\
C_{6,i}&=\frac{\pi^2r^2}{L^2}\frac{t\sigma_{z,i}}{EI_0}r^2
\end{aligned}\right\} \qquad (8)$$

where r is the width of the evenly spaced mesh cells.

When the spacing is not equal, a similar equation is easily obtained with coefficients slightly different from Eq. (8) at mesh points, $i+1$, i and $i-1$.

The boundary conditions for fixed, simply supported and free ends are obtained similarly by replacing the expressions of boundary conditions by finite difference quotients.

Since it has been shown that, to end the plate at the middle of mesh points results in higher accuracy for the same number of mesh cells[29),30)], only the boundaries for a half-integer station $i+1/2$ are shown. It is assumed that the plate thickness is constant near the boundaries.

(1) *Fixed*

$$Y_i=Y_{i+1}=0$$

(2) *Simply supported*

$$Y_{i+1}=-Y_i\,,\qquad Y_{i+2}=-Y_{i-2}$$

(3) *Free*

$$Y_{i+1}=\frac{-\left(k_{3,i+1}\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}+k_{2,i+1}-4k_{4,i+1/2}\right)Y_i+2\dfrac{L^2}{\pi^2r^2}k_{3,i+1/2}Y_{i-1}}{k_{3,i+1}\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}-k_{2,i+1}-4k_{4,i+1/2}}$$

$$Y_{i+2}=\left(1+\frac{k_{2,i+1/2}}{k_{3,i+1/2}}\frac{\pi^2r^2}{L^2}\right)\left(1-\frac{k_{3,i+1}\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}+k_{2,i+1}+4k_{4,i+1/2}}{k_{3,i+1}\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}-k_{2,i+1}-4k_{4,i+1/2}}\right)Y_i$$
$$-\left[1+\frac{2k_{3,i+1/2}\left(1+\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}\right)}{k_{3,i+1}\dfrac{k_{2,i+1/2}}{k_{3,i+1/2}}-k_{2,i+1}-4k_{4,i+1/2}}\right]Y_{i-1}$$

The boundary conditions at the intersection of plates, the compatibility of slope and the equilibrium of moment, which are necessary to obtain the buckling strength of a cross section can be obtained in finite difference forms by replacing the derivatives in their expressions. However, instead of simply transferring the conditions for the differential equation into the difference forms, a different approach may be made. Consider a folded plate as if it is a continuous plate simply supported at the intersection. Then the whole plate can be solved as a single plate with such internal restrains that the deflections are zero at the intersections. The zero deflection implies that the lateral force is in equilibrium with the reaction at the point, regardless of its value. The basic difference equation at a mesh point is the equilibrium equation of lateral force; zero deflection, in turn, implies that no difference equation is to be considered at the point.

When tee- and H-sections are considered, three plates intersect at a point. In this case, half of a flange is considered, of which the flexural rigidity and the thickness are twice as large as actual, because of the symmetry of the shape. Then, the analysis is the same as for an intersection where only two plates intersect.

2.5 Averaging

Since the coefficients of the basic differential equation are not constant, new assumptions must be made concerning their averaging.

It is assumed that the eigenvalue of the set of basic difference equations, Eq. (6), is proportional to coefficient, C_i, and proportional to the reciprocal of the other coefficients.

Based on the above assumption, the averaging of the stress, and material and geometric properties is accomplished as follows:

$$\left.\begin{aligned}&\sigma_{zi}=\frac{1}{\Delta y_i}\int_0^{\Delta y_i}\sigma_z dy\,,\qquad \frac{1}{I_i}=\frac{1}{\Delta y_i}\int_0^{\Delta y_i}\frac{1}{I}dy\,,\\&\frac{1}{k_{j,i}}=\frac{1}{\Delta y_i}\int_0^{\Delta y_i}\frac{1}{k_j}dy\qquad (j=1,2,3\text{ and }4)\end{aligned}\right\}\qquad(9)$$

This averaging method for stress intensity and flexural rigidity follows from their physical meaning. The averaging method of the k's is not straight-forward, however; they are constant in the elastic range and even in the inelastic range where they are variable, no significant difference of numerical value is possible from their definition. Thus, the method of averaging for these variables may not be important. The assumption can be justified from the fact that it is made in order to obtain accurate results as far as possible with the smallest amount of labor, and from the fact that, if a closer mesh is used, the averaging method will loose its significance for this buckling problem.

2.6 Procedure of Numerical Computation

For the buckling analysis of a partially yielded plate, the distribution of stress and the stiffness of the material are functions of the loading and of the residual stress distribution so that it is easier to solve for a critical width-thickness ratio under a known loading rather than for a critical load.

The geometric shape of a plate or a plate assembly must be fixed, for which the analysis will be made. Then, specifying the number of mesh points and giving the magnitude of residual strains, and strains due to thrust at the edges of the mesh cells, the concentrated stress intensity may be computed at each mesh point, as well as the average moduli of the plates (k_1 through k_4), and rigidity, I, at each mesh cell. With these coefficients, Eq. (6) together with suitable boundary conditions forms homogeneous simultaneous equations of the following matrix form

$$AX=\lambda^2 BX \tag{10}$$

where A and B are square matrices and X is an eigenvector. The square of the non-dimensionalized width-thickness ratio, λ^2 is the eigenvalue of the equation, among which only the smallest root is needed for the analysis of the buckling problem. No particular difficulty was encountered in determining the smallest eigenvalue of the above matrix equation employing the iterative method[31),32)].

III. NUMERICAL RESUTTS

3.1 Residual Stress Distribution

Idealized patterns of residual stress distributions are considered, as shown in Fig. 2. The triangular distributions as shown in Fig. 2a and 2b resemble the patterns found in the flange and web of rolled wide-flange shapes in which the magnitudes of the maximum compressive and tensile residual stresses, σ_{rc} and σ_{rt}, respectively, are assumed to be

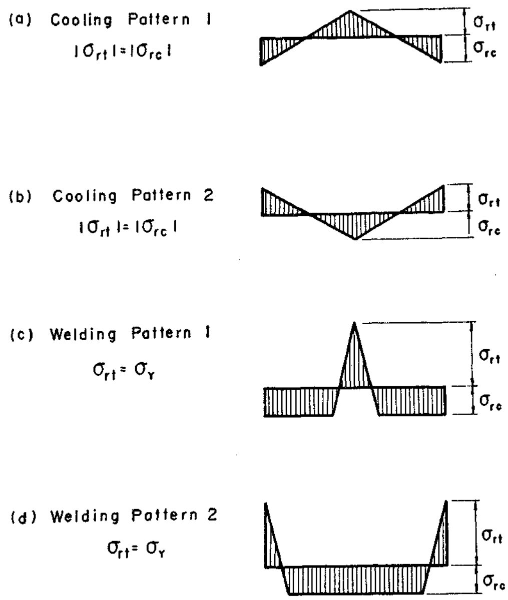


Fig. 2 Idealized residual stress distribution

the same[3),27)]. The patterns shown in Figs. 2c and 2d resemble the patterns found in welded built-up shapes[24),26)]. The tensile residual stress of the weld is assumed to be equal to the yield stress of the parent material at the weld[23),25)].

3.2 Error

An error is inherent in finite difference solutions when the method is used as an approximate method for problems governed by a differential equation. In an engineering problem, it is not essential to have a solution of great accuracy. However, it is necessary to have some idea of the error and the number of cells needed to assure a certain accuracy.

The actual solutions of a finite difference equation for a plate simply supported at all four edges, free of residual stress and with an aspect ratio of 1 are compared with exact solutions and the errors involved are plotted in Fig. 3. Both mesh systems, one with edges on integer stations and the other with edges on half-integer stations, resulted in the identical error for the same number of cells. The similar results on a fixed plate at the unloaded edges, however, show a difference, as seen in Fig. 4. The half-integer system showed the better accuracy. The relationship between error and number of cells of Figs. 3 and 4 were also obtained analytically and presented separately[29)].

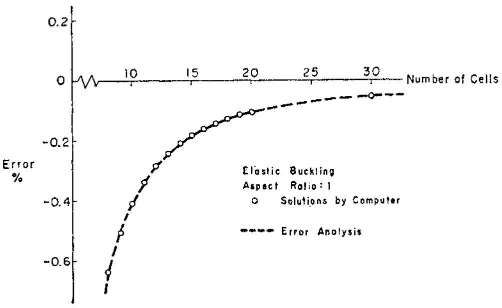


Fig. 3 Error vs. number of cells (S.S. plates at unloaded edges)

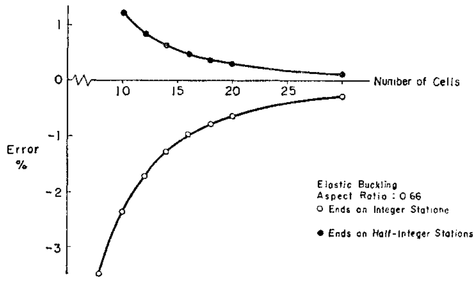


Fig. 4 Error vs. number of cells (fixed plates at unloaded edges)

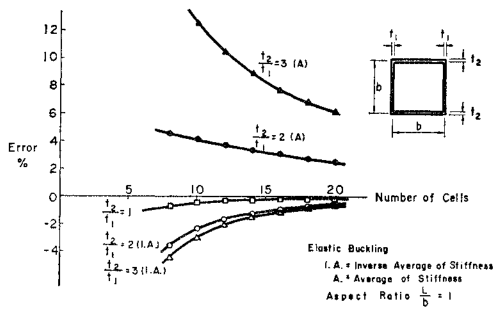


Fig. 5 Difference in error due to different averaging methods of moment of inertia of plates

Fig. 5 shows the errors present in the finite difference solution for the local buckling of square box columns with two pairs of plates with different thicknesses. A mesh system is selected such that the corner of the section is on an integer station, then the stiffnesses of the plates are different at both sides of the point. Two different methods of averaging are considered: one to take the average of inverse stiffnesses defined by the second equation of Eq. (9) and the other is simply to take the average of the stiffnesses. The difference is marked, when a finer mesh is used. The inverse average method shown by open points approaches the exact solution rapidly. Somewhat generalized and detailed analysis and discussions of the errors of this particular problem are also given in Ref. 29.

In the subsequent numerical solution, the number of cells was selected such that the errors in the numerical results would not exceed two percent in any case; in most cases they were less than one percent.

3.3 Aspect Ratio and Critical Width-Thickness Ratio Relationship

It is important for the analysis of local buckling to find the minimum critical width-thickness ratio which is obtained when a plate of a particular aspect ratio is analyzed. Figure 6 shows the variation of the width-thickness ratio for the change of the aspect ratio of a buckling plate simply supported at the un-loaded edges. The loading edges are assumed simply supported. Figure 6a is for a plate without residual stress, in which the minimum width-thickness ratio occurs at the aspect ratio of 1.0 for elastic buckling and at the aspect ratio of 0.7 for plastic buckling. A sudden jump of the aspect ratio, at which the width-thickness ratio is a minimum, is noted between the elastic buckling and plastic buckling. This is because of an abrupt change of material properties due to yielding of the material. Similar relationships for the plates with residual stress distribution of welding type and cooling type are shown in Figs. 6b and 6c, respectively. The relationships for elastic buckling and plastic buckling are similar to those of plates free of residual stress. The sudden jump of the aspect ratio is observed between elastic buckling and elastic-plastic bucklng of a plate with welding type residual stresses; then the aspect ratio changes gradually with the increase of critical strain and approaches the value for plastic buckling. Since no abrupt yielding takes place in a plate with cooling type residual stresses, the change of the aspect ratio for the minimum critical width-thickness ratio is gradual from the value of elastic buckling to that of plastic buckling with increase of the critical strains. The relationships between the critical width-thickness ratio and the aspect ratio for plates fixed at both of the unloaded edges and for plates fixed and free are similar to the relationship for simply supported plates. A plate simply supported and free at the unloaded edges does not have a minimum critical width-thickness ratio; instead, there is an asymptote to a limiting value with increase of the aspect ratio of the buckling plate, as shown in Fig. 7.

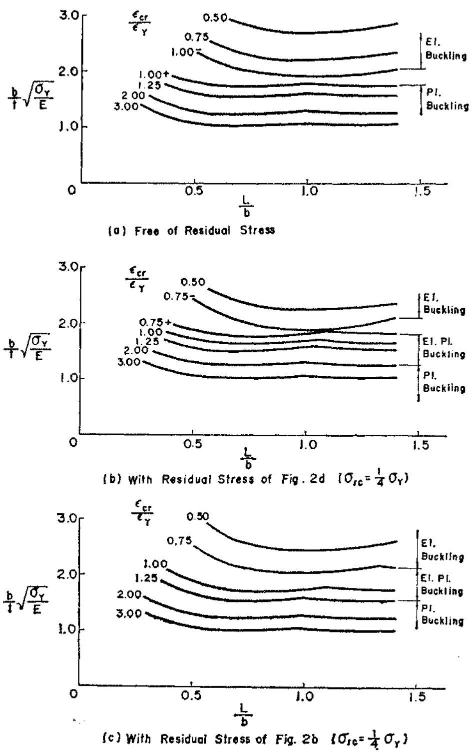


Fig. 6 Width-thickness ratio and aspect ratio relationship (plates simply supported at unloaded edges)

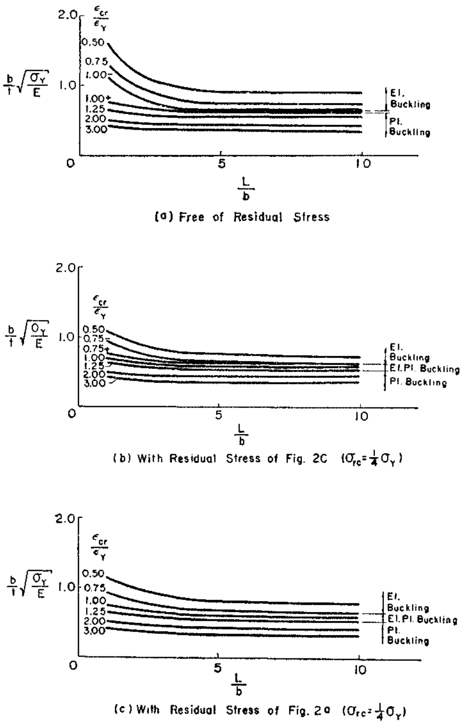


Fig. 7 Width-thickness and aspect ratio relationship (plates simply supported at unloaded edges)

It should be noted that the above figures are results based on the total strain theory of plasticity. The incremental theory of plasticity results in a relationship similar to these figures for elastic-plastic buckling and in an identical relationship for plastic buckling with the plates free of residual stress and buckling at the completion of full yielding.

3.4 Plate Buckling Curve

The relationship between the critical stress and the minimum critical width-thickness ratio was computed for plates with residual stresses. The residual stress patterns of Figs. 2b and 2d are assumed for plates both simply supported and fixed at the unloaded edges so as to resemble the component plates of box-section and web plates for wide-flange and channel-sections. Half of the residual stress patterns of Figs. 2a and 2c is assumed for plates free at one unloaded edge and fixed on simply supported at the other edge so as to resemble outstanding flanges of column cross sections. The complete results are shown in Figs. 8 through 11, where the figures are for the ratio of average critical stress to the static yield stress versus the non-dimensionalized width-thickness ratio, defined as follows.

$$\Lambda=\frac{b}{t}\sqrt{\frac{\sigma_Y}{E}\frac{12(1-\nu^2)}{k\pi^2}} \tag{11}$$

where k is the so-called plate buckling coefficient[7]. The results for elastic-plastic buckling are based on the total strain theory of plasticity unless otherwise noted.

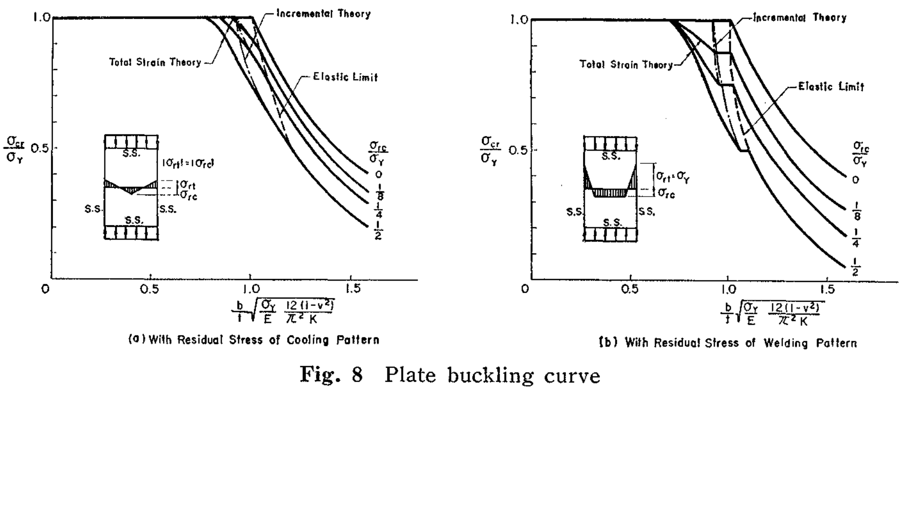


Fig. 8 Plate buckling curve

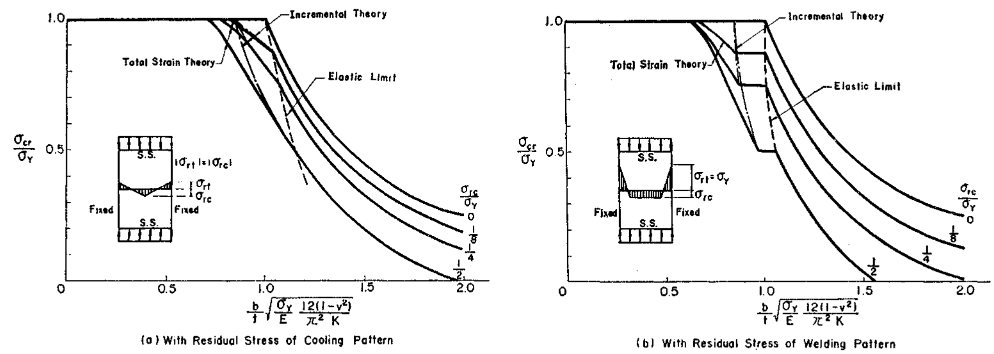


Fig. 9 Plate buckling curve

The assumed residual stress patterns reduce the buckling strength in all cases considered. It is noted that the figures show clearly the effect of residual stress on elastic buckling. The reduction in the elastic buckling strength depends largely on the pattern of the distribution of residual stress and its magnitude. Comparing curves (a) and (b) of Figs. 8 through 11, it is seen that the effect of residual stress is more severe for plates with welding type residual stress than for those with cooling type residual stress and that a larger reduction of buckling strength results for a plate with a larger magnitude of residual stress. The magnitude of the reduction is rather constant for a residual stress pattern re-

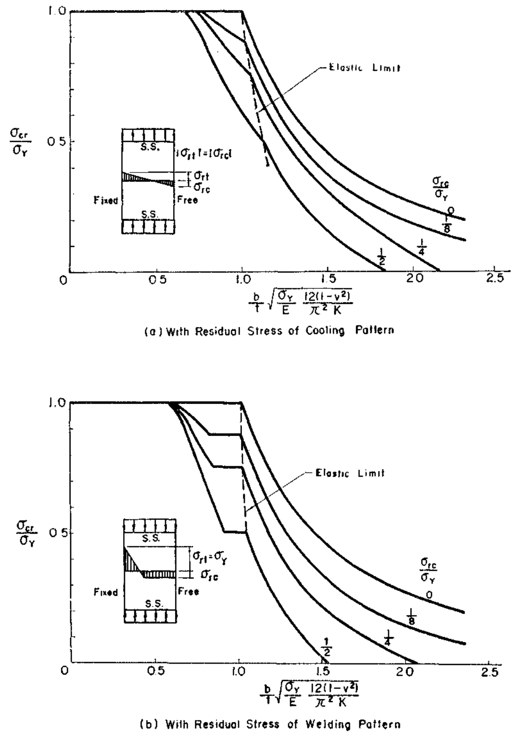


Fig. 10 Plate buckling curve

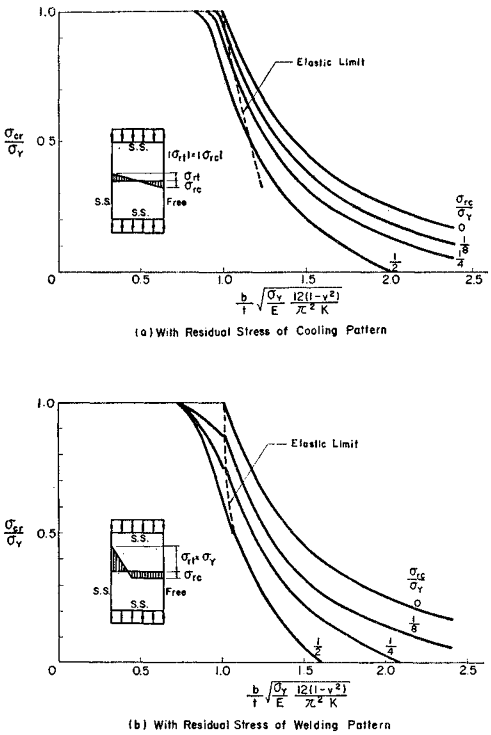


Fig. 11 Plate buckling curve

gardless of the width-thickness ratio. The plate buckling curve for a plate free of residual stress is a hyperbola and does not intersect the abscissa; however, due to the rather constant reduction of elastic buckling strength independent of the width-thickness ratio, the plate buckling curves with residual stress do intersect the abscissa. The intersection shows that buckling of a plate without any external loading is possible, and explains the fact that a plate can distort due to welding alone.

Although in elastic-plastic buckling a larger reduction results due to a pattern of residual stress with a larger magnitude, the reduction depends more on the width-thickness ratio than on the magnitude. The jump discontinuity of the plate buckling curves for plates with the welding type residual stress is due to the penetration of yielding over a large portion of area at the same instant.

A critical value of width-thickness ratio exists in all cases considered; plates with width-thickness ratio less than this critical value sustain the full yield loads. The critical value depends on the magnitude of residual stress for the assumed residual stress distribution of the cooling type, whereas it is constant for practical purposes for the assumed residual stress patterns of the welding type. For plates with welding type residual stress, the critical values vary from 0.55 to 0.7 depending on the conditions at the two unloaded edges; 0.55 for a plate fixed and free, 0.6 fixed at both edges and 0.7 simply supported and free. The current Japanese specification for steel highway bridges[33] and AISC specification[34] assumes the plate buckling curves to reach the yield line at the width-thickness ratio of 0.7, when the abscissa is plotted for the non-dimensionalized width-thickness ratio as defined in Eq. (11). The value of 0.7 is slightly larger than the above values with welding type residual stresses. For buckling stresses of 90, 95 and 100 percent of yield stress, the non-dimensionalized width-thickness ratios are plotted against the maximum magnitude of compressive residual stress in Figs. 12 through 15. Since the realistic magnitude of residual stresses when expressed in terms of σ_{rc}/σ_Y as shown in Fig. 2 are at most 0.3 and 0.5 for rolled shapes and welded built-up shapes, respectively[3),23)~27)], the Figs. 12 through 15 show that the values of 0.7 is a good estimation for a hot-rolled column to sustain its full strength without any local failure; the value may not be large enough for a welded column, so that a welded column may fail locally when the external load reaches close to the yield load of the cross section. The above discus-

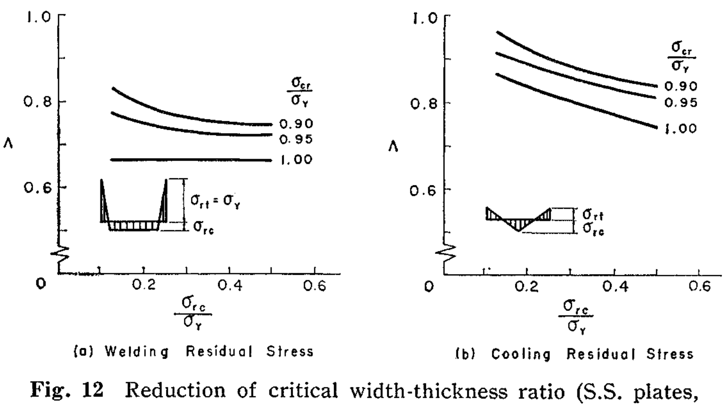
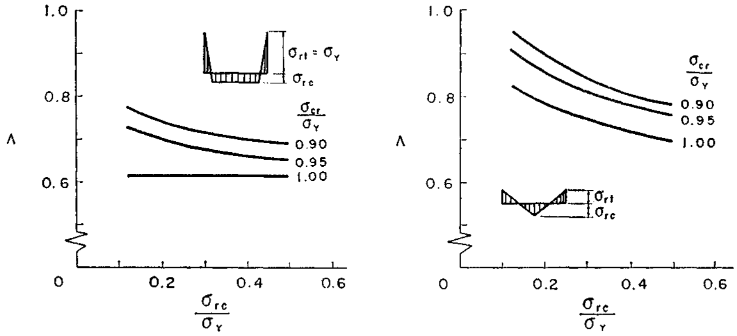
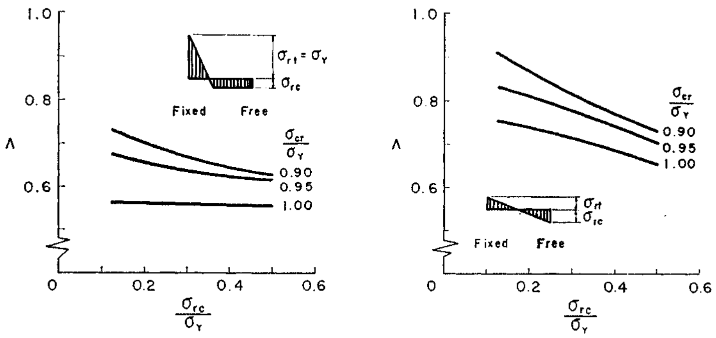


Fig. 12 Reduction of critical width-thickness ratio (S.S. plates, total strain theory)



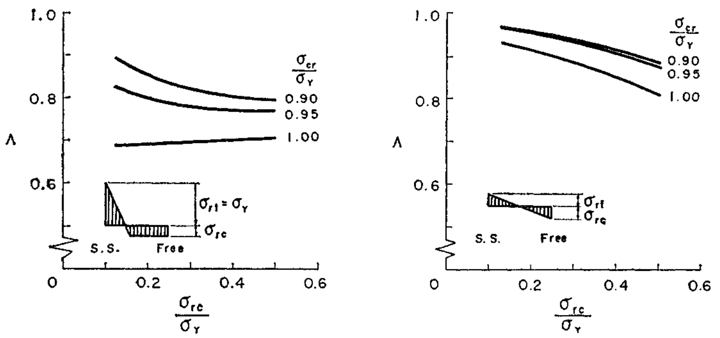
(a) Welding Residual Stress (b) Cooling Residual Stress

Fig. 13 Reduction of critical width-thickness ratio (fixed plates at unloaded edges, total strain theory)



(a) Welding Residual Stress (b) Cooling Residual Stress

Fig. 14 Reduction of critical width-thickness ratio (plates fixed and free at unloaded edges, total strain theory)



(a) Welding Residual Stress (b) Cooling Residual Stress

Fig. 15 Reduction of critical width-thickness ratio (plates S.S. and free at unloaded edges, total strain theory) sion is based only on the buckling analysis of plates with residual stress, while there are other factors such as the out-of-flatness and post-buckling strength to be considered in obtaining a definite conclusion for the values of width-thickness ratio to be used in the design of a steel column. However it can be said from the study of this paper that the introduction of residual stress as a main factor influencing the buckling strength explains the transition parts of the plate buckling curves and substantiates to a certain extent the reduction of 0.7 of width-thickness ratio from the value where the elastic buckling intersects the yield stress as implied in the current design specifications.

3.5 Numerical Results for Column Sections

Employing the finite difference method, the local buckling strengths of column cross sections were obtained with the same simplicity as for plate buckling; the only difference is increase in number of mesh cells for the same accuracy.

Numerical results of the local buckling analysis on column cross sections can be shown in a form similar to the plate buckling curve. However, the fact that there are so many factors, such as geometric shape and residual stress distribution, makes it quite difficult to prepare curves which cover a wide variety of column cross sections.

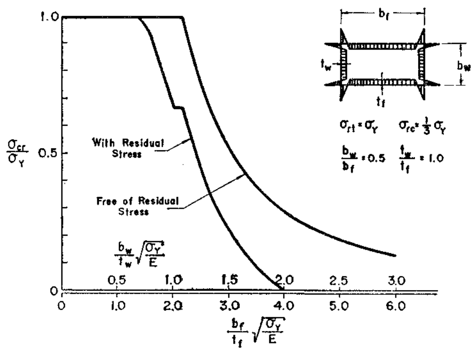


Fig. 16 Local buckling curve

Instead, only one result is presented here in Fig. 16 to illustrate the effect of residual stress for a box cross section with the residual stresses as shown in the figure. It should be mentioned that the reduction of buckling strength due to the presence of residual stress is similar to that found for buckling of plates with residual stresses.

IV. COMPARISON WITH TEST RESULTS

A series of short welded square box-columns of ASTM A7 and T-1* steels were tested. The section was selected to simulate the plates simply supported at the unloaded edges. The width-thickness ratios of the specimens were selected for each steel such that the critical loads were reached in both the elastic range and in the elastic-plastic range. Two identical specimens were cut from a long fabricated piece for each shape, thus a total number of eight specimens were tested. Table 1 shows the detail of the specimens.

The experiments and test results have been reported in detail in a separate paper[35], and only the results are presented here to correlate with the theory.

The measured residual stress distribution closely resembled the pattern assumed for numerical computation and shown in Fig. 17. The ratios between the average compressive residual stress to the yield stress, σ_{rc}/σ_Y are given in Table 1. It is noted that the ratios are smaller in T-1 shapes than those in A7 specimens. All of the test results are shown in Fig. 17, where the theoretical curves

* T-1 constructional alloy steel meets the requirements of ASTM A514 and/or A517 steel.

Table 1 Detail of Specimens

Materials	Specimen No.	b (in)*	t (in)*	b/t	$\frac{t}{b}\sqrt{\frac{\sigma_Y}{E}\frac{12(1-\nu^2)}{\pi^2 k}}$	σ_{rc}/σ_Y
ASTM A-7	S- 1	11.4	0.256	44.5	0.86	0.32
	S-11	11.5	0.256	45.0	0.87	0.32
	S- 2	16.2	0.253	64.0	1.20	0.27
	S-21	16.3	0.254	64.2	1.21	0.27
USS "T-1"	T-1A	11.3	0.256	44.0	1.37	0.10
	T-1B	11.2	0.256	44.0	1.37	0.10
	T-2A	6.77	0.258	26.2	0.85	0.15
	T-2B	6.77	0.258	26.2	0.85	0.15

* Average value of four faces.

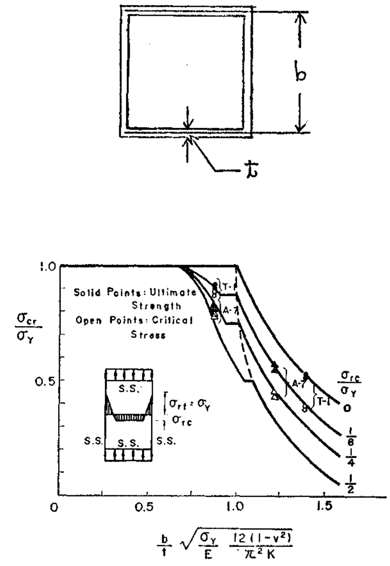


Fig. 17 Local buckling tests of weld- ed square tubes

are identical with Fig. 8b. All four specimens, S-2, S-21, T-1A and T-1B, which buckled in the elastic region, showed good agreement with the prediction with a slightly lower buckling stress. Two theoretical predictions were made for the rest of the specimens which buckled after partial yielding penetrated into the cross section; one is based on the incremental theory predicting no buckling until the yield load is reached as shown by the dotted line and the other based on the total strain theory with results as drawn by solid lines in Fig. 17. Although both predictions were higher than the test results, the difference is very small and a good correlation exists between the test results and the prediction based on the total strain theory as seen in Fig. 17. The test points in Fig. 17 clearly show the difference in the strength of A7 and T-1 steel specimens; the test results for A7 specimens locate below the results for T-1 specimens. This fact can be explained from the theoretical study of this paper together with the difference in σ_{rc}/σ_Y ratios present in test specimens as mentioned earlier.

The ultimate strength of the buckled plates are also shown in Fig. 17 for comparison, although this was not considered in the theoretical study. All four specimens buckled in the elastic range showed a significant post-buckling strength, while the specimens which buckled in the elastic-plastic range had a relatively small reserve of post-buckling strength.

V. SUMMARY AND CONCLUSIONS

This report has presented the local buckling strength of columns containing residual stresses and loaded into the inelastic range of the material. Solutions are obtained on the basis of the finite difference approximation of a differential equation.

The numerical results for plates of various edge conditions are presented in plate buckling curves of non-dimensionalized stress against non-dimensionalized width-thickness ratio.

Theoretical results on plates simply supported at all four edges were correlated with experimental results.

The following conclusions may be drawn from the study of this report:

(1) The finite difference approximation of a differential equation was found to be effective in obtaining the eigenvalue of the basic differential equation governing plate buckling.

(2) Residual stresses present in a column cross section influence the local buckling strength, even in elastic range.

(3) The elastic buckling strength depends largely on the magnitude and distribution of residual stresses.

(4) The effect of residual stresses on the elastic-plastic buckling depends greatly on the width-thickness ratio of the plates.

(5) A critical value of width-thickness ratio exists; plates with width-thickness ratio less than this critical value sustain the full yielding load.

(6) Comparison with the tests shows a correlation between the theoretical results and the test results; for elastic-plastic buckling, the theoretical results based on the total strain theory gives a good correlation with the experimental results, but the results based on the incremental theory predict a much higher critical stress.

VI. ACKNOWLEDGEMENTS

This study has been carried out as a part of the research project on "Welded Built-Up and Rolled Heat Treated T-1 Steel Columns" conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania.

The project was sponsored by the United States Steel Corporation. Charles G. Shilling of that company made many valuable comments, both in the course of the program and in the preparation of this report. Task Group 1 of the Column Research Council acted in an advisory capacity under the chairmanship of John A. Gilligan.

The authors wish to expess their appreciation to the Computer laboratory, Lehigh University for help in programming.

REFERENCES

1) Yang, C. H., Beedle, L. S. and Johnson, B. G.: "Residual Stress and the Yield Strength of Steel Beams," The Welding Journal, Research Supplement, Vol. 31, April 1952.

2) Huber, A. W. and Beedle, L. S.: "Residual Stress and the Compressive Strength of Steel," The Welding Journal, Research Supplement, Vol. 33, No. 12, December 1954.

3) Beedle, L. S. and Tall, L.: "Basic Column Strength," ASCE Proc. Paper 2555, Vol. 86, ST7, July 1960.

4) Fujita, Y.: "Influence of Residual Stresses on the Instability Problems," Journal Zosen Kyokai, Japan, Vol. 107, July 1960. (Japanese with English Abstract).

5) Estuar, F. R. and Tall, L.: "Experimental Investigation of Welded Built-up Columns," The Welding Journal, Vol. 42, April 1963.
6) Bleich, F.: "Buckling Strength of Metal Structures," McGraw-Hill, New York, 1952.
7) Timoshenko, S. and Gere, J. M.: "Theory of Elastic Stability," 2nd Ed., McGraw-Hill, New York, 1961.
8) Bijlaard, P. P.: "Theory of the Plastic Stability of Thin Plates," Vol. 6, Publications, IABSE, Zurich, 1940-41.
9) Bijlaard, P. P.: "Some Contributions to the Theory of Elastic and Plastic Stability," Vol. 8, Publications, IABSE, Zurich, 1947.
10) Bijlaard, P. P.: "Theory and Tests on the Plastic Stability of Plates and Shells," Jour. of Aero. Sci., Vol. 16, No. 9, September 1949.
11) Drucker, D. C.: "A Discussion of the Theories of Plasticity," Jour. of Aero. Sci., Vol. 16, No. 9, September 1949.
12) Bijlaard, P. P.: "On the Plastic Buckling of Plates According to the Flow Theory," Jour. of Aero. Sci., Vol. 17, No. 12, December 1950.
13) Onat, E. T. and Drucker, D. C.: "Inelastic Instability and Incremental Theories of Plasticity," Jour. of Aero. Sci., Vol. 20, No. 3, March 1953.
14) Bijlaard, P. P.: "On Theories of Plasticity and the Plastic Stability of Cruciform Sections," Jour. of Aero. Sci., Vol. 20, No. 11, November 1963.
15) Bijlaard, P. P.: "Discussion to "Inelastic Buckling in Steel" by Haaijer and Thurlimann, Trans. ASCE, Vol. 125, Part I, 1960.
16) Stowell, E. Z.: "A Unified Theory of Plastic Buckling of Columns and Plates," NACA Rep. 898, 1948.
17) Pride, R. A. and Heimerl, G. J.: "Plastic Buckling of Simply Supported Compressed Plates," NACA TN 1817, 1949.
18) Ueda, Y. and Tall, L.: "Inelastic Buckling of Plates with Residual Stresses," Vol. 27, Publications, IABSE, Zurich, 1967.
19) Gerard, G. and Becker, H.: "Handbook of Structural Stability, Part I," NACA TN 3781, 1957.
20) Haaijer, G. and Thurlimann, B.: "Inelastic Buckling in Steel," Trans. ASCE, Vol. 125, Part I, 1960.
21) Yoshiki, M., Fujita, Y. and Kawai, T.: "Influence of Residual Stresses on the Buckling of Plates," Jour. Zosen Kyokai, Japan, Vol. 107, 1960. (Japanese with English Abstract).
22) Handelman, G. H. and Prager, W.: "Plastic Buckling of a Rectangular Plate under Edge Thrusts," NACA Rep. 946, 1949.
23) Nagaraja Rao, N. R. and Tall, L.: "Residual Stresses in Welded Plates," The Welding Journal, Vol. 40, October 1961.
24) Nagaraja Rao, N. R., Estuar, F. R. and Tall, L.: "Residual Stresses in Welded Shapes," The Welding Journal, Vol. 43, July 1964.
25) Odar, E., Nishino, F. and Tall, L.: "Residual Stresses in T-1 Constructional Alloy Steel Plates," WRC Bulletin No. 121, April 1967.
26) Odar, E., Nishino, F. and Tall, L.: "Residual Stresses in Welded Built-up 'T-1' Shapes, WRC Bulletin No. 121, April 1967.
27) Odar, E., Nishino, F. and Tall, L.: "Residual Stress in Rolled Heat-Treated 'T-1' Shapes, WRC Bulletin No. 121, April 1967.
28) Salvadori, M. G.: "Numerical Computations of Buckling Loads by Finite Differences," Trans. ASCE, Vol. 116, 1951.
29) Nishino, F.: "Error in Finite Difference Solutions of Local Buckling Strength," Trans. Japan Society of Civil Engr., No. 127, March 1966.
30) Cyrus, N. J. and Fulton, R. E.: "Finite Difference Accuracy in Structural Analysis," Proc. ASCE, Paper 5023, Vol. 92, ST6, December 1966.
31) Bodewig, E.: "Matrix Calculus," 2nd Ed., North Holland Publishing Co., Amsterdam, Holland, 1959.

32) Faddeeva, V. N.: "Computational Methods of Linear Algebra," Dover Publications Inc., New York, 1959.
33) Japan Road Association: "Specifications for Steel Highway Bridges," Japan Road Association, July 1964. English Translation by Japan Road Association, March 1968.
34) Tall, L. (Editor-in-Chief): "Structural Steel Design," The Ronald Press Co., New York, 1964.
35) Nishino, F., Ueda, Y. and Tall, L.: "Experimental Investigation of the Buckling of Plates with Residual Stresses," ASTM, S. T. P. 419, August 1967.

(*Received June 20, 1969*)