Proc. of JSCE,
No. 167, 1969-7

SOME PROBLEMS OF SUSPENSION BRIDGES UNDER RUNNING OF RAILWAY VEHICLES

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I. INTRODUCTION

A considerable amount of work as to the stability of railway vehicles running through bridges has been carried out. However, so far as long span bridges such as suspension bridges are concerned, there remain many problems on this subject.

Different from other types of bridges, suspension bridges have generally been used only as highway bridges in spite of their wide range of applicable span length. The main reason why suspension bridges have not been used for railway bridges is their comparatively small rigidities which causes a considerable amount of deformation by the large load intensity of railway vehicles. When such loads with a limited length as trains run on long-span bridges, the deflection will be different from that due to static loading. It seems that such influence of deflection will induce several practical problems which tend to limit the running of railway vehicles through bridges.

From the fundamental point of view the writers have been studying the dynamic characteristics of suspension bridges due to the passage of railway vehicles, so as to clarify the relationship between dynamic responses of railway vehicles and suspension bridges. One of these has been reported by M. Ito[1], one of the writers, who discussed mainly the dynamic effects of railway vehicles on suspension bridges by both theoretical and experimental treatments.

The works described in this report deal with the subsidiary problems of deflection resulting from the running of railway vehicles on suspension bridges by extending the same treatments as the paper refered above. These are concerned with the design of railway suspension bridges.

As for the vertical deflection, the dynamic effect of most importance on the stable running of railway vehicles on suspension bridges, is the vibration due to smoothly moving load. Though it is desirable to solve these problems using the whole system including vehicles and bridge, it makes the problem too complicated to obtain much results. Therefore, the dynamic response of suspension bridges to the running of railway vehicles are separately investigated.

II. OUTLINE OF ANALYSIS

The numerical calculations were carried out on the basis of the modal analy-

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sis method, as mentioned in the reference 1). The results of analysis are applied to three specific two-hinged suspension bridges having symmetric side spans, the dynamic characteristics of which are shown in Table 1.

Table 1 Sampled suspension bridges for numerical calculations.

Symbol and Span lengths	Order of Mode	Natural frequencies (c/s) Main span	Side span	a_3/a_1	$\bar{a}_1/a_1$
A	1	0.264	0.264	− 0.0682	−0.1187
	2	0.297	0.755		
Main: 500 m	3	0.606	0.606	16.4723	−1.7617
Side: 147 m	4	1.021	2.840		
B	1	0.152	0.152	− 0.2560	−0.4764
	2	0.134	0.252		
Main: 1000 m	3	0.224	0.224	7.7732	−3.4650
Side: 300 m	4	0.323	0.710		
C	1	0.101	0.101	− 0.1836	−0.7493
	2	0.101	0.143		
Main: 1500 m	3	0.154	0.154	−13.9914	6.6106
Side: 540 m	4	0.215	0.312		

The loadings considered herein are a concentrated moving load weighing 100 ton and a moving train load having the load intensity of $(7.4-L/100)\times 1.2$ t/m* and the maximum length of 400 m, in which L is the length of a train.

In the numerical calculations, only the first three eigenfunctions in each span, as indicated in Eq. (1), were taken into account, and the damping effect of the structure was neglected. These assumptions seemed to be valid within a practical accuracy in a long-span suspension bridges, judging from the previous studies[1].

$$\left.\begin{array}{lll} & \text{main span} & \text{side span} \\ \text{1st mode} & \Phi_1(x)=a_{11}\sin\frac{\pi x}{L}+a_{13}\sin\frac{3\pi x}{L} & \Phi_{11}(x)=\bar{a}_{11}\sin\frac{\pi x}{L_1} \\ \text{2nd mode} & \Phi_2(x)=a_{21}\sin\frac{2\pi x}{L} & \Phi_{12}(x)=\bar{a}_{21}\sin\frac{\pi x}{L_1} \\ \text{3rd mode} & \Phi_3(x)=a_{31}\sin\frac{\pi x}{L}+a_{33}\sin\frac{3\pi x}{L} & \Phi_{13}(x)=\bar{a}_{31}\sin\frac{\pi x}{L_1} \end{array}\right\} \quad (1)$$

where L and L_1 are the span length of center span and side span, respectively.

III. DEFLECTION OF SAMPLED SUSPENSION BRIDGES BY MOVING LOAD

The examples of loci of loading point when a concentrated load moves on

* Tentatively proposed by the Japan Railway Construction Corporation in 1965.

the two-hinged suspension bridges are illustrated in Fig. 1. The curve drawn by a dotted line shows the locus of moving load with infinitesimally low speed. It is obvious that the maximum deflection increases with the velocity of moving load. Though the effect of damping is ignored in the above calculation, it is apparent that damping will generally reduce dynamic effect even in long-span suspension bridges having low structural damping.

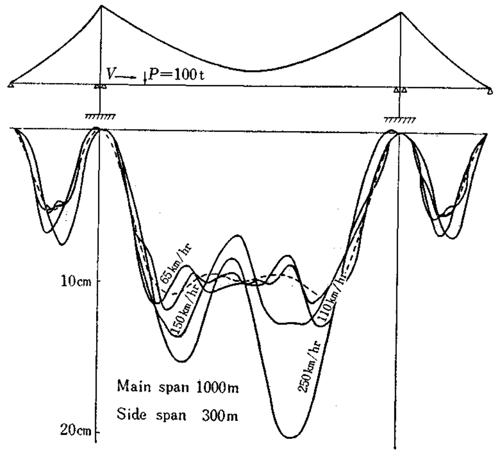

Fig. 1 Loci of deflection under moving load. (Suspension bridge: B)

If the velocity of load increases limitlessly, it finally reaches at the resonance velocity. As for the sampled suspension bridges A, B and C, the load velocity which resonates with the first symmetrical mode is 277 km/hr, 328 km/hr and 363 km/hr respectively. It should be noted that the velocity of existing railway vehicles is usually far below.

In connection with design problems, it is important to consider the difference between the case of a concentrated load and that of distributed load. The calculation shows that the velocity effect on the dynamic deflection due to distributed load is far less than in the case of a concentrated load, and is only 20% maximum deflection under the design live load and the highest temperature.

Table 2 The maximum deflection (accommodated highway and railway)

Type of suspension bridge		A	B	C
Railway load		m	m	m
$(7.4-L/100)\times 1.2$ t/m	$V=$ 0 km/hr	1.92	3.12	3.17
$L\leqq 400$ m	$V=$ 65 km/hr	2.06	2.94	3.20
(single track)	$V=$110 km/hr	1.96	2.98	3.13
V; train speed	$V=$250 km/hr	2.06	4.22	3.94
Highway load	3.0 t/m	0.435	1.166	1.666
Temperature change	±35°C	±0.73	±1.333	±2.129
Total maximum value		3.18	5.44	6.95

IV. TRACK GRADIENT AND RUNNING GRADIENT

One of the problems for the running of railway vehicles on suspension bridges is the gradient related to the deformation of suspension bridges. There are two kinds of gradient: one is the gradient of running direction for railway vehicles, and another is the gradient of the tracks directly under railway vehicles.

Defining the former as the running gradient, the latter, the track gradient,

$$\left.\begin{aligned} \text{Running gradient} &= \frac{d\eta(x,t)}{dx} \\ \text{Track gradient} &= \left\{\frac{\partial(x,t)}{\partial x}\right\}_{t=\text{const.}} \end{aligned}\right\} \quad (2)$$

When Eq. (2) is applied to a simply supported beam, it is found that the running gradient is just twice the track gradient. In other words, if the track gradient is denoted as α, railway vehicles move toward the direction of 2α with the gradient α. The situation is not so simple in case of suspension bridges, but it seems to be similar tendency as seen in Fig. 2. It is obvious that the traction force of railway vehicles is concerned with the running gradient, and the adhesion between vehicles and tracks, with the track gradient. In general, the running and track gradients decreases as span becomes long. Figs. 3 and 4 show the dynamic magnifiers of the running and track gradients defined as the ratio of the maximum running and track gradients to the value when velocity is zero. The track gradient increases smoothly with the increment of load velocity, while the dynamic magnifier of the running gradient becomes large even at low speed.

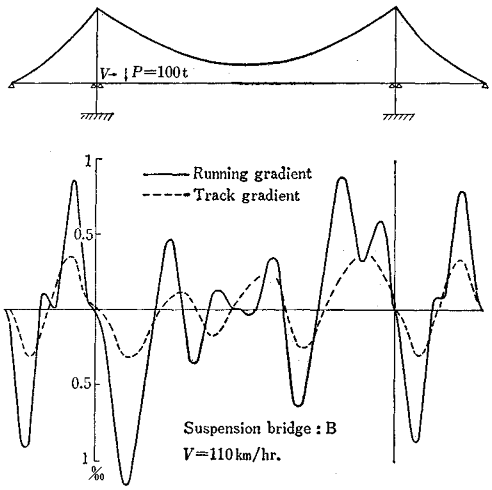

Fig. 2 Running gradient and track gradient.

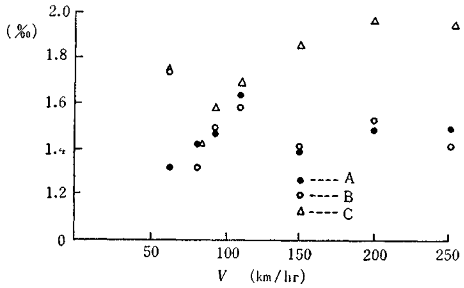

Fig. 3 Dynamic magnifier of running gradient.

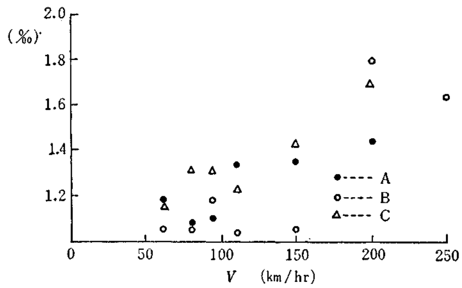

Fig. 4 Dynamic magnifier of track gradient.

V. ANGLE OF TRACKS UNDER TOWERS

When the two-hinged suspension bridge is used as a railway bridge, the track directly under the tower should have a special mechanical device against the angle of tracks between adjacent stiffening trusses due to temparature change and deflection.

In the vertical plane, the angle of tracks under the tower, γ, is expressed as

$$\left.\begin{aligned} \gamma &= \left(\frac{\partial \eta_{\text{II}}}{\partial x}\right)_{x=L_1} + \left(\frac{\partial \eta}{\partial x}\right)_{x=0} \\ \text{or} \\ \gamma &= \left(\frac{\partial \eta}{\partial x}\right)_{x=L} + \left(\frac{\partial \eta_{\text{II}}}{\partial x}\right)_{x=0} \end{aligned}\right\} \qquad (3)$$

The maximum angle of tracks under the tower is shown in Table 3.

Table 3 The maximum angle of tracks (accommodated highway and railway)

Type of suspension bridge		A	B	C
		‰	‰	‰
Railway load $(7.4-L/100)\times 1.2$ t/m $L\leqq 400$ m (single track) V; train speed	$V=$ 0 km/hr	5.4	8.1	7.7
	$V=$ 65 km/hr	5.3	7.3	7.0
	$V=$110 km/hr	5.4	8.0	9.4
	$V=$250 km/hr	6.2	11.5	11.3
Highway load	3.0 t/m	4.0	8.5	12.0
Temperature change	±35°C	5.1	5.5	6.7
Total maximum value		15.3	25.6	30.0

The deflection due to temperature change and highway loading is considered to be repeated with a long period compared with that of railway load. However, the change of the angle of tracks due to railway loading is repeated whenever railway vehicles run through the bridges. According to the specification of the Japanese National Railways, the vertical curve should be inserted when the change of gradient is more than 1%. In case of the suspension bridges mentioned above, it is necessary to moderate the gradient with floating trusses and so forth.

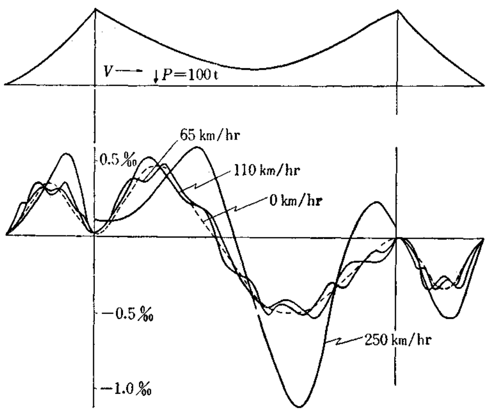

Fig. 5 Angle of tracks under towers.

VI. VERTICAL ACCELERATION

The vibration of railway vehicles on suspension bridges affects not only the stable running of railway vehicles but also the comfort for passengers. Such vibration is caused by the movement of the stiffening truss owing to the passage of railway vehicles in addition to other external forces such as highway loading, wind forces and the irregularity of tracks. Table 4 shows the vertical acceleration of the stiffening truss caused by only the smooth running of concentrated load. Moreover, the influence of the deformation of suspension bridges due to temperature change and highway loading is added. If the velocity of railway vehicles is within 100 km/hr, the vertical acceleration depending on the total deformation of suspension bridge is less than 0.1 g. In case of the distributed load, the vertical acceleration decreases much more.

Table 4 Vertical acceleration for running of single KS-18 loading

Speed \ Suspension bridge	A	B	C
65 km/hr	0.032 g	0.022 g	0.009 g
110 km/hr	0.065 g	0.030 g	0.017 g
250 km/hr	0.177 g	0.162 g	0.086 g

As for the vertical direction, other causes of vibrations in railway suspension bridges are the periodic external forces like hammer blow action of wheels, the spring action of railway vehicles and the impact at railway joints of tracks. These effects, however, are almost negligible compared with that of smoothly running load in long-span suspension bridges.

For example, consider the spring effect of railway vehicles. Taking account of the spring effect of railway vehicles, the generalized force $Q_n(t)$ for n-th mode is expressed as follows:

$$Q_n(t) = (W + Q\cos pt)\Phi_n(vt) \qquad (4)$$

where

W = weight of body + weight of wheel sets
Q = mass of body × the maximum vertical acceleration

Substituting Eq. (4) into the equation of motion of a suspension bridge, it is proved that the influence of the spring action is within a few percent of total dynamic effect. The spring effect of railway vehicles causes small oscillation with high frequencies in case that a concentrated load moves slowly on the suspension bridges A. But as the span increases, it can be almost ignored.

VII. CONCLUSION

It is possible, to some extent, to forcast the vibration of suspension bridges due to the passage of railway vehicles and the subsidiary problems for the running stability of railway vehicles by theoretical analysis. According to the calculation mentioned above, the construction of tracks on long-span suspension bridges is followed by many technical problems to be solved such as the limitation of running and track gradients, the special devices for rail-joints and expansion joints of tracks, and so on.

ACKNOWLEDGEMENTS

The authors wish to acknowledge to the engineers of the Japanese National Railways and the Japan Railway Construction Corporation, who gave the authors useful advices, and also to thank the members of Transportation Engineering Laboratory, Department of Civil Engineering, University of Tokyo for the cooperation to this work.

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(*Received Sept. 3, 1968*)