ON THE ONE-DIMENSIONAL CONSOLIDATION DUE TO THE TWO-DIMENSIONAL DEHYDRATION WITH THE SECONDARY COMPRESSION TAKEN INTO CONSIDERATION

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1. INTRODUCTION

The author proposed a method of analyzing the one-dimensional consolidation due to the three-dimensional dehydration in the previous paper. The method can be applied to the case where a structure is loaded locally on the comparatively wide ground, but it is often unapplicable to the consolidation effect of the ground with the long-stretched structures such as ordinary roads or embankments. In such case, the dehydration accompanying with the consolidation is thought to occur mainly in two directions, namely in the transverse direction of the structure and in the vertical direction of the consolidated layer, and the dehydration is practically observed as the synthetic flow of these two flows.

Therefore, it is necessarily required to regard the phenomena as the one-dimensional consolidation due to the two-dimensional dehydration in the analysis of such cases.

The method of analyzing of the consolidation effects with such cases and its application are mainly reported in the present paper.

2. DERIVATION OF THE BASIC EQUATION OF CONSOLIDATION

With a small element shown in Fig. 1, i_A and i_B may denote hydraulic gradients at faces A and B, u may denote pore water pressure, and γ_w unit weight of water. Then one may get,

$$i_A=-\frac{1}{\gamma_w}\cdot\frac{\partial u}{\partial x}$$

$$i_B=-\frac{1}{\gamma_w}\left(\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x^2}\cdot dx\right)$$

Applying the Darcy's law to this element, amount of water lost per unit time out of the minute element in the x-axis direction is expressed as,

$$\Delta Q_h=k_h i_B B-k_h i_A A=-\frac{k_h}{\gamma_w}\cdot\frac{\partial^2 u}{\partial x^2}\cdot dV$$

where A, B : the surface area of faces A and B $(A=B)$

dV : the volume of the minute element

k_h : the permeability coefficient in the x-axis direction

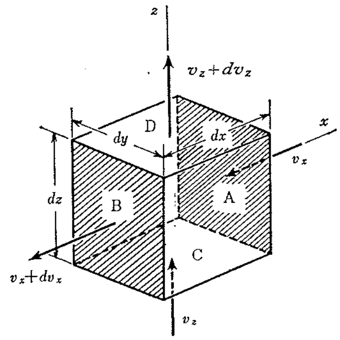

Fig. 1 Mechanism of dehydration from a minute element due to consolidation

In the same manner, the amount of water lost per unit time in the z-axis direction is

$$\Delta Q_v=k_v i_D D-k_v i_C C=-\frac{k_v}{\gamma_w}\cdot\frac{\partial^2 u}{\partial z^2}\cdot dV$$

,where C, D : the surface area of faces C and D $(C=D)$

k_v : the permeability coefficient in the z-axis direction

Therefore, the total amount of dehydration out of the minute element per unit time is

$$\Delta Q=\Delta Q_h+\Delta Q_v=-\left(\frac{k_h}{\gamma_w}\cdot\frac{\partial^2 u}{\partial x^2}+\frac{k_v}{\gamma_w}\cdot\frac{\partial^2 u}{\partial z^2}\right)dV$$

The amount of deformation of soil per unit time is, as shown in the previous paper,

$$\frac{\partial \varepsilon}{\partial t}dV=-\left(vpu+\gamma\frac{\eta p}{p+\eta}u\right)dV$$

Assuming that the amount of deformation equals that of dehydration in the saturated soil, there is obtained the equation

$$\frac{k_h}{\gamma_w}\cdot\frac{\partial^2 u}{\partial x^2}+\frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2}=vpu+\gamma\frac{\eta p}{p+\eta}\cdot u\cdots(2.1)$$

This equation is the basic equation of one-dimensional consolidation due to two-dimensional dehydra tion.

3. DEHYDRATION ONLY BY THE VERTICAL FLOW

The basic equation of consolidation only by the vertical flow is obtained by making $\partial u/\partial x=0$ in the above equation as

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$$\frac{k_v}{r_\omega}\left(\frac{\partial^2 u}{\partial z^2}\right)=vpu+\frac{r\eta p}{p+\eta}\cdot u$$

Next, this equation is analyzed to determine the equation of the pore water pressure u.

Operating $(p+\eta)$ on the both sides and arranging the equation with respect to p, one gets

$$vp^2u+\left\{(v+r)\eta u-\frac{k_v}{r_\omega}\left(\frac{\partial^2 u}{\partial z^2}\right)\right\}p$$
$$-\frac{k_v}{r_\omega}\eta\left(\frac{\partial^2 u}{\partial z^2}\right)=0$$

Assuming that u at any time t is expressed as the product of $\psi(z)$, the function only of z, and $\phi(t)$, the function only of t, and writing

$$\psi''(z)/\psi(z)=-N^2,$$

one obtains two equations independent of each other

$$\phi''(t)+\left\{\frac{\eta}{v}(v+r)+C_vN^2\right\}\phi'(t)+C_v\eta N^2\phi(t)=0 \quad \cdots\cdots(3.1)$$

$$\psi''(z)+N^2\psi(z)=0 \quad \cdots\cdots(3.2)$$

,where $C_v=k_v/v r_\omega$.

The equation (3.1) is a linear differential equation of order 2 and degree 1, and the solution is

$$\phi(t)=C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t}$$

,where

$$\lambda_1,\lambda_2=\frac{1}{2}\left[-\frac{\eta}{v}(v+r)-\frac{k_v}{v r_\omega}N^2 \pm\sqrt{\left\{\frac{\eta}{v}(v+r)+\frac{k_v}{v r_\omega}N^2\right\}^2-\frac{4k_v}{v r_\omega}\eta N^2}\right] \quad \cdots\cdots(3.3)$$

The equation (3.2) is a linear differential equation of order 2 and degree 1, and the solution is

$$\psi(z)=C_3\sin Nz+C_4\cos Nz$$

As $u=\psi(z)\phi(t)$, the general integral of the basic equation becomes

$$u_z=(C_1e^{\lambda_2 t}+C_2e^{\lambda_1 t})(C_3\sin Nz+C_4\cos Nz)$$

The integration constants are determined by the use of the following boundary conditions.

(1) At $0\leqq t\leqq\infty$ and $z=0$, $u=0$

From this condition,

$$C_4=0$$

Accordingly,

$$u_z=C_3\sin Nz(C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t})$$

(2) At $0\leqq t\leqq\infty$ and $z=2H$, $u=0$, where $2H$ denotes the thickness of the consolidated layer.

From this condition,

$$\sin 2NH=0$$

Accordingly, $2NH=m\pi (m=1, 2, 3\cdots)$, then $N=m\pi/2H$ and

$$u_z=\sum_{m=1}^{\infty}(B_1e^{\lambda_1 t}+B_2e^{\lambda_2 t})\sin\frac{m\pi}{2H}z \quad \cdots\cdots(3.4)$$

,where $B_1=C_1C_3$ and $B_2=C_2C_3$.

(3) At $0\leqq z\leqq 2H$ and $t=0$, $u_z=K$, where K denotes the load intensity.

Accordingly,

$$K=\sum_{m=1}^{\infty}(B_1+B_2)\sin\frac{m\pi}{2H}z$$

Multiplying the both sides by $\sin\frac{m\pi}{2H}z$ and integrating the equation with respect to z, one finds

$$B_1+B_2=\frac{2K}{m\pi}(1-\cos m\pi)=\frac{4K}{n\pi} \quad \cdots\cdots(3.5)$$

,where $n=1, 3, 5\cdots$

(4) At $t=0$, $\varepsilon_c=0$.

ε_c is the permanent strain, and is shown by the following equation.

$$\varepsilon_c(t)=r\eta\int_0^t e^{-\eta(t-\tau)}\bar{p}(\tau)d\tau$$

Application of $\bar{p}=K-u_z$ to the above equation gives

$$\varepsilon_c(t)=r\eta\int_0^t e^{-\eta(t-\tau)}\left\{\sum_{n=1,3,5}^{\infty}\frac{4K}{n\pi}\sin\frac{n\pi}{2H}z -\sum_{n=1,3,5}^{\infty}(B_1e^{\lambda_1\tau}+B_2e^{\lambda_2\tau})\sin\frac{n\pi}{2H}z\right\}d\tau$$

The coefficient of $\sin\frac{n\pi}{2H}z$ is

$$r\eta\int_0^t e^{-(t-\tau)}\left\{\frac{4K}{n\pi}-(B_1e^{\lambda_1\tau}+B_2{}^{\lambda_2\tau})\right\}d\tau$$

Excluding the value at $t=0$ by regarding it as 0, and applying the condition that $\varepsilon_c=0$ at $t=0$ to the equation, one obtains

$$\frac{4K}{n\pi}=\eta\left(\frac{B_1}{\lambda_1+\eta}+\frac{B_2}{\lambda_2+\eta}\right) \quad \cdots\cdots(3.6)$$

B_1 and B_2 are determined from the equations (3.5) and (3.6) as

$$B_1=\frac{4K}{n\pi}\cdot\frac{(\lambda_1+\eta)\lambda_2}{(\lambda_2-\lambda_1)\eta}$$

$$B_2=-\frac{4K}{n\pi}\cdot\frac{(\lambda_2+\eta)\lambda_1}{(\lambda_2-\lambda_1)\eta}$$

Applying these values to the equation (3.4) one finds

$$u_z=\sum_{n=1,3,5}^{\infty}\frac{4K}{n\pi}\cdot\frac{1}{(\lambda_2-\lambda_1)\eta} \times[\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}]\sin\frac{n\pi}{2H}z$$

4. DEHYDRATION ONLY BY THE HORIZONTAL FLOW

The basic equation of consolidation only by the horizontal flow is obtained by applying $\partial u/\partial z=0$ to the equation (2.1) as

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial x^2}\right)=vpu+\frac{r\eta p}{p+\eta}\cdot u \quad \cdots\cdots(4.1)$$

The general integral of this equation is obtained in the same manner as in the above-described Section 3 as

$$u=(C_1e^{\lambda'_1 t}+C_2e^{\lambda'_2 t})(C_3\sin Mx+C_4\cos Mx) \quad \cdots\cdots(4.2)$$

,where

$$\lambda_1',\lambda_2'=\frac{1}{2}\left[-(v+r)\frac{\eta}{v}-\frac{k_h}{v r_\omega}M^2 \pm\sqrt{\left\{(v+r)\frac{\eta}{v}+\frac{k_h}{v r_\omega}M^2\right\}^2-4\frac{k_h}{v r_\omega}\eta M^2}\right] \quad \cdots\cdots(4.3)$$

The undetermined constants C_1, C_2, C_3 and C_4 in the above equation (4.2) are determined by the use of boundary conditions.

5. SURFACE CONDITIONS

The consolidation due to the two-dimensional dehydration in the title means the case where dehydration does not occur in the longitudinal direction, or the y-axis direction, of the long-stretched structures, but occurs mainly in its transverse, or the x-axis direction and in the vertical, or the z-axis, direction of the stratum.

The dehydration conditions due to consolidation in the x-axis direction are now examined.

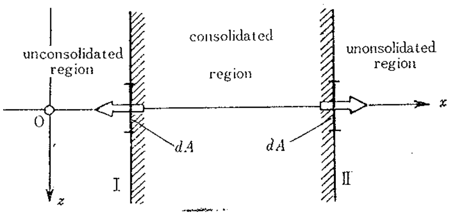


Fig. 2 An explanatory figure of notation of surface conditions

In Fig. 2, the part surrounded by the lines I and II is the part suffering from consolidation, and those outside parts of these lines are those free from consolidation. The x-axis coincides with the transverse direction of the structure on the ground. In other words, dehydration occurs through the boundary faces I and II of the consolidated region in the directions shown by the sign of arrows respectively, and the x-axis is made to point to the right direction.

The amount of water flowing out of the consolidated soil body toward the unconsolidated parts through the faces I and II of the area per time dt is

$$k\left(-\frac{\partial u}{\partial x}\right)dA\cdot dt$$

,where k denotes the permeability coefficient in the x-axis direction.

On the other hand, pore water is thought to flow out through the boundary faces of the consolidated soil body and the unconsolidated soil bodies at the rate proportional to the difference of their pore water pressure. Taking the pore water pressure of the consolidated soil body at the boundary faces I and II shown in Fig. 2 to be u_{I} and u_{II} respectively, and that of the unconsolidated soil bodies to be 0, one gets the amounts of water flowing out of the consolidated soil body into the unconsolidated soil bodies as

$$-a(u_{\mathrm{I}}-0)dAdt \text{ and } a(u_{\mathrm{II}}-0)dAdt$$

respectively, where a is a constant.

The amount of water flowing out of the consolidated soil body and those flowing into the unconsolidated soil bodies can be made equal to each other at their boundary faces.

Accordingly, at the boundary face I,

$$k\left(-\frac{\partial u}{\partial x}\right)_{\mathrm{I}} dAdt=-au_{\mathrm{I}}dAdt$$

$$\therefore \left[-k\left(\frac{\partial u}{\partial x}\right)+au\right]_{\mathrm{I}}=0$$

At the boundary face II,

$$k\left(-\frac{\partial u}{\partial x}\right)_{\mathrm{II}} dAdt=au_{\mathrm{II}}dAdt$$

$$\therefore \left[k\left(\frac{\partial u}{\partial x}\right)+au\right]_{\mathrm{II}}=0$$

Therefore, the equation of the surface condition at the boundary face I becomes

$$\left[-\left(\frac{\partial u}{\partial x}\right)+\beta u\right]_{\mathrm{I}}=0$$

,and that at the boundary face II becomes

$$\left[\left(\frac{\partial u}{\partial x}\right)+\beta u\right]_{\mathrm{II}}=0$$

,where $\beta=\dfrac{a}{k}$

Taking the origin of coordinates 0 of the consolidated part as shown in Fig. 3, one finds two equations of surface conditions of the consolidated soil body at $x=0$ and $x=b$ as

$$\left.\begin{aligned}&\left[-\left(\frac{\partial u}{\partial x}\right)+\beta u\right]_{x=0}=0\\&\left[\left(\frac{\partial u}{\partial x}\right)+\beta u\right]_{x=b}=0\end{aligned}\right\}\cdots\cdots\cdots\cdots(5.1)$$

,respectively.

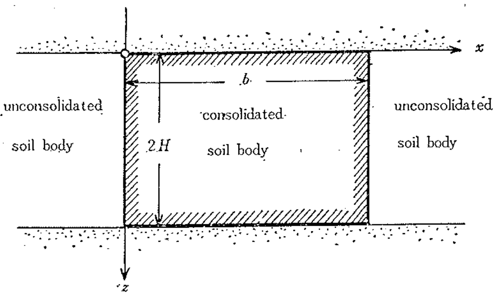


Fig. 3 A figure showing the origin of coordinates of the consolidated soil body

The value of $\left[-\left(\dfrac{\partial u}{\partial x}\right)+\beta u\right]$ is calculated from the equation (4.2) as

$$\left(-\frac{\partial u}{\partial x}+\beta u\right)$$

$$=(C_1e^{\lambda_1't}+C_2{}^{\lambda_2't})\{C_3(\beta\sin Mx-M\cos Mx)+C_4(M\sin Mx+\beta\cos Mx)\}$$

Application of the equation (5.1) to the above equation gives

$$\left(-\frac{\partial u}{\partial x}+\beta u\right)_{x=0}$$

$$=(C_1e^{\lambda_1't}+C_2e^{\lambda_2't})(C_4\beta-C_3M)=0$$

Accordingly, $C_4\beta-C_3M=0$, then $C_4=\dfrac{M}{\beta}C_3$.

Applying this to the equation (4.2), one finds

$$u=(B_1e^{\lambda_1't}+B_2e^{\lambda_2't})\left(\sin Mx+\frac{M}{\beta}\cos Mx\right)$$

,where $B_1=C_1C_3$ and $B_2=C_2C_3$

On the other hand,

$$\left(\frac{\partial u}{\partial x}+\beta u\right)_{x=b}$$

$$=(B_1e^{\lambda_1't}+B_2e^{\lambda_2't})\times\left\{M\cos bM-\frac{M^2}{\beta}\sin bM\right.$$
$$\left.+\beta\sin bM+M\cos bM\right\}=0$$

Accordingly,

$$2\,M\cos bM-\left(\frac{M^2}{\beta}-\beta\right)\sin bM=0$$

$$\therefore\ \tan bM=\frac{2\,M}{\frac{M^2}{\beta}-\beta}=\frac{2\,M\beta}{M^2-\beta^2}=\frac{2(bM)(b\beta)}{(bM)^2-(b\beta)^2} \quad \cdots\cdots\cdots\cdots\cdots(5.2)$$

Letting m_i stand for the values of M that satisfy the above equation, one obtains.

$$u=\sum_{i=1}^{\infty}(B_1e^{\lambda_1't}+B_2e^{\lambda_2't})\left(\sin m_ix+\frac{m_i}{\beta}\cos m_ix\right) \quad \cdots\cdots\cdots\cdots\cdots(5.3)$$

6. INITIAL CONDITION

The load on the saturated soil is thought to be supported by the pore water pressure immediately after the start of consolidation.

Therefore, at $t=0$, $u=K$.

$$K=\sum_{i=1}^{\infty}(B_1+B_2)\left(\sin m_ix+\frac{m_i}{\beta}\cos m_ix\right) \quad \cdots\cdots\cdots\cdots\cdots(6.1)$$

The undetermined constant (B_1+B_2) is determined in the following manner.

If one writes the characteristec function as

$$\sin m_ix+\frac{m_i}{\beta}\cos m_ix=D_i$$

then D_i is the solution of the differential equation obtained by separation of variables in the basic equation of consolidation

$$\varphi''(x)+m_i^2\varphi(x)=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6.2)$$

and further satisfies the equations

$$\left[-\left(\frac{dD_i}{dx}\right)+\beta\,D_i\right]_{x=0}=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(6.3)$$

$$\left[\left(\frac{dD_i}{dx}\right)+\beta\,D_i\right]_{x=b}=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6.4)$$

Now an assumption is made that the characteristic values m_1, m_2, m_3,$\cdots$and the characteristic functions D_1, D_2, D_3,$\cdots$are determined from these conditions, and any two values m_p and m_q, and D_p and D_q ore taken from the series respectively, which naturally satisfy the equation (6.2) to give

$$D_p''+m_p^2D_p=0,\quad D_q''+m_q^2D_q=0,$$

Multiplying the both sides by D_q and D_p, respectively, and subtraction one from the other, one obtains

$$(m_p^2-m_q^2)D_pD_q=D_q''D_p-D_p''D_q$$

Integration of the both sides from $x=0$ to $x=b$ gives

$$(m_p^2-m_q^2)\int_0^b D_pD_q dx$$
$$=\int_0^b D_pD_q''dx-\int_0^b D_qD_p''dx$$
$$=\Big|D_pD_q'-D_qD_p'\Big|_0^b \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6.5)$$

On the other hand, from the equation of the condition (6.3), one gets

$$-D_p'(0)+\beta\,D_p(0)=0$$
$$-D_q'(0)+\beta\,D_q(0)=0$$

From the above two equations, in the cace of $\beta\neq 0$,

$$D_p(0)D_q'(0)-D_q(0)D_p'(0)=0 \quad \cdots\cdots\cdots\cdots(6.6)$$

From the equation of the condition (6.4), one gets

$$D_p'(b)+\beta\,D_p(b)=0$$
$$D_q'(b)+\beta\,D_q(b)=0$$

From the above two equations, in the case of $\beta\neq 0$

$$D_p(b)D_q'(b)-D_q(b)D_p'(b)=0 \quad \cdots\cdots\cdots\cdots(6.7)$$

Application of the equations (6.6) and (6.7) to the right side of the equation (6.5), with the condition $m_p\neq m_q$, gives

$$\int_0^b D_pD_q dx=0$$

Whence, if multiplication of the both sides of the equation (6.1) by D_i and integration of them from 0 to b are made, then all the terms other than the i th one become 0, on the right side.

Accordingly,

$$K\int_0^b D_idx=(B_1+B_2)\int_0^b D_i^2dx$$

$$B_1+B_2=\frac{K\int_0^b D_idx}{\int_0^b D_i^2dx}$$

In the same manner as in the case of dehydration only of the vertical flow, one gets, by letting $\varepsilon_c=0$ at $t=0$,

$$B_1=(B_1+B_2)\frac{\lambda_2'(\lambda_1'+\eta)}{(\lambda_2'-\lambda_1')\eta}$$

$$B_2=-(B_1+B_2)\frac{\lambda_1'(\lambda_2'+\eta)}{(\lambda_2'-\lambda_1')\eta}$$

Applying the above values to the equation (5.3), one obtains

$$u_x=\sum_{i=1}^{\infty}\frac{K}{(\lambda_2'-\lambda_1')\eta}[\lambda_2'(\lambda_1'+\eta)e^{\lambda_1't}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2't}]\left[\frac{D_i\int_0^b D_idx}{\int_0^b D_i^2dx}\right]$$

7. ANALYSIS OF CONSOLIDATION DUE TO A TWO-DIMENSIONAL FLOW

The basic equation for this case is

$$\frac{k_h}{\gamma_w}\cdot\frac{\partial^2u}{\partial x^2}+\frac{k_v}{\gamma_w}\cdot\frac{\partial^2u}{\partial z^2}=vpu+\frac{\gamma\eta p}{p+\eta}\cdot u$$

,and its solution is obtained as a product of respective solutions of one-dimensional dehydration in the x-axis and z-axis directions. Namely,it is expressed as

$$u=K(u_x)_{K=1}(u_z)_{K=1}$$

,where K is the external load, and$(u_x)_{K=1}$ and$(u_z)_{K=1}$ are the values of u_x and u_z when the soil supports the load $K=1$, respectively.

Furthermore, taking $\bar{u}$, $\bar{u}_x$ and $\bar{u}_z$ to be the mean value of u with respect to the x-axis and z-axis, that of u_x with respect only to the x-axis, and that of u_z with respect only to the z-axis respectively, one obtains the equation

$$\overline{u}=K(\overline{u}_x)_{K=1}(\overline{u}_z)_{K=1}$$

,where

$$(\overline{u}_x)_{K=1}=\frac{\int_0^b (u_x)_{K=1}dx}{b}=\sum_{i=1}^{\infty}\frac{1}{(\lambda_2'-\lambda_1')\eta}\{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1' t}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2' t}\}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}$$

and

$$(\overline{u}_z)_{K=1}=\frac{\int_0^{2H}(u_z)_{K=1}\,dz}{2H}=\frac{1}{2H}\left[\sum_{n=1,3,5}^{\infty}\frac{1}{(\lambda_2-\lambda_1)\eta}\cdot\frac{4}{n\pi}\{\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}\}\times\int_0^{2H}\sin\frac{n\pi}{2H}z dz\right]$$

$$=\sum_{n=1,3,5}^{\infty}\frac{8}{(\lambda_2-\lambda_1)\eta}\cdot\frac{1}{(n\pi)^2}\{\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}\}$$

Accordingly, the mean pore water pressure due to the synthetic flow $\overline{u}$ is obtained as

$$\overline{u}=K\left[\sum_{i=1}^{\infty}\frac{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1' t}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2' t}}{(\lambda_2'-\lambda_1')\eta}\cdot\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\right]\times\left[\sum_{n=1,3,5}^{\infty}\frac{\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}}{(\lambda_2-\lambda_1)\eta}\cdot\frac{8}{(n\pi)^2}\right]$$

8. AMOUNT OF SETTLEMENT DUE TO CONSOLIDATION

The amount of settlement at time t due to the synthetic flow is

$$S=\int_0^{2H}\varepsilon\,dz=\int_0^{2H}(\overline{p}v+\varepsilon_c)dz=\int_0^{2H}\left[Kv+r\eta\int_0^t Ke^{-\eta(t-\tau)}d\tau\right]dz-\int_0^{2H}\left[\overline{u}v+r\eta\int_0^t\overline{u}e^{-\eta(t-\tau)}d\tau\right]dz \quad \cdots\cdots(8\cdot1)$$

,where

$$r\eta\int_0^t Ke^{-\eta(t-\tau)}d\tau=\eta r\,Ke^{-\eta t}\left|\frac{e^{\eta\tau}}{\eta}\right|_0^t=r\,K$$

$$r\eta\int_0^t\overline{u}e^{-\eta(t-\tau)}d\tau=r\,e^{-\eta t}\int_0^t\eta\,\overline{u}e^{\eta\tau}d\tau=r\,K\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\cdot\frac{1}{\eta(\lambda_2'-\lambda_1')(\lambda_2-\lambda_1)}\cdot$$

$$\cdot\left[\frac{\lambda_2'\lambda_2(\lambda_1'+\eta)(\lambda_1+\eta)}{\lambda_1'+\lambda_1+\eta}e^{(\lambda_1'+\lambda_1)t}-\frac{\lambda_2'\lambda_1(\lambda_1'+\eta)(\lambda_2+\eta)}{\lambda_1'+\lambda_2+\eta}e^{(\lambda_1'+\lambda_2)t}\right.$$
$$\left.-\frac{\lambda_1'\lambda_2(\lambda_2'+\eta)(\lambda_1+\eta)}{\lambda_2'+\lambda_1+\eta}e^{(\lambda_2'+\lambda_1)t}+\frac{\lambda_1'\lambda_1(\lambda_2'+\eta)(\lambda_2+\eta)}{\lambda_2'+\lambda_2+\eta}e^{(\lambda_2'+\lambda_2)t}\right] \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8\cdot2)$$

and

$$\overline{u}v=vK\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\cdot\frac{1}{\eta^2(\lambda_2'-\lambda_1')(\lambda_2-\lambda_1)}[\lambda_2'\lambda_2(\lambda_1'+\eta)(\lambda_1+\eta)e^{(\lambda_1'+\lambda_1)t}$$
$$-\lambda_2'\lambda_1(\lambda_1'+\eta)(\lambda_2+\eta)e^{(\lambda_1'+\lambda_2)t}-\lambda_1'\lambda_2(\lambda_2'+\eta)(\lambda_1+\eta)e^{(\lambda_2'+\lambda_1)t}+\lambda_1'\lambda_1(\lambda_2'+\eta)(\lambda_2+\eta)e^{(\lambda_2'+\lambda_2)t}] \quad \cdots\cdots\cdots(8.3)$$

Now examination is made with respect to λ_1, λ_2, λ_1' and λ_2'. Developing the $\sqrt{\quad}$ part of the equations (3.3) and (4.3) by the use of the binomial theorem, one obtains λ_1, λ_2, λ_1' and λ_2' as

$$\lambda_1=-\frac{C_v\eta\left(\frac{n\pi}{2H}\right)^2}{\frac{v+r}{v}\eta+C_v\left(\frac{n\pi}{2H}\right)^2}-\frac{\left\{C_v\eta\left(\frac{n\pi}{2H}\right)^2\right\}^2}{\left\{\frac{v+r}{v}\eta+C_v\left(\frac{n\pi}{2H}\right)^2\right\}^3}-\cdots\cdots\cdots$$

$$\lambda_2=-\frac{v+r}{v}\eta-C_v\left(\frac{n\pi}{2H}\right)^2+\frac{C_v\eta\left(\frac{n\pi}{2H}\right)^2}{\frac{v+r}{v}\eta+C_v\left(\frac{n\pi}{2H}\right)^2}+\cdots\cdots\cdots$$

$$\lambda_1'=-\frac{C_h\eta m_i^2}{\frac{v+r}{v}\eta+C_h m_i^2}-\frac{(C_h\eta m_i^2)^2}{\left(\frac{v+r}{v}\eta+C_h m_i^2\right)^3}-\cdots\cdots\cdots$$

and

$$\lambda_2'=-\frac{v+r}{v}\eta-C_h m_i^2+\frac{C_h\eta m_i^2}{\frac{v+r}{v}\eta+C_h m_i^2}+\cdots\cdots\cdots$$

,where $C_v=\dfrac{k_v}{v\gamma_w}$ and $C_h=\dfrac{k_h}{v\gamma_w}$.

Examination is further made for the case where the creep coefficient η is very small compared with the permeability coefficient k and creep lasts very long.

In this case, $\eta \ll C_v\left(\frac{n\pi}{2H}\right)^2$ and $\eta \ll C_h m_i^2$.

Accordingly,

$$\lambda_1 \doteqdot -\eta - \frac{\eta^2}{C_v\left(\frac{n\pi}{2H}\right)^2}$$

$$\lambda_2 \doteqdot -C_v\left(\frac{n\pi}{2H}\right)^2$$

$$\lambda_1' \doteqdot -\eta - \frac{\eta^2}{C_h m_i^2}$$

and $\lambda_2' \doteqdot -C_h m_i^2$

In examining the coefficients in the equations (8·2) and (8·3) by the use of these values, one gets

$$r\eta\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau = \sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\cdot r K e^{-2\eta t}$$

and

$$\bar{u}v = vK\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\cdot e^{-C_h m_i^2 t}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Therefore, the amount of settlement S for this case becomes

$$S = 2HK(v+r) - 2HvK\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}e^{-C_h m_i^2 t}\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

$$-2HrK\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\cdot e^{-2\eta t}$$

The value of $\dfrac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx} = Q_i$ in the above equation is then calculated.

Concerning the numerator in the above expression, there is obtained

$$\left\{\int_0^b D_i dx\right\}^2 = \left\{\int_0^b \sin m_i x dx + \frac{m_i}{\beta}\int_0^b \cos m_i x dx\right\}^2 = b^2\left\{\frac{1}{bm_i}(1-\cos bm_i) + \frac{1}{b\beta}\sin bm_i\right\}^2$$

As to the denominator,

$$b\int_0^b D_i^2 dx = b\left[\int_0^b \sin^2 m_i x dx + \frac{m_i^2}{\beta^2}\int_0^b \cos^2 m_i x dx + \frac{m_i}{\beta}\int_0^b \sin 2m_i x dx\right]$$
$$= \frac{b^2}{2}\left[1+\left(\frac{bm_i}{b\beta}\right)^2 + \frac{1}{2}\left\{\frac{bm_i}{(b\beta)^2} - \frac{1}{bm_i}\right\}\sin 2(bm_i) + \frac{1}{b\beta}\{1-\cos 2(bm_i)\}\right].$$

Accordingly,

$$Q_i = \frac{2\left\{\frac{1}{bm_i}(1-\cos bm_i) + \frac{1}{b\beta}\sin bm_i\right\}^2}{1+\left(\frac{bm_i}{b\beta}\right)^2 + \frac{1}{2}\left\{\frac{bm_i}{(b\beta)^2} - \frac{1}{bm_i}\right\}\sin 2(bm_i) + \frac{1}{b\beta}\{1-\cos 2(bm_i)\}} \quad \cdots\cdots(8\cdot4)$$

Furthermore, the final amount of settlement S_∞ is obtained, by applying $t\to\infty$ to the equation of the amount of settlement S, as $S_\infty = 2HK(v+r)$.

Therefore, the degree of consolidation for this case is

$$U = \frac{S}{S_\infty} = 1 - \frac{v}{v+r}\sum_{i=1}^{\infty}Q_i e^{-C_h m_i^2 t}\cdot\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-C_v\left(\frac{n\pi}{2H}\right)^2 t} - \frac{r}{v+r}\sum_{i=1}^{\infty}Q_i\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}\cdot e^{-2\eta t} \quad \cdots\cdots(8\cdot5)$$

Next, examination is made for the case where the creep coefficient is very large compared with the permeability coefficient and creep finishes in such a short time. In this case,

$\eta \gg C_v\left(\frac{n\pi}{2H}\right)^2$ and $\eta \gg C_h m_i^2$, and then

$$\lambda_1 \doteqdot -\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2$$

$$\lambda_2 \doteqdot -\frac{v+r}{v}\cdot\eta$$

$$\lambda_1' \doteqdot -\frac{v}{v+r}C_h m_i^2$$

and $\lambda_2' = -\dfrac{v+r}{v}\eta$

Examination of the coefficients in the equation (8·2) and (8·3) by the use of these values gives

$$r\eta\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau = rK\sum_{i=1}^{\infty}\frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx}e^{-\frac{v}{v+r}C_h m_i^2 t}\times\sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2}e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

and

$$\bar{u}v = vK \sum_{i=1}^{\infty} \frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx} e^{-\frac{v}{v+r}C_h m_i^2 t} \sum_{n=1,3,5}^{\infty} \frac{8}{(n\pi)^2} e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Therefore, the amount of settlement S for this case is

$$S = 2HK(v+r) - 2HK(v+r)\sum_{i=1}^{\infty} \frac{\left\{\int_0^b D_i dx\right\}^2}{b\int_0^b D_i^2 dx} e^{-\frac{v}{v+r}C_h m_i^2 t} \times \sum_{n=1,3,5}^{\infty} \frac{8}{(n\pi)^2} e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

Accordingly, the degree of consolidation U is expressed by the equation

$$U = \frac{S}{S_\infty} = 1 - \sum_{i=1}^{\infty} Q_i e^{-\frac{v}{v+r}C_h m_i^2 t} \cdot \sum_{n=1,3,5}^{\infty} \frac{8}{(n\pi)^2} e^{-\frac{v}{v+r}C_v\left(\frac{n\pi}{2H}\right)^2 t}$$

This equation means that the delay of creep is negligible in the case where creep coefficient is so large that the permanent deformation occurs in a moment, and so it becomes the same equation of the degree of consolidation as in the case where only the primary consolidation is taken into consideration.

9. RELATION BETWEEN THE DEGREE OF CONSOLIDATION AND THE TIME FACTOR

An equation for calculating the degree of consolidation in the one-dimensional consolidation due to the two-dimensional dehydration with the secondary compression and creep taken into consideration was obtained in the preceding section. The relation between the degree of consolidation and the time factor is now to be obtained by the use of the equation.

Writing $\frac{C_h}{\left(\frac{b}{2}\right)^2}\cdot t = T_h$, $\frac{C_v}{H^2}t = T_v$,

$$\frac{T_h}{T_v} = \frac{C_h}{C_v}\cdot\frac{H^2}{\left(\frac{b}{2}\right)^2} = \alpha, \text{ and } 2\eta t = T'$$

in the equation (8·5), one gets

$$U = 1 - \frac{v}{v+r}\sum_{i=1}^{\infty} Q_i e^{-\frac{(bm_i)^2}{4}\alpha T_v} \cdot \sum_{n=1,3,5}^{\infty} \frac{8}{(n\pi)^2} e^{-\frac{(n\pi)^2}{4}T_v} - \frac{r}{v+r}\sum_{i=1}^{\infty} Q_i \sum_{n=1,3,5}^{\infty}\frac{8}{(n\pi)^2} e^{-T'}$$

$$= \frac{v}{v+r}\left(1 - \sum_{i=1}^{\infty} Q_i e^{-\frac{(bm_i)^2}{4}\alpha T_v} \cdot \sum_{n=1,3,5}^{\infty} \frac{8}{(n\pi)^2} e^{-\frac{(n\pi)^2}{4}T_v}\right) + \frac{r}{v+r}(1 - e^{-T'})$$

$$= U_v + U_r$$

, where U_v and U_r mean the primary consolidation rate and the secondary compression rate respectively. Prior to obtaining the relation between U_v and T_v, the characteristic values bm_i corresponding to $b\beta$ = 0.1, 0.5, ……, 100, 500, ∞ were determined by the use of the equation (5·2), and further the values of Q_i were calculated by applying the values of $b\beta$ and those of bm_i corresponding to them to the equation (8·4).

The values of $Q_i e^{-\frac{(bm_i)^2}{4}\alpha T_v}$ in the equation with respect to U_v were calculated by using the values of bm_i and Q_i corresponding to various values of $b\beta$ obtained in the above calculation and by taking αT_v to be 0.01, 0.02, ……, 50, 60. The values were then summed up in the range of $i=1\sim7$ to obtain $\sum Q_i e^{-\frac{(bm_i)^2}{4}\alpha T_v}$.

Furthermore, the values of $\sum Q_i e^{-\frac{(bm_i)^2}{4}\alpha T_v}$ were calculated by taking T_v=0.0001, 0.0002, ……, 2.0.

The curves indicating the relation between U_v and T_v corresponding to various values of $b\beta$ and α obtained from the results of the above-described calculations are shown in Fig. 4~13.

In these figures, the $U_v - \log T_v$ curves are shown with the case where $\frac{v}{v+r}$=1.0.

Next, the relation between U_r and T' was obtained by calculation by taking $\frac{v}{v+r}$=0.025, 0.050, ………, 0.950, 0.975, 1.0, and the results are shown in Fig. 14.

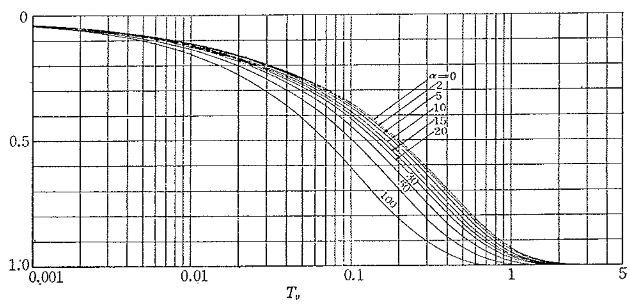


Fig. 4 The $U_v - \log T_v$ curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\beta$=0.1)

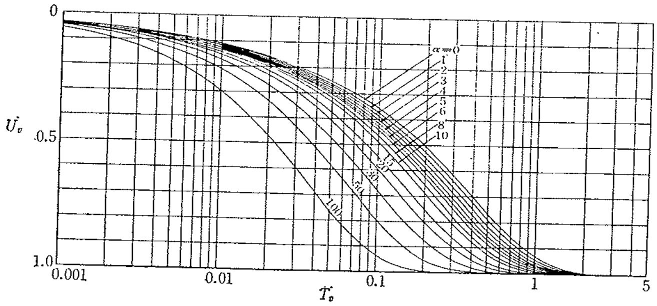


Fig. 5 The $U_v-\log T_v$ curved-ruler by the value of $b\,\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\,\beta=0.5$)

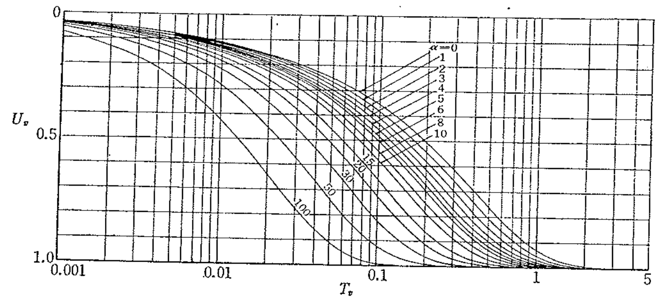


Fig. 6 The $U_v-\log T_v$ curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\,\beta=1.0$)

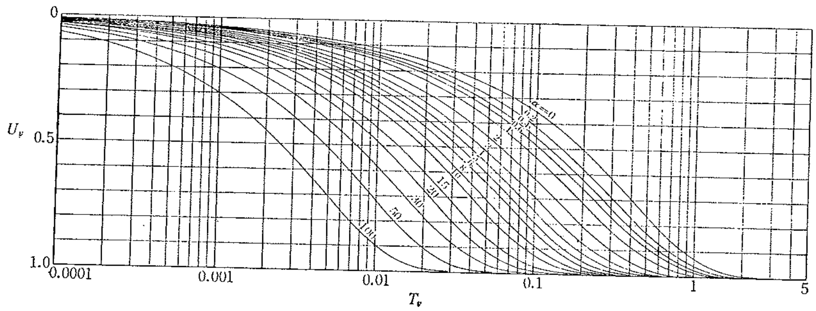


Fig. 7 The $U_v-\log T_v$ curved-ruler by the value of $b\,\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\,\beta=10$)

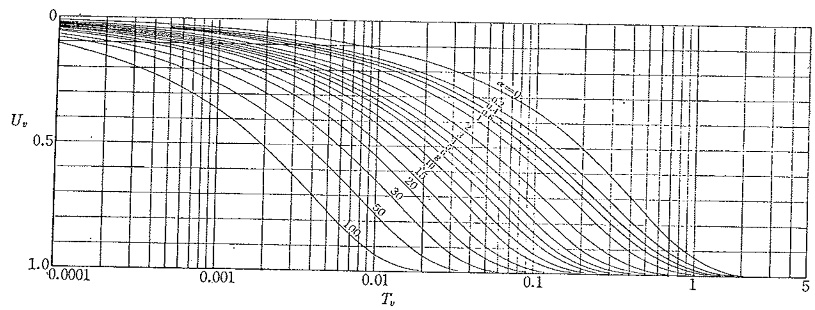


Fig. 8 The $U_v-\log T_v$ curved-ruler by the value of $b\,\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\,\beta=20$)

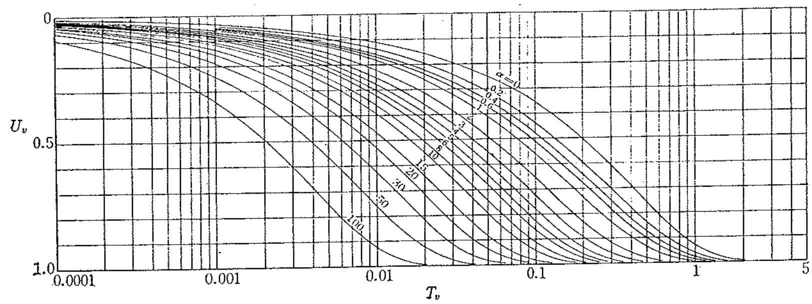


Fig. 9 The U_v−log T_v curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\beta$=30)

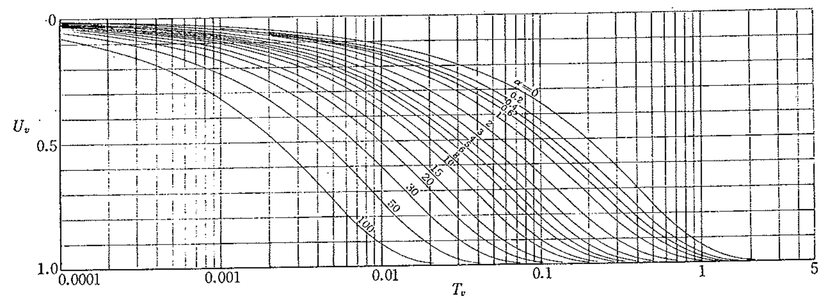


Fig. 10 The U_v−log T_v curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\beta$=50)

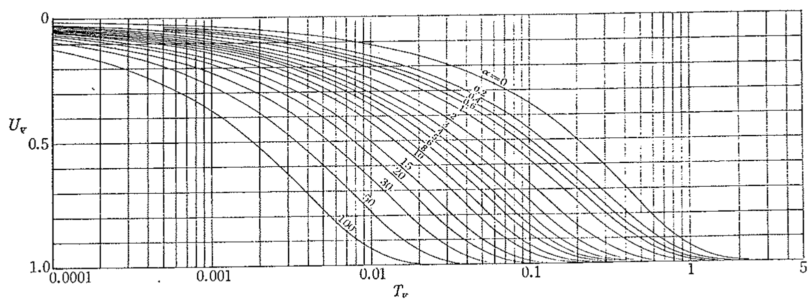


Fig. 11 The U_v−log T_v curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\beta$=100)

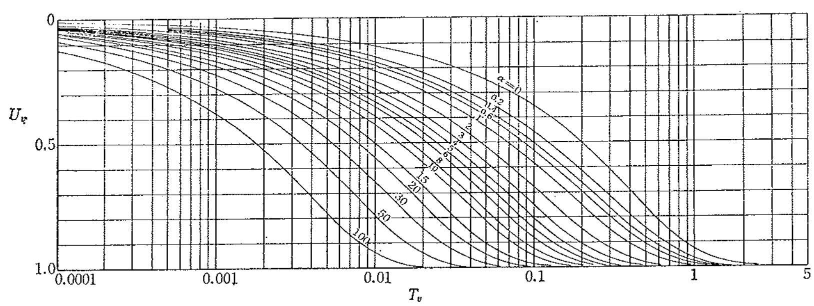


Fig. 12 The U_v−log T_v curved-ruler by the value of $b\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\beta$=500)

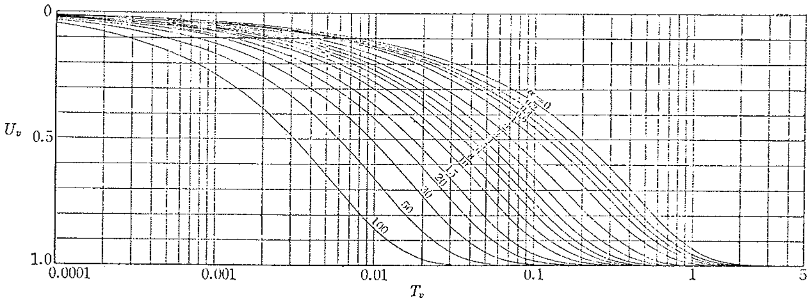
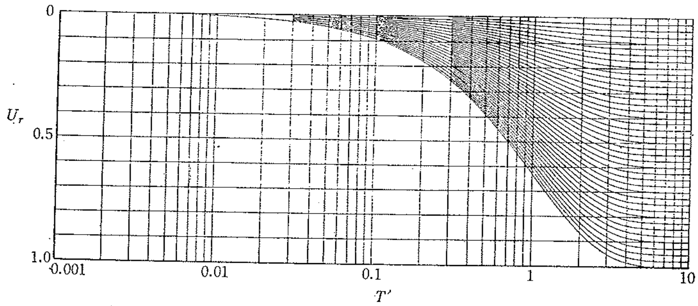


Fig. 13 The U_v—log T_v curved-ruler by the value of $b\,\beta$ of one-dimensional consolidation due to the two-dimensional dehydration (in the case of $b\,\beta=\infty$)



Note: From M. Fukuoka and M. Inada; "Study on the settlement of banks of rivers in the weak ground area." Civil Engineering Research Institute Report No. 87—1 (January 1954)

Fig. 14 The U_r—log T' curved-ruler of one-dimensional consolidation due to the two-dimensional dehydration

10. METHOD OF USING THE FIGURES OF DEGREE OF CONSOLIDATION—TIME FACTOR RELATIONS

The method of obtaining the amount of consolidation settlement in-situ by making use of the figures of $U_v-\log T_v$ curves (Fig. 4~13), results of the above analysis of consolidation due to the two-dimensional dehydration.

As the values of C_v, C_h and β of the consolidated soil layer in site are obtained by a consolidation test due to the three-dimensional dehydration, the value of $b\,\beta$ is obtained by multiplying the loading width of the consolidated layer in site b by β, and α is calculated by

$$\alpha=\frac{C_h}{C_v}\cdot\frac{H^2}{\left(\frac{b}{2}\right)^2}$$

,where H is a half of the thickness of the consolidated layer in the field.

The curve of the degree of consolidation of the primary consolidation part due to the two-dimensional dehydration in site is obtained by finding the $U_v-\log T_v$ curve corresponding to these values of $b\,\beta$ and α in Fig. 4~13.

The curve of the degree of consolidation of secondary compression part is also found in Fig. 14.

Making use of these curve of the degree of consolidation, one can obtain S, the amount of settlement of the consolidated layer in the field with any time t.

$$S=2\,HK(v+r)(U_v+U_r) \quad\cdots\cdots\cdots\cdots\cdots\cdots(10\cdot1)$$

In the equation, K is the intensity of the consolidation load of the consolidated layer in site, and the values of the degree of consolidation U_v and U_r are obtained by making use of the curve of the degree of consolidation of the primary consolidation part (the one found out from the curves in Fig. 4~13) and that of the secondary compression part (Fig. 14) for $T_v=\frac{C_v}{H^2}\cdot t$ and $T'=2\,\eta\,t$, respectively.

The consolidation constants v, r and η are also obtained by the consolidation test.

Therefore, one can get the time-settlement curve of the consolidated layer in site by calculating the amount of settlement corresponding to the time t from the equation (10·1).

11. AN APPLICATION EXAMPLE

Application of the method of analyzing one-dimensional consolidation due to the two-dimensional dehydration to field consolidation settlement.

In this section, examination is made on the analysis and the test results of consolidation settlement in the Lake Hachiro test embankment, which is thought to be the case of the one-dimensional consolidation due to the two-dimensional dehydration.

(1) The state in site

A large-scale test embankment shown in Fig. 15

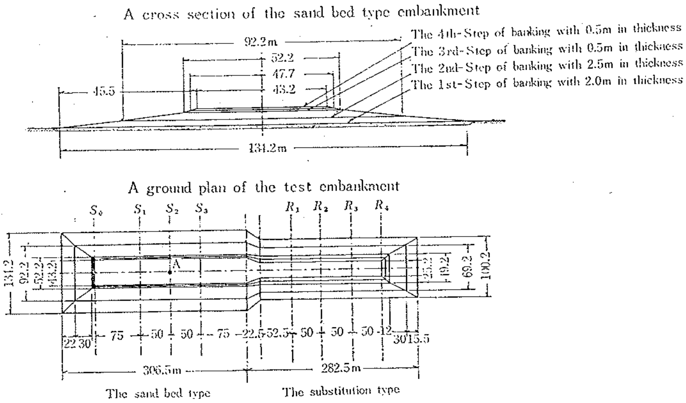


Fig. 15 Explanatory figures of the test embankment

was constructed prior to banking of the Lake Hachi-ro cofferdam embankment for reclamation in order to estimate its amount of settlement and to examine its stability against slide.

This test embankment was planned and constructed in the 33 rd year of Showa (1958), and its settlement was observed for about 3 years. The base ground of the bank is shown in Fig. 16. As seen in the figure, the ground has an almost uniform weak clay layer 29 m thick, with a sand-mixed clay layer from the depth of 18 m to 25 m included. Physical properties of the weak clay were investigated, and the test results of the soil around the depth of 10 m, which gives almost average values, are shown in Table 1.

Table 1 Summarized results of laboratory tests of weak clay at the test embankment section.

Natural moisture content	ω(%)	143.4
Specific gravity of soil particles G		2.58
Liquid limit	L.L. (%)	160.8
Plastic limit	P.L. (%)	47.2
Plasticity index P.I.		113.6
Wet density γ_t	(g/cm^3)	1.375
Dry density γ_d	(g/cm^3)	0.565
Void ratio	e	3.57
Degree of saturation	S (%)	103.6
Texture	Sand (%)	0.7
	Silt (%)	34.4
	Clay (%)	64.9

Table 2 Summarized results of consolidation tests of the above-described soil

Degree of saturation befor consolidation	(%)	97.1
Degree of saturation after consolidation	(%)	107.2
Preconsolidation load	(kg/cm^2)	0.376
Initial void ratio	e_0	4.252
Compression index	C_c	2.360
Consolidation coefficient	C_v (cm^2/sec)	* 5.90×10^{-4}
Coefficient of volume compressibility	m_v (cm^2/kg)	* 6.15×10^{-1}
Permeability coefficient	k_v (cm/sec)	* 1.021×10^{-7}
Rate of elastic deformation	v (cm^2/kg)	* 5.93×10^{-1}
Rate of plastic deformation	r (cm^2/kg)	* 2.86×10^{-1}
Creep coefficient	η (1/sec)	* 4.63×10^{-7}
$v/v+r$		* 0.675
$r/v+r$		* 0.325

The sign * means the value at the consolidation load of 0.4 kg/cm^2

Furthermore, the results of consolidation tests of the soil are shown in Table 2.

As a sand layer of 8 m in thickness lies under the weak layer, the consolidation condition of the ground is thought to be permeable through both face.

Now the method of analysis of the one-dimensional consolidation due to the two-dimensional dehydration is applied to calculation of settlement of the test embankment, to obtain the time-settlement curve and to compare it with the results of observation of settlement for examining applicability of the method.

The settlement curve observed at the center on the observation line S_2 in the sand bed type (the point A in Fig. 15) showed a considerably good results and was used in discussion.

(2) Calculation of consolidation load

Unit weight of soil in the ground is shown in Table 3.

As the groundwater level is at the ground level, the values are thought to coincide with those of the saturated unit weight, which were then used in calculating the submerged unit weight, and the overburden intensity of load for each depth was estimated. The soil profile and the overburden intensity of load observed are shown in Fig. 16.

The overburden intensity of load at the middle point of clay layer (at the depth of 14.5 m) p_0 is 5.66 t/m^2.

The values of intensity of preconsolidation load obtained by consolidation tests are plotted and shown in a dotted line in Fig. 16, which coincides comparatively well with the overburden intensity of load curve. As this suggests the fact that the clay in site may be regarded as normally consolidated, calculation of consolidation was made on the assumption that

Table 3 Unit weight of soil of the ground in-situ

Depth (m)	0~10	10~18	18~25	25~29
Unit weight of soil (t/m^3)	1.35	1.48	1.73	1.62

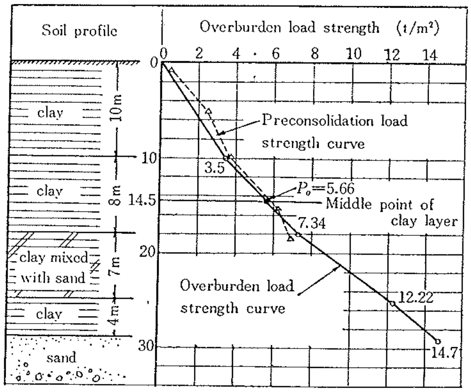


Fig. 16 Figure showing the overburden intensity of load of the original spot

consolidation of clay proceeds depending only on the consolidation load.

Furthermore, examination of the observed settlement curve made it clear that the secondary compression was shown very little effect on the settlement on those days, and therefore calculation of consolidation settlement was made only with the primary consolidation taken into consideration.

(3) Intensity of load due to banking

(i) The intensity of load due to the 1 st-step of banking

The intensity of load in the ground under the center line of the 1 st-step of embankment p_1 was calculated by regarding the banking as a trapezoid with the height of h=2.0 m, the top width of 92.2 m and the bottom width of 134.2 m, and using the unit weight of γ_t=2.0 t/m³.

The value of the dispersion coefficient μ corresponding to trapezoid load was obtained by using twice the modified μ-chart, by transposing the origin of this chart prepared according to the equation of the vertical stress under an infinite terrace,

$$p_z=\frac{\gamma h}{\pi B}(B\beta+x\alpha)=\mu\gamma h$$

,further we got the intensity of load in the ground by multiplying this value by the embankment load γh.

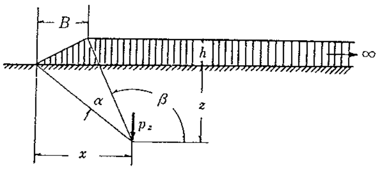


Fig. 17 Explanatory figure of symbols in the equation of normal stress under an infinite terrace

(ii) Intensity of load due to the 2 nd-step of banking

The intensity of load in the ground p_2 was obtained in the same manner as in the case of (i), by regarding the 2 nd-step of banking as a trapezoid with the height of 2.5 m, the top width of 52.2 m and the bottom width of 92.2 m.

(iii) Intensity of load due to the 3 rd and 4 th-steps of bankings

The values of the intensity of load in the ground, p_3 and p_4, were obtained in the same manner as in the case of (i), by regarding the 3 rd and 4 th-steps of bankings as trapezoids with the height of 0.5 m, the top width of 47.7 m and the bottom width of 52.2 m, and with the height of 0.5 m, the top width of 43.2 m, and the bottom width of 47.7 m, respectively.

The dispersion coefficients and the normal stress intensities in the ground corresponding to any bankings obtained in the above manner are shown in Fig. 18 and 19.

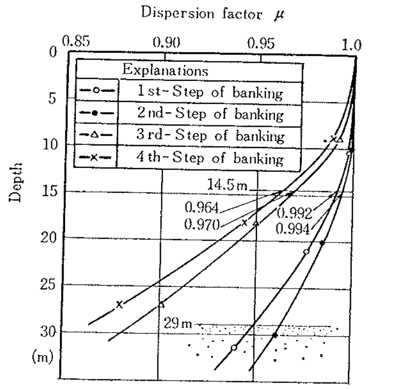


Fig. 18 Figure of stress dispersion factor in the ground

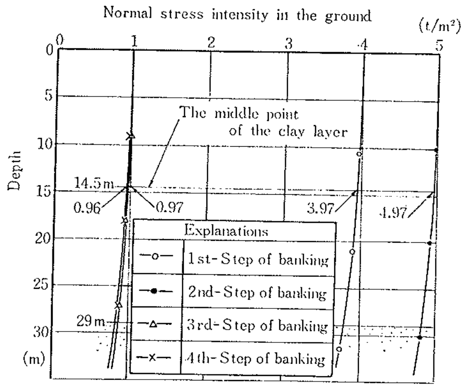


Fig. 19 Figure of normal stress in the ground due to embankment load

The values of the normal stress intensity due to the embankment load at the middle point of the clay layer (at the depth of 14.5 m) obtained from Fig. 19 are 3.97 t/m², 4.97 t/m², 0.97 t/m² and 0.96 t/m², respectively, corresponding to the 1 st-, the 2 nd-, the 3 rd- and the 4 th-steps of banking.

(4) Calculation of the final amount of settlement

(i) The final amount of settlement due to the every-step of banking

The final amout of settlement only caused by the

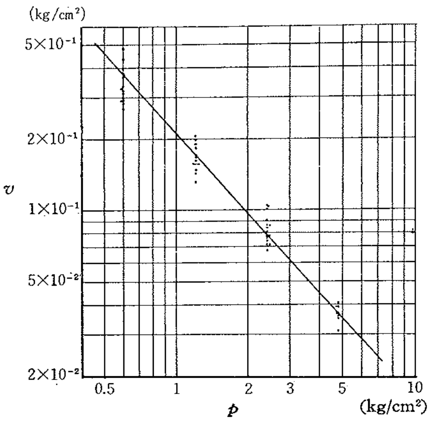


Fig. 20 The $v-p$ curve

Table 4 Calculation table of the final amount of settlement

Step of banking	$p=p_0+\Sigma p_z$ (kg/cm²)	v (cm²/kg)	$2H$ (cm)	$\Delta p=p_z$ (kg/cm²)	S (cm)
The 1st-step	0.963	2.20×10^{-1}	2,900	0.397	252.65
The 2nd-step	1.460	1.37×10^{-1}	2,900	0.497	197.46
The 3rd-Step	1.558	1.25×10^{-1}	2,900	0.098	35.53
The 4th-step	1.655	1.17×10^{-1}	2,900	0.097	32.91

primary consolidation is expressed as

$$S=2v\,\Delta pH$$

In this equation v denotes the rate of elastic deformation, and the $v-\log p$ curve obtained from consolidation tests is shown in Fig. 20.

The final amount of settlement due to any banking calculated by the use of the above equation are shown in Table 4.

The values of the final amount of settlement shown in Table 4 were calculated by neglecting the change of the stress in the ground due to the loads with the proceeding settlement of banking.

In the actual case, however, the stress intensity in the ground decreases as the banking part submerges. Especially, the correction of the final amount of settlement due to the 1 st-step of banking is needed.

(ii) Correction of the final amount of settlement due to the 1 st-step of banking

The rate of decrease of the amount of settlement according to decrease of unit intensity of load caused by the 1 st-step of banking is

$$K_n=\frac{S}{\Delta p}=638.0\ (\text{cm}^3/\text{kg})$$

Taking the bulk density of a submerged embankment to be $\gamma_{\text{sub}}=1.0\ \text{t/m}^3$ one obtains the amount of decrease of the stress in the ground according to the settlement of embankment of 1 m as

$$(\gamma_t-\gamma_{\text{sub}})\mu=(2.0-1.0)\times0.992=0.992\ \text{t/m}^2$$

Therefore, the amount of settlement according to decrease of the unit intensity of load caused by the embankment K_b is

$$K_b=\frac{1}{\mu(\gamma_t-\gamma_{\text{sub}})}=\frac{1}{0.992}\times1\,000$$

$$=1\,008.1(\text{cm}^3/\text{kg})$$

Taking the amount of settlement S_n to be balanced with the intensity of stress p_n, one obtains the following two equation.

$$S_n=K_b(p_z-p_n)$$

$$S-S_n=K_n(p_z-p_n)$$

The corrected final amount of settlement S_n is obtained from the above two equation as

$$S_n=\frac{K_b}{K_b+K_n}S=\frac{1\,008.1}{638.0+1\,008.1}\times252.65$$

$$=154.73\ \text{cm}$$

(5) Calculation of the time of consolidation settlement

The time of consolidation settlement is calculated by the use of the equation $t=\dfrac{H^2}{C_v}\cdot T_v$ and the $U_v-\log T_v$ curve obtained by application of the consolidation due to the two-dimensional dehydration.

As the values of the consolidation coefficient C_v obtained from consolidation tests show variance corresponding to the range of depth, the average consolidation coefficient $\overline{C}_v$ was calculated by using the $C_v-\log p$ curve and the following equation.

$$\overline{C}_v=\left(\frac{\Sigma H_i}{\Sigma\dfrac{H_i}{\sqrt{C_{vi}}}}\right)^2$$

,where H_i and C_{vi} mean the thickness of the i th layer and the coefficient of compressibility of the i th layer.

The $C_v-\log p$ curve and the $\overline{C}_v-\log p$ curve are shown in Fig. 21.

(i) Calculation of settlement due to the 1st-step of banking

The value of $\overline{C}_v$ corresponding to the stress intensity of 0.963 kg/cm² in the ground at the time of loading of the 1st-step of banking was obtained from

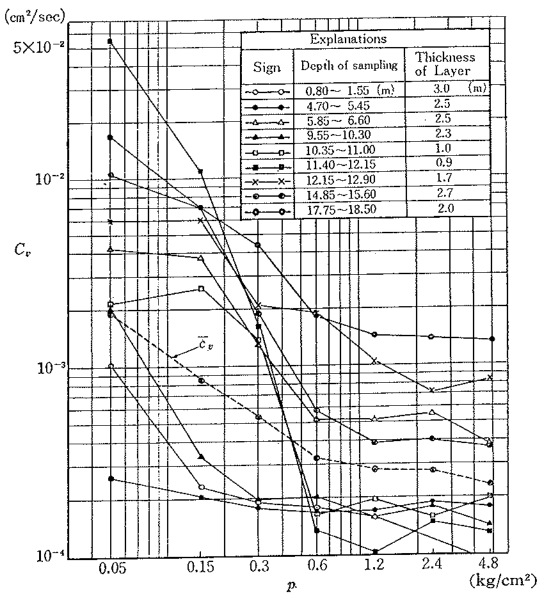


Fig. 21 $C_v-\log p$ figure

Fig. 21 as $\bar{C}_v=2.95\times10^{-4}\text{cm}^2/\text{sec}$.

Therefore the relation between the time of settlement and the time factor for this case becomes

$$t=\frac{H^2}{\bar{C}_v}T_v=\frac{(1\,450)^2}{2.95\times10^{-4}}T_v=7.127\times10^9T_v\ (\text{sec})$$
$$=1.188\times10^8T_v\ (\text{min})$$

The amount of settlement corresponding to the time needed of settlement in this case can be obtained by the use of this equation and the above-described $U_v-\log T_v$ relationship as the final amount of settlement is 154.73 cm. As the slope of the banking in site is as gentle as about 1 : 10 and the difference between the bottom width of the banking of 134.2 m and the top width of the completed banking of 43.2 m is large, the width of the consolidated soil body was assumed as 88.7 m, which is the mean value of the both.

Furthermore, thinking of the soil status in site, the designed section of the banking, the method of execution, and the amount of settlement of the banking caused by those factors, one may assume, $C_h/C_v=1\sim10$, whence

$$\alpha=\left(\frac{C_h}{C_v}\right)\left(\frac{H}{\frac{b}{2}}\right)^2=(1\sim10)\left(\frac{14.5}{\frac{88.7}{2}}\right)^2=0.11\sim1.07$$

So the author used the values of $\alpha=0$, 0.2, 0.4, 0.6, 1.0 in calculation. Further the values of $b\,\beta=$ 10, 20, 50, 100, 500, ∞, were used for the respective values of α and $b\,\beta$ on the calculation for the time needed of settlement.

In this connection, the value of α calculated by the use of the C_h and C_v values obtained from the test results of soil in site and the value of β obtained by the use of the β-value measuring apparatus are to be used afterwards in examining the present theory and its applicability.

(ii) Calculation of settlement due to the 2nd-step of banking

The value of $\bar{C}_v$ corresponding to the stress intensity of 1.46 kg/cm² in the ground at the time of loading of the 2nd-step of banking was obtained from Fig. 21 as $\bar{C}_v=2.80\times10^4\text{cm}^2/\text{sec}$.

As the 2nd-step of banking was constructed 34 days $(=4.90\times10^4\text{min.})$ after the finish of the loading of the 1st-step of banking, the relation between the time of settlement and the time factor for the case becomes

$$(t-4.90\times10^4)=\frac{H^2}{\bar{C}_v}\cdot T_v=\frac{(1\,450)^2}{2.80\times10^{-4}}$$
$$=7.509\times10^9T_v(\text{sec})$$
$$=1.251\times10^8T_v(\text{min})$$

Accordingly, $t=1.251\times10^8T_v+4.90\times10^4$ (min)

The lapsed time to any settlement in this case, may be calculated as the final amount of settlement of 197.46 cm by the use of this equation and the above-described $U_v-\log T_v$ relationship in the same manner as in the case of (i).

(iii) Calculation of settlement due to the 3rd-step of banking

The value of $\bar{C}_v$ corresponding to the intensity of 1.558 kg/cm² in the ground at the time of loading of the 3rd-step of banking was obtained from Fig. 21 as $\bar{C}_v=2.80\times10^{-4}(\text{cm}^2/\text{sec})$. As the 3rd-step of banking was filled 44 days $(=6.34\times10^4\text{min.})$ after the finish of the 1st-step of banking, the relation between the time of settlement and the time factor for this case becomes

$$t-6.34\times10^4=\frac{H^2}{\bar{C}_v}\cdot T_v=\frac{(1\,450)^2}{2.80\times10^{-4}}\cdot T_v$$
$$=7.509\times10^9T_v(\text{sec})$$
$$=1.251\times10^8T_v(\text{min})$$

Accordingly, $t=1.251\times10^8T_v+6.34\times10^4$ (min)

The lapsed time to any settlement in this case, may be calculated as the final amount of settlement of 35.53 cm by the use of this equation and the above-described $U_v-\log T_v$ relationship.

(iv) Calculation of settlement due to the 4th-step of banking

The value of $\bar{C}_v$ corresponding to the stress intensity of 1.655 kg/cm² in the ground at the time of loading of the 4th-step of banking was obtained from Fig. 21 as $\bar{C}_v=2.80\times10^{-4}(\text{cm}^2/\text{sec})$.

As the 4th-step banking was filled 379 days $(=5.46\times10^5\text{min.})$ after the finish of the 3rd-step banking, the relation between the time of settlement and the time factor for the case becomes

$$t-5.46\times10^5=\frac{H^2}{\bar{C}_v}\cdot T_v=1.251\times10^8T_v\ (\text{min})$$

Accordingly, $t=1.251\times10^8T_v+5.46\times10^5$ (min)

The lapsed time to any settlement in this case, may be calculated as the final amount of settlement of 32.91 cm by the use of this equation and the above-described $U_v-\log T_v$ relationship.

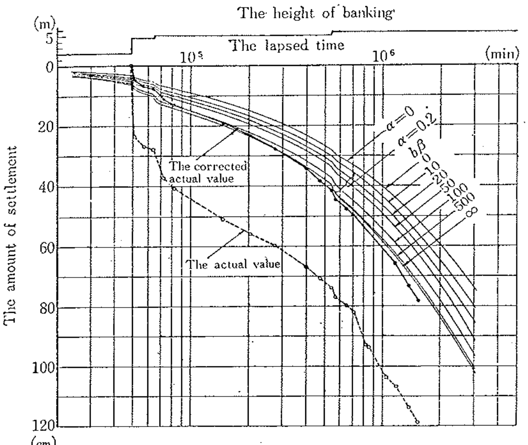
The values of the consolidation constants used in the above calculation of settlement are those measured by the technical staff of the Land Reclamation Office of the Lake Hachiro of those days.

(v) The total settlement due to all steps of the banking

The total settlements due to all steps of the banking were obtained by summing up the amount of settlement due to the respective step of banking based on the results of calculation in the section (i)$\sim$(iv).

They are shown in Fig. 22-a$\sim$Fig. 22-d. The actual settlement data in site are further shown by dotted lines.

According to Fig. 22-a$\sim$22-d, the amounts of settlement actually observed (shown by dotted lines) are 20$\sim$40 cm or so larger than the calculated values. This fact is attributed to the rapid increase of the amounts of settlement actually observed at the time of the banking. This is also proved by the fact that the amounts of settlement actually observed showed to be quite similar to that shown by the calculated



Note : The curve with $\alpha=0$ in the figure shows the values calculated according to Terzaghi's theory of one-dimensional consolidation

Fig. 22-a The settlement curves due to the respective step of embankments and those actually observed (the case of $\alpha=0.2$)

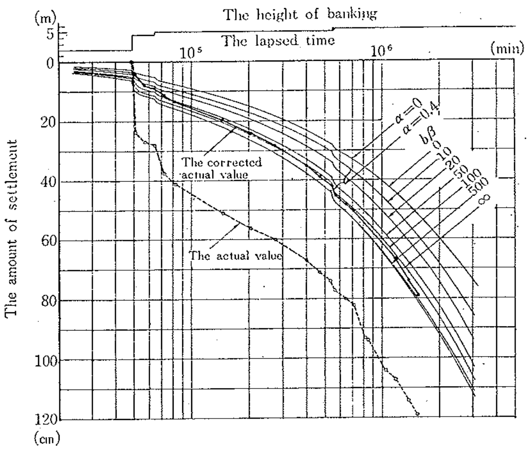


Fig. 22-b The settlement curves due to the respective step of embankments and those actually observed (the case of $\alpha=0.4$)

values after some lapse of time in banking.

The rapid increase of the amount of settlement just after the banking is thought to be attributed to shear deformation due to shear stress in the ground added by the banking, and never be attributed to consolidation deformation due to the decrease of pore water among soil particles.

Accordingly, the amount of the rapid settlement just after the banking is to be studied separately from the consolidation settlement discussed in the present paper.

The corrected actual settlement curve only due to consolidation deformation was therefore obtained by subtracting the amount of the rapid settlement from the amount of settlement actually observed, and is shown by the full lines in Fig. 22-a～22-d.

(vi) Examination of the corrected actual settle-

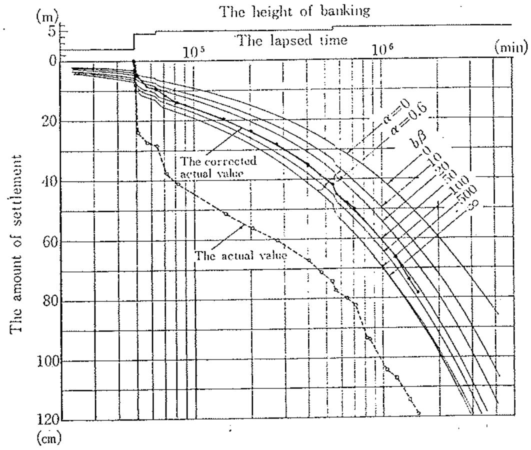


Fig. 22-c The settlement curves due to the respective step of embankments and those actually observed. (the case of $\alpha=0.6$)

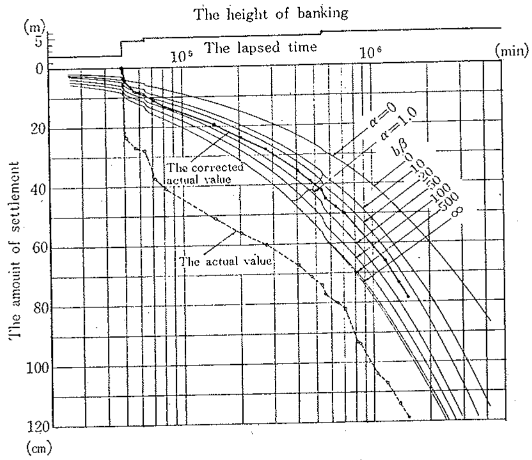


Fig. 22-d The settlement curves due to the respective step of embankments and those actually observed. (the case of $\alpha=1.0$)

ment data with the calculated amounts of settlement

First, the settlement curve in the case of $\alpha=0$ in Fig. 22-a～22-d, namely, in the case where dehydration occurs only in the vertical direction according to Terzaghi's theory, shows such a slow advance of settlement compared with the settlement curve by the corrected actual data, and clearly its advance is not coincident to the corrected actual data.

Next, examination is made by comparison of the calculated values of consolidation settlement due to the two-dimensional dehydration with the corrected actual data.

The calculated values with $\alpha=0.2$ in Fig. 22-a show a slower advance of consolidation even in the case of $b\beta=\infty$ than that shown by the corrected actual date. This suggests that the ground in site does not correspond to the case of $\alpha=0.2$, but to the case of $\alpha>0.2$.

Examining the case of $\alpha=0.4$ in Fig. 22-b, one finds the settlement curve with $b\beta=100$ coinciding well that by the corrected actual data, which suggests that consolidation occurs due to the two-dimensional dehydration. In the same manner, the settlement curves with $\alpha=0.6$, $b\beta=50$ and $\alpha=1.0$, $b\beta=30$ in Fig. 22-c and 22-d are respectively found to coincide with the corrected actual data well.

The above three couples of the α and $b\beta$ values of the cases where the settlement curves by calculation coincide well with the corrected actual data were used for calculation with the horizontal consolidation coefficient C_h and the relative dehydration rate β in site.

The results are shown in Table 5.

Table 5 Calculation table of C_h and β of the case of good coincidence with the actual settlement curve

α	$b\beta$	*C_h (cm²/sec)	β (1/cm)
0.4	100	1.05×10^{-3}	1.13×10^{-2}
0.6	50	1.57×10^{-3}	5.64×10^{-3}
1.0	30	2.62×10^{-3}	3.38×10^{-3}

* $C_h=\alpha C_v\dfrac{(b/2)^2}{H^2}=(2.80\times10^{-4})\times\dfrac{(88.7/2)^2}{14.5^2}\times\alpha$
$=2.62\times10^{-3}\alpha$ (cm²/sec)

The soil sample taken at the depth of 9.50~10.00 m in site (which shows almost the average value of C_v in the layer to the depth of 18.50 m) was subjected to the three-dimensional dehydration consolidation test.

The obtained consolidation constants are shown in Table 6.

As shown in Table 6, the values of $C_v=2.08\times10^{-4}$(cm²/sec), $C_h=9.90\times10^{-4}$(cm²/sec) and $\beta=1.33\times10^{-2}$(1/cm) were obtained with the intensity of consolidation load of 1.6 kg/cm².

The comparison of the experimental results with the actual data shown their good coincidence with the case of $\alpha=0.4$ in Table 5.

Therefore, consolidation settlement in site is expected to advance hereafter to the settlement curve of the case of $\alpha=0.4$ and $b\beta=100$, which is thought to prove applicability of analysis of consolidation due to the two-dimensional dehydration.

12. CONCLUSION

In the present paper, the author showed a method of analysis of the one-dimensional consolidation due to the two-dimensional consolidation with the secondary compression taken into consideration, which is applied to analysis of consolidation of the soil layers under the structures comparatively long in the longitudinal direction, such as banks or roads, constructed on the weak ground, where dehydration is expected to occur in the vertical and transverse direction.

To make an analysis more simply and conveniently, the author showed on the $U_v-\log T_v$ curves and the $U_r-\log T'$ curves and described the method of their application, for the calculation of settlements according to the present method mentioned above.

Furthermore, comparison to the actual settlement data of the test embankment of the Lake Hachiro with the calculated values of settlements by the present method were made, in order to examine applicability of the theory. As a results it was noticed that the amount of settlement actually observed is larger than that calculated by Terzaghi's theory of consolidation due to the one-dimensional dehydration, and that the difference between them becomes larger with the lapse of time of settlement.

On the other hand, the above-described method of analysis based on the theory of the one-dimensional consolidation due to the two-dimensional dehydration was proved to almost satisfy the settlement data actually observed in the field.

And also the present method of analysis might solve some problems on the consolidation which are difficult to be solved by the former method.

The author devotes his gratitude to members of Soil Exploitation Committee for the Reclamation of the Lake Hachiro (Chairman of those days : Dr. Takeo Mogami, Professor of University of Tokyo) who bestowed on the author many precious suggestions in the present study. The author also presents sincere thanks to Messrs. in concern of the Land Reclamation Office of the Lake Hachiro in Ministry of Agriculture & Forestry who gave the author many valuable suggestion and materials on the this research.

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Table 6 Calculation table of C_h and β by the three-dimensional dehydration consolidation test

p (kg/cm²)	C_v (cm²/sec)	$2H_n$(cm)	$\bar{H}=\dfrac{H_n+H_{n+1}}{2}$	* $\dfrac{R}{\bar{H}}$	$\left(\dfrac{R}{\bar{H}}\right)^2$	α_0	*C_h (cm²/sec)	$R\beta$	β (1/cm)
0.1	4.02×10^{-4}	6.00	2.920	1.027	1.056	5.0	2.12×10^{-3}	0.02	6.67×10^{-3}
0.2	2.66×10^{-4}	5.68	2.763	1.086	1.179	3.8	1.18×10^{-3}	0.01	3.33×10^{-3}
0.4	2.13×10^{-4}	5.37	2.578	1.164	1.354	3.2	9.23×10^{-4}	0.02	6.67×10^{-3}
0.8	2.01×10^{-4}	4.94	2.293	1.308	1.712	2.8	9.64×10^{-4}	0.03	1.00×10^{-2}
1.6	2.08×10^{-4}	4.23	1.845	1.626	2.644	1.8	9.90×10^{-4}	0.04	1.33×10^{-2}
3.2	2.05×10^{-4}	3.15	—	—	—	—	—	—	—

* $R=3.0$ cm $\quad C_h=\alpha_0\left(\dfrac{R}{\bar{H}}\right)^2\cdot C_v$

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(Received Dec. 18, 1967)