HYDRAULIC ROLES OF WASH LOAD

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SYNOPSIS

The authors investigated the hydraulic roles and characteristics of wash load in an experimental flume. The main conclusions were as follows:

1) The behavior of wash load is almost the same as the behavior of suspended load.

2) Wash load affects Kármán constant as it relates to the sediment distribution of the suspended load and the velocity profile of the flow.

3) The wash load affects the concentration, C_a, near the bed and, as this concentration is closely connected to the transport rate of the bed-material load, the transport rate is also affected.

4) The wash load is one of the important factors controlling the stability of the stream bed.

1. INTRODUCTION

Sediment load in rivers may be classified into two parts : bed-material load and wash load. Numerous studies of bed-material load have been carried out and many significant results have been obtained[1)2)]. However, except for the work of Einstein and Chien[3)], there are few studies concerning the wash load. This is due to the uncertainity of its origin and the ambiguity of its mechanism. In their experiments, Einstein and Chien investigated the mechanical law by which bed-material load and wash load are controlled, and also the difference in characteristics between them. They revealed that each load was controlled by the same law and that the transport rate of the wash load could be calculated by the bed-load function when the instantaneous bed composition was known. However, as the transport rate of the wash load depends mainly on various factors in the upstream watershed, such as geological and channel conditions, which determine the availability of wash load sediment in the river, they concluded that the wash load could be analyzed statistically only if sufficient information was available.

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When suspended bed material sediment is transported in a given hydraulic condition, it is very important for us to understand what changes take place in the water and sediment flow due to the amount of wash load added to the flow.

Main changes may be expected in the following items:

1) A change in the velocity profile and sediment distribution due to the variation of Kármán constant.

2) A change in the transport rate of the suspended sediment (bed-material load).

3) A change in bed configurations (such as dunes, flat beds, and anti-dunes).

For item 1), following Vanoni's work[4)] in 1944, many researchers have studied the variation of Kármán constant and the change of vertical velocity distribution due to the increase of suspended sediment concentration in the flow. The authors[5)] also obtained similar results by using a wash load as the suspended load.

Vanoni[4)], Ismail[6)] and Einstein and Chien[7)] pointed out that the vertical distribution of sediment concentration in the flow could be sufficiently represented by Rouse's formula[8)] in a practical sense and that the exponent, z, contained in the formula differed slightly from the measured value in experimental flumes and natural streams. Einstein and Chien[7)] reduced the discrepancy between the exponent z obtained from experimental results and that calculated theoretically by reasonably modifying the various assumptions which were introduced in the derivation of the Rouse's formula. However, some difficulties still remain in the practical application.

K. Ashida and M. Michiue[9)] studied the sediment load of nonuniform sands by flume experiments and concluded that the distribution of a suspended load of wellgraded sand could be calculated by Rouse's formula for each sub-divided particle size fraction and that the total suspended load could be calculated by adding together the results of all the calculations for each fraction.

As for item 2), T. Tsubaki[10)] and K. Ashida[9)] predicted that a decrease in the value of Kármán constant due to an increase of the wash load might considerably reduce the transport rate of the suspended load. However, this has not been proved by experimental data.

If, in fact, the wash load does affect the transport rate of the suspended sediment composed of bed material in actual streams, the present conception that the wash load does not affect the stability of the stream bed and does not have an important role in the design of a river channel may be questioned.

By their experimental study, the authors have attempted to provide explanations for the three items mentioned above.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The flume used for experiments is a recirculating type flume with glass walls, which a length of 8 meters and a width of 0.4 meters respectively.

The sand used for bed material is nearly uniform with a mean diameter of 0.018 cm and a specific gravity of 2.65 (sand A). On the other hand, two kinds of very fine sand are used for the wash load, both are nearly uniform silica sand, one with a mean diameter of 0.005 cm and a specific gravity of 2.65 (sand B), and the other with a mean diameter of 0.0015 cm and a specific gravity of 2.65 (sand C). Their grain size distribution curves are shown in Fig. 1.

The procedure for each experiment contains the following four steps and hydraulic conditions are kept uniform throughout the experiment.

1) Clear water (without suspended load and wash load) is circulated and the vertical velocity distribution is measured.

2) The bed-material load is added to the flow and the vertical velocity distribution, the mean sediment concentration and the distribution of sediment concentration are measured.

3) The wash load is added to the flow of 2) and the measurements stated in step 2) are repeated.

4) Then, the wash load concentration is increased and the measurements in step 2) are repeated (see Table 1 for a summary of hydraulic and sediment transport data).

Step 1), using clear water only without suspended load or wash load, is carried out in order to furnish a comparison of hydraulic conditions between a clear water flow and a sediment laden flow.

All of these measurements are made after the steady states of flow are attained by circulating water with the sediment in the system.

These experiments are carried out in non-equilibrium and equilibrium states. Here, non-equilibrium state means the flow state in which the flume bed is exposed without any sediment deposition; that is, the flow does not transport enough sediment. The equilibrium state means the flow and sediment conditions in which the flow is transporting enough sediment without any further deposition and scouring on the sand bed.

In the equilibrium state, sand A is used for bed material and bed-material load and sand B or sand C is used for wash load. Sand A can be easily divided from sand B or C in the sediment mixture by meams of sieve analysis (Fig. 1).

Throughout all experiments, the fine sediment (sand B or C) is kept in suspension without any deposition on the bed and considered sufficiently as wash load.

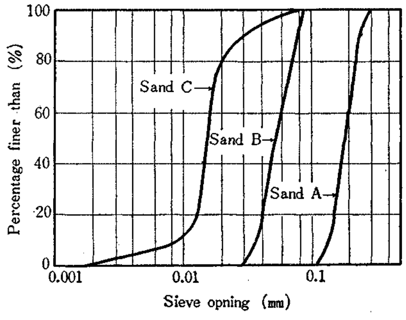

Fig. 1 Grain size distribution curves.

3. DISCUSSION OF RESULTS

3.1 Kármán constant

The authors[5] studied the variation of Kármán constant due to the extremely fine sand (sand C) in suspension and the results have already been published. They are summarized as follows.

The variation of Kármán constant, K, due to the sediment concentration in the flow, is represented by Shimura's formula[11] or Hino's formula[12] by using the proper value of Kármán constant K_0 for clear water.

From the authors' experiment, it is found that K_0 is affected by the Froude number, F_r, of the flow*.

Shimura's formula is

$$\frac{1}{K}=\frac{1}{K_0}+\frac{\beta(\gamma-1)g\bar{C}w_0(h-\delta)}{U_*^3\left(2.3\log\frac{h}{\delta}-1\right)} \quad \cdots\cdots\cdots(1)$$

where,

K_0 : K for clear water flow

$\bar{C}$: suspended sediment concentration (in volu-

* The physical meaning of the variation of K_0 due to Froude number will be discussed in a future publication.

me ratio)

w_0 : fall velocity of sand in still water

h : flow depth

U_* : shear velocity

$$\delta=\frac{11.6\nu}{U_*}$$

ν : kinematic viscosity of water

β : constant (=4.8 given by Shimura)

Eq. (1) was derived theoretically for sediment with uniform grain size. For sediment with a wide range of particle size, Kármán canstant can be also expressed in the form of Eq. (2) if the sediment transfer coefficient ε_s for each grain size of non-uniform sediment is assumed to be constant.

That is, K for sediment mixture can be theoretically expressed as

$$\frac{1}{K}=\frac{1}{K_0}+\frac{\beta(\gamma-1)g(h-\delta)\Sigma\, C_r w_r}{U_*^3\left(2.3\log\frac{h}{\delta}-1\right)} \quad \cdots\cdots(2)$$

where C_r and w_r are the sediment concentration and the fall velocity of the sediment with the grain size d_r respectively.

However, it was found from the authors' study[5] that the values of K calculated from velocity distributino are somewhat less than the values calculated by Eq. (1). After the experimental study, the relation between

$$\frac{1}{K} \text{ and } \frac{(\gamma-1)g\bar{C}w_0(h-\delta)}{U_*^3\left(2.3\log\frac{h}{\delta}-1\right)}$$

is nearly linear. Therefore, these differences are considered to be caused mainly by the value of β.

If sediment deposition does not take place on the flume bed (non-equilibrium state) or the clear water is made to flow over exactly the same bed as the sediment laden flow, as performed by Vanoni and Nomicos[13], the effect of F_r relating to K is almost the same in both the clear water flow and the sediment laden flow because the hydraulic conditions are the same.

Therefore, it is considered that Kármán constant of the sediment laden flow depends only on the turbulent characteristic parameter represented by the second term on the right side in Eq. (2), if $1/K_0$ of the clear water flow, with the same hydraulic conditions and bed configurations as in the sediment laden flow, is subtracted from $1/K$.

As a result, $\beta=9.0$ may be obtained as the proper value by plotting the authors' and Vanoni and Nomicos's experimental data in the relation between

$$\left(\frac{1}{K_v}-\frac{1}{K_0}\right) \text{ and } \frac{(\gamma-1)g(h-\delta)\Sigma\, C_r w_r}{U_*^3\left(2.3\log\frac{h}{\delta}-1\right)}$$

(Fig. 2).

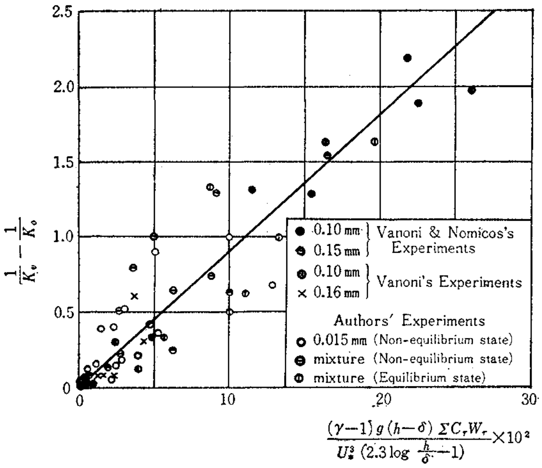

Fig. 2 Relation between K_v and dimensionless parameter of sediment concentration.

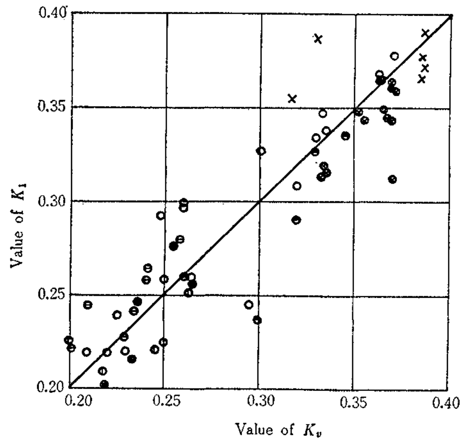

Fig. 3 Comparison between K_v and K_1

Fig. 3 shows a comparison between the values of Kármán constant K_v, estimated from the measured velocity distribution and the values K_1, calculated by Eq. (2) using $\beta=9.0$.

From Fig. 3 it is seen that K-value may be reasonably estimated by Eq. (2) using $\beta=9.0$ for any kind of sediment.

For the equilibrium state it is difficult to determine K_0 for the comparatibly clear water flow over a fixed bed of the same configuration as in the sediment laden flow. Therefore, in this study the values of K_0 obtained from the clear water flow experiments on the smooth bed with the same F_r are substituted for K_0 in Eq. (2). In respect of Vanoni's data[4], however, the values of K_0 indicated in his paper are used.

In Fig. 3, the scattering of the plots in the equilibrium state is slightly larger than that in non-equilibrium state and that of Vanoni and Nomicos's experiment due to the inaccuracy of the estimated

K_0. The value of K, however, may be sufficiently expressed by Eq. (2), if the proper values of K_0 are adopted for both the states of equilibrium and non-equilibrium.

3.2 Vertical distribution of sediment

In the two-dimensional flow, a fundamental equation for the distribution of sediment concentration is given as follows:

$$\frac{d}{dy}\left(\varepsilon_s\frac{dc}{dy}\right)+\frac{d}{dy}(w_0C)=0 \quad\cdots\cdots\cdots\cdots\cdots(3)$$

where ε_s is a sediment transfer coefficient.

Integrating Eq. (3),

$$\varepsilon_s\frac{dc}{dy}+w_0C+A=0 \quad\cdots\cdots\cdots\cdots\cdots(4)$$

where A is an integral constant.

It is assumed that ε_s is equal to the momentum transfer coefficient and the shear stress is proportional to the distance from the bed as Rouse[8] has stated in his theory.

Then,

$$\tau=\rho\varepsilon_s\frac{du}{dy} \quad\cdots\cdots\cdots\cdots\cdots(5)$$

$$\tau=\tau_0\left(1-\frac{y}{h}\right) \quad\cdots\cdots\cdots\cdots\cdots(6)$$

$$\frac{du}{dy}=\frac{U_*}{Ky} \quad\cdots\cdots\cdots\cdots\cdots(7)$$

Therefore,

$$\varepsilon_s=KU_*y\left\{1-\left(\frac{y}{h}\right)\right\} \quad\cdots\cdots\cdots\cdots\cdots(8)$$

Substituting Eq. (8) into Eq. (4), the following relation is obtained:

$$\left.\begin{aligned} C&=\left(\frac{h-y}{y}\cdot\frac{a}{h-a}\right)^z\cdot C_a\\ &\quad+\frac{A}{w_0}\left\{\left(\frac{h-y}{y}\cdot\frac{a}{h-a}\right)^z-1\right\}\\ \frac{A}{w_0}&=\frac{C_b-\left(\frac{h-b}{b}\cdot\frac{a}{h-a}\right)^z\cdot C_a}{\left\{\left(\frac{h-b}{b}\cdot\frac{a}{h-a}\right)^z-1\right\}}\\ z&=\frac{w_0}{KU_*}\end{aligned}\right\} \quad\cdots\cdots\cdots(9)$$

where

C : sediment concentration at y.

y : distance from the bed surface.

a : reference height from the bed.

C_a : concentration at $y=a$.

C_b : concentration at $y=b$.

If it is assumed that the concentration at the water surface is zero, i.e., $A=0$, then Eq. (9) is transformed into

$$\frac{C}{C_a}=\left(\frac{h-y}{y}\cdot\frac{a}{h-a}\right)^z=H^z \quad\cdots\cdots\cdots\cdots\cdots(10)$$

Eq. (10) is Rouse's formula for the distribution of sediment concentration in the two-dimensional flow. (see Appendix)

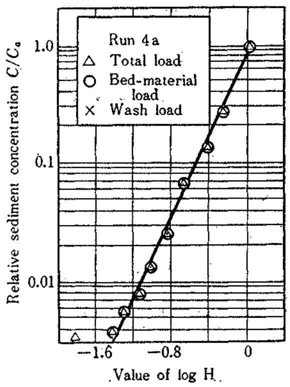

Fig. 4 Relation between measured sediment concentration and the relative height H in Eq. (10).

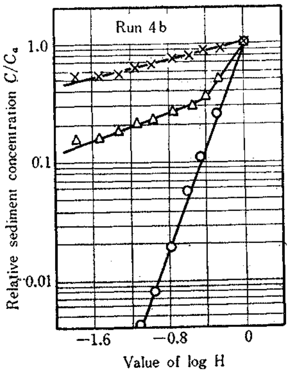

Fig. 5 Relation between measured sediment concentration and the relative height H in Eq. (10).

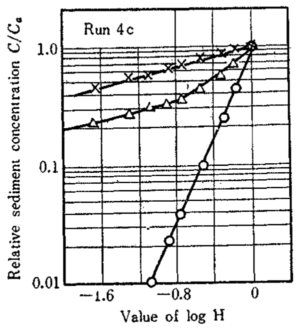

Fig. 6 Relation between measured sediment concentration and the relative height H in Eq. (10).

The applicability of the Rouse type function (the relation between C/C_a and H) is checked by using the observed sediment concentration in Figs. 4, 5 and 6. As a result, the distribution of bed-material load or wash load can be represented by the Rouse type function fairly well as shown in the figures, but the distribution of the total load (bed-material load and wash load) cannot be represented by the

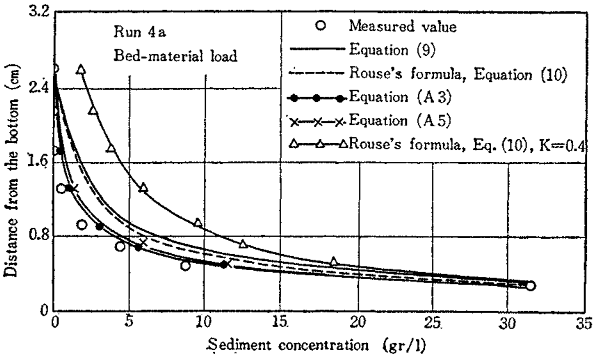

Fig. 7 Comparison between sediment concentration of measured bed-material load and that of bed-material load calculated by Equations (9), (10), (A 3) and (A 5).

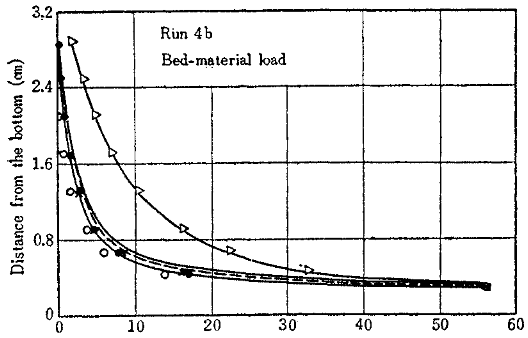

Fig. 8 Comparison between sediment concentration of measured bed-material load and that of bed-material load calculated by Equations (9), (10), (A 3) and (A 5).

Rouse type function when the wash load concentration becomes high, because the wash load is nearly uniformly distributed in the vertical. Therefore, it may be said that the Rouse type function is applicable in representing the distribution of the sediment divided by the particle sizes, even if the sediment is composed of sediment mixture or the flow carries a large amount of wash load.

The comparisons between the distribution curves of sediment concentration measured in the flume and those calculated by Eq. (9) and Eq. (10) are shown in Figs. 7, 8 and 9, where the values of K used in Eq. (9) and Eq. (10) are determined by Eq. (7). From these figures, it can be seen that the constant A can be neglected as the concentration near the water surface is nearly zero for the bed-material load and Eq. (10) is nearly equal to Eq. (9).

However, the vertical distribution of the wash load is nearly uniform and the concentration near the water surface is considerably high. Therefore, Rouse's formula (Eq. (10)) is no longer applicable to very fine sediment such as wash load.

From these facts it can be seen that the distribution of a suspended load with wide range of particle size does not satisfy Eq. (10) because the observed z is not the same as the calculated z. It may not be possible to determine the correct sediment distribution using Eq. (10) even if the concentration C_a near the bed with respect to every particle size is obtained. While

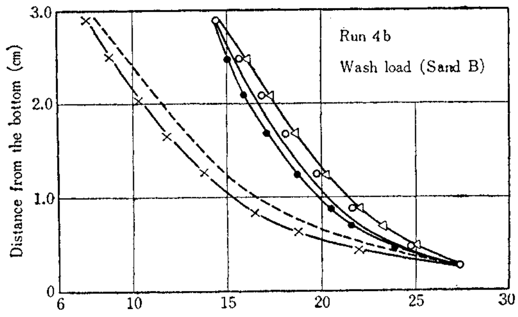

Fig. 9 Comparison between sediment concentration of measured wash load and that of wash load calculated by Equations (9),(10), (A 3) and (A 5).

it is seen from the figures that the values calculated by Eq. (9) agree fairly well with the measured values covering all particle sizes, the concentration near the water surface should be taken into account by introducing the integral constant A.

Figure 10 shows a comparison between the measured concentration of total load and the concentration due to the superposition of bed-material load and wash load calculated by Eq. (9) and Eq. (10).

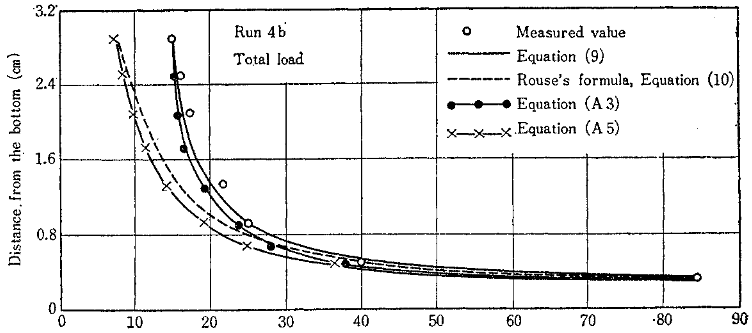

Fig. 10 Comparison between measured values of the concentration of sediment mixture and calculated values of total load.

The sediment distribution of total load can be determined by Eq. (9) with sufficient accuracy, whatever the particle size distribution of sediment mixture may be, if the concentration C_a near the bed and the concentration C_b near the water surface are obtained.

3.3 Variation of bed-material load due to wash load

Where an equilibrium state is attained with regard to the flow and the transported sediment rate, the sediment distribution curves of three experimental cases are shown in Fig. 11, in which the total load is divided into bed-material load and wash load. The three cases are:

ⓐ : Bed-material load only in the flow.

ⓑ : ⓐ+wash load.

ⓒ : ⓑ+wash load in excess.

The authors carried out the experiments for various bed configurations, such as dune, flat bed, and anti-dune, since the suspended load may be affected by the bed configuration. Consequently the concentration distribution of bed-material load increases according to the increase of wash load rate as seen in Fig. 11.

Comparisons among ⓐ, ⓑ and ⓒ should be made under the same bed conditions and the same hydraulic conditions of the flow. However, there were no noticeable changes among them except for runs 5 and 6.

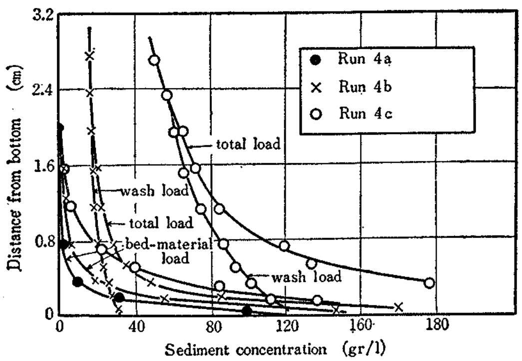

Fig. 11 Sediment concentration of bed-material load, wash load and total load.

Table 1 shows a summary of hydraulic and sediment transport data from the authors' experiments.

It may be said from the data that the transport rate of bed-material load increases by a considerable amount according to an increase in wash load rate.

From the fact mentioned above, it may be considered that the addition of wash load in the flow makes the velocity and concentration gradient steeper.

The following two experiments were carried out

Table 1 Summary of the experimental results.

Run No.	States of sediment	Bed configuration	Discharge Q in l/sec	Depth h in cm	Average velocity U_m in cm/sec	Shear velocity U_* in cm/sec	Froude number F_r	Average concentration of bed material load in gr/l	Average concentration of wash load in gr/l	Kármán constant K	Kinds of Sand
1a	Suspended load	Dune	8	5.0	40.0	4.3	0.58	0.82	0	0.320	Sand A
1b	Suspended and wash load	Dune	8	4.8	41.6	4.2	0.60	1.25	10.2	0.295	Sand A Sand C
1c	Suspended and wash load	Dune	8	4.8	41.6	4.2	0.60	1.78	22.0	0.265	Sand A Sand C
2a	Suspended load	Flat bed	17	5.5	77.2	6.0	1.05	1.10	0	0.272	Sand A
2b	Suspended and wash load	Flat bed	17	5.5	77.2	6.0	1.05	1.40	4.7	0.255	Sand A Sand C
2c	Suspended and wash load	Flat bed	17	5.5	77.2	6.0	1.05	2.74	21.5	0.240	Sand A Sand C
3a	Suspended load	Anti-dune	13	3.9	83.3	3.9	1.34	5.0	0	0.250	Sand A
3b	Suspended and wash load	Anti-dune	13	3.9	83.3	3.9	1.34	5.7	11.0	0.245	Sand A Sand C
3c	Suspended and wash load	Anti-dune	13	3.9	83.3	3.9	1.34	6.6	22.3	0.224	Sand A Sand C
4a	Suspended load	Anti-dune	13.5	3.85	88.0	4.4	1.44	3.2	0	0.267	Sand A
4b	Suspended and wash load	Anti-dune	13.5	3.85	88.0	4.4	1.44	4.3	18.0	0.200	Sand A Sand B
4c	Suspended and wash load	Anti-dune	13.5	3.85	88.0	4.4	1.44	7.7	61.0	0.190	Sand A Sand B
5a	Suspended laod	Dune	8	4.8	42.0	2.4	0.70	0.8	0	—	Sand A
5b	Suspended and wash load	Flat-bed	8	3.3	60.6	3.2	1.05	2.4	5.7	—	Sand A Sand B
5c	Suspended and wash load	Flat-bed	8	3.3	60.6	3.2	1.05	2.9	21.4	—	Sand A Sand B
6a	Suspended load	Flat-bed	17.5	5.1	85.8	6.4	1.20	4.3	0	—	Sand A
6b	Suspended and wash load	Anti-dune	17.5	4.8	91.0	—	1.32	5.2	19.7	—	Sand A Sand B
6c	Suspended and wash load	Anti-dune	17.5	4.8	91.0	—	1.32	7.3	52.3	—	Sand A Sand B

to ascertain the deduction mentioned above.

The first was a measurement of tractive force near the bed by means of a wire-strain gauge. A wire-strain gauge which was completely watertight and insulated was attached to a very thin and elastic stainless steel plate 13 mm long, 3.5 mm wide, and 0.03 mm thick. This plate protruded upwards 4 mm from the bottom of the flume. Clear water was introduced into the flume and the deflection of the gauge was measured by the dynamic strain meter. This deflection may represent the tractive force of the flow near the bed. Subsequently, fine sand(C-sand) was added to the flow and the deflection was measured by the same manner in the state where the fine sand was not deposited on the flume bed.

It was confirmed that the deflection of the gauge with wash load is always larger than that with clear water flow under the same hydraulic conditions.

The second experiment was the direct observation of movement of sand. A coloured uniform sand with a mean size of 1.5 mm and a specific gravity of 2.65 was spread out over the full width of the flume for one meter range in the central part of the flume bed and the same quality, differing only in that it was not coloured, was spread out in the other parts.

Clear water was introduced into the flume in order that the tractive force might be kept critical.

Maintaining the hydraulic conditions constant, the investigation was carried out to determine if the coloured sand would be carried away or not when the wash load was added to the flow.

In this experiment it was found the coloured sand was not moved in the clear water flow, but it was clearly moved when the wash load was added.

According to these two experiments it became clear that the tractive force was increased and the sediment concentration C_a near the bottom was increased as the wash load was increased in the flow.

The reason the tractive force is increased by the addition of wash load under the same hydraulic condition is as follows.

The tractive force τ_0 acting on the bed is expressed by Eq. (11) in a uniform flow.

$$\left.\begin{aligned}\tau_0&=\bar{\rho}\,ghs\\ \bar{\rho}&=\rho_0\{1+(\gamma-1)C\}\end{aligned}\right\}\quad\cdots\cdots(11)$$

where $\bar{\rho}$ is the specific density of the flow. With the addition of the wash load, the following changes are found in the flow.

a) $\bar{\rho}$ is increased, but the flow depth h and the slope of water surface s do not change noticeably.

b) The velocity gradient near the bed becomes steeper as the rate of wash load in the flow is increased. However, there has been very little research regarding this change in velocity gradient near the bed.

From the facts mentioned above, the tractive force on the bed is increased by adding the wash load to the flow.

Now, we will consider the energy dissipation process within a control volume boundary enclosed by line ABCD in Fig. 12. The difference between energy entering the control volume boundary and that leaving it may be expressed as $\tau_0 \cdot U_m$.

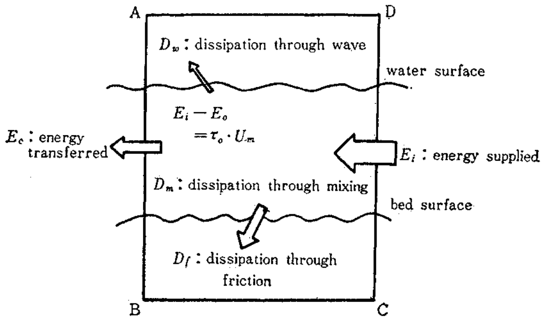

Fig. 12 Energy dissipation process.

This energy balances with total energy composed of the following three parts dissipated within the control volume boundary.

1) Energy dissipation due to friction on the bed, D_f.

2) Energy dissipation due to internal mixing in the fluid, D_m.

3) Energy dissipation due to wave making resistance at the water surface, D_w. Therefore,

$$\bar{\rho}\,ghs\cdot U_m=D_f+D_m+D_w\quad\cdots\cdots(12)$$

The value of the left side in Eq. (12) is increased by adding the wash load as stated before. In the right side, the change of energy dissipation due to wave making resistance need not to be taken into consideration because, according to the experiments*, there is almost no change in D_w whether or not the wash load is added to the clear water flow.

It is found from the experiments that the velocity gradient in the very thin layer near the bed becomes steeper but the velocity distribution above this layer becomes more uniform by adding the wash load to the flow.

As a whole, the energy dissipation, D_m, due to the internal mixing of fluid may be reduced because the energy dissipation in the upper thick layer will be decreased although the dissipation in the lower thin layer may be increased by adding the wash

* The authors will discuss this subject in another publication.

load.

M. Hino[14] has already investigated this subject by his theory and has showed that the decrease of the energy dissipation of the sediment laden flow with the smooth bed is due to internal mixing. Therefore, it may be considered that the energy dissipation D_f on the bed is increased since $\tau_0 \cdot U_m$ is increased and, furthermore, the energy dissipation due to internal mixing is decreased by adding the wash load. The increase of D_f means an increase in the tractive force.

Considering these results and using the measured values, the authors examined the transport rate of bed-material load in the same way as Einstein.

The transport rate, q_s, of the suspended load per unit width is given by Eq. (13)

$$q_s=\int_a^h Cudy \cdots\cdots(13)$$

In Eq. (13), C is expressed by Eq. (10) since it was confirmed that the distribution of the suspended load almost satisfied Rouse's formula. For the velocity profile the logarithmic law was used. Eq. (13) then becomes

$$q_s=4.63\, C_a U_* a \left\{\frac{U_m}{U_*} I_1+\frac{1}{K}(I_1+I_2)\right\} \cdots(14)$$

where I_1 and I_2 are given by Einstein's diagrams.

Shear velocity U_*, depth h and average velocity U_m do not change noticeably with the addition of the wash load except for dune range ($F_r=0.7-0.9$) and flat bed range in the neighbourhood of $F_r=1.2$. If the reference height, a, is assumed to be constant, I_1 and I_2 are functions of z only, where z contains K. Therefore, the inside terms of the bracket in Eq. (14) are functions of K only.

Therefore, q_s in Eq. (14) should be affected by changes of K and C_a.

The results of these computations are shown on Column ⑧ in Table 2. In these computations, the values of C_a and K are calculated from the observed data.

From Table 2, it may be accepted that the transport rate of the bed-material load is increased as the wash load rate is increased.

Column ⑥ in Table 2 shows the results of the same computation in which the constant C_a and the same value for K as in the former case are used. In this case the increase of the wash load makes the transport rate of the bed-material load decrease, which does not agree with the experimental results. The Lane-Kalinske equation[2], which is now in use to express the concentration near the bed, is obtained on the assumption that the falling rate of particles is equal to the rate of particles leaving the bed. It

Table 2 Computations of suspended load.

case	① K	② z	③ $A=\frac{a}{h}$	④ I_1	⑤ I_2	⑥ $\frac{q_s}{4.63 C_a U_* a}$	⑦ C_a (gr/l)	⑧ q_s (gr/sec·cm)
ⓐ	0.267	1.42	8×10^{-2}	0.230	0.400	3.963	31.6	0.76
			1×10^{-1}	0.215	0.350	3.794		0.73
			5×10^{-2}	0.266	0.540	4.294		0.82
ⓑ	0.200	1.90	8×10^{-2}	0.160	0.310	2.450	56.0	0.84
			1×10^{-1}	0.150	0.265	2.425		0.83
			5×10^{-2}	0.180	0.420	2.400		0.82
ⓒ	0.190	2.00	8×10^{-2}	0.150	0.300	2.180	94.0	1.25
			1×10^{-1}	0.138	0.250	2.170		1.25
			5×10^{-2}	0.165	0.390	2.120		1.22

is expressed by Eq. (15).

$$C_a \propto 5.55\, \Delta F(w_0)\left[\frac{1}{2}\left(\frac{U_*}{w_0}\right)\exp\left\{-\left(\frac{w_0}{U_*}\right)^2\right\}\right]^{1.61} \cdots\cdots(15)$$

where $\Delta F(w_0)$ is the portion of a particle size of which the fall velocity is w_0 in the bed material.

It is reasonable that the vertical velocity component which removes sand particles from the bed and keeps them in suspension is due to the turbulent velocity. However, it is a little unreasonable that the vertical velocity $\sqrt{\overline{v^2}}$ is proportional to the shear velocity $U_*\left(=\sqrt{\frac{\tau_0}{\rho}}\right)$ as Lane and Kalinske predicted, because the effect of changes in the velocity gradient by the addition of wash load is not considered.

This gives us the results the concentration near the bed is determined by the bed composition and U_* only and that it is independent of the presence of the wash load. However, it was found that the concentration of the bed-material load near the bed was affected by adding the wash load to the flow. Therefore, the effect of du/dy should be taken into account in Eq. (15). Now, the vertical velocity, v, due to turbulence

$$v=l\frac{du}{dy}=Ky\frac{du}{dy}$$

Reynolds stress is

$$\tau=\rho\, l^2\frac{du}{dy}\left|\frac{du}{dy}\right|$$

Using these relations and calculating Eq. (15) in a similar way as Lane-Kalinske[2], the concentration C_a of the bed-material load near the bed is expressed by Eq. (16).

$$\frac{C_a}{\Delta F(w_0)}\propto\frac{1}{2}\left(\frac{1}{w_0}l_a\frac{du}{dy}\right)\exp\left[-\left(\frac{w_0}{l_a\frac{du}{dy}}\right)^2\right] \cdots\cdots(16)$$

where l_a is a mixing length at reference level, a, and is equal to $K\cdot a$.

Let the bed-shear be τ_1 (run a) for the flow with a bed-material load only, and let it be τ_2 (runs b, c)

when the wash load is added to the flow. The relation $\frac{\tau_2}{\tau_1} > 1$ is obtained because the bed-shear is increased as the wash load is added according to the experimental results.

Therefore,

$$\frac{\tau_2}{\tau_1} = \frac{l_2^2\left[\left(\frac{du}{dy}\right)_2\right]^2}{l_1^2\left[\left(\frac{du}{dy}\right)_1\right]^2} = \left(\frac{K_2}{K_1}\right)^2\left[\frac{\left(\frac{du}{dy}\right)_2}{\left(\frac{du}{dy}\right)_1}\right]^2 > 1 \qquad \cdots\cdots(17)$$

Thus, we obtain the ratio of concentration near the bed C_{a1} and C_{a2}.

$$\frac{C_{a2}}{C_{a1}} = \frac{K_2}{K_1}\frac{\left(\frac{du}{dy}\right)_2}{\left(\frac{du}{dy}\right)_1} \times e^{\left(\frac{w_0}{a}\right)^2 \frac{1}{K_1^2\left(\left(\frac{du}{dy}\right)_1\right)^2}\left[1-\left(\frac{K_1}{K_2}\frac{\left(\frac{du}{dv}\right)_1}{\left(\frac{du}{dy}\right)_2}\right)^2\right]} > 1 \qquad \cdots\cdots(18)$$

The subscripts of C_a and K correspond to the subscript of τ. Eq. (18) shows that the concentration C_a near the bed increases according to the increase of wash load. This is regarded as the increase of the tractive force on the bed.

Therefore, it is important for us to know how to estimate the changes of du/dy resulting form the changes of the sediment concentration.

3.4 The change of bed configurations

Many researchers[16)17)] since Gilbert[15)] have studied the relationship between bed configurations and the hydraulic characteristics of the flow.

But the authors could not find any works on the changes of bed configurations due to the wash load. In the previous section, it was found that if the wash load is added to the flow in the equilibrium state, the tractive force on the bed is increased and the transport rate of bed-material load is also increased.

In this section, the authors investigate the changes of bed configurations before and after the addition of wash load. The authors' experiments are of six kinds. The bed configurations before the addition of wash load are in dune, flat bed, and anti-dune regions.

Before and after the wash load is added to the flow, the changes in bed configurations of dune (except for dune of F_r=0.7-0.9), of flat bed (except for flat bed in the neighbourhood of F_r=1.2) and of anti-dune are not recognized. Therefore, the change of hydraulic characteristics is not also recognized except for the cases stated below.

The bed configuration in the dune region of F_r= 0.7-0.9 changes into a flat bed by a slight increase in the tractive force since dunes in this region are of a relatively long wave length and a small wave height and the flow in this region is unstable.

The change of the bed configuration from dune to flat bed reduces the flow resitance and increases the transport rate of the bed-material load.

The bed configuration is transformed from a flat bed into an anti-dune by adding the wash load to the flow around F_s=1.2.

Although only a few experiments were conducted, all changes of the bed configurations occur at the boundaries between dune and flat bed, or flat bed and anti-dune. This is due to the increase of the tractive force by the addition of the wash load.

Conclusions

1) The wash load does not differ from the bed-material load regarding hydraulic roles on the flow.

2) As the wash load is increased, the velocity gradient in the vertical becomes steeper and the tractive force near the bed is increased. As a result the transport rate of the bed-material load is increased.

3) The wash load as well as bed-material load should be taken into consideration regarding the stability of stream bed and the design of river channel.

4) Kármán constant of the sediment laden flow can be determined by Shimura's formula or Hino's formula when there is a sediment mixture with a wide range of particle size in the flow as well as when there is uniform sediment if the variation of Kármán constant for clear water flow with regard to the Froude number is taken into account.

5) The Rouse type function can be applied to the uniform sediment flow, even if it is very fine sand. For a nonuniform sediment with a wide range of particle size, the Rouse type function is also applicable when the sediment is divided into each sub-divided particle size fraction.

6) Rouse's formula (Eq. (10)) does not give proper values except in the case of bed-material load. For all cases, Eq. (9) should be used instead of Eq. (10). The sediment distribution of the total load can be calculated by superposity the sediment distribution for each particle size.

7) With the addition of the wash load the bed configurations do not change in most cases. However, they do change in some particular cases such as from dunes (F_r=0.7−0.9) to flat beds and from flat beds (nearly F_r=1.2) to anti-dunes.

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Appendix

It should be pointed out that there is an inconsistency in the derivation of Eq. (9) in Section (3.2), i.e., in the diffusion coefficient (Eq. (5)) the shear stress has a linear distribution, while in the velocity gradient given by Eq. (7) the shearing stress is constant throughout the vertical section. This fact has been already pointed out by Hunt[18].

Accordingly, if Eq. (6) is used for the distribution of shear stress, Eq. (7) should be replaced b yEq. (A 1).

$$\frac{du}{dy}=\frac{U_*}{Ky}\left(1-\frac{y}{h}\right)^{1/2} \cdots\cdots\cdots\cdots (A1)$$

Therefore,

$$\varepsilon_s=KU_*y\left(1-\frac{y}{h}\right)^{1/2} \cdots\cdots\cdots\cdots (A2)$$

Eq. (A 3) can be obtained in the same manner from Eq. (4) and Eq. (A 2)

$$\left.\begin{aligned} C&=\left(\frac{\sqrt{1-\frac{y}{h}}+1}{\sqrt{1-\frac{y}{h}}-1}\cdot\frac{\sqrt{1-\frac{a}{h}}-1}{\sqrt{1-\frac{a}{h}}+1}\right)^z\cdot C_a \\ &\quad+\frac{A}{w_0}\left\{\left(\frac{\sqrt{1-\frac{y}{h}}+1}{\sqrt{1-\frac{y}{h}}-1}\cdot\frac{\sqrt{1-\frac{a}{h}}-1}{\sqrt{1-\frac{a}{h}}+1}\right)^z-1\right\} \\ \frac{A}{w_0}&=\frac{C_b-\left(\frac{\sqrt{1-\frac{b}{h}}+1}{\sqrt{1-\frac{b}{h}}-1}\cdot\frac{\sqrt{1-\frac{a}{h}}-1}{\sqrt{1-\frac{a}{h}}+1}\right)^z\cdot C_a}{\left\{\left(\frac{\sqrt{1-\frac{b}{h}}+1}{\sqrt{1-\frac{b}{h}}-1}\cdot\frac{\sqrt{1-\frac{a}{h}}-1}{\sqrt{1-\frac{a}{h}}+1}\right)^z-1\right\}} \end{aligned}\right\} \cdots\cdots\cdots\cdots (A3)$$

In Eq. (A 3), however, K corresponds to K of Eq. (A 1) and is determined by the following equation*.

$$\frac{U_{max}-U}{U_*}=\frac{1}{K}\left[\ln\left\{\frac{\sqrt{1-\frac{y}{h}}+1}{\sqrt{1-\frac{y}{h}}-1}\right\}-2\sqrt{1-\frac{y}{h}}\right] \cdots\cdots\cdots\cdots (A4)$$

where U_{max} is the maximum velocity and U is the velocity at y.

Letting the integral constant A be zero, Eq. (A 3) becomes

$$\frac{C}{C_a}=\left(\frac{\sqrt{1-\frac{y}{h}}+1}{\sqrt{1-\frac{y}{h}}-1}\cdot\frac{\sqrt{1-\frac{a}{h}}-1}{\sqrt{1-\frac{a}{h}}+1}\right)^z=Y^z \cdots\cdots\cdots\cdots (A5)$$

The relation between C/C_a and Y is confirmed by the experimental data, and the results are shown in Figs. 13, 14 and 15. From these figures, the same conclusion may de obtained as in Figs. 4, 5

* This result was given by G.I. Taylor, Proc. Roy. Soc. A. 159 (1937).

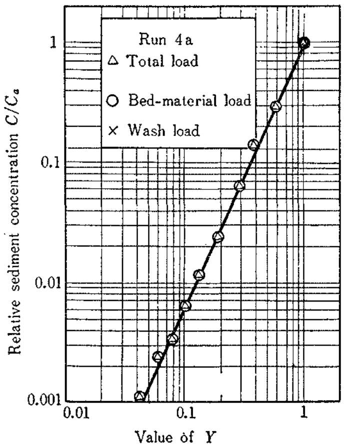

Fig. 13 Relation between measured sediment concentration and the relative height Y in Eq. (A 5).

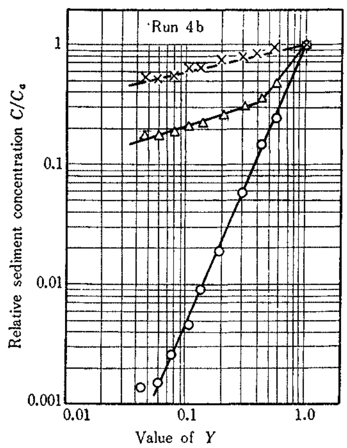

Fig. 14 Relation between measured sediment concentration and the relative height Y in Eq. (A 5).

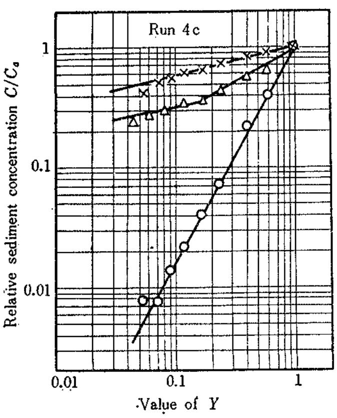

Fig. 15 Relation between measured sediment contration and the relative height Y in Eq. (A 5).

and 6.

Figs. 7, 8 and 9 show the comparisons between measured sediment distributions of the bed-material load and wash load and those calculated by Eq. (A 3) and Eq. (A 5).

Fig. 10 shows the comparison between the measured sediment distributions of the total load and the superposition of sediment distributions of the bed-material load and wash load estimated from Eq. (A 3) and Eq. (A 5).

From these results it is found that Eq. (A 3) and Eq. (A 5) do not differ from Eq. (9) and Eq. (10) respectively.

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