THE FUNDAMENTAL STUDIES ON THE FLEXURAL AND SHEARING PROPERTIES OF CONCRETE BEAMS WITH ARTIFICIAL LIGHTWEIGHT AGGREGATE

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1. INTRODUCTION

Recently many kinds of artificial lightweight aggregates have been developed in order to reduce the own weight of concrete members and to make up shortage of natural aggregates. Some kinds of those products are used in practice as concrete aggregates.

This paper describes the results of the experimental studies on the statical flexure and shear characteristics of lightweight reinforced or prestressed concrete beams.

2. BENDING TEST OF REINFORCED CONCRETE BEAMS

(1) Test specimens

Fourteen beams were fabricated. All beams are of 10×20 cm section and 150 cm long as shown in Fig. 1. Round bar of dia. 6 mm is used as stirrups. As the longitudinal reinforcement two round bars or deformed bars are used in the tension side and two round bars of dia. 9 mm in the compression side, also as shown in Fig. 1 and in Table 1.

Beams of series L and N are made of lightweight concrete and normal weight concrete, respectively.

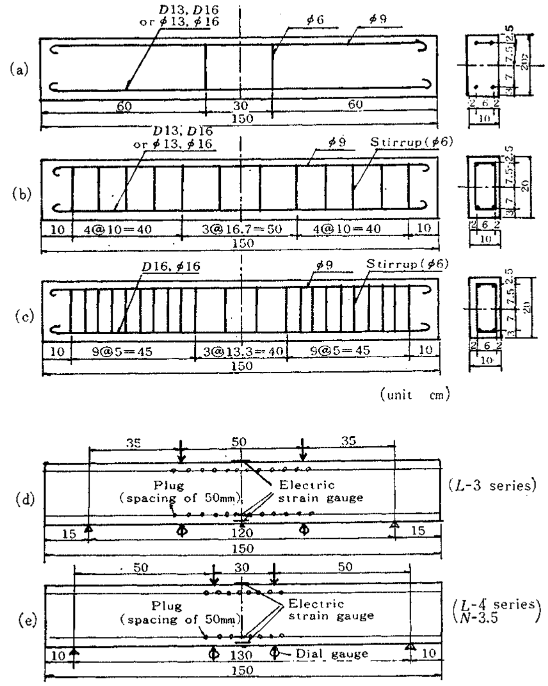

Fig. 1 Dimensions of beam speciments and test procedures.

Table 1 Description of beams for bending test.

Kinds of test beams	Kinds of aggregates	unit cement content (kg/cm²)	Shear span (cm)	tensile reinforcement	Compressive reinforcement	Spacing of Stirrup(φ6) (cm)
L-3-φ13- 5	Lightweight aggregates	300	35	2-φ13	2-φ9	5
-D13- 5		300	35	2-D13		5
-D16-10		300	35	2-D13		10
-D16- 5		300	35	2-D16		5
-D16-10		300	35	2-D16		10
-φ16- 5		300	35	2-φ16		5
-φ16-10		300	35	2-φ16		10
L-4-φ13-10	Lightweight aggregates	400	50	2-φ13		10
-φ16-10		400	50	2-φ16		10
-D13-10		400	50	2-D13		10
-D16-10		400	50	2-D16		10
N-φ13-10	Normalweight aggregates	350	50	2-φ13		10
-D13-10		350	50	2-D13		10
-D16-10		350	50	2-D16		10

The artificial lightweight aggregates used in this study are of expanded shale of pelletized type and those physical properties and grading are shown in Table 2, where are also listed the properties of cement mortar and reinforcing bars used. And the concrete mixes are listed in Table 3.

It was assumed that the artificial lightweight aggregates were in the state of absolute dry, therefore in mixing concrete, some amount of water was added more than specific amount of water, which was calculated from the 24 hours absorption rate. Til-

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Table 2 Properties of materials used

2-(a) Properties of both kinds of aggregates

	Grade	Specific gravity	Absorption (%)	Weight of unit vol. (kg/m³)	Fineness modulus	Screen analysis (total percentage retained)									
						0.15	0.3	0.6	1.2	2.5	5	10	15	20	25 mm
Normal *1 weight aggregates	Fine	2.60	0.98	1 660	2.97	99	88	66	32	11	0	0	0	0	0
	Coarse	2.63	0.98	1 690	7.02	100	100	100	100	100	100	78	49	24	0
Lightweight*2 aggregates	Fine	See table 2-(b)	2.2		2.87	92.5	80.3	69.1	44.2	0.8	0	0	0	0	0
	Coarse		1.7		6.86	100	100	100	100	100	100	85.6	5.2	0.6	0

2-(b)

Granular size (mm)	15~20	10~15	5~10	5~2.5	2.5~1.2	1.2~0.6	0.6~0.3	0.35~0.15	Smaller than 0.15
Sepcific gravity	1.39	1.41	1.38	1.67	1.63	1.75	1.90	2.23	2.34

*1 from the Yasu river

*2 produced by Osaka Cement Co. Ltd.

2-(c) Strength of cement mortar

Strength / Kinds of cement	Flexural strength (kg/cm²)				Compressive strength (kg/cm²)			
	1 day	3 days	7 days	28 days	1 day	3 days	7 days	28 days
Ordinary portland * cement	—	36.4	54.5	74.6	—	148.1	253.1	372.6
Ealy high strength* cement	37.2	50.8	59.8	65.3	109.5	223.1	269.0	338.5

* both kinds of cement are from Osaka Cement Co. Ltd.

2-(d) Properties of reinforcing bars

Bar type*	Diameter (mm)	Area (mm²)	Yield point (kg/mm²)	Tensile Strength (kg/mm²)
SR 24	13	133	31.39	45.60
SR 24	16	201	28.29	40.56
SR 24	9	64	34.56	44.50
	6	28	48.65	52.93
SD 30	12.7	127	41.00	56.27
SD 30	15.9	198	36.40	52.59

* All types of bars are from Kobe Seiko Co. Ltd..

ting type mixer of capacity of 50 liter was used. Besides the beams, 20 standard cylinders of ϕ 10×20 cm were prepared, eleven of which were for compressive strength and elastic modulus tests and others for splitting test, and nine standard beam specimens of 10×10×40 cm were cast for flexural strength test at the same time.

All of these specimens were stripped one day after casting and were exposed to the air in the laboratory until the testing times.

(2) Test procedure

Load was applied to beam as shown in Fig. 1 (d), (e) by 200 ton Amslar type compression testing machine.

Load was usually applied with increment of 0.25 ton but with smaller increment near the cracking load. To measure the deflections at the mid-span and loading points of beam dial gages of 1/100 mm scale were used as illustrated in Fig. 1.

Several electric strain guages were set on the surfaces of the horizontal bars and on the surface and bottom of beam section as also illustrated in Fig. 1. Thus the strains of bars and concrete at the mid-span of beam were measured. Whittemore strain meter was used to measure the width of cracks in the tension side of the beam section. The locations of guage points are indicated in Fig. 1.

(3) Test results and consideration of flexural test

(1) Ultimate bending moment

Both experimental and theoretical ultimate bending moments are listed in Table 4. In calculating the ultimate bending moments, the value of maximum compressive strain of concrete, ε_u, has no significance for under-reinforced beams.

Table 3 Concrete mix.

	Max. size of aggregate (mm)	Slump (cm)	Water cement ratio, w/c (%)	Quantities of ingredients (kg/m³)					Absolute fine agg. percentage s/a
				Cement	Water	Fine aggregate	Coarse aggregate		
							5~10 mm	10~20 mm	
L-3 series	20	5±1	49	300	147	564.4	277.5	277.5	42
L-4 series	20	5±1	34	400	136	482.5	280.1	280.1	38
N-3.5 series	25	6	50	350	175	715	1 115		39

Table 4 Ultimate bending moment

Kinds of test beams	Shear span	Ultimate load	Ultimate bending moment	Theoretical ultimate bending moment				Mode of failure
				$\varepsilon_u=0.3\%$		$\varepsilon_u=0.35\%$		
	a (cm)	p_u (ton)	M_u (t·cm)	M_u' (t·cm)	M_u/M_u'	M_u'' (t·cm)	M_u/M_u''	
L-3 ϕ-13- 5	35	7.65 7.60	133.5	130.8	1.02	125.6	1.06	Flexure
-D13- 5	35	8.67 8.68	151.8	160.2	0.95	160.7	0.94	Flexure
-D13-10	35	7.87 8.45	143.0	160.2	0.89	160.7	0.89	Flexure
-ϕ-16- 5	35	10.00 10.10	176.0	174.2	1.01	174.3	1.01	Flexure ~Shear
-ϕ-16-10	35	9.45 9.45	165.2	174.2	0.95	174.3	0.95	Shear
-D16- 5	35	9.88 9.15	166.5	212.8	0.78	213.3	0.78	Shear
-D16-10	35	7.81 8.03	138.5	212.8	0.65	213.3	0.65	Shear
L-4 -13-10	50	5.00 5.69	133.7	133.6	1.00	133.5	1.00	Flexure
-D13-10	50	6.44 6.62	163.2	164.3	0.99	164.1	0.99	Flexure
-ϕ-16-10	50	7.46 7.10	182.0	178.2	1.02	178.8	1.02	Flexure
-D16-10	50	9.06 8.61	220.5	222.3	0.99	225.1	0.98	Flexure
N -ϕ-13-10	50	5.00 5.35	129.5	129.1	1.00	130.5	0.99	Flexure
-D13-10	50	6.55 6.20	159.0	157.8	1.01	158.5	1.00	Flexure
-D16-10	50	8.65 8.75	217.5	212.0	1.03	212.4	1.02	Flexure

However, for in over-reinforced beams, column loaded with large eccentricity, beams reinforced with bars having no definite yield point, or doubly reinforced beams etc., it is essential to know ε_u. The value of ε_u ever proposed by many researchers is 0.2% to 0.6%, and for concrete using the artificial lightweight aggregate ε_u of 0.3% to 0.35% has been reported. In these measurement, ε_u was estimated as about 0.3%.

Herein, the ultimate bending moments were calculated by the following equation, test specimens being considered as doubly reinforced beams having compression reinforcement of $2\times\phi$ 9 mm bar. (see Fig. 2)

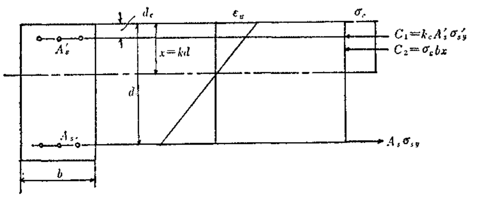

Fig. 2 Strain and stress distributions at failure.

$$M_u=bkd\sigma_c\left(d-\frac{kd}{2}\right)+k_cA_s'\sigma_{sy}'(d-d_c) \qquad \cdots\cdots(1)$$

M_u is calculated for $\varepsilon_u=0.3\%$, 0.35%, respectively and listed in Table 4. The table shows that both values of ε_u give no significant difference and the ratios of the experimental to the theoretical M_u are approximately 1.00 for both $\varepsilon_u=0.3\%$ and 0.35%.

Besides, as the beams are under-reinforced, the ultimate flexural strengths of beams are not significantly affected by the concrete strengths, and nearly same for both lightweight concrete and normal weight concrete beams.

(2) Cracking moment

Cracking moment of beam is calculated by using the following assumption;

1) plane sections remain plane after bending
2) tension side of concrete can be divided into the plastic and elastic region (see Fig. 3)

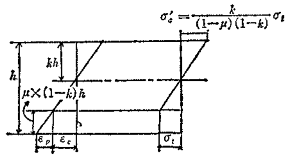

Fig. 3 Strain and stress distributions at cracking.

Herein, the effect of variation of $\varepsilon_p/\varepsilon_e$ on the cracking moment will be considered.

The cracking moments for $\varepsilon_p/\varepsilon_e=0$, 0.5, 1.0 and 1.5 are calculated and compared with the experiments in Table 5. The values of μ, indicating the depth of plastic region of tension side of beam section, are 0, 0.33, 0.50 and 0.60 each corresponding to $\varepsilon_p/\varepsilon_e$ described above.

Table 5 indicates that for lightweight concrete beams, the experimental and theoretical cracking moments are in good agreement when $\varepsilon_p/\varepsilon_e=0.5\sim1.0$($\mu=0.35\sim0.50$) in poor mix (L-3 series) and $\varepsilon_p/\varepsilon_e=1.0$ in rich mix (L-4 series), but for normal weight concrete beams, when $\varepsilon_p/\varepsilon_e=1.5$($\mu=0.6$).

Table 5 Cracking moment

	Shear span (cm)	Cracking load (t)	Cracking moment, M_{cro} (t·cm)	Theoretical craking moments $\varepsilon_p/\varepsilon_e=0$ M_{cro} (t·cm)	M_{cr}/M_{cro}	$\varepsilon_p/\varepsilon_e=0.5$ $M_{cr0.5}$ (t·cm)	$M_c/M_{cr0.5}$	$\varepsilon_p/\varepsilon_e=1.0$ $M_{cr1.0}$ (t·cm)	$M_c/M_{cr1.0}$	$\varepsilon_p/\varepsilon_e=1.5$ $M_{cr1.5}$ (t·cm)	$M_{cr}/M_{cr1.5}$
L-3 -13- 5	35	1.50	26.20	15.68	1.67	21.95	1.19	26.72	0.98	—	—
-D13- 5	35	1.38	24.20	15.42	1.57	21.55	1.12	26.18	0.93	—	—
-D13-10	35	1.25	21.90	15.42	1.42	21.55	1.02	26.18	0.84	—	—
-16- 5	35	1.83	32.00	17.57	1.82	24.94	1.28	30.92	1.03	—	—
-16-10	35	1.75	30.60	17.57	1.74	24.94	1.22	30.92	0.99	—	—
-D16- 5	35	2.00		17.20		24.16		29.85		—	—
-D16-10	35			17.20		24.16		29.85		—	—
L-4 -13-10	50	1.00	25.00	24.28	1.03	33.87	0.74	—	—	—	—
-D13-10	50	1.00	25.00	22.30	1.12	28.49	0.88	—	—	—	—
-16-10	50	1.50	37.50	27.00	1.39	38.07	0.99	—	—	—	—
-D16-10	50	1.00	25.00	29.32	0.86	41.11	0.61	—	—	—	—
N -13-10	50	1.25	31.25	—	—	20.66	1.51	23.15	1.34	26.31	1.18
-D13-10	50	1.00	25.00	—	—	19.53	1.28	22.86	1.09	25.94	0.97
-D16-10	50	1.50	37.50	—	—	21.01	1.78	25.51	1.47	32.19	1.16

The ratio of plastic to elastic tensile strain of lightweight concrete is therefore considerably smaller than that of normal weight concrete. It is also observed that the resistance to cracking of the former beams is inferior to that of the latter.

However, no remarkable difference between two kinds of concrete beams were observed as to the width and the intervals of cracks.

Steel stresses under which the maximum width of cracks is limited to 0.2 mm at the level of the tension reinforcement were about 2 300 kg/cm² for SR 24 and 2 900 kg/cm² for SD 30, respectively.

(3) Steel stress and adoption of elastic modulus ratio (n)

Relation between steel stress and bending moment at the mid-span is shown in Fig. 4. Steel stress calculated with the ordinary design formula, where the tensile resistance of concrete is neglected and elastic modulus ratio (n) is adopted as $n=15$ and 20, are also indicated in the figure.

The figure also shows that steel stress in the lightweight concrete beams is as same as or slightly

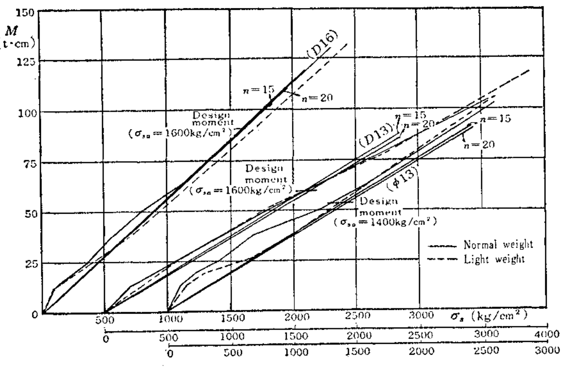

Fig. 4 Relations between moment and steel stress.
(L-4-D 13-10. L-4-D 16-10. L-4-ϕ 13-10
N-D 13-10. N-D 16-10. N-ϕ 13-10.)

larger than that of the normal weight concrete beams for the same bending moment.

In addition, little difference between the theoretical values for $n=15$ and 20 is observed, that is, effect of n-value on the calculated steel stress is quite small, and these theoretical values agree relatively well with the experimental ones at the working load.

The elastic modulus ratio for calculating steel stress at the working load for the lightweight concrete beams may be chosen as $n=15$ with safety as usually done for the normal weight concrete beams.

(4) Deformation characteristics of the flexural beams

(i) Flexural rigidity

Relations between the flexural rigidity calculated by equation (2) and the bending moment are shown in Fig. 5.

$$K=Md/(\varepsilon_c+\varepsilon_s) \quad \cdots\cdots(2)$$

As shown in Fig. 5, the flexural rigidity of the normal concrete beams is considerably larger before cracks occur, but is more rapidly reduced to a certain constant value as load increases than that of the lightwight concrete beams. And the difference of the converged values of rigidities of both kinds of concrete beams is not so appreciable.

This phenomenon is probably due to that the magnitude of movement of neutal axis resulting from the plastic deformation and the development of cracks with increasing load is rather larger in the normal concrete beams than the lightweight beams, and thus, the difference between both flexural rigidities is less than that between both modulus of elasticity.

As to the lightweight concrete beams themselves, the rate of reduction in flexural rigidity is rather larger for the beams made with concrete of higher strength, though the final rigidity is somewhat smaller

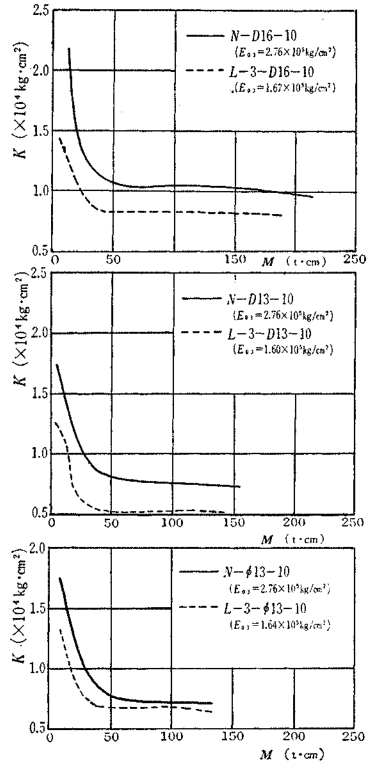

Fig. 5 Relations between moment and flexural rigidity.

for the lower strength concrete beams.

(ii) Deflection

The calculated and observed deflections at the design load (M_{rs}=61, 62 t·cm for $n=15$, $\sigma_{sa}'=1\,600$ kg/cm for SD 30) for the representative beams are shown in Table 6.

The observed deflections of the lightweight concrete beams are larger than that of normal concrete beams by only 20 to 25%, although larger by about 55% theoretically according to the general conception.

Column (1) in Table 6 are deflections computed assuming that a full section of concrete is effective and n values were adopted as 9.9 and 6.4 obtained from each measured elastic modulus of lightweight and normal concrete, respectively. The computation gives smaller deflections for both beams. This method, therefore, cannot be considered as appropriate as the design method.

Table 6 Deflections at design moment.

Kinds of beams	Observed deflections (mm)	Theoretical deflections (mm) ① Full section is effective		② Tension side is neglected	
L-4-D16-10	0.68		0.40(0.59)		0.77(1.13)
L-4-D13-10	0.78	$n=9.9$	0.49(0.63)	$n=9.9$	0.95(1.22)
L-3-D16-10	0.83		0.57(0.69)		1.11(1.34)
N-D16-10	0.52	$n=6.4$	0.27(0.52)	$n=6.4$	0.67(1.29)
N-D13-10	0.66		0.34(0.52)		0.83(1.26)

( ) means the raito to the observed values

Column (2) were computed by the design formula given by ACI Code, in which same values of n are adopted as in column (1), but the tension side of concrete is neglected. The computed values are in conservative side as much as about 23% for the lightweight concrete beams and about 28% for the normal ones.

3. FLEXURAL SHEAR TEST OF REINFORCED CONCRETE BEAMS

(1) Test spfecimens and method

In beam specimens for shearing tests, concretes having cement content of 300 kg/m^3 and 400 kg/m^3 for lightweight beams and 350 kg/m^3 for normal weight beams are used, respectively.

Tension reinforcement are four kind of 2-φ13, 2-φ16, 2-D 13 and 2-D 16 for the former and one kind of 2-D 16 for the latter, while compression reinforcement is one kind of 2-φ9 for both kind of concrete beams.

No stirrups are, however, provided in the test beams. And two kinds of shear span ratio of a/d =2.06 and 2.94 are chosen. (see Table 7)

Table 7 Project of shearing test.

Kinds of beams	kinds of aggregates	effective depth, d (cm)	Shear span, a (cm)	a/d	Tensile reinforcement	Compressive reinforcement	Stirrup
L-3-φ13-0 -D13-0 -φ16-0 -D16-0	Light-weight	17	35	2.06	2-φ13 2-D13 2-φ16 2-D16	2-φ9	not provided
L-4-φ13-0 -D13-0 -φ16-0 -D16-0	Light-weight	17	50	2.94	2-φ13 2-D13 2-φ16 2-D16	2-φ9	not provided
N-D16-0 -D16-0	Normal weight	17	35 50	2.06 2.94	2-D16 2-D16	2-φ9	not provided

Concrete mix proportions and dimensions of the beams were shown previously in Table 3 and Fig. 1, respectively.

Procedures of the test are quite similar to the above mentioned for bending test.

(2) Test results and consideration

Test results are listed in Table 8, where V_c is the load causing the initial diagonal crack.

ACI-ASCE Committee 326 analyses the shearing strength of beam under the following simplified assumptions.

(i) $\sigma_t=\text{const}\times\sqrt{\sigma_c}$, $E_c=\text{const}\times\sqrt{\sigma_c}$

(ii) If $a>2d$, a diagonal tension crack will begin at distance d from the section of maximum moment.

If $a<2d$, a diagonal tension crack will begin at the center of the shear span.

Table 8 Results of shearing test.

Kinds ot beams	Tensile strengh of concrete σ_t (kg/cm²)	Compressive strength of concrete σ_c (kg/cm²)	$\sqrt{\sigma_c}$	Reinforcement ratio p	a/d	Diagonal cracking load. V_c (ton)	Ultimate load (ton)	Shearing strength $v_c=V_c/bd$ (kg/cm²)	Nominal shearing stress $v=V_c/bjd$ (kg/cm²)	$v_c/\sqrt{\sigma_c}$	$1000\times\frac{pVd}{M\sqrt{\sigma_c}}$
L-3-ϕ-13-0	17.4	221	14.9	0.0158	2.06	3.00	4.94	8.83	10.6	0.593	1.002
〃	17.4	221	14.9	0.0158	2.06	3.00	5.28	8.83	10.6	0.593	1.002
L-3-D13-0	17.4	222	14.9	0.0150	2.06	3.00	3.77	8.83	10.5	0.592	0.952
〃	17.4	222	14.9	0.0150	2.06	3.00	3.42	8.83	10.5	0.592	0.952
L-3-ϕ-16-0	17.4	221	14.9	0.0237	2.06	3.35	3.52	9.90	17.7	0.664	1.500
〃	17.4	221	14.9	0.0237	2.06	3.25	5.76	9.60	17.2	0.644	1.500
L-3-D16-0	17.3	257	16.0	0.0233	2.06	3.24	3.74	11.11	19.8	0.692	1.478
〃	17.3	257	16.0	0.0233	2.06	3.50	3.75	10.30	18.4	0.644	1.478
L-4-ϕ-13-0	26.4	427	20.7	0.0158	2.94	3.25	4.22	9.60	11.5	0.464	0.393
〃	26.4	427	20.7	0.0158	2.94	3.00	4.11	8.83	10.6	0.426	0.393
L-4-D13-0	26.4	427	20.7	0.0150	2.94	3.00	3.18	8.83	10.5	0.426	0.374
〃	26.4	427	20.7	0.0150	2.94	2.75	3.90	8.13	9.7	0.393	0.374
L-4-ϕ-16-0	28.0	434	20.8	0.0237	2.94	3.00	3.74	8.83	15.8	0.424	0.588
〃	28.0	434	20.8	0.0237	2.94	2.75	3.73	8.13	14.6	0.391	0.588
L-4-D16-0	31.8	443	21.1	0.0233	2.94	3.50	3.92	10.30	18.4	0.489	0.569
〃	31.8	443	21.1	0.0233	2.94	4.00	4.00	11.78	21.1	0.558	0.569
N-D16-0(35)	17.5	247	15.7	0.0233	2.06	4.13	4.25	12.17	21.7	0.775	1.402
〃	17.5	247	15.7	0.0233	2.06	3.13	4.75	9.21	16.6	0.587	1.402
N-D16-0(50)	17.5	247	15.7	0.0233	2.94	4.25	8.05	12.53	22.4	0.798	0.765
〃	17.5	247	15.7	0.0233	2.94	4.50	7.95	13.24	23.7	0.843	0.765

For the concrete used in these tests are obtained,

$$\sigma_t=1.3\sqrt{\sigma_c} \text{ and } E_c=0.92\times10^4\sqrt{\sigma_c}$$

The locations of diagonal cracks in these tests are schematically shown in Fig. 6. This figure shows

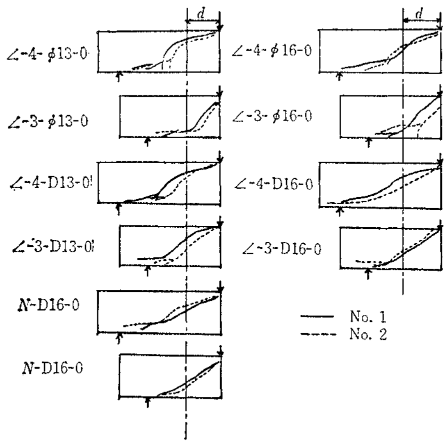

Fig. 6 Locations of diagonal cracks.

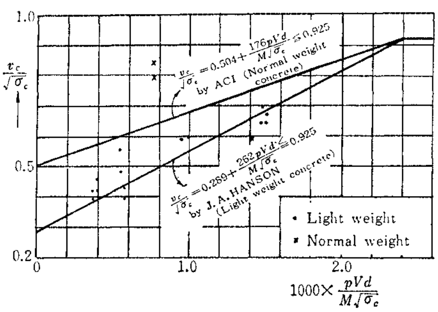

Fig. 7 Relations between $\frac{v_c}{\sqrt{\sigma_c}}$ and $\frac{pVd}{\sqrt{M\sigma_c}}$

that the simplified assumption (ii) made by the Committee agrees fairly well with the experiments.

Thus, the test results are plotted with parameters $v_c/\sqrt{\sigma_c}=V_c/bd\sqrt{\sigma_c}$ as ordinate and $pVd/M\sqrt{\sigma_c}$ as abscissa as shown in Fig, 7.

In Fig. 7, both the formula for non-web-reinforced beam of normal weight concrete proposed by ACI-ASCE Committee 326,

$$v_c\sqrt{\sigma_c}=V_c/bd\sqrt{\sigma_c}$$
$$=0.504+176pVd/M\sqrt{\sigma_c}\leqq0.925$$

and that for the lightweight concrete beams proposed by Mr. Hanson,

$$v_c/\sqrt{\sigma_c}=V_c/bd\sqrt{\sigma_c}$$
$$=0.289+262\,pVd/M\sqrt{\rho_c}\leqq0.925$$

are also plotted.

It appears that the test data are scattered more close to the formula by Mr. Hanson.

On the other hand, the norminal shearing stresses causing the initial diagonal tension crack calculated by $v=V_c/bjd$ $(n=15)$ are as follows.

For lightweight concrete,

$v=10.5\sim17.5\,\text{kg/cm}^2$ for $\sigma_c=220\,\text{kg/cm}^2$
$19.1\,\text{kg/cm}^2$ for $250\,\text{kg/cm}^2$
$10.1\sim19.8\,\text{kg/cm}^2$ for $430\,\text{kg/cm}^2$

For normal weight concrete,

$v=21.1\,\text{kg/cm}^2$ for $\sigma_c=247\,\text{kg/cm}^2$

The shearing strength of lightweight concrete is smaller than that of normal weight concrete by about 30%. Thus, the allowable shearing stress for lightweight concrete should be lower, for instance 70 percent of that for normal weight concrete.

4. TEST ON PRESTRESSED CONCRETE BEAMS USING THE ARTIFICIAL LIGHTWEIGHT AGGREGATE

Test project is listed in Table 9. The properties of cement, artificial lightweight aggregates and reinforcing bars used are same as those used in Table 2, and prestressing bars are from Sumitomo Denko Co. Ltd, haveing dia. of 18 mm, yield strength of 87 kg/cm² and tensile strength of 107 kg/cm².

Concrete mix proportion used in this test is listed in Table 10. These beams were prestressed to predetermined stresses at age of three months with the post-tensioning method, and were not grouted.

Table 9 Test project of prestressed concrete beams.

Kinds of beams	Tensile reinforce-ment	Prest-ressing bar	Effective prestress force (ton)	Spacing of stirrup (φ6) (cm)
P-3.5-D16-A	2-D16	φ18	9.35	5
B	〃	〃	4.87	10
C	〃	〃	6.50	not provided
P-3.5-D13-A	2-D13	〃	9.35	5
B	〃	〃	4.87	10
C	〃	〃	6.50	not provided
P-4.5-D16-A	2-D16	〃	9.85	5
B	〃	〃	4.87	10
C	〃	〃	6.50	not provided
P-3.5- 0-A	0	〃	9.35	5
P-4.5- 0-A	0	〃	9.35	5

Table 10 Concrete mix

	w/c (%)	Cement c (kg/m³)	Water w (kg/m³)	s/a (%)	Fine aggregate s (kg/m³)	Coarse aggregate G (kg/m³) 5~10 mm	Coarse aggregate G (kg/m³) 10~20 mm
P-3.5 series	40.3	35.0	141	40	523.8	279.6	279.6
P-4.5 series	35.1	45.0	158	36	443.5	280.7	280.7

* Aggregates are lightweight (Lionite)

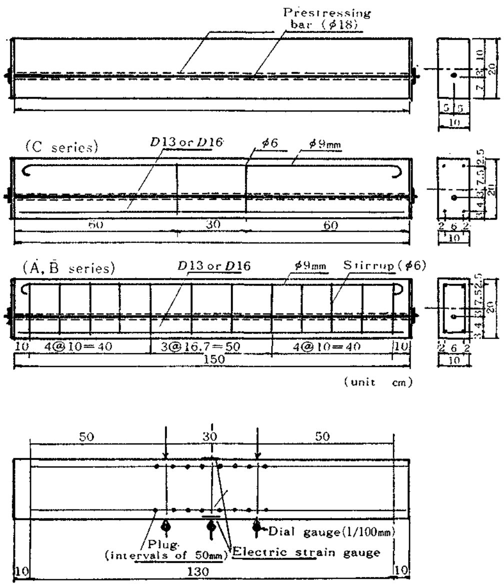

Fig. 8 Dimensions of test beams and test procedures.

The test methods were similar as in 2. Bending Test and then the dimensions of the test beams are shown in Fig. 8.

(1) Cracking properties

Herein, the effects of inserted non-prestressed-reinforcemet and magnitude of average prestress on the appearent flexural strength of beams are examined.

Tensile stress-strain curve of concrete is assumed as shown in Fig. 9.

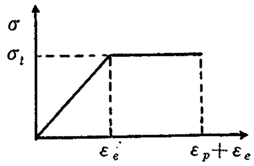

Fig. 9 Tensile stress-strain curve of concrete.

where,

σ_t : tensile strength of concrete

$\varepsilon_p+\varepsilon_e$: maximum tensile strain of concrete at failure

ε_e : critical elastic strain of concrete

Now, μ is designated as $\mu=\varepsilon_p/(\varepsilon_p+\varepsilon_e)$, which is a characteristic value expressing the property of the tensile stress-strain curve of concrete and, as previously described, 0.6 to 0.8 for normal weight concrete.

Consider that the stress distribution in the tension and the compression side of concrete section at occurance of cracks are assumed as shown in Fig. 10.

The stress distribution in Fig. 10 (a) is divided into elements of tension force T and compressive force C. (Fig. 10 (b), (c))

From equibrium of forces, the relations among the valuse of σ_t, σ_g, γ and μ are as follows:

(i) If the tensile reinforcements exist within $\mu\times(\gamma h)$,

$$1+\frac{\sigma_g}{\sigma_t}=\frac{bh}{A_e}\frac{\gamma^2\mu}{2(1-\mu)(1-\gamma)} \quad\cdots\cdots(3)$$

(ii) If the tensile reinforcements exist within $(1-\mu)\times(\gamma h)$

$$1+\frac{\sigma_g}{\sigma_t}=\frac{bh}{A_e}\frac{\gamma\mu}{(1-\mu)(1-\mu)}\left\{\frac{\gamma}{2}+(n-1)p\frac{\gamma-d_1}{\gamma}\right\} \quad\cdots\cdots(4)$$

Further, cracking moment, M_{cr}, is given as follow

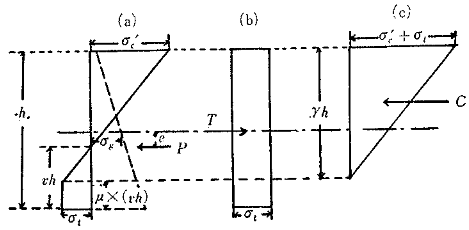

Fig. 10 Stress distributions at occurence of cracks.

$$M_{cr}=C\cdot j+P\cdot e$$
$$=A_e(\sigma_t+\sigma_g)\left(y_e'-\frac{rh}{3}\right)+P\cdot e \quad \cdots\cdots\cdots(5)$$

From eq (5), appearent flexural strength σ_{pB} is given as follow,

$$\sigma_{pB}=\frac{M_{cr}}{Z_e}=\left[A_e(\sigma_t+\sigma_g)\left(y_e'-\frac{rh}{3}\right)+P\cdot e\right]\Big/Z_e \quad \cdots\cdots(6)$$

On the other hand, when no tensile reinforcements exist in the beam, the following formulas are given accoding to the above mentioned method.

$$1+\frac{\sigma_g}{\sigma_t}=\frac{\gamma^2\mu}{2(1-\mu)(1-\gamma)} \quad \cdots\cdots(7)$$

$$M_{cr}=C\cdot j+P\cdot e$$
$$=\left\{(\sigma_g+\sigma_t)\left(\frac{1}{2}-\frac{\gamma}{3}\right)+\sigma_g\cdot\frac{e}{h}\right\}bh^2 \quad \cdots\cdots(8)$$

$$\sigma_{pB}'=\frac{M_{cr}}{Z_c}=\frac{6\,M_{cr}}{bh^2}$$
$$=\left\{\left(1+\frac{\sigma_g}{\sigma_t}\right)(3-2\gamma)+6\frac{\sigma_g}{\sigma_t}\cdot\frac{e}{h}\right\}\sigma_t\cdots \quad (9)$$

In order to calculate the values of σ_{pB} theoretically, firstly σ_t is to be determined for the arbitrarily assumed value of μ. Hereupon, the method determing σ_t from flexural strength of the beam without reinforcements and prestressing bars (σ_{pB}) is according to the following steps.

(i) For the value of μ assumed voluntarily, put $\sigma_g=0$ into eq (7) and calculate γ_1 value by eq (10).

$$\mu\gamma_1^2+2(1-\mu)\gamma_1-2(1-\mu)=0 \quad \cdots\cdots(10)$$

(ii) Put γ_1 from eq (10) into the following equation, which is obtained by putting $\sigma_g=0$ into eq (9).

$$\sigma_{pB}'=(3-2\gamma_1)\sigma_t \quad \cdots\cdots(11)$$

or

$$\sigma_t=\sigma_{pB}'/(3-2\gamma_1) \quad \cdots\cdots(12)$$

After σ_t can be determined for a unique value of μ, the values of γ can be calculated from eq (3) or (4).

Then, σ_{pB} can be determined by putting above

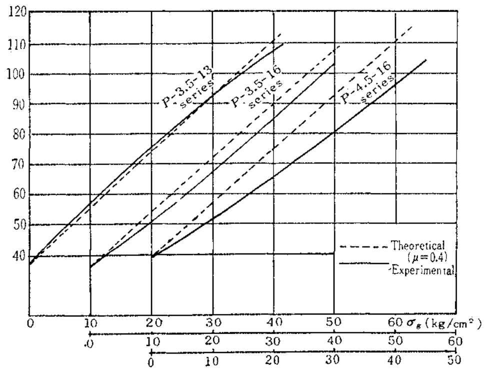

Fig. 11 Relations between flexural strength (σ_{pB}) and prestress stress (σ_g).

values of σ_t, σ_g, γ and others into eq (6).

The experimental and theoretical $\sigma_{pB}-\sigma_g$ curves for $\mu=0.4$ are shown in Fig. 11, which indicates that the theoretical curves are in relatively good agreement with the experimental ones, especially so in low strength series (p-3.5 series).

Since the μ value which have obtained for normal weight concrete is $\mu=0.6\sim0.8$, that of lightweight concrete appears to be somewhat smaller.

Therefore, the tensile plastic ratio μ of lightweight concrete is somewhat smaller than that of normal weight one, that is, the former may be more brittle.

In order words, the ratio of apparent flexural strength to pure tensile strength may be smaller for lightweight concrete than for normal concrete. This fact is also reported by other investigators.

(2) Ultimate bending moment

In calculating the ultimate bending moment, the assumption that the stress and strain distributions of concrete and steels at flexural failure are as shown in Fig. 12 gives the ultimate bending moment as follows,

$$M_u=A_p\sigma_p\left(d_p-\frac{x}{2}\right)+A_s\sigma_s\left(d-\frac{x}{2}\right) \quad \cdots\cdots(14)$$

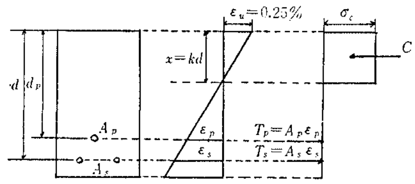

Fig. 12 Strain and stress distributions at failure.

Also, in calculating the ultimate bending moment, the ultimate compressive strain of concrete at the top fiber is assumed as $\varepsilon_u=0.25\%$.

Thus determined theoretical values and the experimental ones are listed in Table 11.

Excepet the beams in *C* series, where all beams failed in shear because of no arrangement of the stirrups, the ratio of the theoretical to experimental values is 1.04 on average.

Morever, the appreciable difference between the ultimate bending capacities of the lightweight and normal weight prestressed concrete beams will not exist, if the results of above calculated values are taken into acount. The calculation of the ultimate bending moment for lightweight concrete beams can therefore made samely as for normal weight concrete beams.

(3) Flexural Rigidity

Some examples of the relation between the moment and the flexural rigidity are given in Fig. 13.

On the test beams having the same quanltity of

Table 11 Test results of prestressed concrete beams.

Kinds of beams	Compressive strength of concrete σ_c (kg/cm²)	Tensile reinforce-ment (SD 30)	Prest-ressing bar (SBPC)	Effective prestress force (t)	Observed Cracking load P_{cr} (t)	Observed Ultimate load P_u (t)	P_u/P_{cr}	Observed Ultimate moment M_u (t·cm)	Theoretical Ultimate moment M_u' (t·cm)	M_u/M_u'	Mode of failure
P-3.5 -0	385	0	ϕ18	9.35	3.75 3.50	6.65 6.30		162.0	203.7	0.80	flexure
P-3.5-D13-A	385	2-D13	ϕ18	9.35	3.50 3.45	11.90 12.03	3.44	299.1	280.1	1.07	flexure
P-3.5-D13-B	385	2-D13	ϕ18	4.87	2.50 2.50	10.70 10.85	4.30	269.4	280.1	0.96	flexure
P-3.5-D13-C	385	2-D13	ϕ18	6.50	2.75 3.25	8.50 9.80		228.8	280.1	0.82	Shear
P-3.5-D16-A	377	2-D16	ϕ18	9.35	3.50 3.75	13.30 13.06	3.64	329.5	308.6	1.07	flexure
P-3.5-D16-B	377	2-D16	ϕ18	4.87	2.50 2.25	12.55 12.55	5.28	313.8	308.6	1.02	flexure
P-3.5-D16-C	377	2-D16	ϕ18	6.50	2.75 3.00	9.00 7.90		211.3	308.6	0.68	flexure
P-4.5 -0	470	0	ϕ18	9.35	3.95 3.75	8.55 9.10		220.6	216.5	1.02	flexure
P-4.5-D16-A	470	2-D16	ϕ18	9.35	3.25 3.25	15.05 15.10	4.64	376.9	344.0	1.09	flexure
P-4.5-D16-B	470	2-D16	ϕ18	4.87	2.40 2.00	14.10 14.80	6.58	361.3	344.0	1.05	flexure
P-4.5-D16-C	470	2-D16	ϕ18	6.50	2.85 2.50	11.50 11.05		281.9	344.0	0.82	Shear

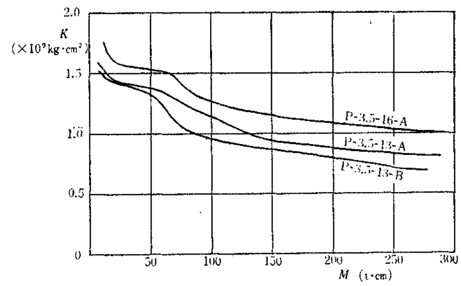

Fig. 13 Relations between moment (M) and flexural rigidity (K).

reinforcement and concrete strength, the figure shows that the larger the prestress or amount of reinforcement the smaller the rate of reduction in the flexural rigidity of beam due to the plastic deformation and occurance of cracks.

Conclusions

Principal results obtained are summarized as follows:

1. The ultimate bending moment is almost same in both lightweight and normal weight concrete beams having the same ratio of reinforcement and concrete strength. The maximum compressive strain of concrete may be chosen with safety as $\varepsilon_u=0.3\%$, if ε_u value is required in calculation.
2. Even if both concrete beams have the same amount of reinforcement and the same concrete strength (pure tensile strength), the cracking moment of the lightweight concrete beams is inferior to that of normal ones. And in calculating the cracking moment, μ value may be adopted as $\mu=0.33\sim0.50$ ($\varepsilon_p/\varepsilon_e=0.5$, 1.0) in the former beames and $\mu=0.6$ ($\varepsilon_p/\varepsilon_e=1.5$) in the

Table 12 Results of tests of concrete strength.

Test series	Kinds of beam specimens	7 days σ_c (kg/cm²)	7 days σ_t (kg/cm²)	7 days σ_B (kg/cm²)	28 days σ_c (kg/cm²)	28 days σ_t (kg/cm²)	28 days σ_B (kg/cm²)	Testing days σ_c (kg/cm²)	Testing days σ_t (kg/cm²)	Testing days σ_B (kg/cm²)	Testing days E_c* (kg/cm²) ×10⁵
L-3	L-3-D16-0, 5, 10	118	14.3	19.0	220	20.4	30.0	257	17.3	34.2	1.67
L-3	L-3-D13-0, 5,1 0	173	17.2	18.7	210	20.4	28.8	222	17.3	43.6	1.60
L-4	L-4-D16-0, 10	312	24.8	34.6	328	26.9	43.2	443	31.8	44.3	2.13
L-4	L-4-D13-0, 10 L-4-ϕ13-0	—	—	—	424	26.7	37.4	427	26.4	46.8	1.93
L-4	L-4-ϕ13-10 L-4-ϕ16-0, 10	278	28.6	31.3	470	24.7	30.2	434	28.0	38.5	1.83
L-3	L-3-ϕ16-0, 5, 10 L-3-ϕ13-0, 5	171	13.0	28.1	230	17.9	31.3	221	17.4	42.8	1.56
N-3.5	N-D16-0 (35, 50) N-ϕ13, D13, D16-10	—	—	—	—	—	—	385	25.3	43.2	2.76
P-3.5	P-3.5-0 P-3.5-D13-A, B, C	384	27.0	23.4	428	29.9	36.0	385	25.3	43.2	1.69
P-4.5	P-3.5-D16-A, B, C	358	24.8	31.7	356	24.5	33.5	377	28.9	42.5	1.79
P-4.5	P-4,5-0 P-4.5-D16-A, B, C	513	24.1	25.9	537	30.5	32.4	470	31.5	46.1	1.95

* E_c; Secant modulus of elasticity at 1/3 of compressive strength

latter.

3. The appreciable differences between the lightweight and normal weight concrete beams were not observed as to the width and frequency of cracks.
4. The steel stress of the lightweight concrete beams is almost same as that of the normal ones, or a little larger at the same bending moment, and in calculation, the elastic modulus ratio (n) may be adopted as $n=15$.
5. The rate of reduction in the flexural rigidity resulting from the plastic deformation and occurance of a crack is rather larger in the normal weight concrete beams, and therefore the difference between the flexural rigidities of both kinds of concrete beams at the final state is not so large as found at the the initial state.
6. Conception that the deflection is in inverse proportion to the elastic modulus of concrete is too errorneous in conservative side, and in calculating the deflection at the design load, a moment of inertia of a transformed section may be used, and the elastic modulus ratio, n, may be adopted as
$$n=\frac{\text{elastic modulus of steel}}{\text{elastic modulus of lightweight concrete}}$$
if the tension side of concrete is neglected.
7. The design formula proposed by Mr. Hanson, $V_c/bd\sqrt{\sigma_c}=0.289+262\,pVd/M\sqrt{\sigma_c}\leqq 0.925$, appears to be useful in the design of shear reinforcement of R.C. members using artificial lightweight aggregates.
8. The nominal shearing stress calculated by $v=V_c/bjd\ (n=15)$ at the initial diagonal cracking load were as follows, for lightweight concrete, $v=10.5\sim19.1$ kg/cm^2 : compressive strength of 220 $\sim$250 kg/cm^2 and $v=10.1\sim19.8$: 430 kg/cm^2, whereas for normal one, $v=21.1$ kg/cm^2 : 247 kg/cm^2.

   Nominal shearing stress causing diagonal cracks in the lightweight concrete beams is therefore as lower as 70% of that in the normal weight concrete beams.
9. The μ value of lightweight concrete may be about 0.4, while that of normal one appears to be 0.6 to 0.8, and its value may be affected also by mix proportion of concrete.
10. In calculation of ultimate bending moment of the prestressed concrete beams using artificial lightweight aggregates, maximum compressive strain of concrete can be assumed as $\varepsilon_u=0.25\%$.
11. The rate of reduction in flexural rigidity is affected mainly by a magnitude of prestress and non-prestressed reinforcement so far as this test is concerned.

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Notation

σ_c : compressive strength of concrete
σ_{pB} : flexural strength of concrete
σ_t : tensile strength of concrete
σ_{sy} : yield point of tensile reinforcement
σ'_{sy} : yield point of compressive reinforcement
σ_{pu} : tensile strength of prestressing bar
E_c : modulus of elasticity of concrete
E_s : modulus of elasticity of steel
n : ratio of E_s to E_c
σ_g : prestress stress at a centroid of a cross section
M : moment
C : compressive resultant force
T : tensile resultant force
P : prestress force
b : width of a cross section
h : full depth of a cross section
d : distance between a top surface and tensile reinforcements
d_c : distance between a top surface and compressive reinforcements
x : distance between a top surface and neutral axis
k : ratio of x to d
A : sectional area ($=bh$)
A_e : transformed sectional area
A_s : sectional area of tensile reinforcement
A_s' : sectional area of compressive reinforcement
A_p : sectional area of prestressing bar
p : steel ratio
γ : ratio of a distance between a top surface and a elastic limitting point in a tension side to a full depth
v : ratio of a distance between a bottom surface and a neutral axis to a full depth
e : eccentricity
Z_c : modulus of a net section
Z_e : modulus of a transformed section
y_e' : distance between a bottom surface and a centroid of a transformed section
d_1 : d/h

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