ON THE ONE-DIMENSIONAL CONSOLIDATION BY THE THREE-DIMENSIONAL DEHYDRATION

—— On the Consolidation Effect due to Sand Pile Drainage Works ——

*By Banjiro Yamada**

I. INTRODUCTION

The analysis of consolidation due to sand pile drainage work is now usually made by means of Barron's method only on the case of the primary consolidation. However, in the weak ground where sand pile drainage works are carried out, the amount of consolidation due to the secondary compression is large, and cannot be neglected. Therefore, the author analyzed consolidation in the above case with the secondary compression taken into consideration.

The author explained the method to gain easily and speedily the consolidation settlement curves, on the primary consolidation part, by composing the relation curves between the degree of consolidation regarding the primary consolidation part and time coefficients, using the ratio of time coefficients of horizontal and vertical direction and the ratio of the diameter of sand pile and that of effective circle, and on the secondary compression part, by composing the relation curves between the degree of consolidation of the secondary compression part and time coefficients on the creep.

Also the author mentioned the method to apply several constants on consolidation to the values of consolidation test by the three-dimensional dehydration.

II. DERIVATION OF THE FUNDAMENTAL EQUATION

The fundamental equation of the consolidation effect due to the three-dimentional dehydration by sand pile is derived by making the amount of dehydration and that of consolidation deformation is equal.

(1) The Amount of Dehydration

In Fig. 1, Q_A and Q_B denote the amount of flow out through the face A and B respectively, one obtains

$$Q_A=k_h i_A A=\frac{k_h}{\gamma_w}\left(\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}dr\right)\times(a+r+dr)d\theta dz$$

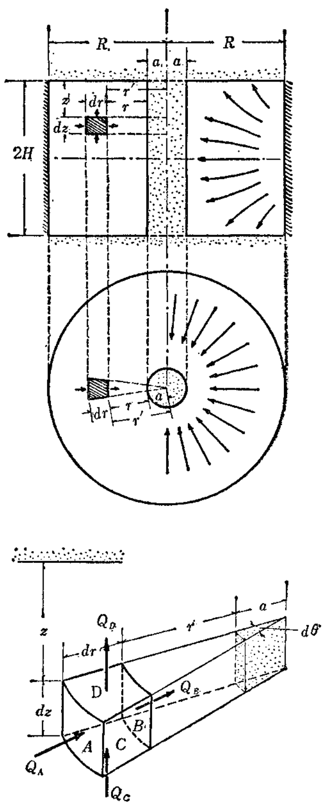


Fig. 1 The explanatory figure of the mechanism of dehydration in a minute element

$$Q_B=k_h i_B B=\frac{k_h}{\gamma_w}\cdot\frac{\partial u}{\partial r}(a+r)d\theta\,dz$$

,where A : area of the face A

B : area of the face B

The amount of flow out of the minute soil element by the horizontal flow per unit time is

$$dQ_h=Q_B-Q_A\fallingdotseq-\frac{k_h}{\gamma_w}\left\{\frac{\partial^2 u}{\partial r^2}(a+r)+\frac{\partial u}{\partial r}\right\}\times dr\,d\theta\,dz$$

In the same manner, letting the amount of flow out through the face C be Q_C and that of the face D be Q_D, respectively. One obtains

$$Q_C=k_v i_c C=\frac{k_v}{\gamma_w}\left(\frac{\partial^2 u}{\partial z^2}dz+\frac{\partial u}{\partial z}\right)(a+r)drd\theta$$

$$Q_D=k_v i_D D=\frac{k_v}{\gamma_w}\cdot\frac{\partial u}{\partial z}(a+r)drd\theta$$

,where C : area of the face C

D : area of the face D

* Dr. Agr., Dr. Eng., Prof., Dept. of Agr. Eng., Fac. of Agr., Utsunomiya University.

The amount of flow out through the minute soil element by the vertical flow per unit time is

$$dQ_v=Q_D-Q_c=-\frac{k_v}{r_\omega}\cdot\frac{\partial^2 u}{\partial z^2}(a+r)dr\,d\theta\,dz$$

Therefore, the total amount of water lost from the minute soil element per unit time is

$$dQ=dQ_h+dQ_v=-\left[\frac{k_v}{r_\omega}\left\{\frac{\partial^2 u}{\partial r^2}(a+r)+\frac{\partial u}{\partial r}\right\}+\frac{k_v}{r_\omega}\cdot\frac{\partial^2 u}{\partial z^2}(a+r)\right]dr\,d\theta\,dz \quad\cdots\cdots\cdots(1)$$

(2) The Amount of Deformation of the Consolidated Soil Body

The total amount of deformation of the soil body per unit time, with the secondary compression taken into considerations, is

$$\frac{\partial(\varepsilon V)}{\partial t}=\left[v\frac{\partial \bar{p}}{\partial t}+r\frac{\partial}{\partial t}\int_0^t\bar{p}(\tau)\times\left\{-\frac{\partial}{\partial\tau}(1-e^{-\eta(t-\tau)})\right\}d\tau\right]\times(a+r)dr\,d\theta\,dz$$

Since $\bar{p}=K-u$, $\frac{\partial\bar{p}}{\partial t}=-\frac{\partial u}{\partial t}$, and by making $\frac{\partial}{\partial t}=p$, there is obtained

$$\frac{\partial(\varepsilon V)}{\partial t}=-\left(vpu+\frac{r\eta p}{p+\eta}u\right)(a+r)dr\,d\theta\,dz \quad\cdots\cdots\cdots\cdots\cdots(2)$$

By the idea that the amount of dehydration and that of deformation of the soil body are made equal, and from the equation (1) and (2), one gets

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+a}\frac{\partial u}{\partial r}\right)+\frac{k_v}{r_\omega}\cdot\frac{\partial^2 u}{\partial z^2}=vpu+\frac{r\eta p}{p+\eta}u \quad\cdots\cdots\cdots(3)$$

This is the fundamental equation of consolidation due to sand drain works.

III. THE FUNDAMENTAL EQUATION OF CONSOLIDATION ONLY BY THE HORIZONTAL FLOW

Making $\frac{\partial u}{\partial z}=0$ in the above equation (3), one gets

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+a}\frac{\partial u}{\partial r}\right)=vpu+\frac{r\eta p}{p+\eta}u \quad\cdots(4)$$

,where $p=\frac{\partial}{\partial t}$

The special solution of the equation (4) is

$$u=(C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t})\{C_3J_0(Mr')+C_4Y_0(Mr')\} \quad\cdots\cdots\cdots\cdots\cdots(5)$$

,where $r'=r+a$, and

$$\lambda_1,\lambda_2=\frac{1}{2}\left[-(v+r)\frac{\eta}{v}-\frac{k_h}{v r_\omega}M^2\pm\sqrt{\left\{(v+r)\frac{\eta}{v}-\frac{k_h}{v r_\omega}M^2\right\}^2-4\frac{k_h\eta}{v r_\omega}M^2}\right] \quad\cdots\cdots\cdots\cdots\cdots(6)$$

$J_0(Mr')$: the Bessel function of the first kind and order 0

$Y_0(Mr')$: the Bessel function of the second kind and order 0

Next, the integration constants C_1, C_2, C_3 and C_4, and the undetermined constant M are determined with the use of the surface conditions and the initial conditions.

From the surface condition, $u=0$ at $r'=a(r=0)$, there is obtained

$$C_3J_0(Ma)+C_4Y_0(Ma)=0 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

And from the boundary condition, $\frac{\partial u}{\partial r'}=0$ at $r'=R(r=R-a)$, there is obtained

$$C_3J_1(MR)+C_4Y_1(MR)=0 \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

From the equations (7) and (8), one gets

$$C_4=-C_3\frac{J_0(Ma)}{Y_0(Ma)}$$

$$\frac{J_0(Ma)}{Y_0(Ma)}=\frac{J_1(nMa)}{Y_1(nMa)} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

,where $n=\frac{R}{a}$

Letting ${}_nm_i$ denotes the ith value that satisfy the equation (9) for a value of M, and substituting this in the equation (5), one obtains

$$u=\sum_{i=1}^{\infty}(C_1'e^{\lambda_1 t}+C_2'e^{\lambda_2 t})\times\left[J_0({}_nm_ir')-\frac{J_0({}_nm_ia)}{Y_0({}_nm_ia)}Y_0({}_nm_ir')\right] \quad\cdots\cdots\cdots\cdots\cdots(10)$$

,where $C_1'=C_1C_3$ and $C_2'=C_2C_3$.

Solving the equation (10), from the initial condition that $u=K$ at $t=0$, there is obtained

$$K=\sum_{i=1}^{\infty}(C_1'+C_2')\times\left[J_0({}_nm_ir')-\frac{J_0({}_nm_ia)}{Y_0({}_nmia)}Y_0({}_nm_ir')\right] \quad\cdots\cdots\cdots\cdots\cdots(11)$$

The undetermined constant $(C_1'+C_2')$ in this equation is determined in the following manner. If one writes the characteristic function as

$$\left[J_0({}_nm_ir')-\frac{J_0({}_nm_ia)}{Y_0({}_nm_ia)}Y_0({}_nm_ir')\right]=D_i$$

,then D_i is the solution of the differential equation (4) and satisfy the conditions

$$[D_i]_{r'=a}=0,\quad\left[\frac{dD_i}{dr'}\right]_{r'=R}=0$$

Now an assumption is made that the characteristic value $m_1, m_2, m_3,\cdots\cdots$ and the characteristic functions belonging to them $D_1, D_2, D_3,\cdots\cdots$ are determined from these conditions, and any two values m_p and m_q, and D_p and D_q are taken from the series respectively which naturally satisfy the equation (4) and give two equations,

$$D_p''+\frac{1}{r'}D_p'+m_p^2D_p=0$$

and

$$D_q''+\frac{1}{r'}D_q'+m_q^2D_q=0$$

Therefore

$$\frac{1}{dr'}\left(r'\frac{dD_p}{dr'}\right)+m_p^2r'D_p=0 \quad\cdots\cdots(12)$$

$$\frac{1}{dr'}\left(r'\frac{dD_q}{dr'}\right)+m_q^2r'D_q=0 \quad\cdots\cdots(13)$$

Multiplying the equation (12) by D_q and the equation (13) by D_p, and subtracting one from the other, one gets

$$(m_p^2-m_q^2)r'\cdot D_pD_q=D_p(r'D_q')'-D_q(r'D_p')'$$

Integration of the both sides of this equation from $r'=a$ to $r'=R$ gives

$$(m_p^2-m_q^2)\int_a^R r'D_pD_qdr'$$
$$=\int_a^R D_p(r'D_q')'dr'-\int_a^R D_q(r'D_p')'dr'$$
$$=\left[\Big|D_p(r'D_q')\Big|_a^R-\int_a^R D_p'(r'D_q')dr'\right]$$
$$-\left[\Big|D_q(r'D_p')\Big|_a^R-\int_a^R D_q'(r'D_p')dr'\right]$$
$$=\Big|r'(D_pD_q'-D_p'D_q)\Big|_a^R$$

Since D_p and D_q shown by the equation (12) and (13) satisfy the conditions $D_p(a)=0$, $D_q(a)=0$, $D_p'(R)=0$ and $D_q'(R)=0$, the right side of the above equation becomes 0.

Therefore

$$(m_p^2-m_q^2)\int_a^R r'D_pD_qdr'=0$$

As $m_p\neq m_q$, $\int_a^R r'D_pD_qdr'=0$

Substituting i for p in the above equation.

$$\int_a^R r'D_iD_qdr'=0$$

Whence, if multiplication of the both sides of the equation (11) by $r'D_i$ and integration of them from a to R are made, then all the terms other than the ith one become 0, to give

$$K\int_a^R r'D_idr'=(C_1'+C_2')\int_a^R r'D_i^2dr'$$

$$\therefore\quad C_1'+C_2'=\frac{K\int_a^R r'D_idr'}{\int_a^R r'D_i^2dr'} \quad\cdots\cdots(14)$$

Next, the plastic deformation rate ε_γ is calculated by the following equation.

$$\varepsilon_\gamma(t)=e^{-\eta t}\int_0^t e^{\eta\tau}\eta\gamma\,\bar{p}(\tau)d\tau$$
$$=\gamma\eta\int_0^t e^{-\eta(t-\tau)}\left\{\sum_{i=1}^{\infty}(C_1'+C_2')D_i-\sum_{i=1}^{\infty}(C_1'e^{\lambda_1\tau}+C_2'e^{\lambda_2\tau})D_i\right\}d\tau$$

In the equation, the coefficient of D_i is

$$\gamma\eta\int_0^t e^{-\eta(t-\tau)}\{(C_1'+C_2')-(C_1'e^{\lambda_1\tau}+C_2'e^{\lambda_2\tau})\}d\tau$$
$$=\gamma\eta\left[\frac{C_1'+C_2'}{\eta}-\left(\frac{C_1'}{\lambda_1+\eta}e^{\lambda_1t}+\frac{C_2'}{\lambda_2+\eta}e^{\lambda_2t}\right)\right]$$

As $\varepsilon_\gamma=0$ at $t=0$, this coefficient value is 0.

Therefore

$$\frac{C_1'}{\lambda_1+\eta}+\frac{C_2'}{\lambda_2+\eta}=\frac{(C_1'+C_2')}{\eta} \quad\cdots\cdots(15)$$

C_1', C_2' are obtained by the equation (14) and (15).

$$C_1'=\frac{(\lambda_1+\eta)\lambda_2}{(\lambda_2-\lambda_1)\eta}\cdot\frac{K\int_a^R r'D_idr'}{\int_a^R r'D_i^2dr'}$$

$$C_2'=-\frac{(\lambda_2+\eta)\lambda_1}{(\lambda_2-\lambda_1)\eta}\cdot\frac{K\int_a^R r'D_idr'}{\int_a^R r'D_i^2dr'}$$

Substituting them for equation (10).

$$u_r=\sum_{i=1}^{\infty}K\left[\frac{(\lambda_1+\eta)\lambda_2}{(\lambda_2-\lambda_1)\eta}e^{\lambda_1t}-\frac{(\lambda_2+\eta)\lambda_1}{(\lambda_2-\lambda_1)\eta}e^{\lambda_2t}\right]\times\left[\frac{D_i\int_a^R r'D_idr'}{\int_a^R r'D_i^2dr'}\right] \quad\cdots\cdots(16)$$

IV. THE FUNDAMENTAL EQUATION OF CONSOLIDATION ONLY BY THE VERTICAL FLOW

The fundamental equation in this cace is as follows.

$$\frac{k_v}{\gamma_\omega}\left(\frac{\partial^2u}{\partial z^2}\right)=vpu+\frac{\gamma\eta p}{p+\eta}\cdot u \quad\cdots\cdots(17)$$

,where $p=\frac{\partial}{\partial t}$

The special solution of this equation is expressed as

$$u=(B_1e^{\lambda_1't}+B_2e^{\lambda_2't})(B_3\sin Nz+B_4\cos Nz)$$

,where

$$\lambda_1',\lambda_2'=\frac{1}{2}\left[-(v+\gamma)\frac{\eta}{v}-\frac{k_v}{v\gamma_\omega}N^2\pm\sqrt{\left[(v+\gamma)\frac{\eta}{v}+\frac{k_v}{v\gamma_\omega}N^2\right]^2-4\frac{k_v}{v\gamma_\omega}\eta N^2}\right] \quad\cdots\cdots(18)$$

Next, the integration constants B_1, B_2, B_3, B_4 and also the undetermined constant N are determined with the use of the surface conditions and the initial conditions.

By applying the surface condition that $u=0$ at $z=0$ to the above equation, there is obtained the equation $B_4=0$.

Furthermore, by applying the condition that $u=0$ at $z=2H$ to the above equation one gets

$$\sin N(2H)=0 \quad\cdots\cdots(19)$$

If one lets n_i denote the ith root that satisfy the equation (19), then on obtains

$$u_i=\sum_{i=1}^{\infty}(B_1'e^{\lambda'_1t}+B_2'e^{\lambda_2't})\sin n_iz \quad\cdots\cdots(20)$$

,where $B_1'=B_1B_3$ and $B_2'=B_2B_3$

Applying the initial condition that $u=K$ at $t=0$ to the equation (20) one obtains the equation

$$K=\sum_{i=1}^{\infty}(B_1'+B_2')\sin n_iz$$

The undetermined constant $(B_1'+B_2')$ in this equation is determined as

$$B_1'+B_2'=\frac{K\int_0^{2H}\sin n_izdz}{\int_0^{2H}\sin^2 n_izdz}=\frac{K}{n_iH}(1-\cos 2n_iH) \quad \cdots\cdots\cdots\cdots\cdots(21)$$

Next, let us assume that $\varepsilon_\gamma=0$ at $t=0$, in this case ε_γ is expressed by the following equation.

$$\varepsilon_\gamma(t)=r\eta\int_0^t e^{-\eta(t-\tau)}\left\{\sum_{i=1}^{\infty}(B_1'+B_2')\sin n_iz - \sum_{i=1}^{\infty}(B_1'e^{\lambda_1'\tau}+B_2'e^{\lambda_2'\tau})\sin n_iz\right\}d\tau$$

In the equation, the coefficient of $\sin n_iz$ is

$$r\eta\int_0^t e^{-\eta(t-\tau)}\{(B_1'+B_2')-(B_1'e^{\lambda_1'\tau}+B_2'e^{\lambda_2'\tau})\}\cdot d\tau$$
$$=r\eta\left[\frac{B_1'+B_2'}{\eta}-\left(\frac{B_1'}{\lambda_1'+\eta}e^{\lambda_1' t}+\frac{B_2'}{\lambda_2'+\eta}e^{\lambda_2' t}\right)\right]$$

Since $\varepsilon_\gamma=0$ at $t=0$, this coefficent value is 0.

Therefore,

$$\frac{B_1'}{\lambda_1'+\eta}+\frac{B_2'}{\lambda_2'+\eta}=\frac{(B_1'+B_2')}{\eta} \quad \cdots\cdots\cdots\cdots\cdots(22)$$

B_1' and B_2' are obtained by the equation (21) and (22),

$$B_1'=\frac{(\lambda_1'+\eta)\lambda_2'}{(\lambda_2'-\lambda_1')\eta}\cdot\frac{K}{n_iH}(1-\cos 2n_iH)$$
$$B_2'=-\frac{(\lambda_2'+\eta)\lambda_1'}{(\lambda_2'-\lambda_1')\eta}\cdot\frac{K}{n_iH}(1-\cos 2n_iH)$$

Applying these to the equation (20), one gets

$$u_z=\sum_{i=1}^{\infty}K\left(\frac{(\lambda_1'+\eta)\lambda_2'}{(\lambda_2'-\lambda_1')\eta}e^{\lambda_1' t}-\frac{(\lambda_2'+\eta)\lambda_1'}{(\lambda_2'-\lambda_1')\eta}e^{\lambda_2' t}\right) \times\left(\frac{1-\cos 2n_iH}{n_iH}\sin n_iz\right)\cdots\cdots\cdots\cdots\cdots\cdots(23)$$

V. THE ANALYSIS OF CONSOLIDATION BY THE THREE-DIMENSIONAL DEHYDRATION

As derived in the section II, the fundamental equation of consolidation by the three dimensional dehydration is

$$\frac{k_h}{r_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+a}\frac{\partial u}{\partial r}\right)+\frac{k_v}{r_\omega}\left(\frac{\partial^2 u}{\partial z^2}\right)$$
$$=vpu+\frac{r\eta p}{p+\eta}\cdot u$$

The solution of this equation u is expressed as a product of the respective solutions for the r- and z-axis, namely,

$$u=K\cdot(u_r)_{K=1}\cdot(u_z)_{K=1}$$

Furthermore, letting $\bar{u}$, $\bar{u}_r$ and $\bar{u}_z$ be the mean values of u with respect to the r- and z-axis at the same time, only to the r-axis, and only to the z-axis, respectively, one obtains the equation,

$$\bar{u}=K\cdot(\bar{u}_r)_{K=1}\cdot(\bar{u}_z)_{K=1}$$

Then $(\bar{u}_r)_{K=1}$ and $(\bar{u}_z)_{K=1}$ are calculated as follows.

$$(\bar{u}_r)_{K=1}=\frac{\int_a^R(u_r)_{K=1}r'dr'}{\int_a^R r'dr'}$$
$$=\frac{2}{R_2-a^2}\left[\sum_{i=1}^{\infty}\left\{\frac{(\lambda_1+\eta)\lambda_2}{(\lambda_2-\lambda_1)\eta}e^{\lambda_1 t}-\frac{(\lambda_2+\eta)\lambda_1}{(\lambda_2-\lambda_1)\eta}e^{\lambda_2 t}\right\}\frac{\left(\int_a^R r'D_idr'\right)^2}{\int_a^R r'D_i^2dr'}\right]$$

$$(\bar{u}_z)=\frac{\int_0^{2H}(u_z)_{K=1}\cdot dz}{2H}$$
$$=\frac{2}{H^2}\left[\sum_{i=1,3,5}^{\infty}\left\{\frac{(\lambda_1'+\eta)\lambda_2'}{(\lambda_2'-\lambda_1')\eta}e^{\lambda_1' t}-\frac{(\lambda_2'+\eta)\lambda_1'}{(\lambda_2'-\lambda_1')\eta}e^{\lambda_2' t}\right\}\frac{1}{n_i^2}\right]$$

Therefore,

$$\bar{u}=K\frac{2}{R^2-a^2}\left[\sum_{i=1}^{\infty}\frac{\left\{\int_a^R r'D_idr'\right\}^2}{\int_a^R r'D_i^2dr'}\right.$$
$$\times\frac{1}{(\lambda_2-\lambda_1)\eta}\{(\lambda_1+\eta)\lambda_2e^{\lambda_1 t}-(\lambda_2+\eta)\lambda_1e^{\lambda_2 t}\}$$
$$\times\frac{2}{H^2}\left[\sum_{i=1,3,5}^{\infty}\frac{1}{n_i^2}\cdot\frac{1}{(\lambda_2'-\lambda_1')\eta}\right.$$
$$\left.\times\{(\lambda_1'+\eta)\lambda_2'e^{\lambda_1' t}-(\lambda_2'+\eta)\lambda_1'e^{\lambda_2' t}\}\right]$$

VI. THE AMOUNT OF CONSOLIDATION SETTLEMENT

The amount of consolidation settlement S at any time is expressed as follows.

$$S=\int_0^{2H}\varepsilon\,dz=\int_0^{2H}\left[\bar{p}v+\eta r\int_0^t\bar{p}e^{-\eta(t-\tau)}d\tau\right]dz$$
$$=\int_0^{2H}\left[Kv+\eta r\,Ke^{-\eta t}\int_0^t e^{\eta\tau}d\tau\right]dz$$
$$-\int_0^{2H}\left[\bar{u}v+\eta r\,e^{-\eta t}\int_0^t\bar{u}e^{\eta\tau}d\tau\right]dz$$

The coefficient values in this equation are obtained in the following manner.

$$\eta r\,Ke^{-\eta t}\int_0^t e^{\eta\tau}d\tau=r\,K$$

$$\eta r\int_0^t\bar{u}e^{-\eta(t-\tau)}d\tau$$
$$=\eta r\,K\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}\cdot\frac{\left\{\int_a^R r'D_idr'\right\}^2}{\int_a^R r'D_i^2dr'}$$
$$\times\sum_{i=1,3,5}^{\infty}\frac{2}{H^2n_i^2}\int_0^t\frac{1}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}$$
$$\times[\lambda_2(\lambda_1+\eta)e^{\lambda_1\tau}-\lambda_1(\lambda_2+\eta)e^{\lambda_2\tau}]$$
$$\times[\lambda_2'(\lambda_1'+\eta)e^{\lambda_1'\tau}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2'\tau}]e^{-\eta(t-\tau)}d\tau$$

$$\bar{u}v=vK\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}\cdot\frac{\left\{\int_a^R r'D_i dr'\right\}^2}{\int_a^R r'D_i^2 dr'}$$
$$\times\sum_{i=1,3,5}^{\infty}\frac{2}{H^2 n_i^2}\cdot\frac{1}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}$$
$$\times[\lambda_2\lambda_2'(\lambda_1'+\eta)(\lambda_1+\eta)e^{(\lambda_1'+\lambda_1)t}$$
$$-\lambda_2'\lambda_1(\lambda_1'+\eta)(\lambda_2'+\eta)e^{(\lambda_1'+\lambda_2)t}$$
$$-\lambda_1'\lambda_2(\lambda_1+\eta)(\lambda_2'+\eta)e^{(\lambda_2'+\lambda_1)t}$$
$$+\lambda_1'\lambda_1(\lambda_2'+\eta)(\lambda_2+\eta)e^{(\lambda_2'+\lambda_2)t}]$$

Now λ_1, λ_2, λ_1' and λ_2' are examined. Writing

$$\frac{k_h}{v\,\gamma_\omega}=C_h \quad \text{and} \quad \frac{k_v}{v\,\gamma_\omega}=C_v$$

in the equations (6) and (18) and making the expansion of the $\sqrt{\ }$ part by means of the binomial theorem, λ_1, λ_2, λ_1' and γ_2' are calculated as follows.

$$\lambda_1=-\frac{C_h\,\eta\, m_i^2}{\frac{v+\gamma}{v}\eta+C_h m_i^2}-\frac{(C_h\,\eta\, m_i^2)^2}{\left(\frac{v+\gamma}{v}\eta+C_h m_i^2\right)^3}-\cdots$$

$$\lambda_2=-\frac{v+\gamma}{v}\eta-C_h m_i^2$$
$$+\frac{C_h\,\eta\, m_i^2}{\frac{v+\gamma}{v}\eta+C_h m_i^2}+\frac{(C_h\,\eta\, m_i^2)^2}{\left(\frac{v+\gamma}{v}\eta+C_h m_i^2\right)^3}-\cdots$$

$$\lambda_1'=-\frac{C_v\,\eta\, n_i^2}{\frac{v+\gamma}{v}\eta+C_v n_i^2}-\frac{(C_v\,\eta\, n_i^2)^2}{\left(\frac{v+\gamma}{v}\eta+C_v n_i^2\right)^3}-\cdots$$

$$\lambda_2'=-\frac{v+\gamma}{v}\eta-C_v n_i^2$$
$$+\frac{C_v\,\eta\, n_i^2}{\frac{v+\gamma}{v}\eta+C_v n_i^2}+\frac{(C_v\,\eta\, n_i^2)^2}{\left(\frac{v+\gamma}{v}\eta+C_v n_i^2\right)^3}+\cdots$$

Next, the examination is made of the case where the creep coefficient η is very small compared with the permeability coefficient k, and creep continues very long.

In this case where $\eta \ll C_v n_i^2$ and $\eta \ll C_h m_i^2$, constants become

$$\lambda_1 \fallingdotseq -\eta-\frac{\eta^2}{C_h m_i^2},\quad \lambda_2 \fallingdotseq -C_h m_i^2$$

and

$$\lambda_1' \fallingdotseq -\eta-\frac{\eta^2}{C_v n_i^2},\quad \lambda_2' \fallingdotseq -C_v n_i^2$$

The above equations are arranged using these values as follows.

$$\eta\gamma\int_0^t \bar{u}e^{-\eta(t-\tau)}d\tau$$
$$=\gamma K\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}\cdot\frac{\left\{\int_a^R D_i r' dr'\right\}^2}{\int_a^R D_i^2 r' dr'}$$
$$\times\sum_{i=1,3,5}^{\infty}\frac{2}{H^2 n_i^2}e^{-2\eta t}$$

$$\bar{u}v=vK\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}\cdot\frac{\left\{\int_a^R D_i r' dr'\right\}^2}{\int_a^R D_i^2 r' dr'}$$
$$\times e^{-C_h m_i^2 t}\sum_{i=1,3,5}^{\infty}\frac{2}{H^2 n_i^2}e^{-C_v n_i^2 t}$$

$$\therefore\quad S=2\,HK(v+\gamma)-2\,HvK\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}$$
$$\times\frac{\left\{\int_a^R D_i r' dr'\right\}^2}{\int_a^R D_i^2 r' dr'}$$
$$\times e^{-C_h m_i^2 t}\sum_{i=1,3,5}^{\infty}\frac{2}{H^2 n_i^2}e^{-C_v n_i^2 t}$$
$$-2\,H\gamma\,K\sum_{i=1}^{\infty}\frac{2}{R^2-a^2}\cdot\frac{\left\{\int_a^R D_i r' dr'\right\}^2}{\int_a^R D_i^2 r' dr'}$$
$$\times\sum_{i=1,3,5}^{\infty}\frac{2}{H^2 n_i^2}\cdot e^{-2\eta t}\quad\cdots\cdots(24)$$

Then the value of the term

$$\frac{\left[\int_a^R D_i r' dr'\right]^2}{\int_a^R D_i^2 r' dr'}$$

in the equation (24) is calculated.

As to the numerator,

$$\left[\int_a^R r'D_i dr'\right]^2=\frac{a^4}{({}_n m_i a)^2 Y_0^2({}_n m_i a)}$$
$$\times\{J_0({}_n m_i a)Y_1({}_n m_i a)-J_1({}_n m_i a)Y_0({}_n m_i a)\}^2$$

As to the denominator,

$$\int_a^R r'D_i^2 dr'$$
$$=\frac{R^2}{2\,Y_0^2({}_n m_i a)}\{J_0({}_n m_i R)Y_0({}_n m_i a)-J_0({}_n m_i a)Y_0({}_n m_i R)\}^2$$
$$-\frac{a^2}{2\,Y_0^2({}_n m_i a)}\cdot\{J_1({}_n m_i a)Y_0({}_n m_i a)-J_0({}_n m_i a)Y_1({}_n m_i a)\}^2$$

Thus there is obtained

$$\frac{\left[\int_a^R r'D_i dr'\right]^2}{\int_a^R r'D_i^2 dr'}=2\,a^2.C$$

, where

$$C=\frac{1}{({}_n m_i a)^2}\cdot\frac{\{J_0({}_n m_i a)Y_1({}_n m_i a)-J_1({}_n m_i a)Y_0({}_n m_i a)\}^2}{n^2\{J_0(n\,{}_n m_i a)Y_0({}_n m_i a)-J_0({}_n m_i a)Y_0(n\,{}_n m_i a)\}^2-\{J_1({}_n m_i a)Y_0({}_n m_i a)-J_0({}_n m_i a)Y_1({}_n m_i a\}^2}$$

and $n=\dfrac{R}{a}$

Applying this to the equation (24), and writing

$$T_h=\frac{C_h t}{(2R)^2}=\alpha T_v.$$

$$\alpha=\frac{C_h}{C_v}\left(\frac{H}{2R}\right)^2 \quad \text{and} \quad T'=2\eta t,$$

one obtains

$$S=2HK(v+r)-2HvK\sum_{i=1}^{\infty}\times\frac{4}{n^2-1}\cdot C\cdot e^{-4n^2({}_nm_ia)^2\alpha T_v}\cdot\sum_{i=1,3,5}^{\infty}\times\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}-2HrKe^{-T'} \cdots\cdots(25)$$

The final amount of consolidation settlement S_∞ is obtained by making $t\to\infty$ in the equation (25), and the degree of consolidation U is calculated by the use of it.

$$U=\frac{v}{v+r}U_v+\frac{r}{v+r}U_r \cdots(26)$$

,where

$$U_v=1-\sum_{i=1}^{\infty}\frac{4}{n^2-1}\cdot C\cdot e^{-4n^2({}_nm_ia)^2\alpha T_v}\times\sum_{i=1,3,5}^{\infty}\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}$$

$$U_r=1-e^{-T'}$$

The values of the undetermined constants n_i and ${}_nm_i$ are calculated from the equations (19) and (9), and the results are shown in the following tables.

Furthermore, the values of $\frac{4}{n^2-1}C$ in the equation (26) are calculated by the use of the (${}_nm_ia$) values in Table 2, and the results are shown in Table 3.

The relation between U_v and T_v in the equation (26) is shown for various values of α in Fig. 2.

Table 1 The values of the undetermined constants $2Hn_i$.

$\sin n_i(2H)=0$						
$2Hn_1$	$2Hn_3$	$2Hn_5$	$2Hn_7$	$2Hn_9$	$2Hn_{11}$	$2Hn_{13}$
π	3π	5π	7π	9π	11π	13π

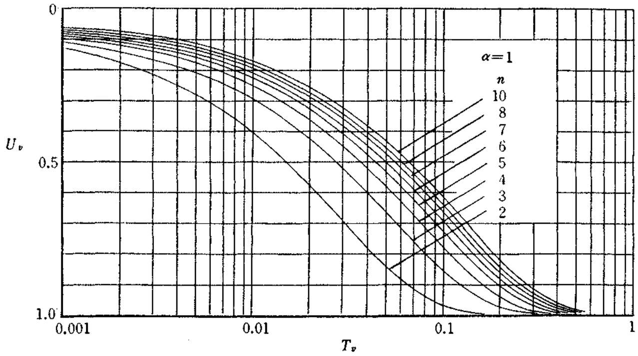


Fig. 2(a) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=1$ and n.

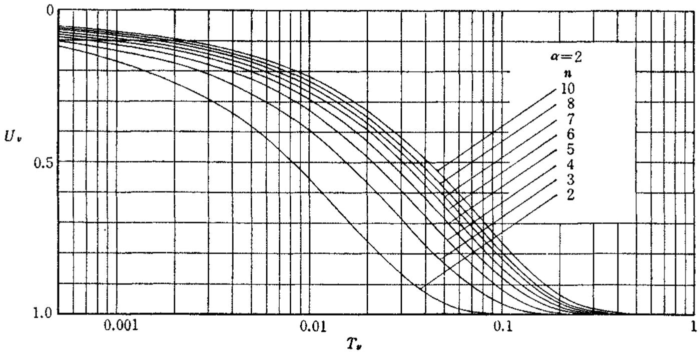


Fig. 2(b) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=2$ and n.

Table 2 The values of the undetermined constants corresponding to the values of n.

$$\frac{J_0({}_nm_ia)}{Y_0({}_nm_ia)}=\frac{J_1(n\cdot{}_nm_ia)}{Y_1(n\cdot{}_nm_ia)}$$

n \ ${}_nm_ia$	m_1a	m_2a	m_3a	m_4a	m_5a	m_6a	m_7a
2	1.347	4.645	7.814	10.967	14.090	17.261	20.406
3	0.625	2.304	3.894	5.475	7.050	8.625	10.195
4	0.393	1.527	2.598	3.637	4.702	5.747	6.796
5	0.282	1.139	1.948	2.731	3.520	4.308	5.095
6	0.218	0.907	1.549	2.183	2.815	3.445	4.076
7	0.176	0.752	1.289	1.817	2.344	2.870	3.395
8	0.147	0.643	1·105	1.556	2.008	2.459	2.909
10	0.110	0.497	0.855	1.210	1.561	1.914	2.262

Table 3 The values of $\frac{4}{n^2-1}C$ corresponding to the values of n.

$$\frac{4}{n^2-1}\cdot C=\frac{4}{(n^2-1)({}_nm_ia)^2}\cdot\frac{\{J_0({}_nm_ia)Y_1({}_nm_2a)-J_1({}_nm_ia)Y_0({}_nm_ia)\}^2}{n^2\{J_0(n_nm_ia)Y_0({}_nm_ia)-J_0({}_nm_ia)Y_0(n_nm_ia)\}^2-\{J_1({}_nm_ia)Y_0({}_nm_ia)-I_0({}_nm_ia)Y_1({}_nm_ia)\}^2}$$

n \ i	1	2	3	4	5	6	7
2	0.8999	0.0632	0.0220	0.0121	0..067	0.0048	0.0032
3	0.9035	0.0491	0.0168	0.0084	0.0051	0.0034	0.0026
4	0.9229	0.0406	0.0135	0.0068	0.0040	0.0027	0.0021
5	0.9355	0.0351	0.0113	0.0058	0.0034	0.0023	0.0016
6	0.9413	0.0300	0.0100	0.0050	0.0029	0.0020	0.0014
7	0.9572	0.0279	0.0089	0.0043	0.0026	0.0017	0.0012
8	0.9669	0.0258	0.0080	0.0039	0.0023	0.0015	0.0011
10	0.9668	0.0225	0.0068	0.0033	0.0019	0.0013	0.0009

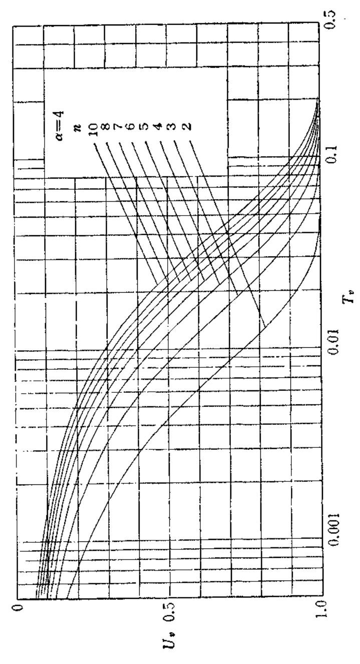


Fig. 2(c) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=4$ and n.

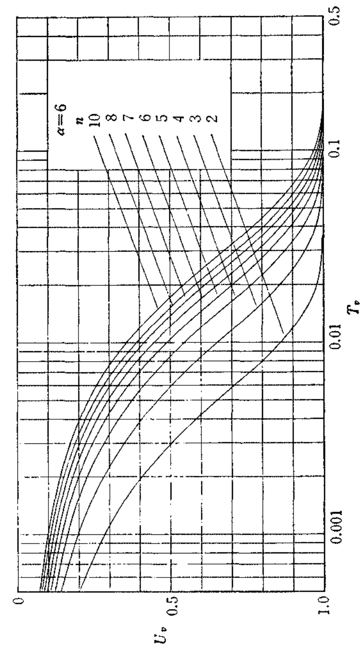


Fig. 2(d) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=6$ and n.

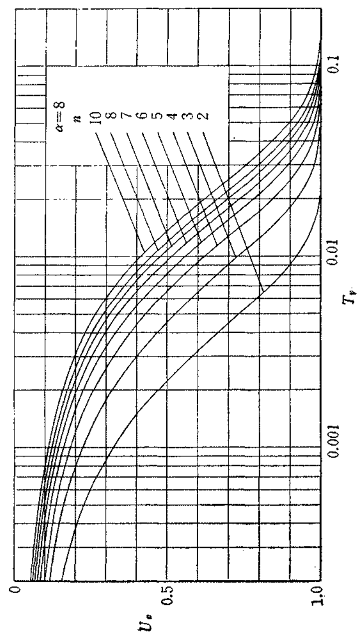


Fig. 2(e) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=8$ and n.

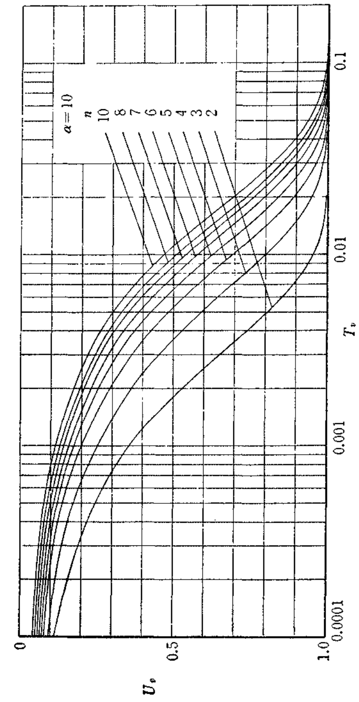


Fig. 2(f) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=10$ and n.

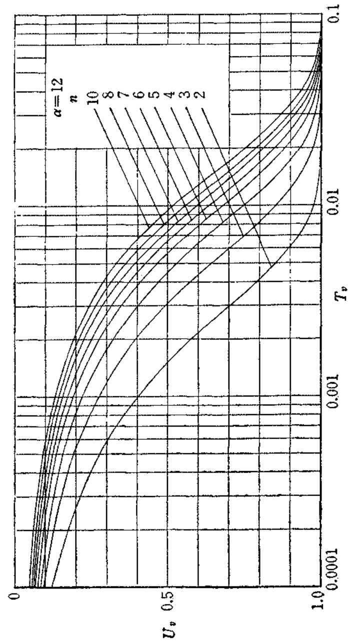


Fig. 2(g) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=12$ and n.

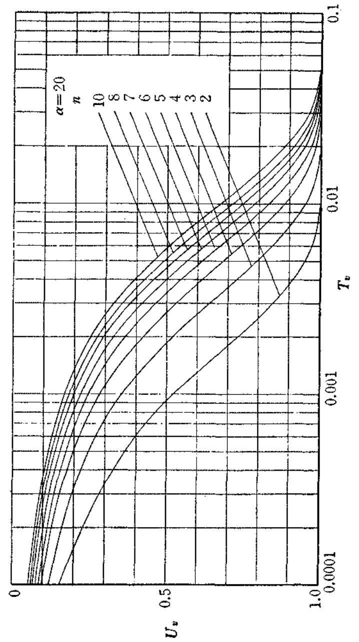


Fig. 2(i) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=20$ and n.

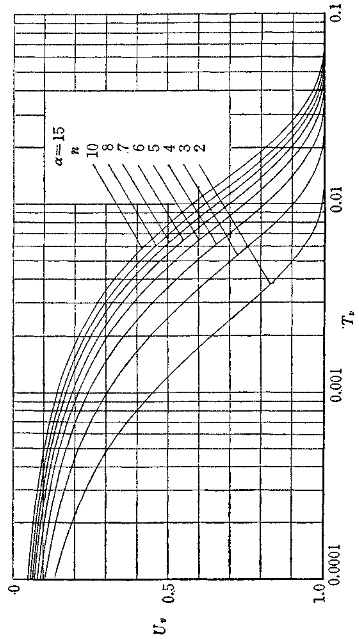


Fig. 2(h) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=15$ and n.

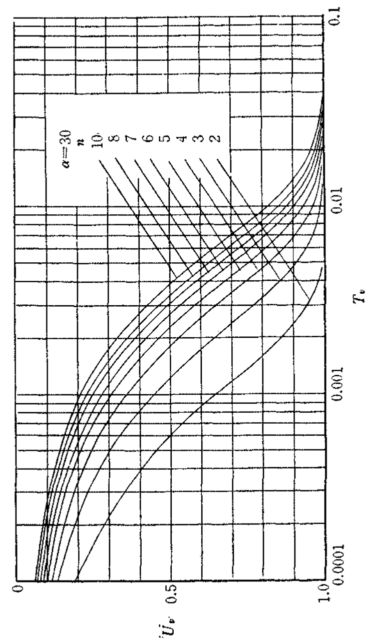
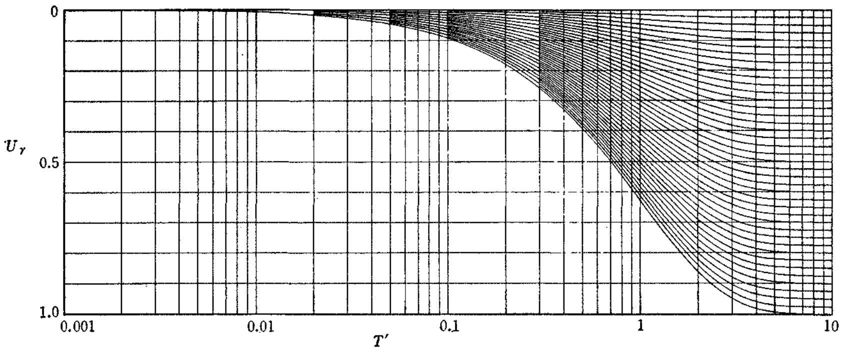


Fig. 2(j) The figure of the $U_v-\log T_v$ curves for the values of $\alpha=30$ and n.



Note : From M. Fukuoka and M. Inada ; "Study on the settlement of banks of rivers in the weak ground area". Civil Engineering Research Institute Report No. 87-1 (January 1954).

Fig. 3 The figure of the $U_r-\log T'$ curves.

Moreover, the present author wrote out a table 10 kinds of $\alpha=1$, 2, 4, 6, 8, 10, 12, 15, 20 and 30.

Here is shown in Fig. 2-a～Fig. 2-j.

Next, the relation between U_r and T' is shown in Fig. 3.

By using each table mentioned above, we can easily estimate the consolidation settlement of the ground due to sand pile drainage works.

VII. THE METHOD OF FORMING THE ESTIMATE-CURVE OF CONSOLIDATION SETTLEMENT IN THE FIELD

The author explains as follows the way to apply the graph of the results of the analysis above mentioned, on estimating the amount of consolidation settlement due to sand pile dranage works.

(1) Determination of the Value of n

The value of n can be obtained from the equation $n=R/a$, when a is a radius of sand pile and R is a radius of effective circle. According to Barron's method, we make $2R$, which denotes a diameter of effective circle, a diameter of a circle with the same area as that in charge of one sand pile. Now assuming that d is the central distance of sand pile, the equation $R=0.564\,d$ is formed when sand pile arranged square, and $R=0.525\,d$ when arranged triangularly.

(2) Determination of the Value of the constant α

The constant α is a ratio of time coefficient T_h of horizontal direction and time coefficient T_v of vertical direction. Thus using this value, we can obtain T_v corresponding to T_h and can denote the degree of consolidation due to the three-dimensional dehydration as a function of only α.

$$\alpha=\frac{T_h}{T_v}=\frac{C_h}{C_v}\left(\frac{H}{2R}\right)^2$$

,where

C_h, C_v : consolidation coefficients of horizontal and vertical direction

$2R$: diameter of effective circle

H : a half of tickness of consolidation layer

As in shown in the above equation, α has different values even in the same soil by the scale of soil body consolidated. So it is not the consolidation constant peculiar to the soil.

Therefore, the value of α of the soil on the field can be obtained from the value of α' which was got from the results of the three-dimensional dehydration consolidation test.

The way to obtain the value of α' was already reported by the author, so it is not referred here. But the value of α' is shown by the next equation when α' is determined on the consolidation data of a cylinder with $2H'$ in height and $2R'$ in diameter.

$$\alpha'=\frac{C_h}{C_v}\cdot\left(\frac{H'}{R'}\right)^2$$

Supposing the value of $\left(\frac{C_h}{C_v}\right)$ to be the consolidation constant of the soil, the relation between α' and α is shown as the next equation.

$$\alpha'\left(\frac{R'}{H'}\right)^2=\alpha\left(\frac{2R}{H}\right)^2$$

$$\therefore\quad \alpha=\frac{\alpha'}{4}\left(\frac{R'H}{RH'}\right)^2$$

Thus we can determined the value of α.

(3) Determination of Elastic Deformation Rate v, Plastic Deformation rate γ and creep coefficient η

The values of v and γ are expressed by the ratio of elastic deformation strain ε_v and plastic deformation strain ε_γ to the increased part Δp of consolidation load. The coefficient η denotes creep coefficient and also the amount corresponding to the speed of plastic deformation strain. Plastic deformation strain is the proportion constant on the assumption that it is produced in proportion to the speed of plastic deformation strain next to happen.

The consolidation constants v, γ and η of the soil can be obtained experimentally. The method is not referred here as it was already reported.

(4) The method of forming the estimate-curve of consolidation settlement

At first, picking up $U_v - \log T_v$ curve, corresponding to the value of n and α determined in the above, from the Fig. 2, we copy the curve on a tracing paper. On that occasion on the abscissa of the tracing paper we write in time t, corresponding to the value of T_v in the Fig. 2, doubly on T_v by getting t from the equation $t=\frac{1}{C_v}H^2T_v$. Thus $U_v - \log t$ curve written on the tracing paper means settlement curve of primary consolidation part corresponding to the lapsed time of consolidation.

Next, we are to get settlement curve of secondary compression part. The amount of final consolidation settlement of secondary compression part is γ/v corresponding to that of primary consolidation part be 1.0. Therefore the settlement curve of secondary compression part corresponds to the curve in which the limitation of U_γ, in the group of $U_\gamma - \log T'$ curves in the Fig. 3, becomes γ/v.

Now putting the tracing paper, written the settlement curve of primary consolidation part, on this curve and also putting together the abscissa of $U_v = 1.0$ and that of U_γ, just as the value of T', obtained from the equation $T'=2\eta t$, comes under time t of the tracing paper, we copy the settlement curve on the tracing paper.

In this way we can get U- $\log t$ curve corresponding to total amount of settlement by primary and secondary consolidation curve written on the tracing paper. Now we can get the amount of final settlement from the equation $S=2HK(v+\gamma)$. So assuming that $U=100\%$ corresponds to the amount of settement of $2HK(v+\gamma)$, we can obtain the estimate-curve of consolidation settlement of the field due to sand pile, by graduating the amount of settlement on the vertical axis.

VIII. THE APPLICATION EXAMPLE FOF THE EFFECT OF THE CONSOLIDATION CONCERNING ON THE IMPROVEMENT WORK OF THE WEAK GROUND IN THE VICINITY OF THE KOISEGAWA RAILWAY BRIDGE OF THE JAPANESE NATIONAL RAILWAYS.

(1) Introduction

The method of analysis on the consolidation effect due to sand pile drainage work, mentioned on the above section, was applied to the widening of the railway embankment in the vicinity of the Koisegawa Railway Bridge between Takahama and Kandachi on the Joban Line of the Japanese National Railways. Prior to the work, the engineering staff of the Japanese National Railways made the investigations to the deformation and settlement of ground caused by the partial prefilling, especially to the consolidation settlement of the ground where the work had been carried out by the Compozer method. The author analyzed these data of measurement by the method of analysis by the three-dimensional dehydration that the author proposed, and here the abstract of the results is described.

(2) The general Aspect of the Ground and Their Consolidation Characteristics.

As to the state of the ground in question, the detailed reports are issued by Mr. Tadahiko Muromachi, the chief research member of the Japanese National Railways, and his co-researchers. Now only the matters having any relation to the object of this report are picked up and described. According to the above report, the ground is a large-scale weak ground forming an alluvial valley about 1.2 km wide.

It consists of a weak clay layer ($N=0\sim1$) that reaches 20 m in the middle of the valley, with a very weak peat layer from the surface to the depth of 3~4 m.

From the results of consolidation tests with the soil sample of the top peat layer and that of the underlying continuous clay layer, it was noticed that the coefficients of consolidation decrease rapidly with increasing intensity of load with the both layers, and that the average values of the coefficient are $2.3\times10^{-2}\sim1.5\times10^{-3}$ cm²/sec and $3.0\times10^{-3}\sim1.7\times10^{-4}$ cm²/sec with the peat layer and the clay layer, respectively in the range of the intensity of load up to 1.6 kg/cm². The value of the coefficient on the clay layer is smaller than that of the peat layer by one order, which suggests that consolidation lasts longer in the clay layer, Furthermore, the thickness of the clay layer is 5~6 times as that of the peat layer, so the amount of consolidation in the clay layer is thought to be remarkably larger. The general information of the ground is as mentioned above. The author analyzed the settlement phenomenon of the clay layer observed by the technical staff of the Japanese National Railways from the standpoint of consolidation due to the three-dimensional dehydration.

In the analysis, the Compozers of 40 cm diameter settled in the triangular disposal with the distance of 1.6 m center to center were regarded as sand drains of the same diameter with the disposal. Furthermore, the clay layer was assumed to be in contact with dehydration layers on the both faces, for the top and the bottom surface of the clay layer are in contact with the sand seam and gravels respectively.

(3) Estimation of the consolidation load

a) The overburden intensity of load p_0

The unit intensity of load at any point in the subsoil are shown in Table 4. As the groundwater is level with the ground surface, these intensities of load are supposed to be the weight of the saturated soils, and the bulk densities of the soils in water are calculated from these values to estimate the overburden intensity of load p_0 towards the depth.

The soil profile of the ground is shown in Fig. 4, and the overburden intensity of load p_0 at the middle point of the clay layer (depth : 10 m) is 2.85 t/m^2.

Table 4 Weight per unit volume of the soil at the state of the ground.

Depth (m)	0~2.5 (Peat layer)	2.5~3.0 (Sand layer)	3.0~13.0 (Clay layer)	13.0~17.0 (Clay layer)
Unit Weight of Soil (t/m^3)	1.20	1.50	1.30	1.40

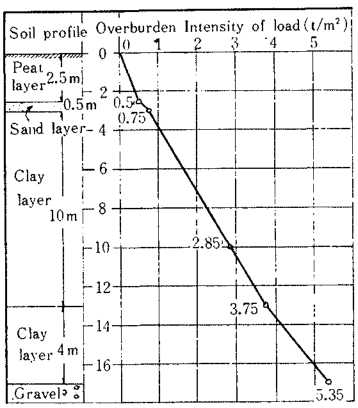


Fig. 4 The chart indicating the overburden load strength per unit area of the working ground.

b) The intensity of load caused by sand mat banking.

The intensity of load in the subsoil under the center line of the sand mat p_1 was calcutated on the assumption that the sand mat treated part as the trial banking is formed a trapezoid with the height h=0.8 m, the top width of 17.2 m and the slope gradient of 1 : 1.5, and that the bulk density is γ=1.65 t/m^3.

The value of the dispersion coeffecient μ corresponding to the trapezoid load was obtained by using twice the modified μ-chart, by transposing the origin of this chart prepared according to the equation of the vertical stress under an infinite terrace,

$$p_z=\frac{\gamma h}{\pi B}(B\beta+x\alpha)=\mu\gamma h$$

,furthermore we got the intensity of load in the ground by multiplying this value by the sand mat load γh.

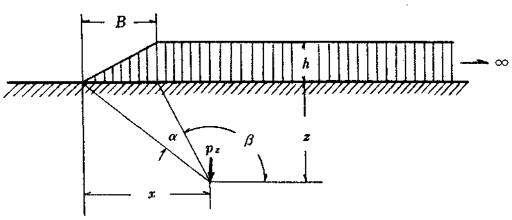


Fig. 5 The explanatory chart of the marks in the equation of the normal stress at the spot under the infinite terrace.

c) The intensity of load caused by the banking part on the sand mat

The intensity of load in the ground caused by the banking part at the same point as in b) p_2 was calculated on the assumption that the banking part on the sand mat is a trapezoid with the height h= 0.7 m, the top width of 15.1 m and the slope gradient of 1 : 1.5, and that the bulk density is γ= 1.65 t/m^3. The method of calculation was the same as in b), and the value of the dispersion coefficient used was that obtained in b), for the values hardly differ from each other with the both cases.

d) The intensity of load caused by the present railway embankment

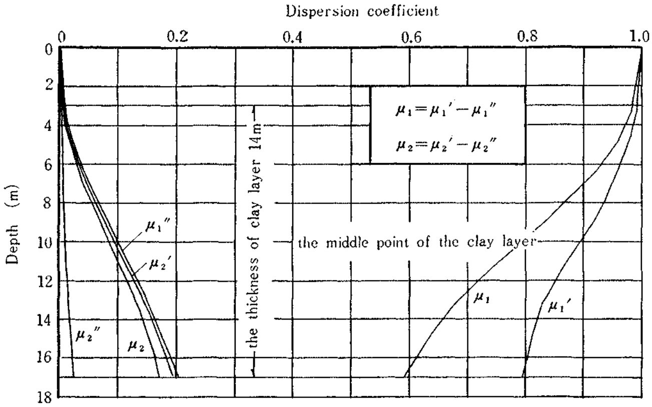
As the above-mentioned trial banking in widening of present railway embankment, the influence of the load strength per unit area caused by the present railway embankment p_3 must be taken into consideration in the estimation of the amount of settlement due to the trial banking.

The present railway banking is a trapezoid with the height h=3.0 m, the top width of 15.1 m and the slope gradient of 1 : 1.5.

The load dispersion coefficient at the point was calculated with which p_1 and p_2 had been calculated. The intensity of load in the ground caused by present railway embankment p_3 was calculated by multiplying the dispersion coefficient by the γh value, which was obtained as

$$\gamma h=1.7\times(3.0+0.6)=6.12 \text{ t/m}^2,$$

taking the reduced height, 0.6 m, of the railway track load into consideration. The intensity of load



where, μ_1 is the dispersion coefficient caused by the sandmat or embankment part.
μ_2 is the dispersion coefficient caused by the present railway banking.

Fig. 6(a) The calculated graph for the dispersion coefficient with the depth of soil due to the loading by the embankment.

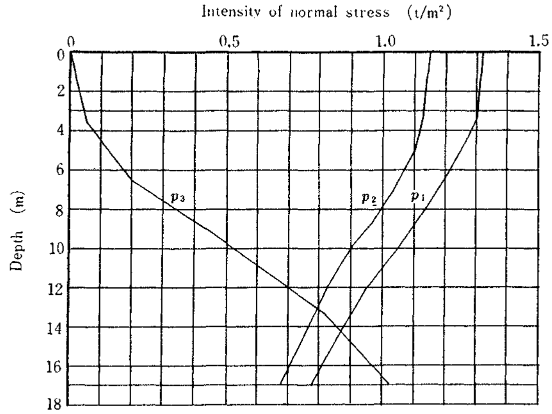


Fig. 6(b) The curve indicating the normal stress p_1, p_2 and p_3 accompany with the depth of subsoil.

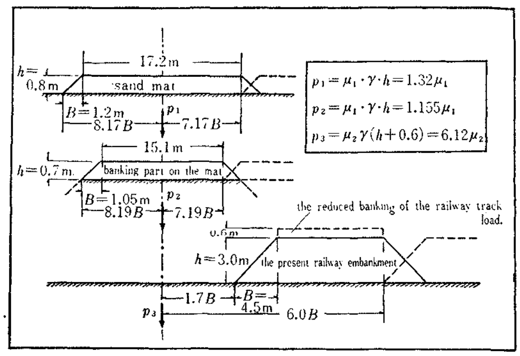


Fig. 6(c) An application Line of normal stress p_1, p_2 and p_3 in the subsoil for calculation.

p_1, p_2 and p_3 as obtained above are shown in Fig. 6.

(4) Calculation of the Final Amount of Settlement.

a) The final amount of settlement due to the sand mat work.

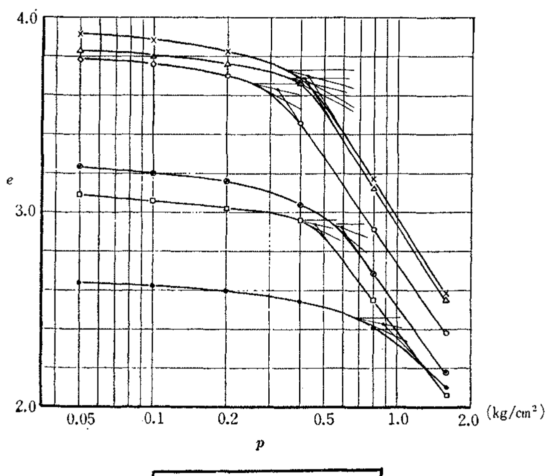
From the results of the consolidation tests with the clay samples, there were obtained the $e-\log p$ curves as shown in Fig. 7. The values of the preconsolidation load, the initial void ratio and the coefficient of compressibility obtained from the figure are shown in Table 5.

In Fig. 8 is shown the comparison of the precompression load vs. the depth of sampling points plotted from the data shown in Table 5 with the pressure curve of the intensity of load (p_0+p_3) in the ground prior to the sand mat work. This figure is shown that the clay layer is over-consolidated.

The nearest point of the plotted ones to the (p_0+p_3) pressure curve is picked

Table 5 The calculated table of the consolidation constants.

Depth of sampling (m)	Precompression load (t/m²)	Difference of void ratios (e_a-e_b)	Difference of the intensity of load $(\log p_b-\log p_a)$	Coefficient of compressibility	Initial void ratio
4.20	3.20	0.53	0.30103	1.76	3.63
6.20	4.30	0.59	〃	1.96	3.70
9.19	4.15	0.57	〃	1.89	3.69
11.17	4.60	0.49	〃	1.63	2.94
14.19	6.00	0.48	〃	1.49	2.92
16.19	8.80	0.38	〃	1.26	2.43



Explanations

Sort of sample	Depth of sample	Thickness of layers
—○—○—	4.20m	2.8m
—×—×—	6.20	2.0
—△—△—	9.19	2.2
—□—□—	11.17	3.0
—●—●—	14.19	1.5
—•—•—	16.19	2.5

Σ 14.0m

Fig. 7 $e-\log p$ curves.

up and the precompression load is assumed to be expressed by the parallel line to the pressure curve passing through this picked-up point. From this line, the precompression load at the middle point of the clay layer (depth : 10 m) is obtained as $\sigma_0=4.10$ t/m^2.

Furthermore, the average initial void ratio $\bar{e}_0$, the average precompression load $\bar{\sigma}_0$ and the average coefficient of compressibility $\bar{c}_c$ are calculated from Table 5 as follows.

$$\bar{e}_0'=\frac{3.63+3.70+3.69+2.94+2.92+2.43}{6}$$

$$=3.22$$

The standard deviation of $\bar{e}_0$

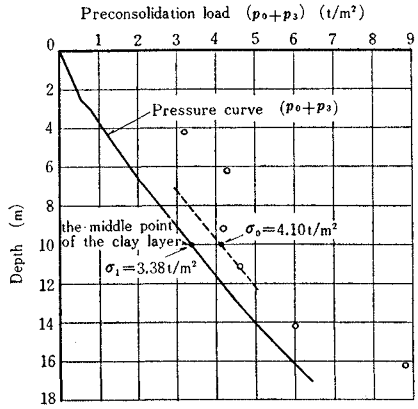


Fig. 8 The chart of relationship between the precompression load and the depth

$$=\sqrt{\frac{(3.63-3.22)^2+(3.70-3.22)^2+(3.69-3.22)^2+(2.94-3.22)^2+(2.92-3.22)^2+(2.43-3.22)^2}{6-1}}$$

$$=0.54$$

$$\bar{e}_0=\bar{e}_0'-0.54=2.68$$

(the initial void ratio for calculation)

In the sameway, $\bar{\sigma}_0'=5.18$ t/m^2, the standard deviation of $\bar{\sigma}_0=1.99$ t/m^2 and $\bar{\sigma}_0=\bar{\sigma}_0'-1.99=3.19$ t/m^2 (the precompression intensity of load for calculation), and also $\bar{c}_c'=1.67$, the standard deviation of $\bar{c}_c=0.26$, $\bar{c}_c=\bar{c}_c'+0.26=1.93$ (the coefficient of compressibility for design) Therefore the initial void ratio for the design use at the middle point of the clay layer is obtained as

$$e_0=\bar{e}_0-\bar{c}_c\log\left(\frac{\sigma_0}{\bar{\sigma}_0}\right)=2.47$$

From the above results, the final amount of settlement S is calculated from the equation

$$S=\frac{2H}{1+e_0}\bar{c}_c\log\left(\frac{\sigma_1+\Delta\sigma}{\sigma_0}\right)$$

and is shown in Table 6.

where symbols stand for

σ_1 : the effective loading pressure strength prior to banking at the middle point of the clay layer ($=p_0+p_3$ t/m^2)

$\Delta\sigma$: increase of stress in the ground at the same point caused by the banking work ($=p_1$t/m^2)

σ_0 : the precompression load at the same point (t/m^2).

The above value of the final amount of settlement was calculated on the assumption that the stress in the ground caused by the sand mat does not vary with proceeding settlement of the sand mat. In the actual case, however, the stress in the ground decreased according as the sand mat settles itself to get dipped into water, and so it is necessarily required to make correction in the case of a large amount of settlement.

The rate of decrease of the amount of settlement according to decrease of unit intensity of load caused by sand mat is

$$K_n=\frac{S}{(\sigma_1+\Delta\sigma)-\sigma_0}=0.794(\text{m}^3/\text{t})$$

The bulk density of the dipped sand mat γ_{sub} is $\gamma_{sub}=\frac{G-1}{1+e}\gamma_w$, and the term e in the equation is obtained by $e=\frac{G}{\gamma_t}(1+\omega)-1$.

Taking the wet bulk density $\gamma_t=1.65$ t/m^3, the water content $\omega=0.1$ and the specific gravity of soil particles $G=2.60$, one obtains $e=0.73$, and then $\gamma_{sub}=0.92$ t/m^3.

The decrease of the stress in the ground according to the settlement of the sand mat of 1.0 m is, therefore, $(\gamma_t-\gamma_{sub})\mu_1$. Then the amount of settlement according to decrease of unit intensity of load caused by the sand mat, K_b is

$$K_b=\frac{1}{(\gamma_t-\gamma_{sub})\mu_1}=\frac{1}{(1.65-0.92)\times0.79}$$

$$=1.734(\text{m}^3/\text{t})$$

Now, in case the intensity of load caused by the sand mat decreases according as the a part of sand mat settles itself to get dipped into water, and the same intensity of load became σ_n at that time, there is obtained the final amount of settlement S_n, and therefore S_n is expressed by the following equation.

$$S_n=K_b\{(\sigma_1+\Delta\sigma)-\sigma_n\} \quad \cdots\cdots(27)$$

Table 6 The calculation table for the final amount of settlement.

$2H$ (m)	p_0(t/m^2)	p_3(t/m^2)	$\sigma_1=p_0+p_3$(t/m^2)	$\Delta\sigma=p_1$(t/m^2)	$\sigma_1+\Delta\sigma$(t/m^2)	σ_0(t/m^2)	$\bar{c}_c$	e_0	S(m)
14.0	2.85	0.53	3.38	1.05	4.43	4.10	1.93	2.47	0.262

and

$$S-S_n=K_n\{(\sigma_1+\Delta\sigma)-\sigma_n\} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(28)$$

From the equation (27) and (28), one may get the corrected final amount of settlement S_n as follows

$$S_n=\frac{K_b}{K_n+K_b}\cdot S=\frac{1.734}{2.528}\times 26.2=17.3\text{ cm}$$

b) The final amount of settlement only caused by the banking part on the sand mat.

First, the final amount of settlement caused by the sand mat and the banking part together is calculated in the same manner as in a).

$$S=\frac{2H}{1+e_0}\bar{c}_c\log\left(\frac{\sigma_1+\Delta\sigma}{\sigma_0}\right)$$

In the above equation,

$$\sigma_1=p_0+p_3=3.38\text{ t/m}^2$$
$$\Delta\sigma=p_1+p_2=1.95\text{ t/m}^2$$

and

$$S=\frac{14.0}{1+2.47}\times 1.93\log\left(\frac{5.33}{4.10}\right)=0.887\text{ m}$$

The difference of the amount of settlement according to $\sigma_n'=5.0\text{ t/m}^2$, S_n', and S is

$$S-S_n'=\frac{2H}{1+e_0}\bar{c}_c[\log(\sigma_1+\Delta\sigma)-\log\sigma_n']$$
$$=7.787(\log 5.33-\log 5.0)=0.216\text{ m}$$

On the assumption that the amount of settlement varies linearly in the range of the intensity of load of $5.33\sim 5.0\text{ t/m}^2$, the rate of variance of the intensity of load of $5.53-5.0=0.33\text{ t/m}^2$ is

$$K_n'=\frac{0.216}{0.33}=0.655(\text{m}^3/\text{t})$$

Now, the corrected final amount of settlement S_n of this case is calculated in the same manner as in a), one then obtains

$$S_n=\frac{K_b}{K_n'+K_b}S=\frac{1.734}{2.389}\times 88.7=64.2\text{ cm}$$

From the above calculations, there is obtained the final amount of settlement only caused by the embankment part on the sand mat as

$$S=64.2-17.3=46.9\text{ cm}$$

(5) Calculations of the Time of Consolidation Settlement.

The Compozers of 40 cm diameter were settled in the triangular disposal with the center to center distance of 1.6 m. The author examined further the amount of consolidation settlement by the analysis method of consolidation due to the three-dimensional dehydration, regarding these Compozers as sand drains with the same diameter and disposal.

The diameter of the effective circle of drains $2R$ is taken to be the same as that of the circle having the same area as that affected by one sand pile, according to Barron's method.

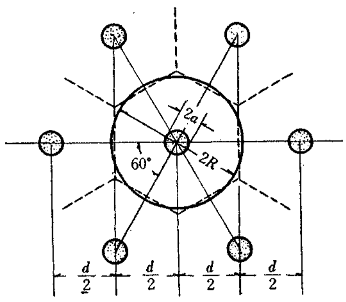


Fig. 9 The chart for the disposition of sand piles.

Taking d to be the center to center distance of the regular triangular disposal of sand piles, one gets

$$\frac{3}{2}d^2\tan 30°=\pi R^2$$

$$R=\sqrt{\frac{\sqrt{3}}{2\pi}}d=0.525\,d$$

Therefore the ratio of the radius of the effective circle R to that of drains $a(=20\text{ cm})$, n is

$$n=\frac{R}{a}=\frac{0.525\times 160}{20}=4.2$$

The time of settlement is calculated by the equation $t=\frac{H^2}{\bar{C}_v}T_v$. In the equation, $\bar{C}_v$ is the average coefficient of consolidation with the consolidated clay layer, and is calculated from the data based on the $\log C_v-\log p$ curves of the consolidation tests with soil samples at various depth in the clay layer, by the use of the equation

$$\bar{C}_v=\left[\frac{\Sigma H_i}{\Sigma\frac{H_i}{\sqrt{C_{vi}}}}\right]^2$$

where H_i = the thickness of the i th layer

C_{vi} = the coefficient of consolidation of the i th layer.

The $\log C_v-\log p$ curves and the $\log\bar{C}_v-\log p$

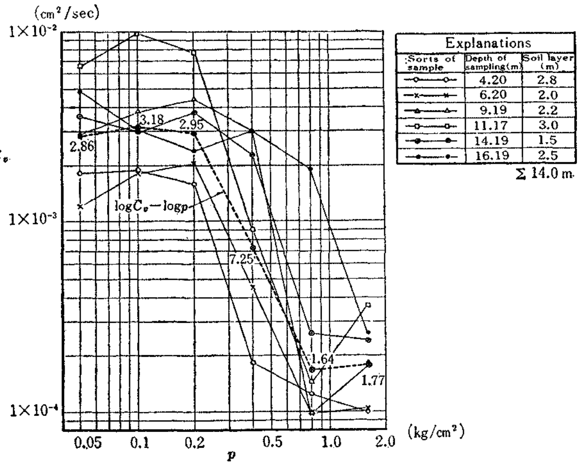


Fig. 10 log. C_v-log p curves.

curve are shown in Fig. 6.

a) The settlement curve only caused by the load of the sand mat after the settling work of Compozers.

In the calculation of the time of settlement, the relation of the degree of consolidation to the time factor is obtained from the $U_v-\log T_v$ chart with the value of $\alpha=\frac{C_h}{C_v}\left(\frac{H}{2R}\right)^2$ and n, for $n=4.2$ and $\alpha=1$, 2, 4, 8, 10, 12, 15, 20, 30, 40, 50 and 60, and then the relation of the time factor to the time of settlement is calculated by the use of the equation $t=\frac{H^2}{\bar{C}_v}T_v$.

The value of $\bar{C}_v$ in the equation is obtained in Fig. 10 as that corresponding to the mean value 3.86 t/m² of the intensity of load prior to loading

$\sigma_1(=p_0+p_3)=3.38$ t/m²

and the intensity of load after loading

$\sigma_n=4.33$ t/m², i.e. $\bar{C}_v=7.60\times10^{-4}$(cm²/sec).

The relation of the time factor to the time of settlement is, therefore,

$$t=\frac{H^2}{\bar{C}_v}\cdot T_v=\frac{\left(\frac{1}{2}\times1400\right)^2}{7.60\times10^{-4}}\cdot T_v$$
$$=6.447\times10^8T_v \text{ (sec)}$$
$$=1.075\times10^7T_v \text{ (min)}$$

As the amount of settlement caused by the sand mat prior to the settling work of Compozers is observed to be 6 cm, the amount of settlement only caused by the sand mat after the settlement work of Compozers is estimated to be 17.3−6.0=11.3 cm. Then the lapsed time corresponding to the amount of settlement is calculated by the use of the degree consolidation—the time factor relationship and the above equation.

The results are applied to make of Fig. 11.

b) The settlement curve only caused by the load of the banking part on the sand mat after the settling work of Compozers.

As the treated part on the sand mat was applied 8.4×10^4(min) after the settling work of Compozers, the equation of the time of settlement becomes

$$t-8.4\times10^4=\frac{H^2}{\bar{C}_v}T_v$$

The value of $\bar{C}_v$ in the equation was obtained in Fig. 10 which corresponds to the mean value 4.65 t/m² of the intensity of load prior to banking 4.33 t/m² and that of after banking 4.96 t/m², i.e. $\bar{C}_v=5.00\times10^{-4}$(cm²/sec). Therefore,

$$t-8.4\times10^4=\frac{\left(\frac{1}{2}\times1400\right)^2}{5.00\times10^{-4}}\cdot T_v$$
$$=9.8\times10^8T_v \text{ (sec)}$$
$$=1.63\times10^7T_v \text{ (min)}$$

The lapsed time of settlement corresponding to the amount of settlement was calculated for the final amount of settlement of 46.9 cm by the use of the degree of consolidation—the time factor relationship and the above equation.

The results are also applied to make of Fig. 11.

We can obtain the amount of settlement S_1 and S_2 at the same time by the relation at the time t between the amount of settlement S_1 caused by only the load of sand mat and the amount of settlement S_2 caused by only the load of the banking part as is above mentioned. Now we add them and suppose them to be the whole amount of settlement S_{1+2} at the time. $S_{1+2}-\log t$ curves on each value of α are

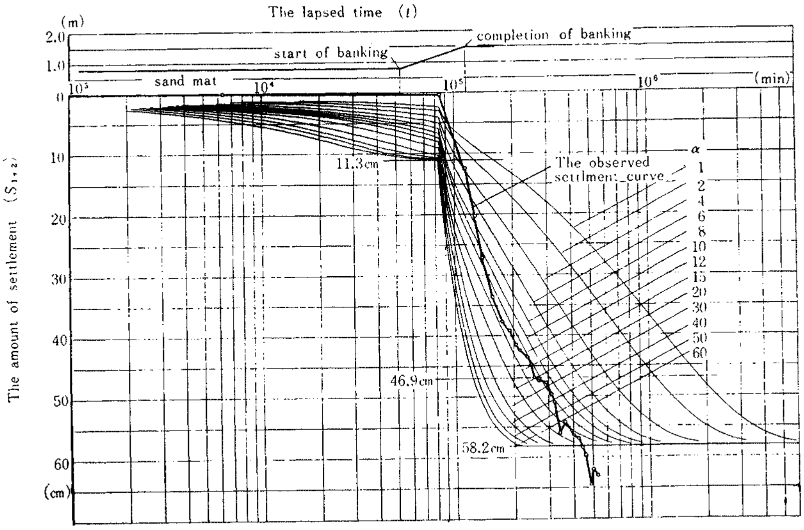


Fig. 11 The consolidation settlement curves due to the sand mat load and the load of the banking part (Compozer-treated zone).

illustrated in Fig. 11.

The observed settlement curve is also shown in Fig. 11. The calculated settlement curve is a little defferent from the observed one. Namely, the calculated settlement curve shows settlement of about 3~11 cm prior to the banking work on the sand mat, while settlement by the observed settlement curve until the time of banking is very little. The difference may be attributed to the remarkable compression of the ground caused by settling of Compozers. The calculation for settlement caused by the banking is to be made, with such a ground state taken into consideration.

From the Fig. 11, assuming that a moment loading of the banking was conducted at a time on the average lapsed time between the start of the observed remarkable settlement (lapsed time 8.78×10^4min) and the completion of banking (lapsed time 1.15×10^5min), one obtains the equation for calculation of the time of settlement as

$$t-1.01\times10^5=\frac{H^2}{\bar{C}_v}\cdot T_v=1.63\times10^7T_v \text{ (min)}$$

$$\therefore\ t=1.63\times10^7T_v+1.01\times10^5 \text{ (min)}$$

The lapsed time of settlement corresponding to the amount of settlement is then calculated for the final amount of settlement of 46.9 cm, by the use of the degree of consolidation—the time factor relationship and the above equation. The results are shown in Fig. 12, which is compared with the observed settlement curve. The observed settlement curve is in good accordance with the calculated settlement curve with $\alpha=40$.

Therefore the horizontal coefficient of consolidation C_h of the ground is calculated from the equation

$$\alpha=\frac{C_h}{\bar{C}_v}\left(\frac{H}{2R}\right)^2$$

and

$$\bar{C}_v=5.00\times10^{-4}\ (\text{cm}^2/\text{sec})$$

$$R=0.525\,d=84.0\text{ cm}$$

,where

$$H=\frac{1}{2}\times1400=700\text{ cm}$$

then,

$$C_h=\alpha\bar{C}_v\left(\frac{2R}{H}\right)^2=1.15\times10^{-3}(\text{cm}^2/\text{sec})$$

Furthermore, in order to confirm the reliability of the method of analysis based on the theory of consolidation due to the three-dimensional dehydration, the author estimated the C_h value from the results of analysis of the observed settlement curve in the non-treated ground, for comparison with the above value.

(6) The Calculation of the Time of Settlement due to the three-dimensional dehydration in the non-treated Ground.

Taking the vertical and horizontal coefficients of consolidation to be C_v and C_h respectively, the thickness of the consolidated layer to be $2H$, and the radius of the effective circle of the theree-dimensional dehydration to be R, one obtains the ratio of the time factor in the vertical direction T_v to that in the horizontal derection T_h, α, as

$$\alpha=\frac{T_h}{T_v}=\frac{C_h}{C_v}\left(\frac{H}{R}\right)^2$$

If the values are taken as shown in Fig. 13, i.e.

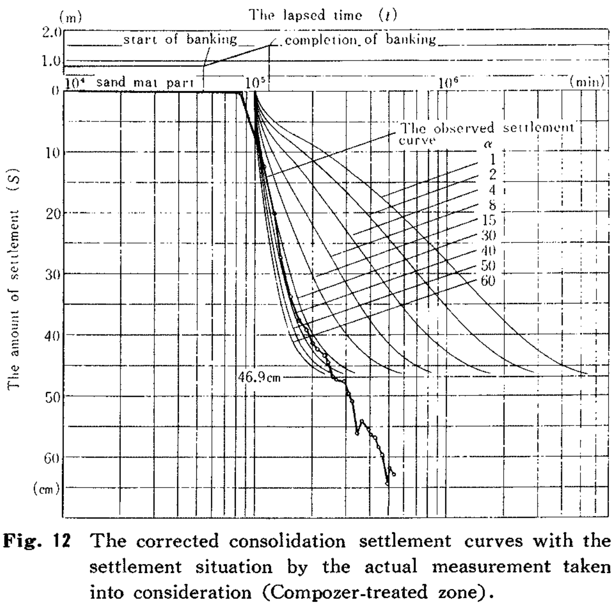


Fig. 12 The corrected consolidation settlement curves with the settlement situation by the actual measurement taken into consideration (Compozer-treated zone).

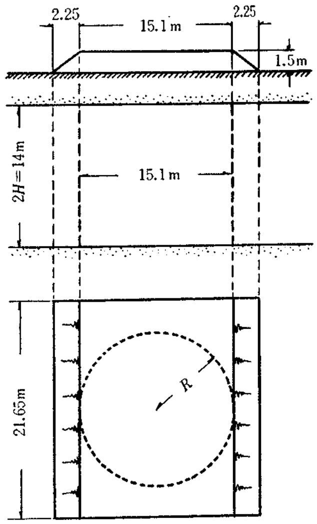


Fig. 13 The hypothetical chart of the consolidated soil body part in case of the consolidation analysis due to three-dimentional dehydration.

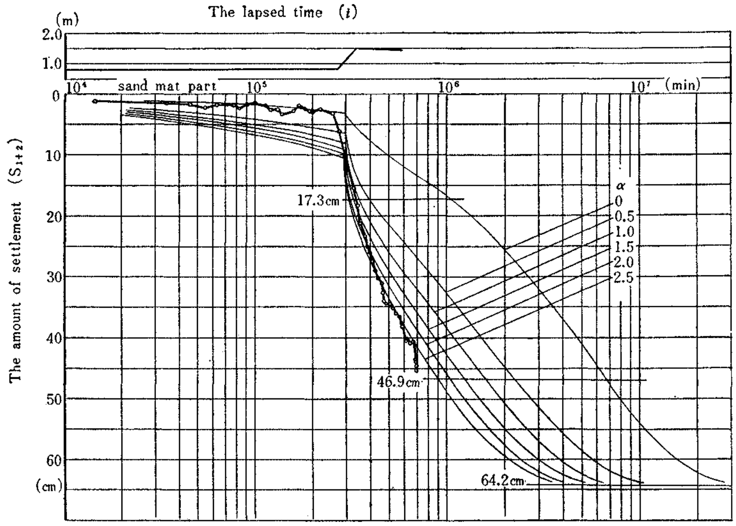


Fig. 14 The chart for the consolidation settlement curves due to the sand mat load and the load of the banking part (non-treated zone).

$$H=\frac{14}{2}=7\text{ m} \quad \text{and} \quad R=\frac{15.10}{2}=7.55\text{ m}$$

then there is obtained

$$\alpha=\left(\frac{7.00}{7.55}\right)^2\frac{C_h}{C_v}=0.86\left(\frac{C_h}{C_v}\right)$$

The relation of U_v to T_v depending on the α value according to the equation

$$U_v=1-\sum_{i=1,3,5}^{\infty}\frac{8}{(2\,Hn_i)^2}e^{-\frac{(2Hn_i)^2}{2}\cdot T_v}\times\sum_{i=1}^{\infty}\frac{4}{(Rm_i)^2}e^{-(Rm_i)^2\alpha T_v}$$

was obtained by the use of the α-chart, and then the required time of settlement corresponding to the amount of settlement caused by p_1 and p_2 was calculated.

The amount of settlement corresponding to the time t, S_1 and S_2 are plotted (the figure is omitted).

The total amount of settlement corresponding to the time $S_{1+2}(=S_1+S_2)$ was then calculated, which is shown in Fig. 14 for each value of α. In this figure is also shown the observed settlement curve.

As shown in the figure, the observed settlement curve is in good accordance with the calculated settlement curve with $\alpha=2.5$. Therefore the C_h value is calculated under the condition of $\alpha=2.5$. Since $H=\frac{14}{2}=7$ m and $R=\frac{15.10}{2}=7.55$ m in this case as previously described, one gets

$$\alpha=\left(\frac{7.00}{7.55}\right)^2\frac{C_h}{\overline{C}_v}=0.86\left(\frac{C_h}{\overline{C}_v}\right)$$

$$\therefore\quad C_h=\frac{\alpha}{0.86}\cdot\overline{C}_v=\frac{2.5}{0.86}\times 5.00\times10^{-4}$$

$$=1.45\times10^{-3}\ (\text{cm}^2/\text{sec})$$

This value is good accordance with that obtained in the case of the Compozer-treated ground 1.15×10^{-3}(cm²/sec), and is thought to be a reasonable value for C_h in this ground.

Furthermore, since the observed settlement curve is in accordance with the settlement curve with $\alpha=2.5$, the value of the coefficient β in the equation of consolidation described in the previous report, is thought to be ∞ (completely permeable on side faces in the case).

As seen from the above description, the analysis by the use of the equation of consolidation proposed by the author is said to have consistently explained in contrast to the observed values obtained by the Japanese National Railways engineering staff with the consolidation settlements in the two separate zones of the non-treated ground and the Compozer-treated ground.

The author wishes to develop further the study on this problem, by many field investigations and experiments.

IX. CONCLUSION

Here are reported some basic ideas for the calculation of consolidation settlement by the three-dimensional dehydration of the soil due to sand pile drainage works. And also the method of calculation is explained with the figures which is convenient to make calculation of consolidation easily and speedily on this method.

The method of using the figures is not referred to, because the author already explained it in the treatise of "On the one-dimensional consolidation by the three-dimensional dehydration, with the secondary compression taken into consideration".

We have been calculating the consolidation settlement due to sand pile drainage works by using mainly the calculation of dehydration to the horizontal direction. But the method of this paper may make one possible to get the approximate value more exactly, because one can calculate the consolidation of sand pile drainage by the three-dimensional dehydration with secondary compression taken into condideration, by using the consolidation constants which are obtained by the soil-experiment using the consolidation testing apparatus by the three-dimensional dehydration of soil. And also this method might solve some problems on the consolidation which are difficult to be solved by the former method.

The author is especially grateful to an excellent scholar Michitaka Saito, the Head of Soil Mechanics Laboratory, Railways Technical Research Institute of the Japanese National Railways (J.N.R.), to Mr. Tadahiko Muromachi, a Chief Research Member, and also to the persons concerned of Mito Railways Superintendent Bureau of J.N.R. who bestowed on the author many valuable suggestions and materials on this research.

Reference

1) The commetee for technical investigation of Osaka Bay : Study on the ground settlement in Osaka. (July, 1949).
2) Masami Fukuoka and Mashiho Inada : Study on the settlement of banks of rivers in the weak ground area. Civil Engineering Research Institute Report. No. 87-1 (January, 1954).
3) Takeo Mogami, et al. : "Soil Mechanics" Kyoritsu Publishing Co. Inc. (August, 1964).
4) R.A. Barron : The influence of Drain Wells on the Consolidation Fine Grained Soils. (July, 1944).
5) Hiroshi Nakano : On the trial manufacture of triaxial consolidation testing machine for weak clay. Soil Mechanics and Foundation Engineering. Vol. 8, No. 3. Japanese Society of Soil Mechanics and Foundation Engineering. (June, 1960).
6) Syunsuke Takagi : A graph and its application on the sand pile drainage work. Soil Mechanics and Foun dation Engineering. Vol. 1, No. 12. Japanese Society of Soil Mechanics Foundation Engineering. (November, 1955).
7) Tadahiko Muromachi, et al. : Research of the weak ground in the vicinity of Koisegawa Railway Bridge between Kandachi and Takahama of Joban Line (First report). Railways Technical Research Institute. Prompt Report No. 65-1049. Japanese National Railways. (May, 1965).
8) Tadahiko Muromachi : A study on the measure to counter the weak ground by the trial embankment (I)……in the vicinity of the Koisegawa Railway Bridge between Takahama and Kandachi of Joban Line……Railway Civil Engineering. Vol. 8, No. 2 (February, 1966).
9) Tadahiko Muromachi : A study on the measure to counter the weak ground by the trial embankment (II)……in the vicinity of the Koisegawa Railway Bridge between Takahama and Kandachi of Joban Line……Railway Civil Engineering. Vol. 8, No. 3 (March, 1966).
10) Toshio Ikeda and Hideo Takagi : "The investigation, design and calculation of the embankment on the weak ground". Gihodo Publishing Co. Inc. (March, 1965).
11) Banjiro Yamada : Study on the settlement of the Base Foundation caused by Consolidation, in consideration of the Secondary Consolidation caused by Creep. Special Bulletin of the College of Agriculture Utsunomiya University, No. 14 (March, 1962).
12) Banjiro Yamada : On the one-dimensional consolidation by the three-dimensional dehydration with secondary compression taken into consolidation. Transactions of the Japan Society of Civil Engineers. No. 142 (June, 1967).
13) Banjiro Yamada : On the one-dimensional consolidation by the three-dimensional dehydration.……Consolidation analysis in the case of defferent consolidation permeability in the soil strata directly surrounding a consolidated soil body with special consideration of the secondary compression. Transactions of the Japan Society of Civil Engineers. No. 143 (July, 1967).

(Received April 6, 1967)