TWO-DIMENSIONAL PHOTOELASTIC EXPERIMENT MADE BY GELATINE GEL

By Minoru Yamamoto and Shigeaki Morichi***

1. INTRODUCTION

Gelatne gel has so high photoelastic sensitivity that we can not find the other materials comparable to it for the moment. Therefore, it is very suitable for the investigation of the stresses in a body produced by its own weight. This characteristic has been known for a long time, but we find only few examples[1)~4)] applied to the research of plane-strain problems for the sake of the difficulty in the treatment of gelatine gel.

In the field of civil engineering, we have many problems in which the effect of gravity should be considered in the stress analysis. So we regret that the photoelastic experiment by gelatine gel has not been developed, for we recognize it as one of the most useful methods for researching the influence of gravity. Under this circumstance, we check the reliability of the data obtained from some photoelastic experiments made by gelatine gel under the field of gravity, to discuss the merit of gelatine gel as a photoelastic material.

Hereupon we make some experiments on a semi-infinite plate with a circular hole or a square-like hole situated near the horizontal boundary and some similar experiments in which a circular hole is situated near the inclined boundary of a semi-infinite plate. Then we examine the characteristics of these experiments by comparing the experimental data with the results obtained from theory. We already have some theories*** of a semi-infinite plate containing a hole under the field of gravity. But we need not use theoretically too exact values in comparison with the experimental data, for it is rather inadequate to require as high accuracy as in an experiment easily made by another hard photoelastic material. Therefore, we decide to utilize Dr. Yamaguchi's approximate theory, which is very simple. In the case of a square-like hole, we have an approximate[7)] theory given by Dr. Okamoto and so our experiments are so arranged as to be able to compare directly with it. On the other hand, in the case of a semi-infinite plate of an inclined boundary containing a circular hole, we compare the experimental data with Dr. Yamaguchi's theory extended by the authors from the above mentioned view-point.

According to the literatures already reported, we find that the experiments have been made in the state of plane-strain due to the softness of gelatine gel. However, since it seems to be more advantageous for us to deal with the experiments as those in the state of plane-stress in order to raise the accuracy of the experiments, we will deal with them only in the state of plane-stress.

2. COURSE OF EXPERIMENT

2.1 Properties of gelatine gel

In this article, we do not mention the general properties of gelatine gel, but only those which are very important as a photoelastic material.

(a) Gelatine gel being extremely soft in comparison with the other photoelastic materials, a model made by gelatine gel is easily injured and when it has stains on its surface, it is difficult to remove them. But the surface of a model incident of polarized light is desirable to be plain and flawless, so we have to be careful of the production and the treatment of a model.

(b) A defective model is apt to be made, if we do not pay any attention to the agitation and the adjustment of temperature during heating for the dissolution of the saturated gelatine into the designated volume of water.

(c) The transparency, one of the most important qualities as a photoelastic material, varies considerably according to the manufacturing process of gelatine on the market. But if we select the pure products of high jelly-strength, extracted at a low temperature, we need not care about this property.

(d) Gelatine gel rots easily and its rot gives the direct influence on both the strength and the transparency of a model, so that we must prevent it by using an antiseptic.

(e) We have to consider the influence of the deformation upon the experimental data, as the deformation produced by the weight of a gelatine

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*** For example, we find the theories given by Dr. Yamaguchi[5)] and also by Dr. Ito[6)] even in our country.

model is very large, because of its small Young's modulus. Therefore, when we make a model, we must arrange its shape so as to take the form which we expect after we put it in the field of gravity through standing it up.

(f) The greater part of the water contained in a model is free water fixed mechanically in gelatine gel. If we leave the model alone in the air, the water begins to evaporate from the surface of the model, this inducing the dry-shrinkage which causes shrinkage-stress. So we have to take a photograph of fringe pattern as early as possible.

(g) The melting point and the freezing point of gelatine gel are about 31°C and 29°C respectively, so that during the treatment of a model we had better keep the laboratory at a moderate tempera ture, for instance, the one between 10°C and 15°C.

(h) The photoelastic sensitivity and the physical properties of a model are delicately influenced by the process of its production, so that we have to inspect them either with a part of the model used in the experiment or with the material of the same batch as for the model. For reference, we show those data measured by us. On the model made of 10% aqueous solution, we obtain the values between 0.04~0.06 cm/gr as the photoelastic sensitivity and the values between 0.4~0.6 kgr/cm^2 as Young's modulus. And also we obtain 0.5 as Poisson's ratio and 1.02 gr/cm^3 as the density.

2.2 Production of a model

(a) Making gelatine gel

Gelatine plates on the market are first cut into pieces with scissors and then the designated volume of water with antiseptic is added to a definite amount of gelatine to get the required consistency. After saturating about for 20 hours, the compound of the saturated gelatine and the remains of the water is heated to a temperature between 40°~50°C in a hot water-bath until the gelatine dissolves perfectly. Then the solution is poured into a mold through gauze and cured about for 20 hours in a cool room to be gelatinized. The matters to be considered concerning to the production of gelatine gel are as follows.

1) The larger Young's modulus of gelatine gel is, the easier its treatment becomes, so that the higher consistency of the solution is desirable. But, when the consistency is too high, it becomes difficult to dissolve gelatine perfectly even after very many hours of saturation and also the model is liable to leave unhomogeneous parts in itself during gelatinization. On the other hand, with the increase of viscosity due to the increase of consistency of the solution, the fine bubbles produced during agitating and casting the solution become liable to remain in the model on the way to arise up to its surface. According to these circumstances, it is preferable to choose the consistency of the solution between 9~11%, giving consideration to its elasticity. Thus the consistency of 10% is adopted throughout our experiments.

2) Penta-chloro-phenor (p.c.p.) is used as the antiseptic in our experiments. If we add p.c.p. between 0.1~0.2% of the weight of gelatine to the solution, we can protect the model from rot and that does not give any influence upon both the transparency and the elastic properties of the model.

3) The saturated gelatine is dissolved in a hot water-bath at either the temperature or a little higher one, which the gelatine was extracted at. We must keep this rule by all means in order to preserve the native properties of the gelatine. When we dissolve it at a relatively low temperature, we sometimes find such a defect as the loss of homogenity in a model, resulting from the partial consolidation of the solution during casting. Therofore, we had better determine the profitable temperature of dissolution by test. The author's experiences teach us that the proper temperature is 46~47°C at a room temperature between 10~15°C, for the gelatine whose extracted temperature and jelly-strength are about 40°C and 900 gr respectively.

All the models in our experiments are made according to the above-mentioned description.

(b) Mold

As gelatine is isinglass of high purity, we have to take care of the model when we remove the mold. When the mold is removed by force, the flakes of the material are easily scraped away from the surface of the model, because of the low strength of gelatine gel. To solve this unfavourable problem, we put tin-plates coated by grease inside the side-planes of a mold parallel to the axis of polarized light and also put a clean vinyl chloride plate of about 0.2 mm thick inside one of the two side-planes through which polarized light passes, i.e., on the bottom of the mold, thus enabling to seperate easily the parts of the mold from the model through their flexibility. We can easily make the surface of the model stainless and flat in this way. Then the tin-plates and the vinyl chloride plate are backed by veneer boards in order to keep the shape of the model right. These boards and plates are bound only by Scotch tape so as to be easily seperated each other. Photo-

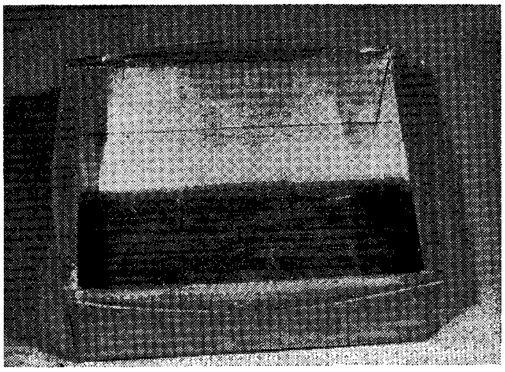


Photo 1

1 shows one of the molds used in our experiments. After a mold is set up, the outsides of the mold are charged with white wax in order to stop the leakage of gelatine solution and to keep each part of the mold tight at the right position.

(c) Casting

The gelatine solution generally contains many fine babbles, when it is poured into a mold. But if we tap the outsides of the mold, they soon go up near the upper surface. Therefore, if we let the solution overflow from the mold while we are tapping the outsides of the mold and additionally we take off the bubbles by the help of a thin flat bar, we can easily get a model almost free from bubbles. However, it is impossible for us to completely remove the bubbles which remain just beneath the surface in this way. Any few bubbles give so important influence upon the experiment that we have to remove all of them. Accordingly we submerge a thin cover plate of vinyl chloride in the solution along the brim of the mold, as shown in Photo-2. In this case we must be careful not to leave the buubbles beneath the cover plate.

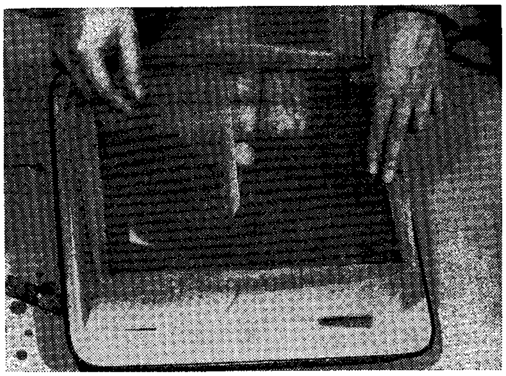


Photo 2

(d) Working of modle

The description is limited only to the method of making a hole through a model of our experiments, for the general method of working a model seems to be analogized from this. We have two methods to make a hole.

1) To cut out a hole after having stripped the model from a mold.
2) To insert a bar at the designated position before gelatinization instead of cutting out a hole.

The first method causes a model large deformation when cutting out a hole, as Young's modulus of the model is very small. So we have to take a method to cut out a hole under the restraint of deformation. But, we encounter a new problem that it is difficult to keep the surface of the model clean. Consequently we can not but give up the first method and adopt the second method.

The bar used in the second method has the special section which is so shaped as to offset the deformation occured around the hole and get a normal shape of the hole, when the model is stood up. Also the end brims of the bar are slightly rounded off so as not to injure the edge of the hole, when it is pulled out from the model. But here we have another problem of the initial stress resulting from the shrinkage of the model which is prevented by the insertion of the bar during gelatinization. However, even if the initial stress arises, it could be considered as an experimental error, because any influence of the shrinkage upon the experimental data will be canceled by the large deformation during the experiment. Therefore, we may neglect such a problem.

2.3 Photoelastic sensitivity

The photoelastic sensitivity depends on the various conditions in the process of making a model to say nothing of the temperature of the model during an experiment. So we have to make it a rule to measure the photoelastic sensitivity in every experiment.

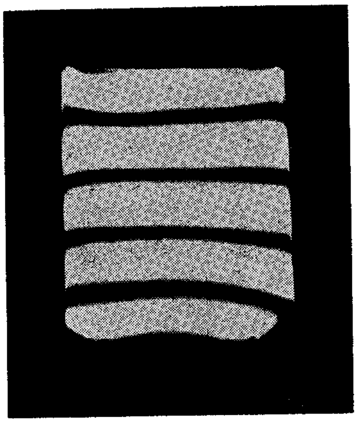


Photo 3

It is practical for us to measure the photoelastic sensitivity from the stresses produced by standing up a rectangular parallelopided block. Photo-3 shows an example of fringe pattern taken by this method, where the thickness of the block $d=9.98$ cm, the density 1.02 gr/cm^3, the mean distance between the fringes $h=2.09$ cm, the room temperature during the experiment 21.2°C. From these data, the photoelastic sensitivity is calculated as

$$\alpha=\frac{1}{\rho hd}=\frac{1}{1.02\times 2.09\times 9.98}$$
$$=0.0469(\mathrm{cm/gr}).$$

In the experiments made by the authors, the block for inspecting the photoelastic sensitivity is not made of the material in the same batch as that of the model used in the experiment, but it is cut out directly from the model itself. This practice is also very convenient, as the influence of creep upon the sensitivity is simultaneously considered.

3. EXAMPLES OF TWO-DIMENTIONAL PHOTOELASTIC EXPERIMENTS IN THE STATE OF PLANE-STRESS

3.1 Semi-infinite plate with a horizontal boundary

(a) A circular hole

According to the above-mentioned means, a model is made by pouring gelatine solution into a retangular mold laid horizontally, inside dimensions of which are 28×22×10 cm. The minimum dimension 10 cm is assigned to the thickness of a model. The elliptical section of the bar which is used for making a circular hole of about 2.7 cm in diameter is determined through pre-tests for each experiment. The depths of about one, two and three times the diameter of the circular hole are adopted as those of covering over the hole in our experiments.

We put a model into the field of gravity by standing it up, setting a side-plane of 28×10 cm as a bottom. We can imagine the scene of the experiment by Photo-4. The fringe patterns taken in our experiments are shown in Photo-5.

The stress distribtions around the hole are shown together with the theoretically calculated values in Fig. 1. Poisson's ratio, ν of gelatine gel is 0.5, so that it is enough to compare the experimental data with the theoretical values for $\nu=0.5$ in order to discuss the accuracy of the experiments. But there are some cases in which we cannot neglect the influence of Poisson's ratio upon the stress distribution as in these experiments, when we apply the results of a photoelastic experiment made by gelatine gel directly to a practical problem Therefore, we give the theoretical values for $\nu=0.25$, too in the figures as the examples to show the influence of variation of Poisson's ratio.

(b) A square-like hole

Dr. Okamoto puts the following relation between the orthogonal rectangular co-ordinates, $z=x+iy$ and the orthogonal curvilinear co-ordinates $\omega=\alpha+i\beta$ to prescribe the shape of a hole with the curve

$$z=a(e^{\omega}+be^{-\omega}+ce^{-n\omega})$$

where a, b and c are arbitrary constants of real

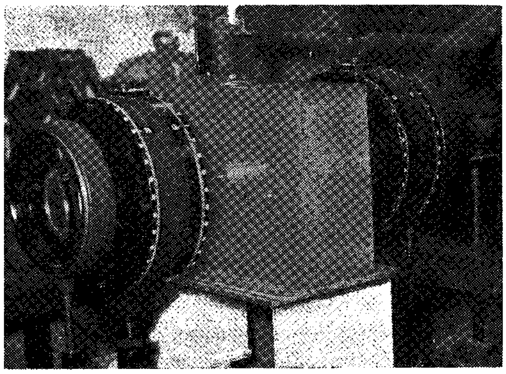
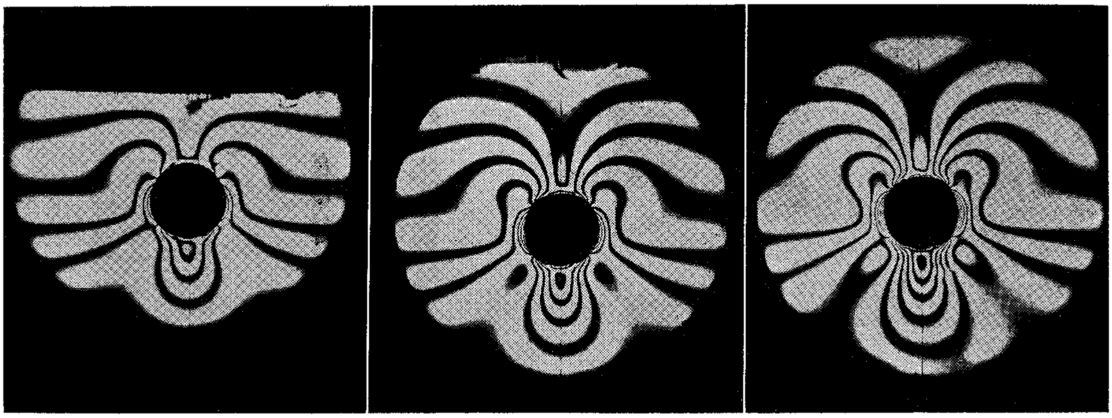


Photo 4



(a) (b) (c)

Photo 5

number and n is a positive integer. The shape and the dimensions of the hole are determined by substituting $a=1.1$ cm, $b=0$, $c=-0.1$ and $n=3$

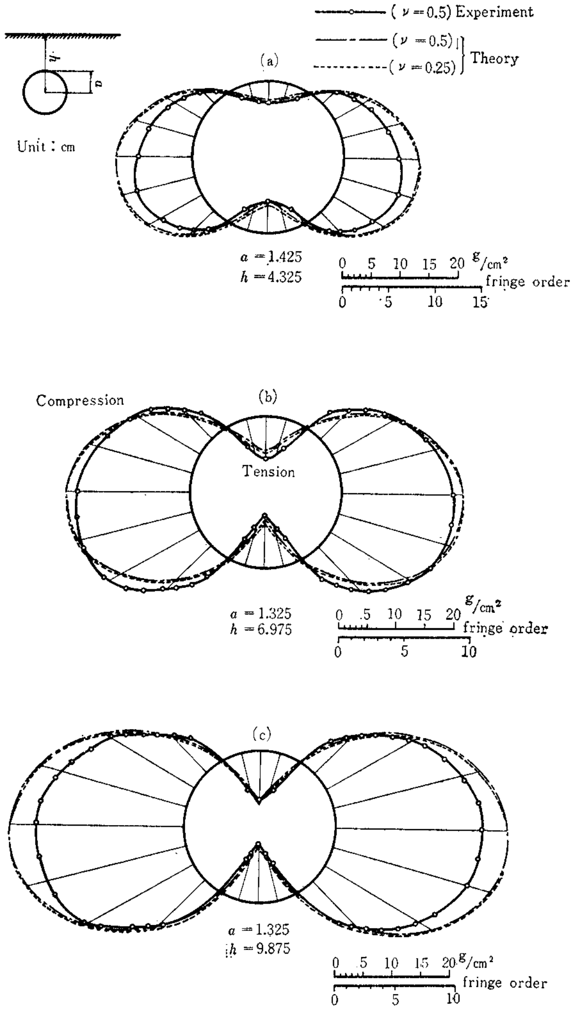


Fig. 1

into the formula and the depths of about three, five and seven times the height of the hole are adopted as those of covering over the ceiling of the hole in our experiments. The other means of the experiments are the same as in (a).

Photo 6 shows the fringe patterns taken in our experiments. Fig. 2 also shows the results of the experiments and the theoretical calculations related to the stresses around the hole.

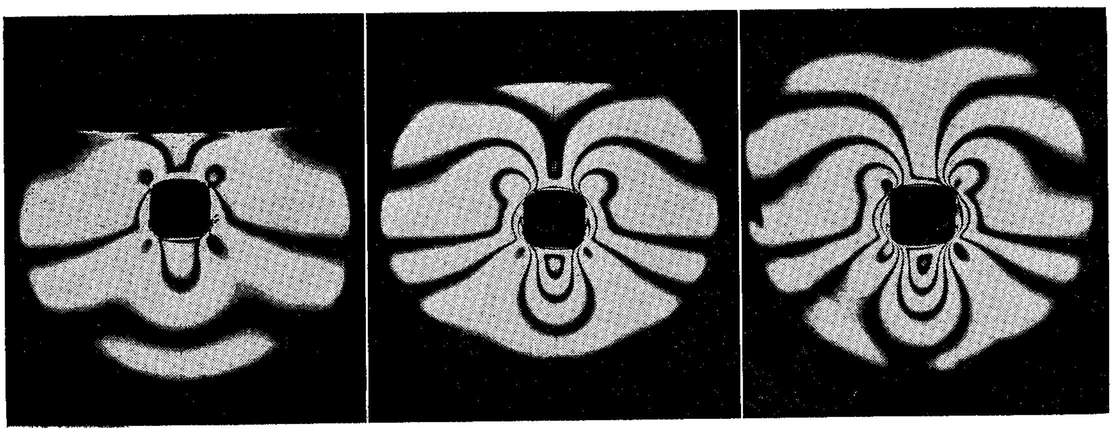
(c) Semi-infinite plate with an inclined boundary

The stress produced by the weight of a semi-infinite plate with an inclined boundary must be indicated by a linear function of the vertical depth from the boundary. Photo 7 is the fringe pattern which is taken from the 10 cm thick model in the shape of a right-angled, isosceles triangle whose isosceles sides are 30 cm long respectively. It is obvious from the photo that the isochromatics are parallel to the boundary, so that we can see the expected stress condition near the central part of the inclined boundary. Thus we determine to put a circular hole at this part of the model in our experiments. The diameter of the hole is about 2.0 cm and the smallest depths of covering over the hole are about one, two and three times the diameter of the hole.

Photo 8 shows the fringe patterns and Fig. 3 shows the stress distributions around the hole, obtained experimentally and theoretically.

4. ACCURACY OF EXPERIMENTS

Let us investigate the accuracy of the experiments from the above-mentioned examples. We give such a rather large dimension as 10 cm to



(a) (b) (c)

Photo 8

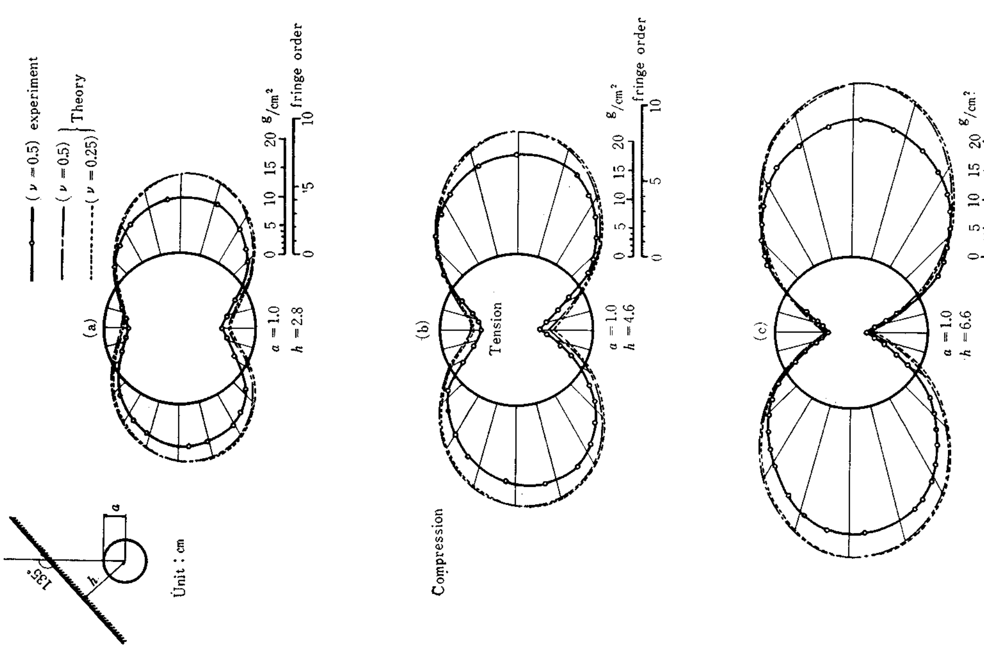


Fig. 3

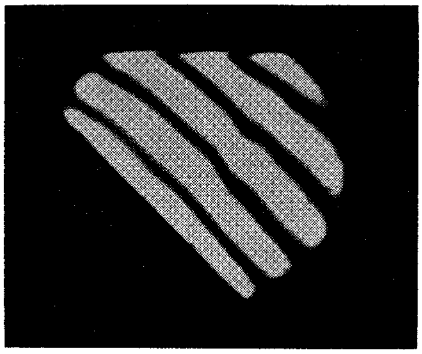


Photo 7

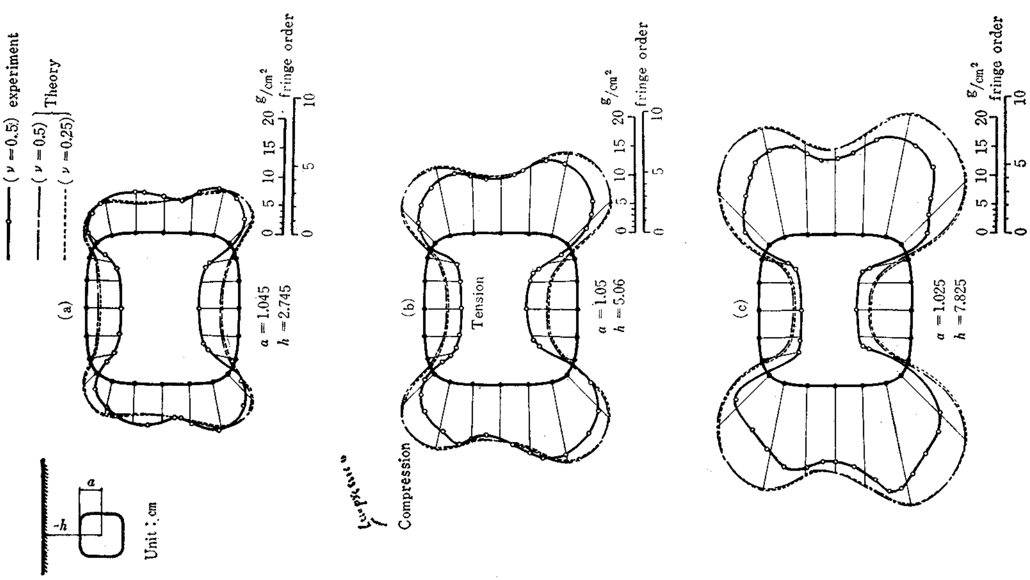
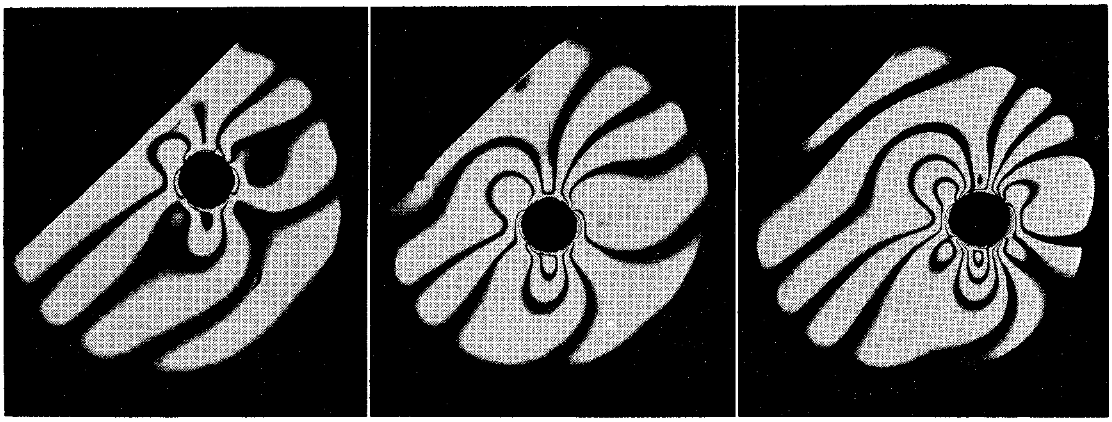


Fig. 2



(a) (b) (c)

Photo 8

the thickness of a model for the size of a hole, considering the stability of a model while it stands by itself and the numbers of fringe desired from an analytical view-point. Thus the following problems come into question.

1) Whether the stress is in the state of plane-stress or not?

2) It is difficult to parallel the axis of polarized light to the inner surface of the hole.

3) The isochromatics are disturbed by the dispersion of polarized light on the distorted surfaces of a model resulting from the deformation in the direction of the thickness of a model, which is large in such a part as the periphery of a hole where stress concentration occurs.

According to the literature[8], it is reported that the deviation of the average stresses in all the thickness of a model from the stresses in its middle layer does not exceed 12% when the ratio of the thickness of a model to the diameter of a hole contained in it is about 4. Therefore, if we carefully determine the suitable size of a hole in relation to the thickness of a model from this result, we can neglect the first problem with the consideration that there is a fairly large error in an experiment by gelatine gel. The second problem may be regarded as to be settled mainly by technical devices of an experiment according to the experience of our experiments. The third problem, substantially based on the fact that Young's modulus is small, causes an error which is as inevitable as the error originating from the distortion of a model owing to the large deformation. Thereupon we intend to judge the usefulness of gelatine gel as an experimental material by investigating the influence of these problems including the initial stress already mentioned upon the circumferential stresses around the hole in our experiments.

When we compare the experimentally determined curves with those theoretically obtained in the figures, we can see that the formers are comparable to the latters as a whole except the moderate error contained in the first problem and furthermore find the tendency that the more correct the dimensions of a model after loading are, the more the formers approach the latters. These facts seem to show that the error of an experiment made by gelatine gel is mainly related to the accuracy of the dimensions except the error based on the first problem and also that the error resulting from the initial stress may be considered to be small as already mentioned. Therefore, all the more fine devices of arranging the shape and the dimensions of a model are desired in order to get a better accuracy of an experiment.

On the other hand, in such a case as Poisson's ratio has any influence upon the stress distribution, there is a problem of error which originates from the difference of Poisson's ratio between an actual object and its model, though this has no relation to the error of an experiment itself. Thus, comparing the theoretical values for $\nu=0.25$ with those for $\nu=0.5$ in the figures, the difference betweem them lies within the error of our experiment, so that, as a matter of fact we need not take account of the difference of Poisson's ratios in the stress distribution in the case of an experiment made by a gelatine model.

As mentioned above, the error is controlled by various factors. However, if we allow a moderate quantity of an unavoidable error resulting from the native properties of glatine gel, we may consider that the error is controlled simply by the dimensions of a model. Therefore, if we make efforts to reduce these sorts of error by any devices on a model, it may be possible for us to make an experiment with a sufficient accuracy from the practical view-point.

5. CONCLUSION

We believe that the avilability of gelatine gel should be recognized in the field of civil engineering, because the influence of gravity is very easily introduced in the photoelastic experiment made by a gelatine model. But this sort of experiment seems not have been applied too much for its worth, owing to the only reason why the treatment of gelatine gel is troublesome. According to the experiments found in few literatures already reported, it seems to us that the state of plane-strain makes the experiments all the more difficult. Thereupon, we deal with the general course of the experiment through several experiments made under the state of plane-stress to improve the method of an experiment made by gelatine gel. At the same time we show that we can make the experiment with a sufficient accuracy from the practical view-point, by discussing the accuracy of our experiments. However, the difficulty in the treatments of the model is inevitable owing to the gelatine gel. From this standpoint, the usefulness of gelatine gel can not be displayed unless we apply it to the problems which make the most of its merit, for instance, to the problems of the influence of gravity upon the stress distribution.

On the other hand, there is the stress-freezing method under the application of centrifugal force, this being another method of photoelastic experiment in which the influence of body force is able to be taken into account. But, comparing this with the above-mentioned method, the latter has various merits, for instance, easiness in planning of an experiment, capability of observing the stress distribution during an experiment etc. Furthermore, since we can make an experiment by a model composed of several kinds of gelatine gels whose Young's moduli are different each other, we can find the possibility that the special field of experiment is developed by gelatine gel.

As to isoclinics which are not described in this report, it seems to be profitable to investigate them by another thin model instead of directly using the model for isochromatics. In this case, we have to make the experiment, in which we exchange the compression field under gravity into the tension field through suspending the model instead of standing it up by itself. We are now studying whether this method is adequate or not.

At the end, we cordially thank Mr. Kinya Suzuki, who helped us as a good assistant during our experiments.

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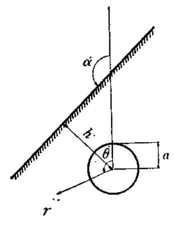
Appendix

Stress near a Circular Hole Contained in a Semi-infinite Plate with an Inclined Boundary under the Field of Gravity.

We analyze the stresses near a circular hole which is contained in a semi infinite plate with an inclined boundary under the field of gravity by the extension of Dr. Yamaguchi's theory. But here we show only the formulas of the stress components.

We use the co-ordinat system as shown in Fig. and following notations.



r, θ : Polar co-ordinates

σ_r, σ_θ : Radial and tangential normal stress components in polar co-ordinates

$\tau_{r\theta}$: Shearing stress component in polar co-ordinates

a : Radius of a hole
ρ : Density
g : Gravitational acceleration
ν : Poisson's ratio
k : Ratio of horizontal earth pressure to vertical one at the depth h in a semi-infinite plate

Then the stress componenst near a hole are given by following formulas.

$$\sigma_r=k_1W+k_2V$$
$$\sigma_\theta=l_1W+l_2V$$
$$T_{r\theta}=m_1W+m_2V$$

where

$$W=\rho gh$$
$$V=\rho ga$$

and

$$k_1=\frac{\sec\alpha}{2(k-\tan^2\alpha)}\left\{(k+1)\cdot\tan\alpha\cdot\left(1-\frac{a^2}{r^2}\right)+2k\cdot\left(1-4\cdot\frac{a^2}{r^2}+3\cdot\frac{a^4}{r^4}\right)\sin 2\theta\right.$$
$$\left.+(1-k)\cdot\tan\alpha\cdot\left(1-4\cdot\frac{a^2}{r^2}+3\cdot\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$k_2=\frac{1}{4(k-\tan^2\alpha)}\left[(k+1)\cdot\tan\alpha\cdot\left(\frac{r}{a}-\frac{a^3}{r^3}\right)\sin\theta+(1-3k)\cdot\tan\alpha\cdot\left(\frac{r}{a}-5\cdot\frac{a^3}{r^3}+4\cdot\frac{a^5}{r^5}\right)\sin 3\theta\right.$$
$$+\left\{(2k-\overline{3+k}\cdot\tan^2\alpha)\cdot\frac{r}{a}+(3+\nu)(\tan^2\alpha-k)\cdot\frac{a}{r}+(k\cdot\overline{1+\nu}+\overline{k-\nu}\cdot\tan^2\alpha)\cdot\frac{a^3}{r^3}\right\}\cos\theta$$
$$\left.+\{(k-1)\tan^2\alpha-2k\}\left(\frac{r}{a}-5\cdot\frac{a^3}{r^3}+4\cdot\frac{a^5}{r^5}\right)\cos 3\theta\right]$$

$$l_1=\frac{\sec\alpha}{2(k-\tan^2\alpha)}\left\{(k+1)\cdot\tan\alpha\cdot\left(1+\frac{a^2}{r^2}\right)-2k\cdot\left(1+3\cdot\frac{a^4}{r^4}\right)\sin 2\theta+(k-1)\cdot\tan\alpha\left(1+3\cdot\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$l_2=\frac{1}{4(k-\tan^2\alpha)}\left[(k+1)\cdot\tan\alpha\cdot\left(3\cdot\frac{r}{a}+\frac{a^3}{r^3}\right)\sin\theta+(3k-1)\cdot\tan\alpha\cdot\left(\frac{r}{a}-\frac{a^3}{r^3}+4\cdot\frac{a^5}{r^5}\right)\sin 3\theta\right.$$
$$-\left\{(2k+\overline{3k+1}\cdot\tan^2\alpha)\cdot\frac{r}{a}+(1-\nu)\cdot(\tan^2\alpha-k)\cdot\frac{a}{r}+(k\cdot\overline{1+\nu}+\overline{k-\nu}\cdot\tan^2\alpha)\cdot\frac{a^3}{r^3}\right\}\cos\theta$$
$$\left.+\{2k-(k-1)\cdot\tan^2\alpha\}\cdot\left(\frac{r}{a}-\frac{a^3}{r^3}+4\cdot\frac{a^5}{r^5}\right)\cos 3\theta\right]$$

$$m_1=\frac{\sec\alpha}{2(k-\tan^2\alpha)}\left\{(k-1)\cdot\tan\alpha\cdot\left(1+2\cdot\frac{a^2}{r^2}-3\cdot\frac{a^4}{r^4}\right)\sin 2\theta+2k\cdot\left(1+2\cdot\frac{a^2}{r^2}-3\cdot\frac{a^4}{r^4}\right)\cdot\cos 2\theta\right\}$$

$$m_2=\frac{1}{2(k-\tan^2\alpha)}\left[\left\{-(2k+\overline{k-1}\cdot\tan^2\alpha)\cdot\frac{r}{a}-(1-\nu)\cdot(\tan^2\alpha-k)\cdot\frac{a}{r}+(k\cdot\overline{1+\nu}+\overline{k-\nu}\cdot\tan^2\alpha)\frac{a^3}{r^3}\right\}\sin\theta\right.$$
$$+\{2k-(k-1)\cdot\tan^2\alpha\}\cdot\left(\frac{r}{a}+3\cdot\frac{a^3}{r^3}-4\cdot\frac{a^5}{r^5}\right)\sin 3\theta+(k+1)\cdot\tan\alpha\cdot\left(-\frac{r}{a}+\frac{a^3}{r^3}\right)\cdot\cos\theta$$
$$\left.+(1-3k)\cdot\tan\alpha\cdot\left(\frac{r}{a}+3\cdot\frac{a^3}{r^3}-4\cdot\frac{a^5}{r^5}\right)\cdot\cos 3\theta\right]$$

We can get Dr. Yamaguchi's formulas, when we transfer the above formulas to those for the problem of a semi-infinite plate with a horizontal boundary, containing a circular hole under the state of plane-strain by means of substituting $\frac{\nu}{1-\nu}$ and $\frac{\pi}{2}$ for K and α respectively.

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