ON THE ONE-DIMENSIONAL CONSOLIDATION BY THE THREE-DIMENSIONAL DEHYDRATION

——CONSOLIDATION ANALYSIS IN THE CASE OF DIFFERENT CONSOLIDATION PERMEABILITY IN THE SOIL STRATA DIRECTLY SURROUNDING A CONSOLIDATED SOIL BODY WITH SPECIAL CONSIDERATION OF THE SECONDARY COMPRESSION——

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I. INTRODUCTION

The consolidation effect of soil by three-dimensional dehydration depends not only on the nature of the soil layer, but also on those of layers vertically or horizontally next to the very layer. The consolidation test and its analysis are usually conducted under the condition that the consolidation permeability state of the surrounding soil is that of perfect drainage or that of no drainage. However, there are in general intermediate drainage conditions that lie between them. Furthermore, there are generally many cases where the secondary compression of soil is not taken into consideration in the analysis of three-dimensional consolidation of soil, but this effect cannot be neglected in the case of the weak ground. So the author describes here the method of analysis of three-dimensional consolidation with the above-mentioned effects taken into consideration, and its application.

II. DERIVATION OF THE BASIC EQUATION OF DEHYDRATION ONLY BY THE HORIZONTAL FLOW

The basic equation of three-dimensional dehydration is

$$\frac{k_h}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)+\frac{k_v}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial z^2}\right)=vpu+\frac{\gamma\eta p}{p+\eta}u \qquad \cdots\cdots(1)$$

From this basic equation, the basic equation only by the horizontal flow is obtained as

$$\frac{k_h}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)=vpu+\frac{\gamma\eta p}{p+\eta}u \qquad \cdots\cdots(2)$$

A particular solution of the equation is

$$,u=(C_1e^{\lambda_1 t}+C_2e^{\lambda_2 t})\{C_3J_0(Mr)+C_4Y_0(Mr)\} \qquad \cdots\cdots(3)$$

,where C_1, C_2, C_3, and C_4 are integration constants and

$$\lambda_1,\ \lambda_2=\frac{1}{2}\left[-(v+\gamma)\frac{\eta}{v}-\frac{k_h}{v\gamma_\omega}M^2 \pm\sqrt{\left\{(v+\gamma)\frac{\eta}{v}+\frac{k_h}{v\gamma_\omega}M^2\right\}^2-4\frac{k_h}{v\gamma_\omega}\cdot\eta M^2}\right] \qquad \cdots\cdots(4)$$

$J_0(Mr)$ is the Bessel function of first kind and order 0, $Y_0(Mr)$ is the Bessel function of second kind and order 0, and M is an undetermined constant.

The values of these constants are determined by surface conditions and initial conditions.

III. SURFACE CONDITIONS (I)

The excess pore water pressure in a soil body in the consolidation process must satisfy, other than the differential equation previously described inside the consolidated soil body, a certain equation on

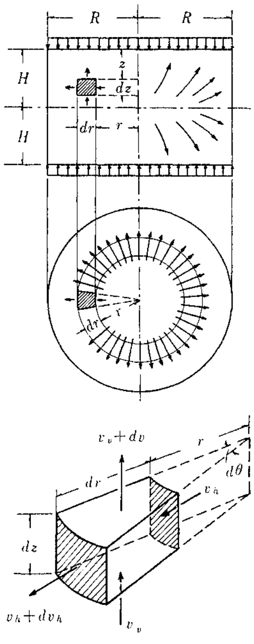

Fig. 1 An explanatory figure of the mechanism of three-dimensional dehydration

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the surface of the consolidated soil body in general.

This equation is a so-called surface condition. Generally speaking, letting u_s and u_0 be the excess pore water pressure on the surface of the consolidated soil body and that outside the body respectively, one may assume that the amount of water flowing out of the surface is proportional to (u_s-u_0), and get the amount of water flowing out of unit surface per unit time as $a(u_s-u_0)$, where a is the outflow index, a constant determined by the nature of soil that surrounds the consolidated soil body. Now on these assumptions, an equation of the surface condition is derived.

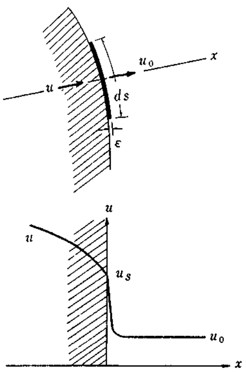

Fig. 2 An explanatory figure of the variation of the pore water pressure in the boundary face

First, letting ε and ds be the thickness and the area of a minute sheet-figured element respectively, one obtains as the amount of water flowing into this element in time dt

$$-k\left(\frac{\partial u}{\partial x}\right)_s \cdot ds \cdot dt$$

,where k is the permeability coefficient of soil, and $(\ )_s$ denotes the value of the term in ( ) on the surface.

Next, the amount of water flowing out of this element in time dt is

$$a(u_s-u_0)\cdot ds \cdot dt$$

,and outside the consolidated soil body, where soil is not consolidated and the excess pore water pressure $u_0=0$, this term becomes

$$a(u)_s\, ds \cdot dt.$$

By making the thickness of the surface element $\varepsilon \to 0$, one can make the amount of water flowing in equal that of water flowing out, as this element comes to have no volume. Therefore,

$$\left(\frac{\partial u}{\partial x}\right)_s + \frac{a}{k}(u)_s = 0$$

Introducing $\frac{a}{k}=\beta$ (relative radiation index),

$$\left(\frac{\partial u}{\partial x}+\beta u\right)_s = 0 \quad \cdots\cdots(5)$$

This is the equation that gives the surface condition.

Now in the central axis of the consolidated soil body ($r=0$), where $\beta=0$ and one can make $\frac{\partial u}{\partial r}=0$, one obtains by differentiating the equation (3) and taking $r=0$,

$$\frac{\partial u}{\partial r} = (C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t})(-M)\{C_3 J_1(0) + C_4 Y_1(0)\} = 0$$

As $Y_1(0) = -\infty$, C_4 must be 0 in order to satisfy the above equation. Then equation (3) becomes

$$u = (C_1' e^{\lambda_1 t} + C_2' e^{\lambda_2 t})\, J_0(Mr)$$

,where $C_1' = C_1 \cdot C_3$ and $C_2' = C_2 \cdot C_3$

Furthermore on the surface of the consolidated soil body ($r=R$), one obtains by applying the equation (5),

$$\left(\frac{\partial u}{\partial r}+\beta u\right)_R = (C_1' e^{\lambda_1 t} + C_2' e^{\lambda_2 t}) \times \{-MJ_1(MR) + \beta J_0(MR)\} = 0$$

Therefore

$$\beta J_0(MR) = MJ_1(MR) \quad \cdots\cdots(6)$$

If the i th root of the equation (6) is expressed as m_i,

$$u = \sum_{i=1}^{\infty} (C_1' e^{\lambda_1 t} + C_2' e^{\lambda_2 t})\, J_0(m_i r) \quad \cdots\cdots(7)$$

IV. INITIAL CONDITIONS (I)

The excess pore water pressure in the soil body in the consolidation process is in the so-called unsteady state. So it is important and must be made definite solving the problem how the distribution of the excess pore water pressure in the consolidated soil body was at a specific time.

As this specific time is selected as the zero point of time usually, this definite condition is called the initial condition.

Now the following condition is used as the initial condition. At $t=0$,

$$\varepsilon_e = v\bar{p} = v(K-u) = 0, \text{ namely, } u = K.$$

In these equations, ε_e, K, v and $\bar{p}$ are elastic strain, strength of load, elastic deformation rate and effective stress respectively.

Making $t=0$ and $u=K$ in the equation (7),

$$K = \sum_{i=1}^{\infty} (C_1' + C_2')\, J_0(m_i r)$$

By multiplying the both sides by $J_0(m_i r)\, r$ and integration, one obtains

$$C_1' + C_2' = \frac{2KJ_1(m_i R)}{Rm_i\{J_1^2(m_i R) + J_0^2(m_i R)\}}$$

Applying $J_0(m_i R) = \frac{m_i}{\beta} J_1(m_i R)$, which is derived from the equation (6), to the above equation,

$$C_1'+C_2'=\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)} \quad \cdots\cdots\cdots(8)$$

Next, at $t=0$, $\varepsilon_c=0$, where ε_c is the permanent strain and expressed by the following equation.

$$\varepsilon_c=e^{-\eta t}\int_0^t e^{\eta\tau}\,\eta\gamma\,\bar{p}\,d\tau$$

Substituting $\bar{p}$ in the equation for $\bar{p}=K-u$

$$\varepsilon_c=\gamma\eta\int_0^t e^{-\eta(t-\tau)}\left\{\sum_{i=1}^{\infty}\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)}\right.$$

$$\left.\times J_0(m_ir)-\sum_{i=1}^{\infty}(C_1'e^{\lambda_1\tau}+C_2'e^{\lambda_2\tau})J_0(m_ir)\right\}d\tau$$

The coefficient term of $J_0(m_ir)$ is

$$\gamma\eta\int_0^t e^{-\eta(t-\tau)}\left\{\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)}-(C_1'e^{\lambda_1\tau}\right.$$

$$\left.+C_2'e^{\lambda_2\tau})\right\}d\tau=\gamma\eta\left\{\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)\eta}\right.$$

$$\left.-\left(\frac{C_1'}{\lambda_1+\eta}e^{\lambda_1 t}+\frac{C_2'}{\lambda_2+\eta}e^{\lambda_2 t}\right)\right\}$$

As $\varepsilon_c=0$ at $t=0$, this coefficient term becomes 0. Therefore

$$\frac{C_1'}{\lambda_1+\eta}+\frac{C_2'}{\lambda_2+\eta}=\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)\eta} \quad \cdots\cdots\cdots(9)$$

C_1' and C_2' are calculated from the equations (8) and (9) as

$$C_1'=\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)}\cdot\frac{\lambda_2(\lambda_1+\eta)}{\eta(\lambda_2-\lambda_1)}$$

$$C_2'=\frac{-2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)}\cdot\frac{\lambda_1(\lambda_2+\eta)}{\eta(\lambda_2-\lambda_1)}$$

So the excess pore water pressure u_r, when the load K is supported only by the horizontal flow, becomes

$$u_r=\sum_{i=1}^{\infty}\frac{2K\beta^2}{Rm_i(\beta^2+m_i^2)J_1(m_iR)}\cdot\frac{1}{\eta(\lambda_2-\lambda_1)}$$

$$\times\{\lambda_2(\lambda_1+\eta)e^{\lambda_1 t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2 t}\}J_0(m_ir) \quad \cdots\cdots\cdots(10)$$

V. DERIVATION OF THE BASIC EQUATION OF DEHYDRATION ONLY BY THE VERTICAL FLOW

As the excess pore water pressure in the cylindrical soil body only by the vertical flow is independent of the horizontal flow, the basic equation in this case is obtained, by making $\frac{\partial u}{\partial r}=0$ and $\frac{\partial^2 u}{\partial r^2}=0$ in the equation (1), as

$$\frac{k_v}{\gamma_\omega}\left(\frac{\partial^2 u}{\partial z^2}\right)=vpu+\frac{\gamma\eta p}{p+\eta}u \quad \cdots\cdots\cdots(11)$$

A particular solution of the equation (11) is

$$u=(B_1e^{\lambda_1' t}+B_2e^{\lambda_2' t})(B_3\sin Nz+B_4\cos Nz) \quad \cdots\cdots\cdots(12)$$

,where B_1, B_2, B_3 and B_4 are integration constants, and N is an undetermined constant and in which,

$$\lambda_1',\ \lambda_2'=\frac{1}{2}\left\{-(v+\gamma)\frac{\eta}{v}-\frac{k_v}{v\gamma_\omega}N^2\right.$$

$$\left.\pm\sqrt{\left\{(v+\gamma)\frac{\eta}{v}+\frac{k_v}{v\gamma_\omega}N^2\right\}^2-4\frac{k_v}{v\gamma_\omega}\eta N^2}\right\} \quad \cdots\cdots\cdots(13)$$

Next, the values of these constants are determined by applying the surface conditions and initial conditions to the equation.

VI. SURFACE CONDITIONS (II)

As $\beta'=\infty$, on the top surface of the consolidated soil body ($z=0$), which is perfectly pervious, one obtains $B_4=0$ by applying the definite condition of pore water pressure $u=0$, then the equation (12) becomes

$$u=(B_1'e^{\lambda_1' t}+B_2'e^{\lambda_2' t})\sin Nz$$

,where $B_1'=B_1\cdot B_3$ and $B_2'=B_2\cdot B_3$.

And on the bottom surface of the consolidated soil body ($z=2H$), one obtains by applying the equation (5)

$$\left(\frac{\partial u}{\partial z}+\beta' u\right)_{2H}=(B_1'e^{\lambda_1' t}+B_2'^{\lambda_2' t})$$

$$\times(N\cos 2NH+\beta'\sin 2NH)=0$$

Namely

$$\tan 2NH=-\frac{N}{\beta'} \quad \cdots\cdots\cdots(14)$$

If one expresses the ith root of the equation (14) as n_i, one gets

$$u=\sum_{i=1}^{\infty}(B_1'e^{\lambda_1' t}+B_2'e^{\lambda_2' t})\sin n_iz \quad \cdots\cdots\cdots(15)$$

VII. INITIAL CONDITIONS (II)

In the same way as in the case of initial conditions in Section IV, one can obtain by applying the condition that $u=K$ at $t=0$ to the equation (15)

$$K=\sum_{i=1}^{\infty}(B_1'+B_2')\sin n_iz$$

Multiplying both sides by $\sin n_iz$ and integration them, one obtains

$$B_1'+B_2'=\frac{2K(1-\cos 2n_iH)(1+\tan^2 2n_iH)}{2Hn_i(1+\tan^2 2n_iH)-\tan 2n_iH}$$

By applying the relation $\tan 2n_iH=-\frac{n_i}{\beta'}$, which is derived from the equation (14), to the above equation, one obtains

$$B_1'+B_2'=\frac{2K(\beta'^2+n_i^2)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\times(1-\cos 2n_iH)\cdots\cdots\cdots\cdots\cdots\cdots(16)$$

Next the condition that $\varepsilon_c=0$ at $t=0$ is used.
In the equation,

$$\varepsilon_c=r\eta\int_0^t e^{-\eta(t-\tau)}\Big[\sum_{i=1}^{\infty}\frac{2K(\beta'^2+n_i^2)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\times(1-\cos 2n_iH)\sin n_iz-\sum_{i=1}^{\infty}(B_1'e^{\lambda_1'\tau}+B_2'e^{\lambda_2'\tau})\sin n_iz\Big]d\tau$$

The coefficient term of $\sin n_iz$ is

$$r\eta\int_0^t e^{-\eta(t-\tau)}\Big[\frac{2K(\beta'^2+n_i^2)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\times(1-\cos 2n_iH)-(B_1'e^{\lambda_1'\tau}+B_2'e^{\lambda_2'\tau})\Big]d\tau$$

$$=r\eta\Big[\frac{2K(\beta'^2+n_i^2)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\cdot\frac{1-\cos 2n_iH}{\eta}-\Big(\frac{B_1'}{\lambda_1'+\eta}e^{\lambda_1't}+\frac{B_2'}{\lambda_2'+\eta}e^{\lambda_2't}\Big)\Big]$$

As $\varepsilon_c=0$ at $t=0$, this term becomes 0.
Then

$$\frac{B_1'}{\lambda_1'+\eta}+\frac{B_2'}{\lambda_2'+\eta}=\frac{2K(\beta'^2+n_i^2)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\times\frac{1-\cos 2n_iH}{\eta}\cdots\cdots\cdots\cdots\cdots(17)$$

From the equations (16) and (17) are obtained

$$B_1'=\frac{2K(\beta'^2+n_i^2)(1-\cos 2n_iH)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\cdot\frac{\lambda_2'(\lambda_1'+\eta)}{\eta(\lambda_2'-\lambda_1')}$$

$$B_2'=\frac{-2K(\beta'^2+n_i'^2)(1-\cos 2n_iH)}{\{2H(\beta'^2+n_i^2)+\beta'\}n_i}\cdot\frac{\lambda_1'(\lambda_2'+\eta)}{\eta(\lambda_2'-\lambda_1')}$$

By applying these values of B_1' and B_2' to the equation (15), one obtains the excess pore water pressure u_z in the case where load K is supported only by the vertical dehydration as

$$u_z=\sum_{i=1}^{\infty}\frac{2K(\beta'^2+n_i^2)(1-\cos 2n_iH)}{[2H(\beta'^2+n_i^2)+\beta']n_i}\cdot\frac{1}{\eta(\lambda_2'-\lambda_1')}\times\{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1't}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2't}\}\sin n_iz\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

VIII. THE ANALYSIS OF CONSOLIDATION BY THREE-DIMENSIONAL DEHYDRATION

The basic equation of consolidation by three-dimensional dehydration is, as described before,

$$\frac{k_h}{r_\omega}\Big(\frac{\partial^2u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\Big)+\frac{k_v}{r_\omega}\Big(\frac{\partial^2u}{\partial z^2}\Big)=vpu+\frac{r\eta p}{p+\eta}\cdot u$$

The solution of this equation is obtained as a product of respective solutions of one-dimensional problems with respect to r-axis and z-axis, namely,

$$u=K(u_r)_{K=1}\cdot(u_z)_{K=1}$$

And letting $\bar{u}$, $\bar{u}_r$ and $\bar{u}_z$ be the mean value of u with respect to r-axis and z-axis, that of u_r with respect to r-axis and that of u_z with respect to z-axis respectively, one obtains

$$\bar{u}=K(\bar{u}_r)_{K=1}\cdot(\bar{u}_z)_{K=1}$$

$(\bar{u}_r)_{K=1}$ and $(\bar{u}_z)_{K=1}$ are calculated as follows.

$$(\bar{u}_r)_{K=1}=\frac{2}{R^2}\int_0^R(u_r)_{K=1}\,r\,dr$$

$$=\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2m_i^2(\beta^2+m_i^2)}\cdot\frac{1}{\eta(\lambda_2-\lambda_1)}\times\{\lambda_2(\lambda_1+\eta)e^{\lambda_1t}-\lambda_1(\lambda_2+\eta)e^{\lambda_2t}\}$$

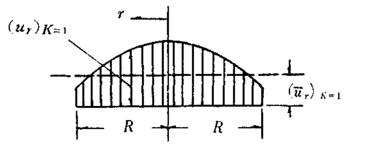

Fig. 3 An explanatory figure of the mean pore water pressure in the horizontal direction

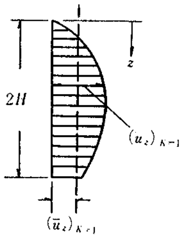

Fig. 4 An explanatory figure of the mean pore water pressure in the vertical direction

In the same manner,

$$(\bar{u}_z)_{K=1}=\frac{1}{2H}\int_0^{2H}(u_z)_{K=1}\cdot dz$$

$$=\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{Hn_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}\times\frac{1}{\eta(\lambda_2'-\lambda_1')}\cdot\{\lambda_2'(\lambda_1'+\eta)e^{\lambda_1't}-\lambda_1'(\lambda_2'+\eta)e^{\lambda_2't}\}$$

The signs (+) or (−) corresponds to the case where the value of $2n_iH$ is within the second or the fourth quadrant respectively.

IX. THE AMOUNT OF CONSOLIDATION SETTLEMENT

The amount of consolidation settlement S at any time t is evaluated as follows.

$$S=\int_0^{2H}(\bar{p}v+\varepsilon_c)dz$$

$$=\int_0^{2H}(Kv+\eta r\,Ke^{-\eta t}\int_0^t e^{\eta\tau}d\tau)\,dz-\int_0^{2H}(\bar{u}\cdot v+\eta r\,e^{-\eta t}\int_0^t\bar{u}\,e^{\eta\tau}\,d\tau)dz\cdots\cdots(19)$$

Next, the coefficient terms in the above equation are calculated.

$$\eta r K e^{-\eta t}\int_0^t e^{\eta\tau}\, d\tau = \eta r K e^{-\eta t}\left|\frac{e^{\eta\tau}}{\eta}\right|_0^t = r K$$

$$\eta r e^{-\eta t}\int_0^t \bar{u}\, e^{\eta\tau}\, d\tau$$

$$= r K \sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}\cdot\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}\cdot\left[\frac{\lambda_2\lambda_2'(\lambda_1+\eta)(\lambda_1'+\eta)}{\eta(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')(\lambda_1+\lambda_1'+\eta)}e^{(\lambda_1+\lambda_1')t}\right.$$

$$-\frac{\lambda_1\lambda_2'(\lambda_2+\eta)(\lambda_1'+\eta)}{\eta(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')(\lambda_2+\lambda_1'+\eta)}e^{(\lambda_2+\lambda_1')t}-\frac{\lambda_2\lambda_1'(\lambda_1+\eta)(\lambda_2'+\eta)}{\eta(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')(\lambda_1+\lambda_2'+\eta)}e^{(\lambda_1+\lambda_2')t}$$

$$\left.+\frac{\lambda_1\lambda_1'(\lambda_2+\eta)(\lambda_2'+\eta)}{\eta(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')(\lambda_2+\lambda_2'+\eta)}e^{(\lambda_2+\lambda_2')t}\right]$$

$$\bar{u}v = vK\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}\left[\frac{\lambda_2\lambda_2'(\lambda_1+\eta)(\lambda_2'+\eta)}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}e^{(\lambda_1+\lambda_1')t}\right.$$

$$\left.-\frac{\lambda_2\lambda_1'(\lambda_1+\eta)(\lambda_2'+\eta)}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}e^{(\lambda_1+\lambda_2')t}-\frac{\lambda_1\lambda_2'(\lambda_2+\eta)(\lambda_1'+\eta)}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}e^{(\lambda_2+\lambda_1')t}+\frac{\lambda_1\lambda_1'(\lambda_2+\eta)(\lambda_2'+\eta)}{\eta^2(\lambda_2-\lambda_1)(\lambda_2'-\lambda_1')}e^{(\lambda_2+\lambda_2')t}\right]$$

Now an examination is made of λ_1, λ_2, λ_1' and λ_2'. Writing $\frac{k_h}{v\gamma_\omega}=C_h$ and $\frac{k_v}{v\gamma_\omega}=C_v$ and applying the binomial theorem to the term of $\sqrt{\ }$ in the equations (4) and (13)

$$\lambda_1 = -\frac{C_h\eta m_i^2}{\frac{v+r}{v}\eta+C_h m_i^2}-\frac{(C_h\eta m_i^2)^2}{\left(\frac{v+r}{v}\eta+C_h m_i^2\right)^3}-\cdots\cdots$$

$$\lambda_2 = -\frac{v+r}{v}\eta-C_h m_i^2+\frac{C_h\eta m_i^2}{\frac{v+r}{v}\eta+C_h m_i^2}+\frac{(C_h\eta m_i^2)^2}{\left(\frac{v+r}{v}\eta+C_h m_i^2\right)^3}+\cdots\cdots$$

$$\lambda_1' = -\frac{C_v\eta n_i^2}{\frac{v+r}{v}\eta+C_v n_i^2}-\frac{(C_v\eta n_i^2)^2}{\left(\frac{v+r}{v}\eta+C_v n_i^2\right)^3}-\cdots\cdots$$

$$\lambda_2' = -\frac{v+r}{v}\eta-C_v n_i^2+\frac{C_v\eta n_i^2}{\frac{v+r}{v}\eta+C_v n_i^2}+\frac{(C_v\eta n_i^2)^2}{\left(\frac{v+r}{v}\eta+C_v n_i^2\right)^3}+\cdots\cdots$$

Then investigation is made into the case where the creep coefficient η is very small compared with the permeability coefficient k, and creep lasts very long. Namely,

when $\eta \ll C_h m_i^2$ and $\eta \ll C_v n_i^2$,

$$\lambda_1 \fallingdotseq -\eta-\frac{\eta^2}{C_h m_i^2},\quad \lambda_2 \fallingdotseq -C_h m_i^2$$

and

$$\lambda_1' = -\eta-\frac{\eta^2}{C_v n_i^2},\quad \lambda_2' \fallingdotseq -C_v n_i^2$$

On the basis of these results, the coefficient values in the equation of $\eta r e^{-\eta t}\int_0^t \bar{u}\, e^{\eta\tau}\, d\tau$ are evaluated, and the equation is arranged to become

$$\eta r e^{-\eta t}\int_0^t \bar{u}\, e^{\eta\tau}\, d\tau = r K\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}e^{-2\eta t} \fallingdotseq r K e^{-2\eta t}$$

In the same manner the coefficient values in the equation of $\bar{u}v$ are examined and there is obtained

$$\bar{u}v = vK\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}e^{-C_h m_i^2 t}\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{(\beta'^2+n_i^2)+\beta'\}}e^{-C_v n_i^2 t}$$

The above results are applied to the equation (19) to give

$$S = 2HK(v+r) - 2HKv\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}e^{-C_h m_i^2 t}\times\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}e^{-C_v n_i^2 t} - 2HKre^{-2\eta t}\quad\cdots\cdots(20)$$

Furthermore, investigation is made into the case where the creep coefficient η is very great compared with the permeability coefficient k, and creep is completed in a very short time. Namely, when $\eta \gg C_h m_i^2$ and $\eta \gg C_v n_i^2$

$$\lambda_1 \fallingdotseq -\frac{v}{v+r}\cdot C_h m_i^2,\quad \lambda_2 \fallingdotseq -\frac{v+r}{v}\eta$$

and

$$\lambda_1' \fallingdotseq -\frac{v}{v+r}\cdot C_v n_i^2,\quad \lambda_2' \fallingdotseq -\frac{v+r}{v}\eta$$

By making use of these results, the amount of settlement S in this case is

$$S = 2HK(v+r) - 2HK(v+r)\times\sum_{i=1}^{\infty}\frac{4\beta^2}{R^2 m_i^2(\beta^2+m_i^2)}e^{-\frac{v}{v+r}C_h m_i^2 t}\times\sum_{i=1}^{\infty}\frac{(\sqrt{\beta'^2+n_i^2}\pm\beta')^2}{H n_i^2\{2H(\beta'^2+n_i^2)+\beta'\}}e^{-\frac{v}{v+r}C_v n_i^2 t}\quad\cdots\cdots(21)$$

This equation explains the case where the permanent deformation occurs in a moment and the delay of creep is negligible, only with the primary consolidation taken into consideration.

X. THE DEGREE OF CONSOLIDATION

The amount of the final settlement S_∞ by consolidation is calculated by making $t \to \infty$ in the equation (20). Namely,

$$S_\infty = 2HK(v+r)$$

Now one may write

$$\frac{C_v}{H^2}t = T_v,\ \frac{C_h}{R^2}t = T_h,\ \eta t = T'$$

and

$$T_h = \frac{C_h}{R^2}t = \left\{\frac{C_h}{C_v}\left(\frac{H}{R}\right)^2\right\} T_v = \alpha T_v$$

,then the degree of consolidation U is evaluated as follows.

$$U = \frac{S}{S_\infty}$$

$$= \frac{v}{v+r}\left[1 - \sum_{i=1}^{\infty} \frac{4(R\beta)^2}{(Rm_i)^2\{(R\beta)^2 + (Rm_i)^2\}} \times e^{(Rm_i)^2 \alpha T_v} \times \sum_{i=1}^{\infty} \frac{2\{\sqrt{(2H\beta')^2 + (2Hn_i)^2} \pm 2H\beta'\}^2}{(2Hn_i)^2\{(2H\beta')^2 + (2Hn_i)^2 + 2H\beta'\}} \times e^{-\frac{(2Hn_i)^2}{4}T_v}\right] + \frac{r}{v+r}(1 - e^{-2T'})$$

In this equation, the first term is the ratio of the amount of the elastic part consolidation to that of the total consolidation, which is denoted by the primary consolidation rate U_v, and the second term is the ratio of the amount of the plastic part consolidation to that of the total consolidation, which is denoted by the secondary compression rate U_r. Then U_v is a function of T_v, with $\frac{v}{v+r}$, β', β and α involved as consolidation constants, and U_r is a function of T' with $\frac{r}{v+r}$ involved as a consolidation constant.

In the case where the secondary compression can be neglected, namely where the equation (21) can be used, one obtains

$$U = 1 - \sum_{i=1}^{\infty} \frac{4(R\beta)^2}{(Rm_i)^2\{(R\beta)^2 + (Rm_i)^2\}} \times e^{-\frac{v}{v+r}(Rm_i)^2 \alpha T_v} \times \sum_{i=1}^{\infty} \frac{2\{\sqrt{(2H\beta')^2 + (2Hn_i)^2} \pm 2H\beta'\}^2}{(2Hn_i)^2\{(2H\beta')^2 + (2Hn_i)^2 + (2H\beta')\}} \times e^{-\frac{v}{v+r}\cdot\frac{(2Hn_i)^2}{4}\cdot T_v}$$

,which is the same equation as derived only with the primary consolidation taken into consideration.

As to the double sign $\pm$ of the part with respect to $(2H\beta')$ in the equation of U_v, it is defined, as described in Section Ⅷ, to be $(+)$ in case the value of $(2Hn_i)$ is in the second quadrant, and to be $(-)$ in case in the fourth quadrant.

XI. THE DEGREE OF CONSOLIDATION CURVE ON VARIOUS SURFACE CONDITIONS

The progress of consolidation depends on the surface conditions of the consolidated soil body, and on the degree of consolidation curve characteristic of the consolidated layer. Namely, it is because of the fact that the values of the consolidation constants are characteristic of the very layers consolidated.

Now examination is made of some cases with either of the two extreme surface conditions, the one when the surface of the consolidated soil body is in contact with a perfectly pervious layer and the other when that is in contact with a perfectly impervious layer.

(1) The case where $\beta = \infty$ and $\beta' = \infty$

This is the case where the consolidated soil body is in contact with a perfectly pervious layer both in the vertical and in the horizontal direction. This is also the case in the equation (5), where the value of the outflow index a to the outside of the consolidated soil body is very great compared with the value of the permeability coefficient k inside the consolidated soil body

$$\left(\frac{\partial u}{\partial x} + \beta u\right)_s = 0$$

This leads to $\left(\frac{\partial u}{\beta \partial x} + u\right)_s = 0$

As $\beta = \infty$, $\frac{\partial u}{\beta \partial x} = 0$

Hence $u_s = 0$

In other words, this is the case where the amount of consolidation-dehydration from the inside of the consolidated soil body is absorbed in a moment by its outside, and the pore water pressure always remains zero on its surface. The equation of U_v in this case is as follows.

$$U_v = \frac{v}{v+r}\left[1 - \sum_{i=1}^{\infty} \frac{4}{(Rm_i)^2} e^{-(Rm_i)^2 \alpha T_v} \times \sum_{i=1,3,5}^{\infty} \frac{8}{(2Hn_i)^2} e^{-\frac{(2Hn_i)^2}{4}T_v}\right]$$

The values of Rm_i and $2Hn_i$ are what satisfy the equations $J_0(Rm_i) = 0$ and $\tan(2Hn_i) = 0$, as readily seen from the equations (6) and (14).

(2) The case where $\beta = \infty$ and $\beta' = 0$

This is the case where the amount of consolidation-dehydration from the inside of the consolidated soil body is absorbed in a moment by its outside on its side surface, but consolidation-dehydration is completely suppressed on its bottom surface. In other words, this is the case where the bottom surface of the body is in contact with an impervious layer, and its top and side surfaces are in contact with a pervious layer. The equation of U_v in this case is as follows.

$$U_v = \frac{v}{v+r}\left[1-\sum_{i=1}^{\infty}\frac{4}{(Rm_i)^2}e^{-(Rm_i)^2\alpha T_v}\right.$$
$$\left.\times\sum_{i=1,3,5}^{\infty}\frac{2}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}\right]$$

The values of Rm_i and $2Hn_i$ are, as seen in the equations (6) and (14), which satisfy the equations $J_0(Rm_i)=0$ and $\tan(2Hn_i)=-\infty$.

(3) The case where $\beta=0$ and $\beta'=0$

This is the case where both on the side surface and on the bottom surface of the consolidated soil body consolidation-dehydration is suppressed.

The suppression of dehydration on the side surface means that no water flows in the horizontal direction and $\alpha=0$, and that dehydration is conducted only in the vertical direction irrespective of the horizontal direction. Therefore the equation of U_v in this case is

$$U_v = \frac{v}{v+r}\left\{1-\sum_{i=1,3,5}^{\infty}\frac{2}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}\right\}$$

Furthermore, the values of $2Hn_i$ in the case are the same as shown in (2).

(4) The case where $\beta=0$ and $\beta'=\infty$

This is the case where consolidation-dehydration is suppressed on the side surface of the consolidated soil body, and dehydration is conducted only on its top and bottom surfaces. And this case corresponds to that obtained by making $\alpha=0$ in the case of (1). Therefore the equation of U_v is

$$U_v = \frac{v}{v+r}\left\{1-\sum_{i=1,3,5}^{\infty}\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}\right\}$$

and the values of $2Hn_i$ are the same as shown in (1).

Now the degree of consolidation curves in the above cases with a condition $\frac{v}{v+r}=1$ are shown below.

As easily seen in the figure, comparison of (3) with (4), namely, comparison of the case where the bottom surface is perfectly impervious ($\beta'=0$) with the case where it is perfectly pervious ($\beta'=\infty$), both cases being those of dehydration only by the vertical flow with no horizontal flow, shows that it needs the consolidation time in the former case four times longer than in the latter case.

Furthermore comparison of (3) with (2), namely comparison of the case where the side surface is perfectly impervious ($\beta=0$) with the case where it is perfectly pervious ($\beta=\infty$), both cases being those where dehydration out of the bottom surface does not occur, shows that it needs the consolidation time in the former case about 13 times longer than in the latter case when $\alpha=1$, and about 50 times when $\alpha=5$. These comparisons are those of the very extreme cases, but at least they suggest that β, β' and α have so great an influence on the consolidation time needed.

When the consolidated layer is present between sand layers, and the value of β' is so great compared with that of β that dehydration is thought to be conducted only in the vertical direction, its consolidation curve is expected to belong to the case (4). However, to the weak ground where weak clay extends from the earth's surface to the considerable depth, the above conventional interpretation can not be applied, in point of surface conditions in the analysis of consolidation.

Namely, when the consolidated soil body is in the weak layer, the values of β, β' and α of the weak soil are expected to have a great influence on the consolidation time, and determination of these consolidation constants becomes the necessary condition for the analysis of consolidation settlement in the weak ground. In the following are described the method of determination of these consolidation constants.

XII. DETERMINATION OF CONSOLIDATION CONSTANTS OF SOIL

The values of the consolidation constants described above are to be determined experimentally.

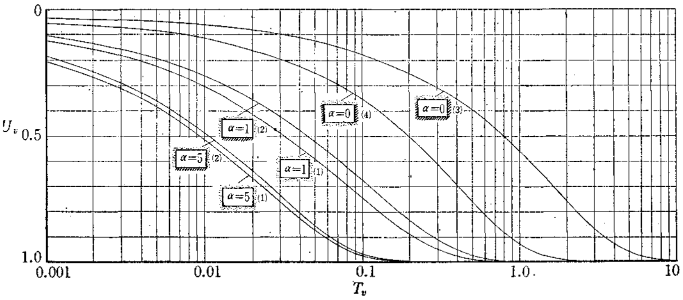

Fig. 5 The figure for comparison of U_v-log T_v curves on various surface conditions with one another.

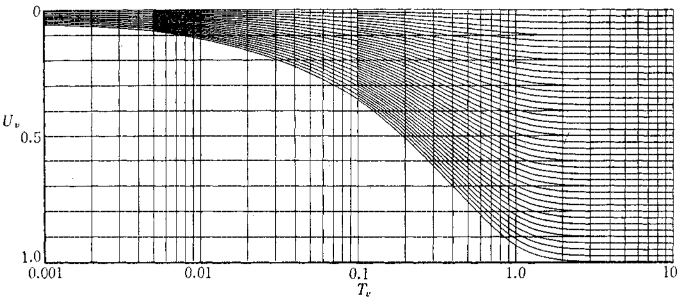

Fig. 6 The figure of U_v-log T_v curves with variation of $\frac{v}{v+\gamma}$ in the case where dehydration is conducted only in the vertical direction

$$U_v=\frac{v}{v+\gamma}\left[1-\sum_{i=1,3,5}^{\infty}\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}\right]$$

(the case where $\alpha=0$ and $\beta'=\infty$)

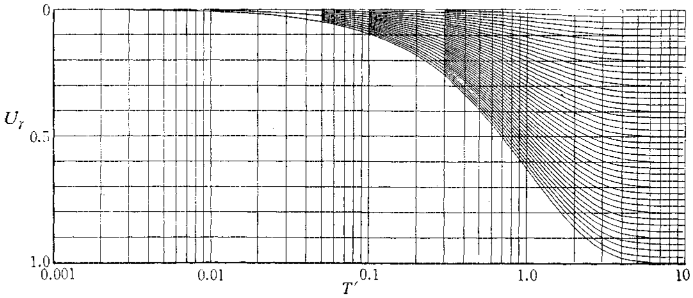

Fig. 7 The figure of U_γ-log T' curves with variation of $\frac{\gamma}{v+\gamma}$ in the case where dehydration is conducted only in the vertical direction

$$U_\gamma=\frac{\gamma}{v+\gamma}[1-e^{-T'}]$$

The method of determination of these are explained in the following.

(1) Determination of $\frac{v}{v+\gamma}$ and $\frac{\gamma}{v+\gamma}$

These are the constant values of soil which show the ratio of the elastic deformation rate and that of the permanent deformation rate to the total deformation rate by consolidation.

These constant values are determined by the analysis of experimental results, making use of the surface conditions described in the preceding section (4).

Namely, the equations of U_v and U_γ are as follows.

$$U_v=\frac{v}{v+\gamma}\left[1-\sum_{i=1,3,5}^{\infty}\frac{8}{(2Hn_i)}e^{-\frac{(2Hn_i)^2}{4}T_v}\right]$$

$$U_\gamma=\frac{\gamma}{v+\gamma}(1-e^{-T'})$$

In the equation of U_v, the values of $(2Hn_i)$ are π, 3π, 5π, 7π, ...

The relation of U_v-log T_v and that of U_γ-log T' are shown in Fig. 6 and 7 respectively.

Now it is explained how to use Fig. 6 and 7. A consolidation test is carried out on the surface condition described in the preceding section (4), and the U-log t curve is drawn on a sheet of tracing paper according to the same scale as in the figure of U_v-log T_v curves. Then this sheet is put on the figure of U_v-log T_v curves, and is moved right and left with the T_v-axis and t-axis just fitted to each other, until the U_v-log T_v curve that coincides well with the U-log t curve is found in the figure, from which the value of $\frac{v}{v+\gamma}$ is obtained. The value of $\frac{\gamma}{v+\gamma}$ is then obtained by calculating $1-\frac{v}{v+\gamma}$.

Furthermore, the ratio of any time t in the U-log t curve to the time T_v in the U_v-log T_v curve that coincides with it, $\frac{T_v}{t}$ is obtained.

Next, the line of $\frac{v}{v+\gamma}$ is drawn parallel to the t-axis on the U-log t curve on the sheet of tracing paper, and this sheet is put on the figure of U_γ-log T' curves.

Then the sheet is moved right and left with the T'-axis and the $\frac{v}{v+\gamma}$ line just fitted to each other, until the U_γ-log T' curve that coincides well with the U-log t curve is found in the figure, then the values of t and T' that coincide with each other give the ratio T'/t. Thus the vertical consolidation coefficient C_v and the creep coefficient η are obtained by the equations,

$$C_v=\left(\frac{T_v}{t}\right)H^2$$

$$\eta=\left(\frac{T'}{t}\right)$$

And the elastic deformation rate v and the permanent deformation rate γ are evaluated by making use of the equations

$$v=\frac{\varepsilon_v}{\Delta p}=\frac{1}{\Delta p}\cdot\frac{d_v-d_s}{2H}=\left(\frac{d_{v+\gamma}-d_s}{\Delta p\cdot 2H}\right)\frac{v}{v+\gamma}$$

$$\gamma=\frac{\varepsilon_\gamma}{\Delta p}=\frac{1}{\Delta p}\cdot\frac{d_{v+\gamma}-d_v}{2H}=\left(\frac{d_{v+\gamma}-d_s}{\Delta p\cdot 2H}\right)\frac{\gamma}{v+\gamma}$$

where the meaning of symbols is as follows.

- ε_v, ε_γ : the amount of strain of the primary and the secondary part of consolidation respectively
- Δp : the amount of increase in load
- $2H$: the thickness of the consolidated soil boby
- d_v : the amount of settlement until the finish of the primary consolidation
- $d_{v+\gamma}$: the amount of settlement until the finish of the total consolidation
- d_s : the amount of settlement on the moment of loading

(2) Determination of α

The value of α, which is the ratio of the time factor in the vertical dehydration T_v to that in the horizontal dehydration T_h, is the important constant value of consolidation relating the vertical consolidation coefficient C_v with the horizontal consolidation coefficient C_h. In determination of this value, the surface condition in the preceding section (1) is used. Namely, U_v is obtained by the following equation.

$$U_v=\frac{v}{v+\gamma}\left\{1-\sum_{i=1,3,5}^{\infty}\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{4}T_v}\times\sum_{i=1}^{\infty}\frac{4}{(Rm_i)^2}e^{-(Rm_i)^2\alpha T_v}\right\}$$

In this equation, the values of $(2Hn_i)$ and (Rm_i) are those shown in the preceding section (1).

Now in Fig. 8 is shown the relation of U_v-log T_v with variation of α on the condition that $\frac{v}{v+\gamma}=1$. The method of using this figure is then explained. The U-log t curve of three-dimensional dehydration is obtained using the consolidometer which assures that the pore water pressure on the top, bottom and side surfaces of the cylindrical consolidated soil body always remains zero. Then the curve is separated into the part of primary consolidation and that of secondary compression by the method described in the paragraph (1).

The curve of the part of primary consolidation and the U_v-log t curve of one-dimensional dehydration obtained in the paragraph (1) are drawn on a sheet of tracing paper according to the same scale as in this figure (the α-figure), which sheet is put on this figure.

The U_v-log t curve of one-dimensional dehydration on the sheet of tracing paper is then fitted to the curve of $\alpha=0$ in the figure (the α-figure) with the T_v-axis just fitted to the t-axis at the same time. In this case, the U_v-log t curve of one-dimensional dehydration on the sheet of tracing paper should coincide well with the curve of $\alpha=0$ in this figure (the α-figure), as it was drawn in advance according to the curve of ($\alpha=0$, $\beta'=\infty$) in the paragraph (1). Then there is obtained the value of α of the curve in the figure, with which the U_v-log t curve of three-dimensional dehydration on

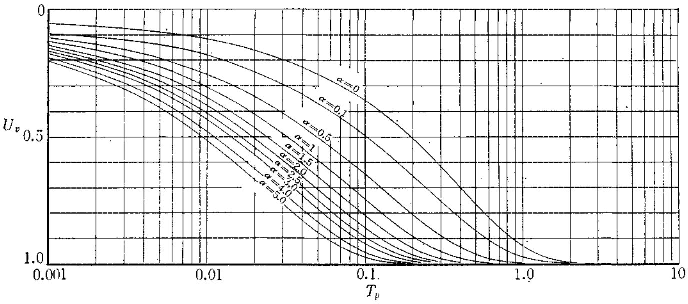

Fig. 8 The figure showing the U_v-log T_v curves with variation of α in three-dimensional dehydration (the α-figure)

$$U_v=1-\sum_{n=1,3,5}^{\infty}\frac{8}{(2Hn_i)^2}e^{-\frac{(2Hn_i)^2}{2}T_v}\cdot\sum_{i=1}^{\infty}\frac{4}{(Rm_i)^2}e^{-(Rm_i)^2\alpha T_v}$$

$$\left(\text{the case where } \frac{v}{v+\gamma}=1,\ \beta=0 \text{ and } \beta'=\infty\right)$$

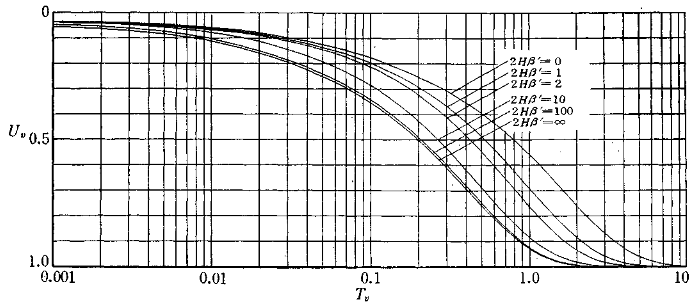

Fig. 9 The figure showing the U_v-log T_v curves with variation of $(2H\beta')$ in the case where only the vertical dehydration is conducted

$$U_v=1-\sum_{i=1}^{\infty}\frac{2[\sqrt{(2H\beta')^2+(2Hn_i)^2}\pm 2H\beta']^2}{(2Hn_i)^2[(2H\beta')^2+(2Hn_i)^2+2H\beta']}e^{-\frac{(2Hn_i)^2}{4}T_v}$$

$\left(\text{the case where } \alpha=0 \text{ and } \frac{v}{v+r}=1\right)$

the sheet of tracing paper coincides well. In the process described above the value of α is determined. Then the horizontal consolidation coefficient C_h is evaluated by the equation

$$C_h=\alpha\, C_v\left(\frac{R}{H}\right)^2$$

(3) Determination of β'

In order to determine the relative outflow index of the pore water pressure in the vertical direction β' of the consolidated soil body, the condition of $\alpha=0$, namely the state of no dehydration on the side surface of the consolidated soil body, is applied to the equation of U_v to give

$$U_v=\frac{v}{v+r}\times\left[1-\sum_{i=1}^{\infty}\frac{2\{\sqrt{(2H\beta')^2+(2Hn_i)^2}\pm 2H\beta'\}^2}{(2Hn_i)^2\{(2H\beta')^2+(2Hn_i)^2+2H\beta'\}}\right]\times e^{-\frac{(2Hn_i)^2}{4}T_v}$$

Now the U_v-log T_v curves with variation of $(2H\beta')$ on the condition that $\frac{v}{v+r}=1$ is obtained and shown in Fig. 9.

For consolidation test to determine β', in the container and under the porous stone on the bottom surface of the cylindrical consolidated soil body is put the formed soil body of the same kind, in such a way that the latter body may not be subject to the consolidation load. Furthermore, it shall be characteristic of this consolidometer that the pore water pressure in the consolidated soil body should dissipate gradually in the compression process, not only through the top surface but also through the lower soil body. By making use of this consolidometer is obtained the U-log t curve, which is to be separated into the part of primary consolidation and that of secondary compression according to the values of $\frac{v}{v+r}$ and $\frac{r}{v+r}$ obtained in the paragragh (1). Then the curve of this part of primary consolidation and the U_v-log t curve of one-dimensional dehydration obtained in the paragraph (1) are drawn on a sheet of tracing paper according to the same scale as in this figure (the β'-figure), which sheet is put on this figure.

The U_v-log t curve on the sheet of tracing paper obtained in the paragraph (1) is then just fitted to the curve of $2H\beta'=\infty$ in the figure (the β'-figure) with the T_v-axis well fitted to the t-axis, and the value of $2H\beta'$ of the curve in the figure (the β'-figure) is obtained, with which the U_v-log t curve on the sheet of tracing paper obtained by making use of the consolidometer to determine β' coincides well.

Thus the value of $2H\beta'$ is obtained, which make it possible to determine β'.

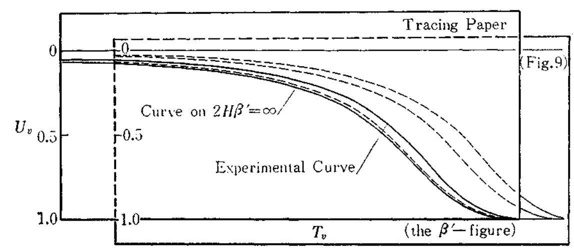

Fig. 10 The figure for analysis of the coefficient β'

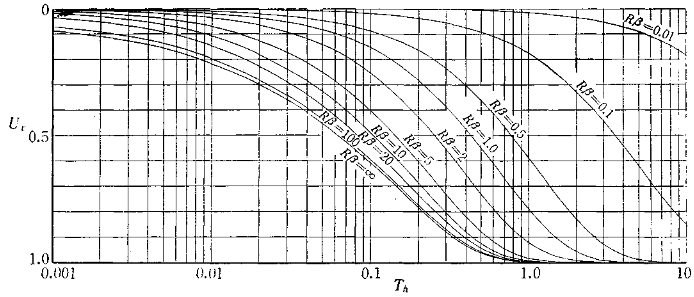

Fig. 11 The figure showing the U_v-log T_h curves with variation of $(R\beta)$ in the case where dehydration is conducted only in the horizontal direction

$$U_v=1-\sum_{i=1}^{\infty}\frac{4(R\beta)^2}{(Rm_i)^2[(R\beta)^2+(Rm_i)^2]}e^{-(Rm_i)^2T_h}$$

$$\left(\text{the case where } \beta'=0 \text{ and } \frac{v}{v+r}=1\right)$$

(4) Determination of β

For determination of the relative outflow index of the pore water on the consolidated soil body in the horizontal direction β, the analysis is made by making use of the equation of consolidation only by the horizontal flow. The equation of U_v in this case is

$$U_v=\frac{v}{v+r}\left[1-\sum_{i=1}^{\infty}\frac{4(R\beta)^2}{(Rm_i)^2\{(R\beta)^2+(Rm_i)^2\}}\times e^{-(Rm_i)^2T_h}\right]$$

Now the figure of the U_v-log T_h curve with variation of $(R\beta)$ on the condition that $\frac{v}{v+r}=1$ is obtained and is shown in Fig. 11.

For consolidation test to determine the value of β, in the container and outside the porous stone on the side surface of the cylindrical consolidated soil body is put the formed soil body of the same kind. It shall be characteristic of the consolidometer used that dehydration should be conducted only through the outside soil.

The U-log t curve is then obtained by making use of this consolidometer, which curve is to be separated into the part of primary consolidation and that of secondary compression according to the values of $\frac{v}{v+r}$ and $\frac{r}{v+r}$ obtained in the paragraph (1).

On the other hand, the U-log t curve in the case where dehydration is conducted only through the side surface of the consolidated soil body ($R\beta=\infty$) is obtained, which is separated into the part of primary consolidation and that of secondary compression in the same manner as described before.

The above two U_v-log t curves are drawn on a sheet of tracing paper accoding to the same scale as in the figure (the β-figure), which sheet is put on the figure. After the U_v-log t curve of the case of $R\beta=\infty$ on the sheet of tracing paper is just fitted to the curve of $R\beta=\infty$ in the figure (the β-figure), with the T_h-axis well fitted to the t-axis, the value of $R\beta$ of the curve in the figure (the β-figure) is obtained, with which the other U_v-log t curve on the sheet of tracing paper coincides well. In the process described above, the value of $R\beta$ is obtained and so β is determined.

XIII. AN APPLICATION EXAMPLE

The author already explained the method of determining the consolidation constants about the consolidation test due to three-dimensional dehydration and showed its example in the last report. So here the author intends to explain the method of the consolidation analysis considered the value of β' which was omitted in that report, using an example.

(1) The synopsis of the testing method

The value of β' is a coefficient which represents a degree of hydraulic gradient to let flow water, wrung from the consolidated soil body, outside the consolidated soil body. We shall name it relative dehydration index. This dimension is (1/cm). This value is not a characteristic of the soil itself of the consolidated soil body but it is determined by both the soil of the consolidated soil body and the soil that touches it. Here the author has measured the value of β' when the soil of the consolidated soil body is the same as the soil that touches it, marking the vertical flow in the case of one-dimensional dehydration. The sample is the weak soil collected from central zone of the reclamation land of the Lake Hachiro H-1 spot 0.86~1.59 m in depth. As is shown in Fig. 12, the sample is set up to the consolidometer, shaped to 6 cm in diameter 2 cm thick and as an external material of the

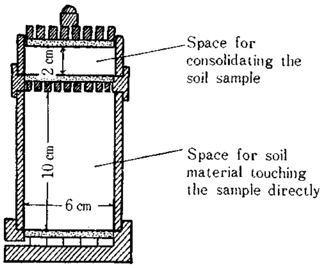

Fig. 12 Apparatus for measuring the value of β'

consolidated soil body shaped to 6 cm in diameter 10 cm thick.

And also as a case corresponding to $\beta'=0$ and $\beta'=\infty$, two kinds of samples 6 cm in diameter 2 cm thick, are set up; one is perfectly closed to dehydration from the bottom surface and the other made possible of a perfect dehydration. On these three kinds of samples the consolidation test has been made on four stages of load strength per unit area 0.1, 0.2, 0.4 and 0.8 kg/cm^2, with drainage condition from the top surface made possible of a perfect dehydration.

(2) The analysis of the test results

Among these samples with three kinds of the drainage conditions, U-$\log t$ curve, which is made possible of a perfect dehydration from both the top and the bottom surface, is shown in Fig. 13. In

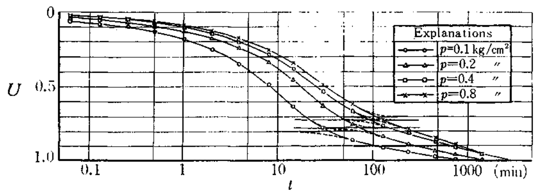

Fig. 13 U-$\log t$ curves

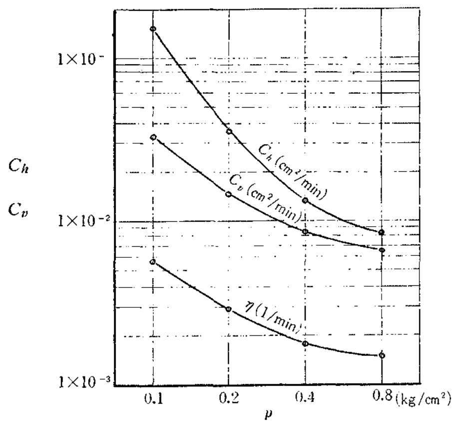

Fig. 14 The variation curves of C_v, C_h and η by load strength per unit area

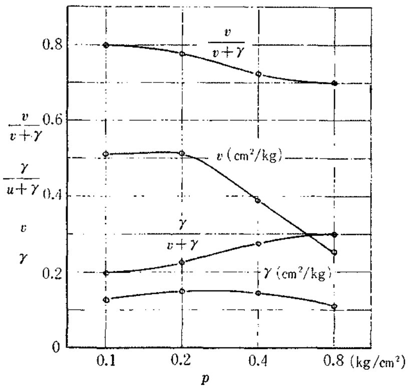

Fig. 15 The variation curves of $\frac{v}{v+\gamma}$, $\frac{\gamma}{v+\gamma}$, v, and γ by load strength per unit area

Fig. 14 and Fig. 15 are shown the consolidation constants by load strength per unit area of the sample which are obtained from the settlement curves above mentioned, the curves of the consolidation tests due to three dimensional dehydration, and the records of the consolidation tests of the same sample.

By the way the analytical method of these consolidation constants is not referred here as was already reported in the last paper.

Next the author explains the analytical method of the value of β'. Now comparing U_v-$\log t$ curves of the consolidation sample possible of a perfect dehydration from the top and the bottom surface, with those from the samples for measuring β', the value of β' can be obtained in β'-figure (Fig. 9) from differences of the time needed for the consolidation of both samples.

At first we devide U-$\log t$ curves of three kinds of samples with different drainage conditions into two parts of the primary consolidation and the secondary compression, by using the value of $\frac{v}{v+\gamma}$ and $\frac{\gamma}{v+\gamma}$ which are already in use.

And we write the primary consolidation part, that is, U-$\log t$ curve on a tracing paper graduated as β'-figure.

From Fig. 16 to Fig. 19 are shown U_v-$\log t$ curves on each stage of loading.

In these figures, No. 1 is a sample number of a

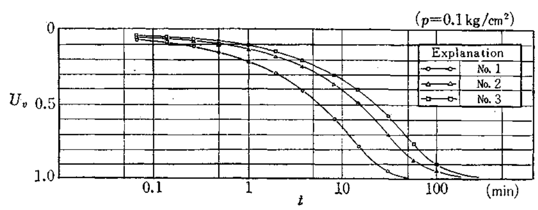

Fig. 16 U_v-$\log t$ curves

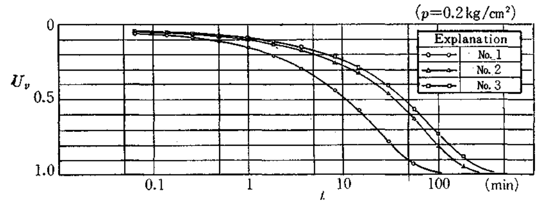

Fig. 17 U_v-log t curves

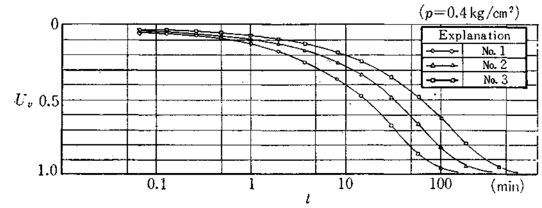

Fig. 18 U_v-log t curves

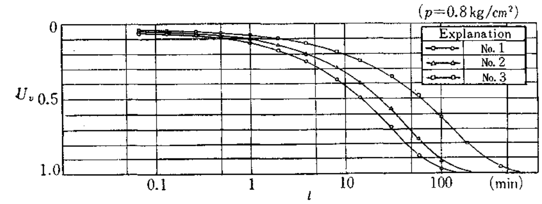

Fig. 19 U_v-log t curves

case of dehydration of both the top and the bottom surface, No. 2 a case of a measurement apparatus for the value of β' (Fig. 12 cf) and No. 3 a case of dehydration of the only the top surface.

Now we put these figures on β'-figure, overlapping the curve No. 1 on the curve $2H\beta'$ in β'-figure, with the result that the curve No. 3 overlaps the curve $2H\beta'=0$. And finding out the value of $2H\beta'$ in corresponding to the position of the curve No. 2, we calculate the value of β'.

Table 1 is the calculation table of the value of β'. The transformation in the value of β' towards load strength per unit area is shown in Fig. 20.

In this report the author has explained the analytical method and its result trying the consolidation test due to one-dimensional dehydration on the weak soil of central zone of the reclamation land of the lake Hachiro, also showing the analytical result of the consolidation constants of the soil in a case considered the secondary compression, and also trying to measure concretely the value of β'.

According to the test result by the measurement apparatus for the value of β' explained here, it is recognized that the value of β', in the case that the consolidated soil body is the same as the soil that touches it, increases as load strength per unit area grows bigger. And also it is pointed out that three-dimensional dehydration can be fully made in the case that the consolidation layer is overweighted with big loading on the part of the thick layer.

Table 1. Calculation table of the value of β'

p (kg/cm²)	$2H\beta'$	$2H$ (cm)	β' (1/cm)
0.1	0.8	1.930	0.42
0.2	0.6	1.796	0.33
0.4	2.0	1.640	1.22
0.8	8.0	1.434	5.58

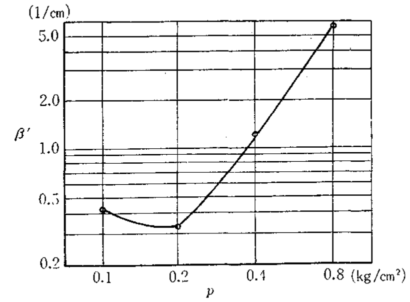

Fig. 20 The value of β' towards load strength per unit area

Furthermore, it is recognized that when we investigate the settlement-estimate by the consolidation analysis due to three-dimensional dehydration, we must use not only C_v, C_h towards load strength per unit area but also the proper value of β'.

XIV. CONCLUSION

The analysis of consolidation by three-dimensional dehydration is made, with the secondary compression taken into consideration. The examination is made with the cases of any intermediate dehydration conditions that lie between the case where the surface of the consolidated soil body is in contact with a perfectly pervious layer and the case where it is in contact with a perfectly impervious layer, and the equation of the surface condition containing the relative outflow index is derived on the assumption that the amount of dehydration through the surface of the consolidated soil body is proportional to the difference of the excess pore water pressure inside and outside the consolidated soil body, which is applied to the analysis.

The author described further the method of determination of consolidation constants according to the present analysis method, and simple solutions by making use of figures.

As described above, introduction of the relative outflow index to the analysis of consolidation by three-dimensional dehydration and the method of determination of consolidation constants thereby are characteristic of the present study. Furthermore, consideration of the secondary compression is thought to make the calculation of the amount of consolidation settlement more accurate than that by conventional methods.

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