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ON THE CRITICAL TRACTIVE VELOCITY OF A RIVER-FLOW

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Synopsis In spite of the fact that the water flowing on a river bed consisted of laden sands or gravels is often nonuniform flow, there are few papers desting with the critical tractive force on that flow. And in arranging the experimental data, it is specially necessary that the critical tractive velocity is not uniform. Under such consideration the author induces an equation for that critical nonuniform velocity, and under some assumptions he arranges his experiments of water flowing on small sands. Such results show that consumptions of inner energy of the flow increase by $(\bar{d})^{-\frac{1}{2}}$ and critical tractive forces are proportional to $\bar{d}$, where $\bar{d}$ is a mean diameter of sands or gravels. But in an actual river there are many cases for nonuniform flow which is assumed as a uniform one. And then from the above experiment the author deems that the Chezy's coefficient (C) is not larger than 35 (by units of m-sec). Then, as an example by using of those results and Kramer's and O'Brien's equations on the critical tractive forces, the author estimates the water discharge under the critical tractive force in a cross-section of Yoshino River (in Shikoku District).

I. Tractive force due to a nonuniform flow.

When a flow of river acts on the river-bed tracting grains of sands or gravels, the critical tractive force T_c is given by an equation of DuBoys (1879) as follows:

$$T_c=\gamma_w R\ J \qquad (1)$$

where, notation of γ_w is a unit weight of river water (about 1 000 kg/m³), R a hydraulic mean depth of the river and J an inclination of surface of water. But eq. (1) is formed by a steady and uniform flow. And if a notation of U_c is given, the critical tractive velocity in such cases, from the Chezy's formula of velocity, U_c is shown by,

$$U_c=C\sqrt{R\ J} \qquad (2)$$

Therefore from eqs. (1) and (2),

$$U_c^2=C^2 T_c/\gamma_w \qquad (3)$$

is obtained. These relations are checked by the author's experiment at the following section.

If a flow has nonuniform velocity, it seems that the eq. (3) cannot realize such case and then a next eq. (4) by the author is rational. That is,

$$\frac{d}{dx}\{y_0+y_1+U_c(1+\alpha_w)/2g+C_w U_c^3/R\}=0 \qquad (4)$$

where a notation x is an ordinate for the direction of flow, d/dx a derivative, y_0 a height of river bed, y_1 an average depth at a cross-section of flow, g gravity accerelation, α_w a coefficient of consumption of inner energy by flowing down and C_w a coefficient of friction on the bed of river, but α_w and C_w are not constant in a steady and nonuniform flow. But from an energy, the author thinks that T_c is very likely,

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$$T_c=\gamma_w U_c^2/C_w^2 \qquad \cdots\cdots(5)$$

Then, using a notation of inclination of surface (J) and eq. (5), and if velocity in a small distance reaches a value of critical tractive one, eq. (4) is reformed as follows:

$$-J+(1/2\,g)\left\{\frac{d}{dx}U_c^2(1+\alpha_w)\right\}+(1/\gamma_w)\frac{d}{dx}\frac{T_cU_c}{R}=0$$

assuming that a value of$(1+\alpha_w)$and T_c in this part are constant with respect to J, or

$$J=\frac{1+\alpha_w}{2g}\frac{dU_c^2}{dx}+\frac{T_c}{\gamma_w}\frac{d}{dx}\frac{U_c}{R} \qquad \cdots\cdots(6)$$

By eq. (6) a problem on a steady and nonuniform flow of critical tractive velocity can be solved rationally as described in the following section.

II. An experiment of critical tractive force on fine sands.

An experiment on the following points was executed by the author in the summer of 1950. That is,

Collection of sands; —

Place: — a part of distance 42 km at the understream of the Yoshino River (Shikoku District)

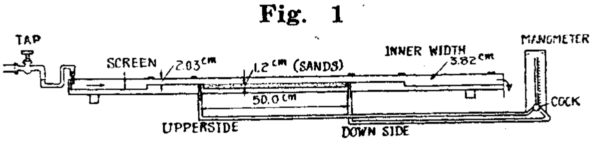
Mean diameter of sand grains; —

Notation of this diameter, $\bar{d}$

Reading of a grain of typical sample by a micrometer.

Water channel, (**Fig. 1**); —

Fig. 1



Horizontal bed.

The manometer of one glass tube having a cock.

By gradually opening of a tap of water supply.

Measuring; —

Three times continuously.

And near values of the two results being averaged.

Results, (**Table. 1**); —

From the experiment the author concluded that, (1) Near values were obtained in such cases that the diameters of grains are large and uniform, (2) Reading of surface slope had more or less errors, and (3) Surface slope of water on sands was larger than that of other parts.

And from the above results of the experiment, by uses of the graphical method, author obtains that,

Table 1

Experimental results of critical tractive force on fine sands

Mean dia d(mm)	Upper: Depth h(cm)	Upper: Velocity u_1(cm/s)	Upper: Wetted peri R_1(cm)	Down: Depth h_2(cm)	Down: Velocity u_2(cm/s)	Down: Wetted peri R_2(cm)	Surface Inclination: Measured J	Surface Inclination: Calculated J_c
0.59	0.42	11.0	0.34	0.32	14.4	0.27	0.0020	0.00106
0.17	0.44	6.4	0.36	0.33	8.5	0.28	22	96
0.28	0.51	9.5	0.40	0.40	12.1	0.33	22	148
0.18	0.44	5.7	0.36	0.34	7.4	0.29	20	77
0.11	0.70	11.3	0.51	0.52	15.2	0.41	36	319
0.15	0.73	12.3	0.52	0.51	17.6	0.40	44	422
0.22	0.72	10.6	0.52	0.49	15.6	0.39	46	321
0.45	0.60	9.1	0.46	0.45	12.9	0.36	30	214
1.96	0.81	12.8	0.56	0.58	18.0	0.44	46	621
0.55	1.09	15.0	0.68	0.74	22.2	0.53	70	370
2.22	1.05	16.4	0.67	0.72	20.6	0.52	66	564
0.73	0.94	12.9	0.62	0.56	21.7	0.43	76	385
1.54	0.67	11.4	0.49	0.51	15.0	0.40	32	472
0.16	0.44	7.2	0.36	0.41	7.8	0.33	6	73
0.22	0.71	11.7	0.51	0.52	16.0	0.41	38	291
0.57	0.90	18.6	0.60	0.63	22.5	0.47	54	320
0.76	0.80	11.3	0.56	0.45	20.1	0.36	70	629
1.05	1.14	15.1	0.70	0.70	24.7	0.51	88	727
1.48	0.95	17.3	0.62	0.81	19.8	0.56	28	213
1.10	1.05	15.6	0.67	0.69	23.2	0.50	72	608
1.00	0.97	14.4	0.63	0.69	20.2	0.50	56	427
0.44	0.81	13.4	0.56	0.51	21.6	0.40	60	586
1.22	0.93	15.0	0.62	0.53	26.3	0.41	80	1063
0.85	0.92	15.8	0.61	0.69	21.1	0.56	46	458
0.75	1.43	19.9	0.82	1.01	28.2	0.65	84	628
0.17	0.95	16.1	0.62	0.61	25.0	0.46	68	910
0.45	0.58	10.6	0.44	0.44	13.9	0.36	28	205
1.70	0.92	14.0	0.61	0.58	22.2	0.44	68	858
1.54	0.89	17.1	0.60	0.68	22.4	0.50	42	496
1.60	1.33	18.3	0.78	0.84	28.9	0.57	98	1030
1.62	0.85	15.2	0.58	0.57	22.6	0.44	56	582
0.94	0.82	15.4	0.57	0.61	20.7	0.46	42	418
0.66	1.07	14.6	0.67	0.55	28.4	0.43	104	1139
1.28	0.93	15.6	0.62	0.65	21.9	0.48	56	553
1.11	0.81	13.1	0.56	0.58	18.2	0.44	46	418
0.33	0.73	12.5	0.52	0.44	20.7	0.36	58	601
0.43	0.79	17.2	0.55	0.46	21.6	0.36	66	412
0.44	0.91	14.7	0.61	0.60	22.3	0.46	62	534
0.76	0.71	13.0	0.51	0.51	18.1	0.40	40	376
0.20	1.10	20.5	0.69	0.90	25.2	0.60	40	481
0.17	0.30	10.1	0.26	0.20	15.2	0.18	20	396
0.45	0.47	12.0	0.38	0.33	17.1	0.28	28	386
0.33	0.62	10.5	0.47	0.38	17.1	0.31	48	442
0.43	0.70	17.7	0.51	0.51	24.3	0.40	38	546
0.44	0.61	12.4	0.46	0.41	18.4	0.33	40	432
0.76	0.53	11.3	0.41	0.39	15.3	0.32	28	326
0.45	0.35	8.4	0.29	0.27	16.2	0.24	16	514
0.53	0.88	13.3	0.59	0.58	20.2	0.44	60	415
0.43	0.84	14.5	0.57	0.52	23.3	0.41	64	651
0.59	0.90	14.5	0.60	0.62	20.0	0.47	56	403
1.17	0.90	15.2	0.60	0.63	27.7	0.47	54	383
0.42	0.82	13.1	0.57	0.62	18.9	0.47	40	404
0.22	0.64	11.9	0.48	0.50	15.3	0.40	28	167
2.80	0.90	11.6	0.60	0.63	16.8	0.47	54	716
1.54	0.92	15.2	0.61	0.74	18.9	0.53	36	320
0.65	1.02	15.3	0.65	0.75	20.8	0.53	54	370
0.30	1.34	18.8	0.78	1.05	24.0	0.67	58	423
2.18	1.14	15.9	0.70	0.93	19.6	0.62	42	330
1.18	1.00	16.4	0.65	0.75	21.9	0.53	50	444
0.11	0.71	12.8	0.51	0.53	17.2	0.41	36	302
0.20	0.72	14.9	0.52	0.52	20.6	0.41	40	471
1.47	0.84	14.3	0.57	0.54	20.9	0.42	60	679
1.29	0.99	14.7	0.64	0.59	24.6	0.45	80	892
1.53	1.00	17.3	0.65	0.70	23.6	0.51	60	605
1.49	1.23	17.8	0.74	0.85	24.2	0.58	76	567
0.72	0.71	11.7	0.51	0.51	16.2	0.40	40	320

$$T_c/\gamma_w = K_1\bar{d} \quad \cdots\cdots (7)$$

$$(1+\alpha_w)/2g = K_2(\bar{d})^{-1/2} \quad \cdots\cdots (8)$$

then, eq. (6) may be shown as follows by these eqs. (7) and (8); that is,

$$J\Delta x = K_1\bar{d}(U_2/R_2 - U_1/R_1) + K_2(\bar{d})^{-1/2}(U_2^2 - U_1^2)$$

And also by its experiment, Δx is 0.5 m, and then,

$$J = 0.000\,069\,\bar{d}(U_2/R_2 - U_1/R_1) + 0.465(\bar{d})^{-1/2}(U_2^2 - U_1^2) \quad \cdots\cdots (9)$$

And unit of $\bar{d}$ is mm and others are m-sec. Then,

$$T_c = K_1\gamma_w\bar{d} = 0.069\,\bar{d}\ (\text{kg/m}^2) = 69\,\bar{d}\ (\text{gr/m}^2) \quad \cdots\cdots (10)$$

Now from many researches on the critical tractive force[2],

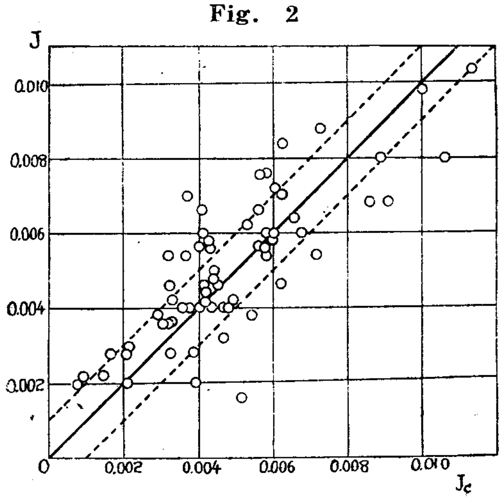
Kramer;	$T_c = 27.5\,\bar{d}/M$	(gr/m)	
Krey;	$T_c = 52.1$ or $68.4\,\bar{d}$	(〃)	……(11)
O'Brien;	$T_c = 107.2\,\bar{d}$	(〃)	

Therefore, eq. (10) is near to the coefficient by Krey and M in the Kramer's equation is 1/2.35. And from

$$(1+\alpha_w)/2g = 0.046\ (d)^{-1/2} \quad \cdots\cdots (12)$$

it is shown that rates of consumption of inner energy of water flow are the smaller on the larger diameter of grains at the critical tractive state. In other words, turbulent effects which existed already in its inner flow, are the smaller for the smaller grains, and therefore the energy of layer of bed in the flow (or shearing energy) are much consumed for the traction of the small sands.

Fig. 2



In the next figure (**Fig. 2**) is shown the actual (J) and calculated (J_c) inclinations of water by above experiments and eq. (7).

If these values are coincident, that has perfect correlation between J and J_c, but errors in reading the average diameters of grains and the heights of water surfaces are most included. If 0.05 cm of reading of water level is an allowable error, more points will be contained in this domain, because the dotted lines shown in the same figure are obtained by calculation. Thus, the experiment by the eq. (6) is carried out and the above mentioned conclusions can be obtained.

III. Critical tractive velocity assuming a uniform flow.

When water uniformly flows on the same size of grains of sands or gravels, it is impossible that the critical tractive velocity reaches the same value in comparatively long distance. This can be proved by the eq. (4), but its method are omitted in this paper. But problems of an actual river are often solved obtaining the critical tractive velocity assuming a uniform flow at the short distance in short time; for an example, a flood flow in unit time at the short distance. Therefore, in the experiment in this paper, if an average value of velocities of flow at upper and lower stream sides is assumed a uniform velocity, the average velocity may be correlated with the diameter of grains of sands or gravels. Then, author shows **Fig. 3**. If the velocity correlated with the diameter of grains, nearly,

$$U_c^2 = 0.033\,05\,\bar{d}^{0.2772} \qquad \cdots\cdots(13)$$

therefore, since it is not uniform flow, a usual expression as $U_c^2 \propto \bar{d}$ does not coincide with eq. (13). But tendency of $U_c^2 \propto \bar{d}$ may be accepted in **Fig. 3**. And if we adopt the Chezy's equation and use the average value of wetted perimeters of upper and lower stream sides, the Chezy's coefficient C is shown in **Fig. 4** from results of the above experiment. From this figure, it is clear that, (1) if d is zero, C is larger than 35, (2) of course, when $\bar{d}$ is the larger, C is the smaller a little 〔$C \fallingdotseq 35 - (1/2.7)\,\bar{d}$〕, but (3) C is nearly constant and independent of the diameter of grains of sands or gravels. If we can assume the Chezy's formula and $C \fallingdotseq 35$ (constant), or,

$$U_c = 35\sqrt{RJ}$$

and if we use eq. (11),

Fig. 3

Correlation between U_c and $\bar{d}$

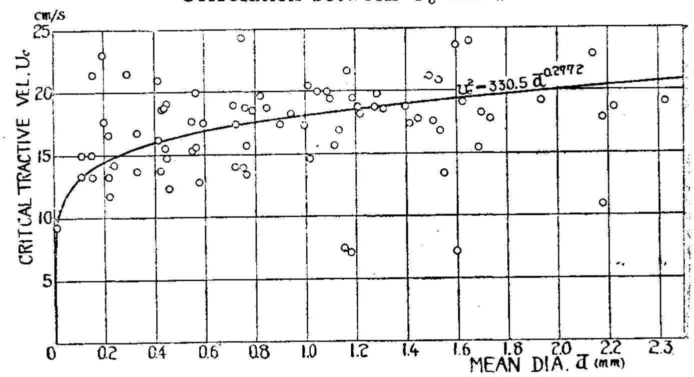
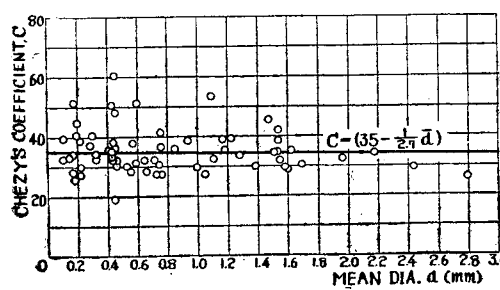


Fig. 4

Chezy's coefficient and mean dia of sands

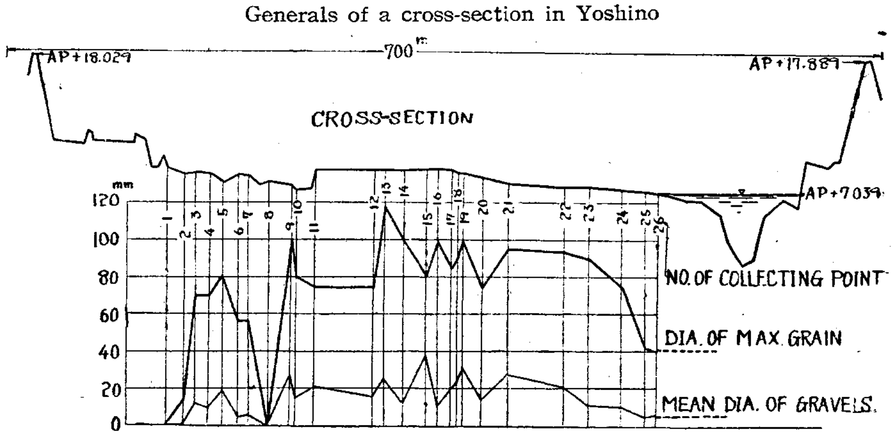


$$\left.\begin{array}{ll} \text{Kramer;} & U_c = 0.318(\bar{d})^{\frac{1}{2}}, (M \fallingdotseq 1/3) \\ \text{Krey;} & U_c = (0.252 \text{ to } 0.290)\,(\bar{d})^{\frac{1}{2}} \\ \text{O'Brien;} & U_c = 0.362(\bar{d})^{\frac{1}{2}} \end{array}\right\} \qquad \cdots\cdots(15)$$

By the method of that eq. (15), we can estimate the discharge of an actual river in the critical tractive force. For an example in a cross section of the Yoshino River

Fig. 5

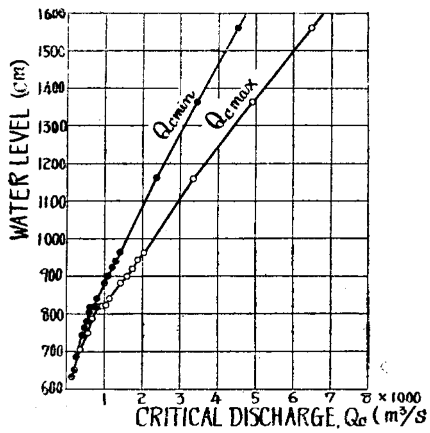
Generals of a cross-section in Yoshino



(in Shikoku) on 20th Oct. 1951 (after the Typhoon "Lise"), author obtained the result of survey of cross section, the variation of water level due to the Typhoon and collection of sands or gravels on the surface of river bed (about 20 or 30 kg for a point, and at 26 points). **Fig.** 5 shows the general picture of its result. From the variation of water level and the cross section, it seems that the increasing water overflowed the high detritus of gravels in the central part and the decreasing water was separated both right and left sides of bank by its detritus. Then, under such consideration, author seeked the mean diameter of grains of deposited sand or gravels (by screening method) and the cross sectional area at the each water level. **Fig. 6** shows that result, in which author uses $U_{min}=0.252(\bar{d})^{\frac{1}{2}}$ (Krey) (for decreasing discharge) and $U_{max}=0.36(\bar{d})^{\frac{1}{2}}$ (O'Brien) (for increasing discharge). Then, if the actual discharge is larger than that of this curve, the bed consisted of sands may move and be scoured.

Fig. 6

Curve of critical discharge and water level



IV. Conclusion

In this paper author studied a critical tractive force of the nonuniform flow, and applied their equations to an experiment. Its result contained more or less errors, but showed a comparatively good correlation. But although its experiment was not related to the uniform flow, of course Chezy's coefficient C was close to 35. Then, by the use of that value, in a section of the Yoshino River, the author estimated the discharge in critical tractive state. If the actual discharge in the Yoshino River is larger than that estimated value, scouring force acts on its bed of the river and the stream transports sands or gravels.

The author wishes to express his gratitue for many conveniences given by Mr. Kichizo Kikkawa (in the Construction Bureau) and also to expess his desire for having more actual data of the Yoshino River given.

Notes

1). T. Kuboo, "On the stream of an alluvial river", Journal of J.S.C.E., Vol. 36, No. 5, 1951.
2). S. Shultis and W. E. Corfitzen, "Bed-load transportation and the stable problem", Trans. Am. Geo. Union, p. 456, 1937.

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