SALT-WATER WEDGE IN UNCONFINED COASTAL AQUIFERS

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SYNOPSIS

Approximate expressions for the equilibrium position of the intruded wedge and the movement of the interface in an unconfined coastal aquifer are derived. The earlier works of R.R. Rumer and D.R.F. Harleman, and J. Bear and G. Dagan are extended giving the attention to the thickness of the fresh-water layer at sea line.

Experiments are performed in the laboratory on a two-dimensional model containing isotropic, homogeneous porous media.

INTRODUCTION

To satisfy the increasing demands of water supply, many schemes for development of water resources are in argument. The main point of these plans rests on not the multi-purpose dams at the upper reaches of rivers but the dams at river mouths. On the exploitation of water resources in the coastal regions we should have the knowledge of the behavior of sea water intruded into porous media. Using this knowledge we can protect the fresh-water resources from contamination by salt-water.

Most of the earlier works deal with the steady-state flow or with idealized boundary conditions.

H.R. Henry gave a theoretical treatment of the steady-state problem of sea-water intrusion in a semi-infinite confined aquifer. R.R. Rumer and D.R.F. Harleman[1] experimentally confirmed that Henry's solution was applicable for a finite confined aquifer and gave an approximate solution in more simple form.

They also dealt with the unsteady-state problem. But they succeeded in solving only the particular case of no fresh-water flow to the sea with the outcrop depth of half a thickness of the aquifer.

P. Kochina[2] investigated on the unsteady-state flow in porous media, but dealt scarcely with two-layered system.

The study by J. Bear and G. Dagan[3] increased the knowledge on the unsteady-state motion of the intruding salt-water wedge. They, however, assumed the depth of fresh-water at sea line was negligible. Dealing with a confined aquifer, they assumed an abrupt change of fresh-water discharge.

On the other hand, giving the change to the water level of reservoir and to the yield of wells, in an unconfined aquifer the free surface is gradually settled towards the new equilibrium level and fresh-water dischage to the sea gradually changes corresponding with the movement of the free surface.

Considering the steady-state and the unsteady-state motion in an unconfined aquifer, we are going to try to relate the flow régime with the head difference between fresh-water and salt-water.

THEORETICAL CONSIDERATIONS

The following assumptions are adopted for the presentation in this paper :

1. The aquifer is unconfined and the flow pattern is two-dimensional.
2. The aquifer is isotropic and homogeneous.
3. No mixing occurs at the interface.
4. The length of the intruded wedge is sufficiently large compared with the thickness of the aquifer; therefore the Dupuit assumption may be applicable.

Equilibrium position of wedge

A schematic view of a coastal aquifer and nomenclatures are shown in Fig. 1. The equilibrium position of the intruded wedge can be determined from Darcy's law using the Dupuit assumption.

The fresh-water discharge per unit width, Q_L, can be written as,

$$Q_L = -\frac{K_1}{2L}\frac{\rho_1+\Delta\rho}{\Delta\rho}\Delta h^2 \quad\cdots\cdots(1)$$

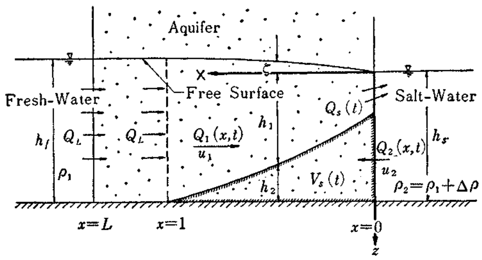

Fig. 1 Schematic view of a coastal aquifer.

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where $K_1=k\rho_1 g/\mu_1$ is the Darcy permeability for the upper laper, L is the length of the porous medium, $\Delta\rho$ is the density difference between salt-water and fresh-water, ρ_1 is the density of fresh-water and Δh is the head difference between fresh-water and salt-water. Note that any positive discharge denotes the flow in the $+x$ direction. Strictly speaking, Δh denotes the head difference between fresh-water at the wedge front and salt-water. But we put approximately $\Delta h=h_f-h_s$, considering the fourth assumption above-mentioned.

At the sea line, however, vertical velocity component prevails, and the Dupuit assumption most differs from real flow régime. Using the solutions obtained under Dupuit assumption as the first approximation, we can write the outcrop opening as,

$$h_{10}=\frac{|Q_L|}{\sqrt{2\dfrac{\Delta\rho}{\rho_1}}K_1}\cdots\cdots(2)$$

Determining the constant of integration by eq. (2), the equilibrium position of the intruded wedge is given by,

$$\frac{\varepsilon K_1}{|Q_L|}h_1=\left(2\frac{\varepsilon K_1}{|Q_L|}x+0.5\right)^{1/2}\cdots\cdots(3)$$

where h_1 is the distance between the salt-water level and the wedge interface and ε is $\Delta\rho/\rho_1$.

Eq. (3) which shows the stationary interface of an unconfined flow pattern is very much similar to that of a confined flow pattern.

Movement of the intruding wedge toe

In the unsteady-motion, Darcy's law is still applied assuming that inertia terms do not play a principal role.

(a) Method of development in series

The equation of motion for the upper and the lower layer can be written as follows respectively :

$$\begin{cases} u_1=-\dfrac{k}{\mu_1}\dfrac{\partial P_1}{\partial x}\cdots\cdots(4)\\ u_2=-\dfrac{k}{\mu_2}\dfrac{\partial P_2}{\partial x}\cdots\cdots(5)\end{cases}$$

where k is the intrinsic permeability of a porous medium and μ_1 and μ_1 are viscosities of fluid of the upper and the lower layer, respectively.

Considering the Dupuit assumption, we can put,

$$\begin{cases} P_1=\rho_1 g(z+\zeta)\cdots\cdots(6)\\ P_2=\rho_1 g(h_1+\zeta)+\rho_2 g(z-h_1)\cdots\cdots(7)\end{cases}$$

Substituting eqs. (6) and (7) to eqs. (4) and (5) respectively, we obtain the following relations :

$$\begin{cases} u_1=-\dfrac{k\rho_1 g}{\mu_1}\dfrac{\partial\zeta}{\partial x}\cdots\cdots(8)\\ u_2=-\dfrac{k\rho_1 g}{\mu_2}\left(\dfrac{\partial\zeta}{\partial x}-\dfrac{\partial h_2}{\partial x}\right)-\dfrac{k\rho_2 g}{\mu_2}\dfrac{\partial h_2}{\partial x}\cdots\cdots(9)\end{cases}$$

Based on eqs. (8) and (9), we can write,

$$\left|\frac{\partial\zeta}{\partial x}\Big/\frac{\partial h_2}{\partial x}\right|<\frac{|u_1|}{|u_1|+u_2}\cdot\frac{\Delta\rho}{\rho_1}\leqq\varepsilon\cdots\cdots(10)$$

with $\mu_1=\mu_2$, $u_1<0$, $u_2\geqq 0$ and $\varepsilon=\Delta\rho/\rho_1$. Because of the small variation of the free surface compared with that of the wedge interface, we neglect the contribution of the thickness of the fresh-water layer above the sea level, ζ, to the unit discharge, Q_L. Thus, we approximately consider the aquifer as confined one with the thickness of h_s.

Hence, the equations of continuity for the upper and the lower layer may be reduced to the following forms respectively :

$$\begin{cases} \theta\dfrac{\partial(h_s-h_2)}{\partial t}+\dfrac{\partial}{\partial x}\left\{u_1(h_s-h_2)\right\}=0\cdots\cdots(11)\\ \theta\dfrac{\partial h_2}{\partial t}+\dfrac{\partial}{\partial x}(u_2 h_2)=0\cdots\cdots(12)\end{cases}$$

where θ is porosity of a given porous medium.

Introducing eqs. (8) and (9) to eqs. (11) and (12) respectively, summing the resulting two equations and eliminating terms relating to ζ, we obtain,

$$\frac{\theta}{K}\frac{\partial h_2}{\partial t}-\frac{\varepsilon}{2}\frac{\partial^2 h_2^2}{\partial x^2}+\frac{\varepsilon}{2h_s}\frac{\partial}{\partial x}\left(h_2\frac{\partial h_2^2}{\partial x}\right)=0\cdots\cdots(13)$$

as the basic equation of the lower layer, where K is $k\rho_1 g/\mu_1=k\rho_1 g/\mu_2=K_1$.

Considering that h_2 depends only upon $\xi=x/(m\sqrt{t})$, where m is a constant, and introducing the following substitutions such as,

$h_{20}=h_2(0,t)$, $r=h_2/h_{20}$, $\beta=h_{20}/h_s$ and

$\xi=x\sqrt{\theta}/\sqrt{2\beta\varepsilon Kh_s t}$

eq. (13) is transformed into the non-linear ordinary differential equation as follows :

$$\frac{d^2r^2}{d\xi^2}+2\xi\frac{dr}{d\xi}-\beta\frac{d}{d\xi}\left(r\frac{dr^2}{d\xi}\right)=0\cdots\cdots(14)$$

Assuming that in the case under consideration the point of intersection of the integral curve with the abscissa axis exists, we denote its abscissa by the letter e. From this stand-point, we find the series development in powers of the difference $(\xi-e)$:

$$\begin{aligned} r=&-e(\xi-e)+\left(-\frac{1}{4}+\frac{1}{2}\beta e^2\right)(\xi-e)^2\\ &+\left(-\frac{1}{72}+\frac{5}{18}\beta e-\frac{1}{2}\beta^2e^3\right)(\xi-e)^3\\ &+\left(\frac{1}{576e^2}+\frac{5}{96}\beta+\frac{29}{144}\beta^2e^2+\frac{5}{8}\beta^3e^4\right)(\xi-e)^4\\ &+\left(-\frac{11}{86\,400e^3}+\frac{7\beta}{4\,320e}-\frac{2\,437}{10\,800}\beta^2e\right.\\ &\left.+\frac{1}{3}\beta^3e^3-\frac{7}{8}\beta^4e^5\right)(\xi-e)^5+\cdots\cdots(15)\end{aligned}$$

In the course of this work no strict discussion of convergence of the series used is attempted. In practice, however, we suppose that the condition that $|\beta e|$ is less than unity is sufficient for convergence. And the evaluation of the value of e hereafter shows that this condition may be possible.

Putting $r=1$ at $\xi=0$ and neglecting the rapidly vanishing terms in brackets, we can find the values of e corresponding with the parameter β :

Table. 1

β	0.500	0.600	0.700	0.800	0.900	1.000
e	1.048	1.033	1.014	1.009	0.996	0.986

Using the values in Table 1 and eq. (15), the interfacial forms of the intruding wedge can be calculated as shown in Fig. 2.

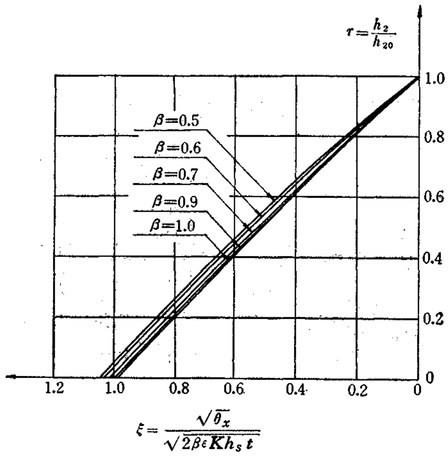

Fig. 2 Interfacial forms of the intruding wedge.

Consider the equation $\xi=e$ and we can obtain the abscissa of the wedge toe :

$$x=e\sqrt{\frac{2\beta\varepsilon Kh_s}{\theta}}\cdot\sqrt{t} \quad \cdots\cdots(16)$$

In this method, however, by the additional restriction with the transformation, the solutions are applicable only to the case of no fresh-water flow into the sea. In such cases eq. (16) shows good agreement containing the parameter β with the approximate solutions which were offerd respectively for various values of β by some authors as follows :

Table 2

β	c	Authors
0.500	1.00	Rumer and Harleman
0.500	1.05	Eq. (16)
1.000	1.32	Bear and Dagan
1.000	1.39	Eq. (16)

Where C is a constant, $x=C\sqrt{\frac{K\varepsilon h_s}{\theta}}\cdot\sqrt{t}$

(b) Method of assuming the distribution of fresh water discharge

By the Dupuit assumption and the result of eq. (10), we can write horizontal velocity components as follows :

$$u_1=\frac{Q_1(x,t)}{h_1},\quad u_2=\frac{Q_2(x,t)}{h_2} \quad \cdots\cdots(17)$$

where Q_1 and Q_2 are the discharge of the upper and the lower layer respectively

Substituting eq. (17) to eqs. (8) and (9), and considering the continuity, the foundamental equation of the upper layer is oftained :

$$\frac{\partial h_1}{\partial x}=\frac{Q_L}{K\varepsilon(h_S-h_1)}-\frac{Q_1 h_S}{K\varepsilon h_1(h_S-h_1)} \quad \cdots(18)$$

Let $V_S(t)$ denote the volume of the region occupied by salt-water.

By continuity,

$$\begin{cases} \theta\dfrac{dV_S(t)}{dt}=Q_L-Q_S & \cdots\cdots(19) \\ V_S(t)=\displaystyle\int_{h_{10}}^{h_S} x(h_1,t)dh_1 & \cdots\cdots(20) \end{cases}$$

where Q_S is Q_1 at the sea line, or $Q_S\equiv Q_1(0,t)$.

§1. Two layers initially separated

In this case the barrier is removed at the initial time and salt-water begins to intrude into the aquifer. We assume the distribution of the fresh-water discharge to the sea has the linear relationship with h_1 shown as the straight line in Fig. 3.

As for the absolute value of Q_L, it gradually increases and reaches to the equilibrium value. However, the rate of change is large at initial period. At the same time, $|Q_S|$ is infinite initially and its derivative with respect to time is very large. Hence, neglecting the alteration of $|Q_L|$, we assume abrupt change for the fresh-water discharge at the wedge toe.

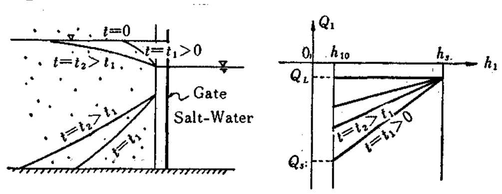

Fig. 3 Approximate distribution of fresh-water discharge.

The expression of these straight lines is,

$$Q_1=Q_L+\frac{Q_L-Q_S}{h_S-h_{10}}(h_1-h_S) \quad \cdots\cdots(21)$$

Substituting eq. (21) to eq. (18), and integrating with $h_1=h_{10}$ at $x=0$ we obtain,

$$h_1^2-h_{10}^2=-2\frac{h_S Q_S-h_{10}Q_L}{K\varepsilon(h_S-h_{10})}x \quad \cdots\cdots(22)$$

Substitution of eq. (22) into eq. (20) gives,

$$\bar{V}_S=-\frac{(1-\alpha)^3(1+2\alpha)}{6(\bar{Q}_S-\alpha)} \quad \cdots\cdots(23)$$

where $\bar{Q}_S=Q_S/Q_L$ $(Q_L\neq 0)$, $\alpha=h_{10}/h_S$ and $\bar{V}_S=Q_L V_S/(K\varepsilon h_S^3)$.

From eq. (19) we obtain,

$$\frac{d\bar{V}_S}{d\bar{Q}_S}\frac{d\bar{Q}_S}{d\bar{t}}=1-\bar{Q}_S \quad \cdots\cdots(24)$$

where $\bar{t}=Q_L^2 t/(\theta K\varepsilon h_S^3)$.

For simplicity we consider α does not rely upon time. Substituting eq. (23) to eq. (24) and integrating under the initial condition that $\bar{Q}_S=\infty$ at $\bar{t}=0$

we obtain,

$$\bar{t}=\frac{(1-\alpha)(1+2\alpha)}{6}\times\left[\ln\left|\frac{\bar{Q}_S-\alpha}{\bar{Q}_S-1}\right|-(1-\alpha)\frac{1}{\bar{Q}_S-\alpha}\right] \quad\cdots\cdots(25)$$

Putting $h_1=h_S$ in eq. (22) to evaluate the position of the wedge toe, we obtain,

$$\bar{x}=-\frac{(1-\alpha)^2(1+\alpha)}{2}\frac{1}{\bar{Q}_S-\alpha} \quad\cdots\cdots\cdots\cdots\cdots(26)$$

where $\bar{x}=Q_L x/(K\varepsilon h_S^2)$.

Connecting eq. (25) with eq. (26) by parameter $\bar{Q}_S$, we may obtain the information on the movement of the intruding wedge toe.

Calculations of eqs. (25) and (26) for various values of α are shown in Fig. 4.

§2. Further movement towards landward from the equilibrium position

The change of yield of wells and that of the water level in the fresh-water region induce a new movement of the saline wedge. For the flow system with free surface, it is convenient if the movement is described in relation to the head difference between fresh-water and salt-water. After steady-state conditions were established, the fresh-water level is suddenly dropped.

The wedge front moves towards landward searching for the new equilibrium position. In this case, a fresh-water discharge Q_{L0} exists initially. In other words abrupt changes do not occur in reality and a transition period must exist when the relationship takes a form similar to the dashed lines in Fig. 5.

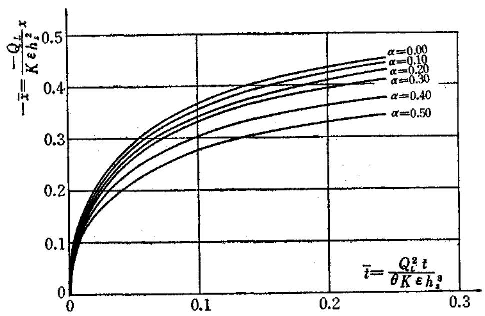

Fig. 4 Variation of the movement of the intruding wedge toe.

These curves are approximated by the broken lines as shown in Fig. 5.

The equation which describes these lines is,

$$Q_1=(1-a)(Q_L-Q_S)\frac{h_1-h_{10}}{h_S-h_{10}}+Q_S \quad\cdots\cdots(27)$$

where "a" is a dimensionless constant to be determined later.

From eqs. (18) and (27), we obtain,

$$h_S^2\frac{\partial\lambda}{\partial x}=\frac{(a-\alpha)(Q_L-Q_S)}{K\varepsilon(1-\alpha)(1-\lambda)}-\frac{Q_S}{K\varepsilon\lambda}+\frac{\alpha(1-a)(Q_L-Q_S)}{K\varepsilon(1-\alpha)(1-\lambda)\lambda} \quad\cdots\cdots\cdots\cdots(28)$$

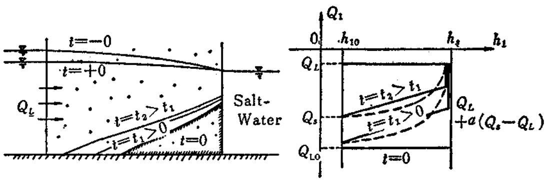

Fig. 5 Approximate distribution of fresh-water dischage.

where $\lambda=h_1/h_S$.

From eq. (28) we may describe several characteristics.

1. In the vicinity of the coast line, λ tends to α and the second term on the right side of eq. (28) is the dominant one. The corresponding shape of the interface is close to a parabolic one. Similarly for $\lambda\to 1$, the salt-water flow forms a parabolic shape of the interface near its toe.

2. At $\lambda=1$, $\partial\lambda/\partial x\to\infty$, i.e., the interface is perpendicular to the impervious bottom.

3. An inflection point exists along the interface. There $\partial^2\lambda/\partial x^2=0$, and,

$$\lambda_B=\frac{\{(1-\alpha)Q_S-\alpha(1-a)(Q_L-Q_S)\}-\sqrt{a(1-2\alpha)(Q_S-Q_L)Q_S+\alpha a(Q_L-Q_S)^2}}{(a-\alpha)(Q_L-Q_S)+(1-\alpha)Q_S} \quad\cdots\cdots(29)$$

where α is small compared with unity, so higher order terms of α are neglected.

According to J. Bear and G. Dagan, experiments show that $\lambda_B>0.75$ where $Q_L=0$. Assuming $\alpha=0.15$, from eq. (29) we obtain $a<0.13$ for the corresponding value of a. Since small variations in "a" have relatively small effects on the value of λ_B, we use the value of $a=0.1$ herein.

Considering both a and α are small, we may neglect higher order terms of these values and their products compared with unity.

Integration of eq. (28) with $\lambda=\alpha$ at $x=0$ gives,

$$x=\frac{K\varepsilon(1-\alpha)h_s^2}{(a-\alpha)Q_L+(1-a)Q_S}\left\{\frac{1}{2}(\alpha^2-\lambda^2)-(1+\gamma)(\alpha-\lambda)-\gamma(1+\gamma)\ln\left|\frac{\lambda+\gamma}{\alpha+\gamma}\right|\right\} \quad\cdots\cdots(30)$$

where $\gamma=(\alpha Q_L-Q_S)/\{(a-\alpha)Q_L+(1-a)Q_S\}$.

Substituting eq. (30) to eq. (20) we obtain the expression for $\bar{V}_S$ as follows like as in section 1,

$$\bar{V}_S=\frac{1-\alpha}{a-\alpha+(1-a)\bar{Q}_S}\left[-\frac{1}{6}+\frac{a(1-\bar{Q}_S)\{a(1-\bar{Q}_S)+\alpha-\bar{Q}_S\}}{2\{a-\alpha+(1-a)\bar{Q}_S\}^2}+\frac{a^2(\alpha-\bar{Q}_S)(1-\bar{Q}_S)^2}{\{a-\alpha+(1-a)\bar{Q}_S\}^3}\ln\left|\frac{\alpha-(1-\alpha)\bar{Q}_S}{a(1-\bar{Q}_S)}\right|\right] \quad\cdots\cdots(31)$$

Considering the relation explained in eq. (24) and obtaining $d\bar{V}_S/d\bar{Q}_S$ from eq. (31), we can get a differntial equation to determine $\bar{Q}_S$.

$$\frac{1-\alpha}{6}\frac{(1-a)d\bar{Q}_S}{(1-\bar{Q}_S)\{a-\alpha+(1-a\bar{Q}_S)\}^2}-\frac{a}{2}\frac{1-\alpha}{1-\bar{Q}_S}\frac{\bar{Q}_S^2-2(1+3a)\bar{Q}_S+2(\alpha+3a)}{\{a-\alpha+(1-a)\bar{Q}_S\}^4}d\bar{Q}_S$$
$$+a^2(1-\alpha)\frac{(1-a)\bar{Q}_S^2+(\alpha-3)\bar{Q}_S+(a+3\alpha)}{\{\alpha-\alpha+(1-a)\bar{Q}_S\}^5}\ln\left|\frac{\alpha-(1-\alpha)\bar{Q}_S}{a(1-\bar{Q}_S)}\right|d\bar{Q}_S=d\bar{t} \quad \cdots\cdots(32)$$

Putting $\delta=(a-\alpha)/(1-a)$ and integrating eq. (32) with the initial condition $\bar{Q}_S\equiv Q_{L0}/Q_L=Q_f$ at $\bar{t}=0$, we obtain the following relation. As above-mentioned α is treated as an invariable with time.

$$\bar{t}=\frac{1}{6(1-a)^2(1-\alpha)}\ln\left|\frac{Q_f-1}{Q_f+\delta}\frac{\bar{Q}_S-\delta}{\bar{Q}_S-1}\right|+\frac{(1-\alpha)a^2}{(1-a)^4}\left\{\frac{1}{2}+\frac{\alpha-3-2\delta}{3(1-a)(\bar{Q}_S+\delta)}+\frac{a+3\alpha+3\delta}{4(1-a)(\bar{Q}_S+\delta)^2}\right\}$$
$$\times\frac{1}{(\bar{Q}_S+\delta)^2}\ln\left|\frac{(1-\alpha)(\bar{Q}_S+\delta)}{a(\bar{Q}_S+\delta-1)}\right|-\frac{(1-\alpha)a^2}{(1-a)^4(Q_f+\delta)^2}\left\{\frac{1}{2}+\frac{\alpha-3-2\delta}{3(1-a)(Q_f+\delta)}+\frac{a+3\alpha+3\delta}{4(1-a)(Q_f+\delta)^2}\right\}$$
$$\times\ln\left|\frac{(1-\alpha)(Q_f+\delta)}{a(Q_f+\delta-1)}\right|-\frac{(1-2\alpha)a^2}{(1-a)^4}\left\{\frac{1}{2}+\frac{3a+13\alpha+\delta-12}{12(1-a)}\right\}\ln\left|\frac{Q_f+\delta-1}{Q_f+\delta}\frac{\bar{Q}_S+\delta}{\bar{Q}_S+\delta-1}\right|$$
$$-\frac{1}{(1-a)^3}\left[\frac{1}{6}-\frac{(1-2\alpha)a^2}{(1-a)}\left\{\frac{1}{2}+\frac{3a+13\alpha+\delta-12}{12(1-a)}\right\}\right]\left(\frac{1}{\bar{Q}_S+\delta}-\frac{1}{Q_f+\delta}\right)$$
$$-\frac{a}{2(1-a)^2}\left[\frac{1+\delta-\alpha}{1-\alpha}-\frac{(1-2\alpha)a^2}{(1-a)^2}\left\{\frac{1}{2}+\frac{3a+13\alpha+\delta-12}{12(1-a)}\right\}\right]\left\{\frac{1}{(\bar{Q}_S+\delta)^2}-\frac{1}{(Q_f+\delta)^2}\right\}$$
$$+\frac{a}{3(1-a)^3}\left\{3a+\alpha+\delta+\frac{(1-2\alpha)(\alpha-3-2\delta)a}{3(1-a)^2}+\frac{(1-2\alpha)(a+3\alpha+3\delta)a}{4(1-a)^2}\right\}\left\{\frac{1}{(\bar{Q}_S+\delta)^3}-\frac{1}{(Q_f+\delta)^3}\right\}$$
$$+\frac{(1-2\alpha)(a+3\alpha+3\delta)}{4(1-a)^5}a^2\left\{\frac{1}{(\bar{Q}_S+\delta)^4}-\frac{1}{(Q_f+\delta)^4}\right\} \quad \cdots\cdots(33)$$

At the salt-water wedge toe, h_1 is h_s. Hence, substitution of $\lambda=1$ to eq. (30) gives the relation explaining the position of the wedge toe :

$$\bar{x}=-\frac{(1+\alpha)(1-\alpha)^2}{2\{a-\alpha+(1-a)\bar{Q}_S\}}+\frac{a(1-\alpha)^2(1-\bar{Q}_S)}{\{a-\alpha+(1-a)\bar{Q}_S\}^2}$$
$$-\frac{a(1-\alpha)(\bar{Q}_S-\alpha)(\bar{Q}_S-1)}{\{a-\alpha+(1-a)\bar{Q}_S\}^3}\ln\left|\frac{a(\bar{Q}_S-1)}{(1-\alpha)\bar{Q}_S-\alpha}\right| \quad \cdots\cdots(34)$$

where $\bar{x}=Q_Lx/(K\varepsilon h_s^2)$, $a=0.1$ and $\alpha=h_{10}/h_s$.

In the case of § 2 assuming that the distribution of the fresh-water discharge to the sea has the linear relatinonship with h_1 as in § 1, instead of eq. (33) we obtain corresponding equation as follows :

$$\bar{t}=\frac{(1-\alpha)(1+2\alpha)}{6}\ln\left|\frac{\bar{Q}_S-\alpha}{\bar{Q}_S-1}\frac{Q_f-1}{Q_f-\alpha}\right|$$
$$-\frac{(1-\alpha)^2(1+2\alpha)}{6}\left(\frac{1}{\bar{Q}_S-\alpha}-\frac{1}{Q_f-\alpha}\right) \quad \cdots\cdots(35)$$

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus is shown in Fig. 6. The unconfined aquifer was constructed of lucite. The aquifer dimensions were 200 cm in length, 50 cm in depth and 20 cm in width.

Fresh-water was injected through a venturi-meter into the fresh-water reservoir. Part of fresh-water passed through the aquifer. The rest spilled over the weir in the head control weir and was measured.

The sea water density was kept within a determined range by recirculation. This was accomplished by pumping salt-water into the model ocean from a 0.5 m³ reservoir and then allowing the excessed water to flow back to the reservoir by gravity. Slow dilution of the ocean by the fresh-water flow required the periodic addition of salt to maintain a constant density in the ocean and the reservoir. For all runs, salt-water was marked with potassium permanganate. The ocean level was controlled by a weir.

The aquifer was packed with a dry medium. The porosity of each packing was determined from the weight of the medium added, its specific gravity and the volume of the aquifer in sample rods. When the medium was in place, fresh-water was supplied into the fresh-water reservoir in order to saturate the aquifer gradually.

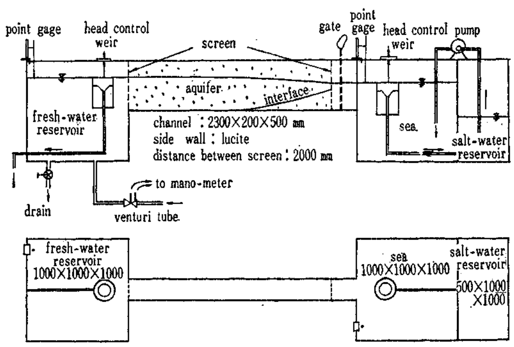

Fig. 6 Schematic view of experimental apparatus.

The media used in the aquifer are plastic spheres (d_m=0.92 mm, K=0.50 cm/sec, θ=0.375) and a coarse sand (d_m=2.40 mm, K=1.45 cm/sec, θ= 0.397).

The condition of sea is established in the following manner. A gate was settled against the sea at the end of the aquifer. This provided a barrier that prevented salt-water from invading the aquifer before experiments.

With the barier in place, the density of the ocean was controlled into a predetermined amount. The movement of the wedge toe was recorded for comparison with the movement predicted by eqs. (25) and (26). From preliminary calculations using eq. (3) a length of intruded wedge was determined. A long period of adjustment was necessary for the fresh-water flow and saline wedge to reach equilibrium. When no-further changes in the position of the interface took place, steady-state conditions were assumed to exist, and the position of the interface was recorded.

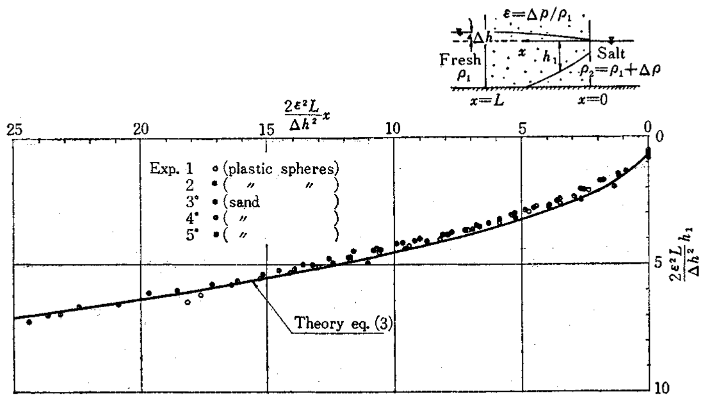

Fig. 7 Equilibrium position of the intruded wedge.

Then giving the sudden drop to the head by operating the control weir of the fresh-water reservoir, further movement of the salt-water wedge was induced.

EXPERIMENTAL RESULTS AND CONSIDERATIONS

Equilibrium position of wedge

The experimental results for the interface position in an un confined aquifer with a vertical outcrop are shown in Fig. 7. The results show good agreement with eq. (3).

The experimental verification of eq. (3) indicates that we can approximately take an unconfined aquifer for a confined aquifer with a thickness of h_s which is the depth of the sea.

Movement of the intruding wedge toe

We did not perform the experiment with no fresh-water flow to the sea. Concerning this cose, we showed the comparison in Table 2. Therefore, the following results are all related to the case where the fresh-water flow exists, or $Q_L \neq 0$.

§1. Two layers initially separated

The typical ones of the experimental results are shown in Fig. 8. with the medium of plastic spheres and in Figs. 9. and 10. with that of a coarce sand.

Experimental resultsshow good agreement with eqs. (25) and (26) with the medium of a coarse sand. With regarding to the medium of plastic spheres, however, the experiment shows the slower movement than the theoretical prediction.

One of the causes is the assumption that α does not rely upon time. In experiments with plastic spheres the thickness of the salt-water layer at the sea line gradually increases to the final value and takes about 2,000 seconds to reach 90% of the final value. On the contrary, in experimets with sand it takes about 300 seconds. It is supposed that this difference is caused by the difference of the permeability of media.

The other cause is the fact that both the Dupuit

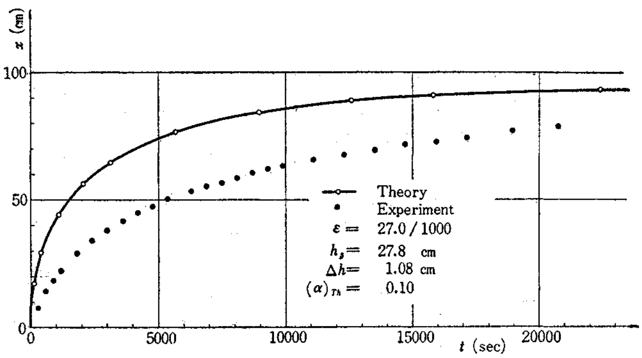

Fig. 8 Movement of the wedge toe-experimental result with plastic spheres.

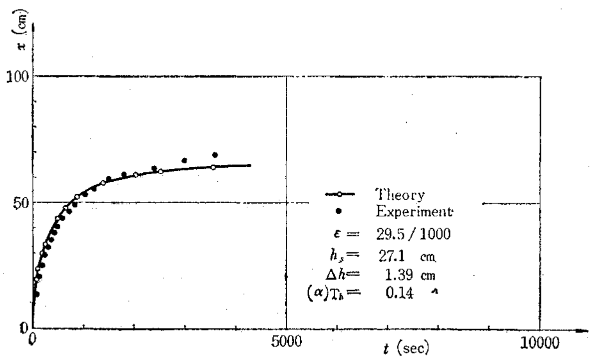

Fig. 9 Movement of the wedge toe-experimental result with sand.

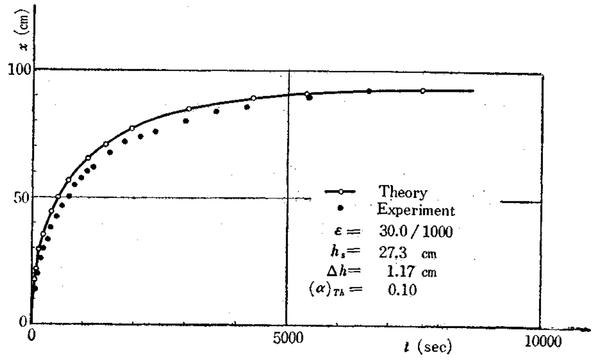

Fig. 10 Movement of the wedge toe-experimental result with sand.

assumption and the equation of continuity may coexist only for the sufficiently large values of permeability.

§ 2. Further movement towards the landward from the equilibrium position

In this case experiments were performed only with sand media. The experimental results are exhibited in Figs. 11. and 12. From these results, we understand that neither eq. (33) nor eq. (35) describes the phenomenon accurately.

The calculation of the initial position of the further intrusion differs from the experimental result.

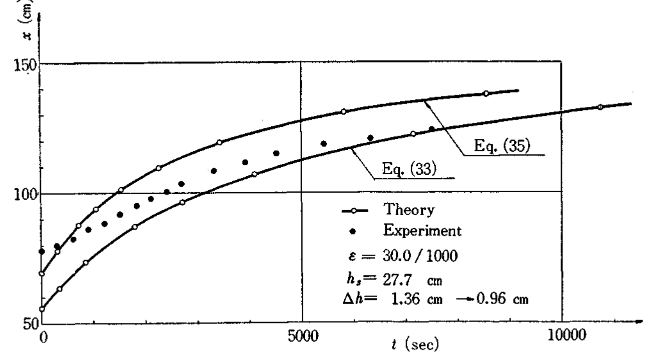

Fig. 11 Further movement of the wedge toe-experimental result with sand.

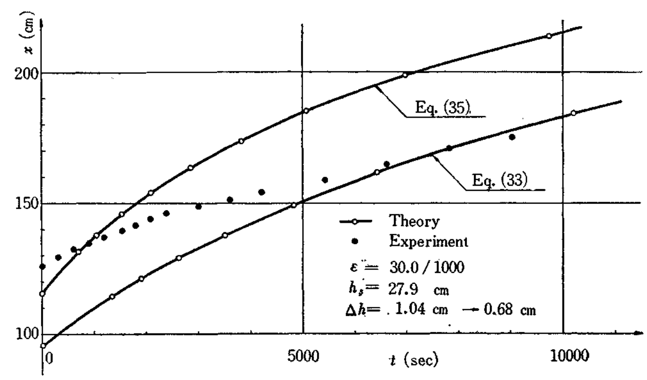

Fig. 12 Further movement of the wedge toe-experimental result with sand.

It is derived from the assumption that the steady-state discharge of fresh-water Q_{L0} changes to Q_L abruptly at the wedge toe. In theoretical prediction the process that the fresh-water discharge changes from Q_{L0} to Q_L consists of only a part of the process that it changes from infinity to Q_L. Hence the real process that the fresh-water discharge reaches to its equilibrium condition with Q_{L0} and then reduces to Q_L differs from the simplified assumption. But it is supposed that eqs. (33) and (35) are useful as a guide.

In the problem of sudden change of the fresh-water level, the behavior of the free surface itself will be important. For example, when the fresh-water level is dropped at upstream reservoir a reversal gradient of the free surface seems to appear. We hope that we are able to give the clear insight of this phenomenon through the further investigation.

CONCLUSIONS

1) Flow in an unconfined aquifer may be approximately considered as that in a confined aquifer if the aquifer is sufficiently thin.

2) The difference for the wedge toe velocity in the earlier approximate solutions can be explained by the consideration of the thickness of the outcrop opening.

3) In unsteady motion problems, the Dupuit assumption may be applicable only for the media with the large Darcy permeability.

ACKNOWLEDGEMENTS

This study represents a portion of the research program on the salt-water intrusion into an estuary reservoir, sponsored by the Ministry of Education. The research was also sponsored by the Committee of Estuary Reservoir Development.

The study presented here is a part of the thesis to be submitted by N. Tamai to the Graduate School, University of Tokyo in partial fulfilment of the requirements for the degree of M. Eng..

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NOMENCLATURES

The following letter symbols have been adopted in this paper, the values with suffix 1 and 2 refering to the upper and the lower layer, respectively :

h : thickness of the layer,

ζ : thickness of the fresh-water layer above the sea level,

h_f : depth of the fresh-water reservoir,

h_s : depth of the sea,

h_{10} : depth of the outcrop opening,

h_{20} : thickness of the salt-water layer at the sea line,

Δh : head difference,

Q : discharge per unit width,

Q_L : discharge of fresh-water at the wedge toe,

Q_S : discharge of fresh-water at the sea line,

Q_{L0} : initial value of Q_L,

V_S : volume of the region occupied by salt-water wedge,

L : length of the porous medium,

K : Darcy permeability,

u : seepage velocity given by Darcy's law,

ρ : density,

$\Delta\rho$: density difference,

ε : $\Delta\rho/\rho_1$,

μ : viscosity,

θ : porosity,

α : h_{10}/h_s $(=1-\beta)$ and

β : h_{20}/h_s.

(Received June 30, 1966)

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昭和42年3月15日印刷
昭和42年3月20日発行
土木学会論文集 第139号 定価200円（〒20円）

編集兼発行者 東京都新宿区四谷一丁目 社団法人 土木学会 羽田巌
印刷者 東京都港区赤坂1-3-6 株式会社 技報堂 大沼正吉

発行所 社団法人 土木学会 振替東京 16828番
東京都新宿郵便局区内 新宿区四谷一丁目 電話（351）代表 5138番