A THEORETICAL INVESTIGATION ON AERODYNAMIC STABILITY OF LONG-SPANNED SUSPENSION BRIDGE

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Abstract

This paper presents a theoretical consideration on the flutter wind velocity taking into an account the various structural parameters on the base of the theory developed by F. Bleich, which is originally concerned with the two-dimensional air stream. We investigate here, to some extent, a few fundamental cases of the three dimensional wind velocity variations in the horizontal direction and its influence on the critical wind speed. Numerical illustrations are proceeded for the 1st plan of the proposed Akashi Straits Bridge and compared with the results by A. Selberg's empirical formula for corresponding torsional and deflectional characteristics of the proposed suspension bridge.

Preface

In recent years, the construction of long-spanned suspension bridges is considered to connect the Honshu Island with the Shikoku Island from the national demand for industrial and regional developments in Japan. In connection with this plan, it is easily understood that the structural investigation on possibility of so large scale bridges is closely related with the problems of the aerodynamic instability and wind resistant behavior of a suspension bridge.

In spite of numbers of experiments and theoretical investigations about these themes, it remains still. uncertain to be solved with satisfactory accuracy because of mathematical complexity in the fundamental equations and of mechanical difficulty in determination of aerodynamic forces acting on the structural members of suspension bridge.

In this paper the behavior and the instability of a suspension bridge are considered three-dimensionally by taking an account the steady and partially distributed wind in spanwise direction. The theoretical investigations thus follows that the aerodynamic characteristics of such flexible structures as suspension bridge are characterized not only by mechanical factors as the circular frequencies of deflectional or torsional modes of vibrations but by the geometrical factors of the cross sectional shape of stiffening floor.

The numerical method developed here results in fairly good agreement with the empirical formula by Selberg for the critical velocity of a suspension bridge.

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1. Fundamental Equation for Flutter Speed.

The unsteady aerodynamic force acting on a thin airfoil in unsteady motion in a two-dimensional incompressible fluid was obtained by Wagner, Küsser, von Kármán, Shears, etc[1]. For the structural point of suspension bridge, F. Bleich[2] applied for the first time the thin airfoil theory for determination of the critical wind velocity for flutter phenomenon where the lift and the torque moment are written as

$$\left.\begin{aligned} L &= -sV^2\left[f_1\left(\phi+\frac{b}{V}\right)\dot{\eta}+f_2\frac{b}{V}\dot{\phi}\right] \\ M &= \frac{sbV^2}{2}\left[f_1\left(\phi+\frac{b}{V}\right)\dot{\eta}-f_3\frac{b}{2V}\dot{\phi}\right] \end{aligned}\right\} \quad \cdots\cdots(1)$$

where $s=2\pi\rho b,\quad f_1=C(k),\quad f_2=1+C(k),$
$f_3=1-C(k)$

and

$$C(k)=\frac{H_1^{(2)}(k)}{H_1^{(2)}(k)+iH_0^{(2)}(k)} \quad \cdots\cdots\cdots\cdots\cdots(2)$$

In eq's (2), k is termed as the reduced velocity to be equal to $\omega b/V$ and $C(k)$ is called the Theodorsen's function in which $H(k)$ are the Hankel functions.

In eq. (1) the center of gyration is agreed with the mid-chord. The aerodynamic response of a suspension bridge is given as a coupled vibration because of inclusion of both vertical and torsional modes in lift and torque moment.

The virtual work done by virtual displacements is

$$\delta W=-sb\int_L V^2\Big[f_1\Big(q_2\Phi_2\Phi_1+\frac{b}{V}\dot{q}_1\Phi_1^2\Big)+f_2\frac{b}{2V}\dot{q}_2\Phi_1\Phi_2\Big]dx\,\delta q_1+\frac{sb}{2}\int_L V^2\Big[f_1\Big(q_2\Phi_2^2+\frac{b}{V}\dot{q}_1\Phi_1\Phi_2\Big)-f_3\frac{b}{2V}\dot{q}_2\Phi_2^2\Big]dx\,\delta q_2 \quad\cdots\cdots(3)$$

where $q_1=e^{i\omega_1 t}$, $q_2=e^{i\omega_2 t}$ and L indicates the span length of wind loading, hence, the differential equations of motion is given as

$$(\ddot{q}_1+\omega_1^2q_1)\int_L\Phi_1^2dx+\frac{sbV}{mb}\Big[f_1q_2V\int_{\alpha_2}^{\alpha_2}\Phi_1\Phi_2dx+f_1b\dot{q}_1\int_{\alpha_1}^{\alpha_2}\Phi_1^2dx+f_2\frac{b}{2}\dot{q}_2\int_{\alpha_1}^{\alpha_1}\Phi_1\Phi_2dx\Big]=0 \quad\cdots\cdots(4)$$

$$(\ddot{q}_2+\omega_2^2q_2)\int_L\Phi_2^2dx-\frac{sbV}{2mr^2}\Big[f_1q_2V\int_{\alpha_2}^{\alpha_2}\Phi_2^2dx+f_1b\dot{q}_1\int_{\alpha_1}^{\alpha_2}\Phi_1\Phi_2dx-f_3\frac{b}{2}\dot{q}_2\int_{\alpha_1}^{\alpha_1}\Phi_2^2dx\Big]=0 \quad\cdots\cdots(5)$$

where Φ_1 : normalized vertical deflection mode

Φ_2 : normalized torsional mode

α_1, α_2 : parameters to indicate the region where the wind acts

ω_1 : natural frequency of vertical vibration

ω_2 : natural frequency of torsional vibration

Introducing the reduced frequency $k=\dfrac{\omega b}{V}$ and the flutter frequency ω, we have

$$q_1=u_1e^{i\omega t},\quad q_2=u_2^{\,i\omega t}\quad\cdots\cdots(6)$$

and adding the abbreviation

$$\int_{\alpha_1}^{\alpha_2}\Phi_i\Phi_jdx=D_{ij}$$

above equations (4), (5) are written in more convenient form as,

$$-\omega^2u_1+\omega_1^2u_1+\frac{sb\omega^2}{mk^2}\Big[f_1D_{12}u_2+if_1kD_{11}u_1+\frac{if_2k}{2}D_{12}u_2\Big]=0\quad\cdots\cdots(7)$$

$$-\omega^2u_2+\omega_2^2u_2-\frac{sb}{2mr^2}\frac{\omega^2b^2}{k^2}\Big[f_1D_{22}u_2+if_1kD_{12}u_1-\frac{if_3k}{2}D_{22}u_2\Big]=0\quad\cdots\cdots(8)$$

The frequency equation derived from eq's (7) and (8) comprises both the real part and imaginary part containing two indeterminate parameters, namely the flutter frequency and the critical wind velocity. In equating the real part and imaginary part of the frequency equation to zero respectively, two equations with real coefficients are obtained as follows,

$$\begin{aligned}&\omega^4\Big[\{1+Sf_{1i}kD_{11}\}\Big\{1+\frac{S}{2}\Big(\frac{b}{r}\Big)^2D_{22}\Big(f_{1R}+\frac{k}{2}f_{3R}\Big)\Big\}+\frac{1}{2}S^2f_{1R}kD_{11}D_{12}\Big(\frac{b}{r}\Big)^2\Big(f_{1i}-\frac{k}{2}f_{3i}\Big)\\&-\frac{1}{2}SkD_{12}^2\Big\{f_{1R}\Big(f_{1i}+\frac{k}{2}f_{2i}\Big)+f_{1i}\Big(f_{1R}+\frac{k}{2}f_{2R}\Big)\Big\}\Big]\\&-\omega^2\Big[\omega_1^2\Big\{1+\frac{S}{2}\Big(\frac{b}{r}\Big)^2D_{22}\Big(f_{1R}+\frac{k}{2}f_{3R}\Big)\Big\}+\omega_2^2(1+Skf_{1i}D_{11})\Big]+\omega_1^2\omega_2^2=0\quad\cdots\cdots(9)\end{aligned}$$

for real part, and

$$\begin{aligned}&\omega^2\Big[\frac{S}{2}\Big(\frac{b}{r}\Big)^2kD_{12}^2\Big\{f_{1R}\Big(f_{1R}+\frac{k}{2}f_{2R}\Big)-f_{1i}\Big(f_{1i}+\frac{k}{2}f_{2i}\Big)\Big\}+\frac{S}{2}\Big(\frac{b}{r}\Big)^2D_{22}\Big(f_{1i}-\frac{k}{2}f_{3i}\Big)(1+Sf_{1i}kD_{11})\\&-Skf_{1R}D_{11}\Big\{1+\frac{S}{2}\Big(\frac{b}{r}\Big)^2D_{22}\Big(f_{1R}-\frac{k}{2}f_{3R}\Big)\Big\}\Big]+Sf_{1R}kD_{11}\omega_2^2-\frac{S}{2}\Big(\frac{b}{r}\Big)^2D_{22}\Big(f_{1i}-\frac{b}{2}f_{3i}\Big)\omega_1^2\\&=0\quad\cdots\cdots(10)\end{aligned}$$

for imaginary part in which

$$S=\frac{sb}{mk^2},\quad f_1=f_{1R}+if_{1i},\quad f_2=f_{2R}+if_{2i}$$

and $f_3=f_{3R}+if_{3i}$

These two equations are transcendental equations about parameters k and ω, which solutions may not be obtained in the closed form by means of the ordinary algebraic calculations. In above equations S, b, r and natural frequencies ω_1^2, ω_2^2 can be calculated from given design factors. Consequently to solve these equations the values for f_j in Table 1 are used interactively corresponding to any k. Substituting the values of f_j for a certain k, the remaining parameter ω is determined from one of the equations. Using the obtained parameter ω for the second equation, the parameters are examined whether or not these satisfy both expressions consistently, and the critical velocity V is obtained from the relation $k=\omega b/V$.

Table 1

k	f_{1r}	$-f_{1i}$	f_{2r}	$-f_{2i}$	f_{3r}	f_{3i}
0.00	1.0000	0.0000	2.0000	0.0000	0.0000	0.0000
0.02	0.9637	0.0752	1.9637	0.0752	0.0367	0.0752
0.04	0.9267	0.1160	1.9267	0.1160	0.0733	0.1160
0.06	0.8920	0.1426	1.8920	0.1426	0.1080	0.1426
0.08	0.8604	0.1604	1.8604	0.1604	0.1395	0.1604
0.10	0.8319	0.1723	1.8319	0.1723	0.1681	0.1723
0.12	0.8063	0.1801	1.8063	0.1801	0.1937	0.1801
0.14	0.7834	0.1849	1.7834	0.1849	0.2166	0.1849
0.16	0.7628	0.1876	1.7628	0.1876	0.2372	0.1876
0.18	0.7443	0.1887	1.7443	0.1887	0.2557	0.1887
0.20	0.7276	0.1886	1.7276	0.1886	0.2724	0.1886
0.22	0.7125	0.1877	1.7125	0.1877	0.2875	0.1877
0.24	0.6989	0.1862	1.6989	0.1862	0.3011	0.1862
0.26	0.6865	0.1842	1.6865	0.1842	0.3135	0.1842
0.28	0.6753	0.1819	1.6753	0.1819	0.3247	0.1819
0.30	0.6650	0.1793	1.6650	0.1793	0.3350	0.1793
0.32	0.6556	0.1766	1.6556	0.1766	0.3444	0.1766
0.34	0.6469	0.1738	1.6469	0.1738	0.3531	0.1738
0.36	0.6390	0.1709	1.6390	0•1709	0.3610	0.1709
0.38	0.6317	0.1679	1.6317	0.1679	0.3683	0.1679
0.40	0.6250	0.1650	1.6250	0.1650	0.3750	0.1650
0.42	0.6187	0.1621	1.6187	0.1621	0.3813	0.1621
0.44	0.6130	0.1592	1.6130	0.1592	0.3870	0.1592
0.46	0.6076	0.1563	1.6076	0.1563	0.3924	0.1563
0.48	0.6026	0.1535	1.6026	0.1535	0.3974	0.1535
0.50	0.5979	0.1507	1.5979	0.1507	0.4021	0.1507
0.55	0.5866	0.1444	1.5866	0.1444	0.4134	0.1444
0.60	0.5788	0.1378	1.5788	0.1378	0.4212	0.1378
0.65	0.5713	0.1319	1.5713	0.1319	0.4287	0.1319
0.70	0.5648	0.1264	1.5648	0.1264	0.4352	0.1264
0.75	0.5591	0.1213	1.5591	0.1213	0.4409	0.1213
0.80	0.5541	0.1165	1.5541	0.1165	0.4459	0.1165
0.85	0.5498	0.1121	1.5498	5.1121	0.4502	0.1121
0.90	0.5459	0.1078	1.5459	0.1078	0.4541	0.1078
0.95	0.5425	0.1039	1.5425	0.1039	0.4575	0.1039
1.00	0.5394	0.1003	1.5394	0.1003	0.4606	0.1003

It should be noticed that the external forces of lift and torque moments are derived for the two dimensional flow which means correspond to the infinite Aspect Ratio, $AR=\infty$, while the theoretical consideration here are made three dimensionally taking an account of spanwise variation of wind velocity in eq's (4) and (5). The smaller the Aspect Ratio (usually less than 5), the more different the two-dimensional lift-curve slope from the actual ones. However for so long structure as a suspension bridge, the aspect ratio varies from 10 to 40 and the correction may remain so small as to be negligible[1)].

2. Numerical illustrations.

Structural characteristics relating the aerodynamic stability are governed by various factors such as the shape of the floor system, the width-depth ratio b/d, existence of slots, etc.

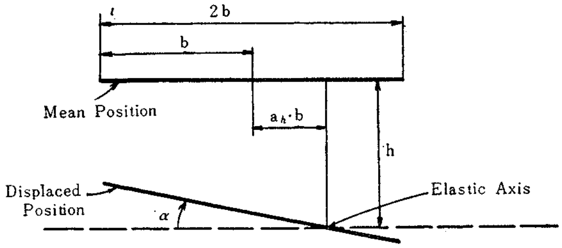


Fig. 1 Notations.

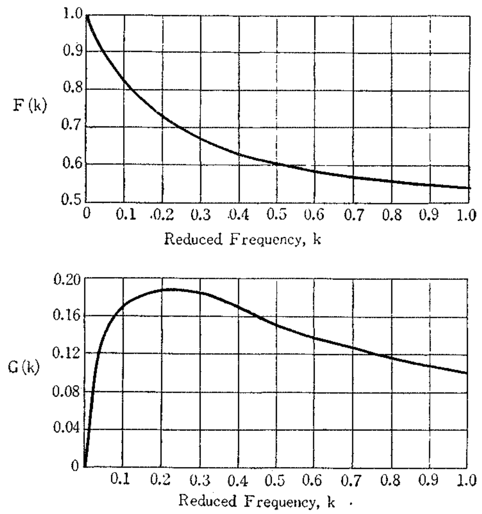


Fig. 2 The Real and Imagenary Parts of Theodorsen's Function $F(k)$ and $G(k)$.

In this paragraph, we take notice of b/d and consider the variation of flutter velocity with respect to this geometric parameter (b/d).

Dimensions used in this calculation are as follows:

Span length

$l=1300$ m

Cable sag

$f=108$ m

Dead load of floor

$W_f=40.678$ t/m

Weight of cables

$W_c=2\times 9.661$ t/m

Horizontal component of the cable tension due to dead load

$H_W=5.841\times 10^4$ t/cable

Cross-sectional area of cable

$A_c=1.232$ m^2/cable

Elastic modulus

$E=E_c=2.1\times 10^7$ t/m^2

Cross-sectional area of Upper chord

456 cm^2

Table 2

Case	d	$2b$	$2b/d$	I_y	I_z
1	23.717	23.717	1.00	12.656	12.656
2	18.605	27.908	1.50	7.788	17.524
3	16.641	29.122	1.75	6.231	19.082
4	15.000	30.000	2.00	5.000	20.250
5	13.622	30.650	2.25	4.175	21.137
6	12.457	31.142	2.50	3.491	21.821
7	10.607	31.820	3.00	2.531	22.781
8	9.215	32.251	3.50	1.910	23.403
9	8.135	32.540	4.00	1.489	23.823

Cross-sectional area of lower chord
395 cm^2

The b/d values are exchanged in ten cases, maintaining the mass radius of gyration to be constant as $r=16.830$ m (refer to the Table 2).

The natural frequencies of vertical and torsional vibration ω_1^2, ω_2^2 and the corresponding mode functions for free vibrations are obtained on the basis of the method developed by F. Bleich[3] which is described as follows.

Setting up the deflectional mode and torsional mode as

$$\eta=a_1 \sin\frac{\pi x}{l}+a_3 \sin\frac{3\pi x}{l} \qquad \text{for the main span,}$$

$$\eta_1=\bar{a}_1 \sin\frac{\pi x}{l_1} \qquad \text{for the side span,}$$

$$\phi=b_1 \sin\frac{\pi x}{l}+b_3 \sin\frac{3\pi x}{l} \qquad \text{for the main span,}$$

$$\phi_1=b_1 \sin\frac{\pi x}{l_1} \qquad \text{for the side span.}$$

The frequency equation of vertical vibration is obtained by eliminating a_1, $\bar{a}_1$ and a_3 from the following expression,

$$\left.\begin{aligned} &Aa_1-\frac{k^*}{p}\left(a_1+\frac{a_3}{3}+2\alpha\beta\,\bar{a}_1\right)=0 \\ &Ba_3-\frac{k^*}{3p}\left(a_1+\frac{a_3}{3}+2\alpha\beta\,\bar{a}_1\right)=0 \\ &C\bar{a}_1-\frac{k^*}{p}\alpha^2\beta\left(a_1+\frac{a_3}{3}+2\alpha\beta\,\bar{a}_1\right)=0 \end{aligned}\right\} \quad \cdots(11)$$

where

$$A=S^*\omega^2-\lambda-H_W$$

$$B=S^*\omega^2-81\,\lambda-9\,H_W$$

$$C=\alpha^2\beta\,S^*\omega^2-\frac{\gamma}{\alpha^2}\lambda-H_W$$

and

$$\alpha=l_1/l,\quad \beta=W_1/W,\quad \gamma=I_1/I,\quad S^*=Wl^2/g\,\pi^2$$

$$\lambda=\frac{\pi^2EI}{l},\quad k^*=\frac{32f}{\pi^3},\quad p=\frac{\pi l}{16}\frac{L_E}{E_cA_c}$$

The frequency equation of torsional vibration is similarly obtained from

$$\left.\begin{aligned} &(\tilde{S}\,\omega^2-\Lambda-R)a_1-K\left(a_1+\frac{a_3}{3}+2\,\alpha\,\bar{a}_1\right)=0 \\ &(\tilde{S}\,\omega^2-81\Lambda-9\,R)-\frac{K}{3}\left(a_1+\frac{a_3}{3}+2\,\alpha\,\bar{a}_1\right)=0 \\ &\left(\alpha\,\tilde{S}\,\omega^2-\frac{\Lambda}{\alpha^3}-\frac{R}{\alpha}\right)\bar{a}_1-\alpha\,K\left(a_1+\frac{a_3}{3}+2\,\alpha\,\bar{a}_1\right)=0 \end{aligned}\right\} \quad \cdots\cdots(12)$$

where

$$\Lambda=EY\pi^4/2\,l^3$$

$$R=\left(E\beta\,bd+\frac{1}{4}H_Wb^2\right)\frac{\pi^2}{2\,l}$$

$$K=\frac{E_cA_c}{L_E}\,\frac{64\,f^2b^2}{\pi^2l^2}$$

$$\tilde{S}=\frac{\bar{m}l}{4}$$

In the case of suspension bridge without side spans, coefficients relating the side span such as C, α, β, γ and $\bar{a}_1$ vanish and above two equations are thus reduced to more simple form as follows.

For vertical vibration

$$\left.\begin{aligned} &\left(A-\frac{k^*}{p}\right)a_1-\frac{k^*}{3p}a_3=0 \\ &-\frac{k^*}{3p}a_1+\left(B-\frac{k^*}{9p}\right)a_3=0 \end{aligned}\right\} \quad \cdots\cdots(13)$$

For torsional vibration

$$\tilde{S}\,\omega^2-\Lambda-R)a_1-K\left(a_1+\frac{a_3}{3}\right)=0$$

$$(S\,\omega^2-81\Lambda-9\,R)a_3-\frac{K}{3}\left(a_1+\frac{a_3}{3}\right)=0$$

the derived values are shown in Table 3.

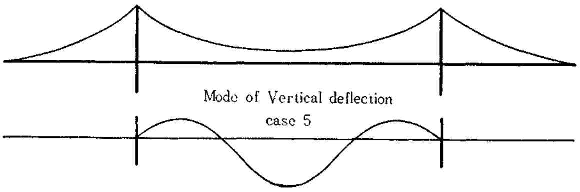
To examine the effect of distributed wind acting on the suspension bridge, we consider the four types of load conditions indicated in Fig. 3 as fully loaded, 80%-loaded, 60%-loaded

Table 3

Vertical mode $\sin\frac{\pi x}{L}-A\sin\frac{3\pi x}{L}$

Torsional mode $\sin\frac{\pi x}{L}-B\sin\frac{3\pi x}{L}$

Case	$2b/d$	ω_1	A	ω_2	B
1	1.00	1.0123	1.3915	1.2287	0.0979
2	1.50	0.9808	1.5009	1.3571	0.1192
3	1.75	0.9702	1.5358	1.3763	0.1337
4	2.00	0.9616	1.5652	1.3833	0.1529
5	2.25	0.9558	1.5855	1.3797	0.1757
6	2.50	0.9509	1.6013	1.3722	0.2041
7	3.00	0.9439	1.6234	1.3461	0.2781
8	3.50	0.9394	1.6393	1.3125	0.3786
9	4.00	0.9362	1.6478	1.2726	0.5004



and 50%-loaded cases.

As the result of calculations the final solutions for two parameters k and ω^3 are obtained graphically as the interesting points of two $k-\omega^2$ curves corresponding to equations (9) and (10) respectively (Fig. 4).

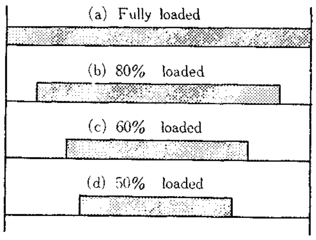


Fig. 3 Load Condition.

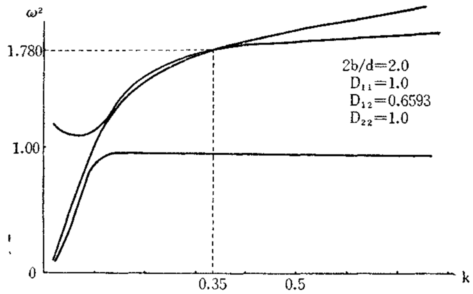


Fig. 4.1 Case 4, Fully Loaded.

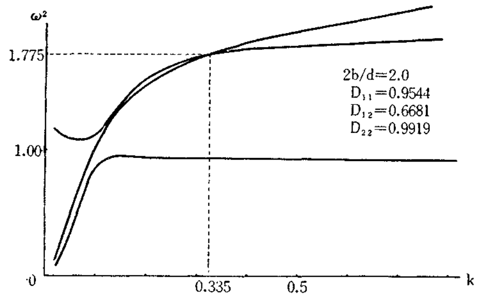


Fig. 4.2 Case 4, 80% Loaded.

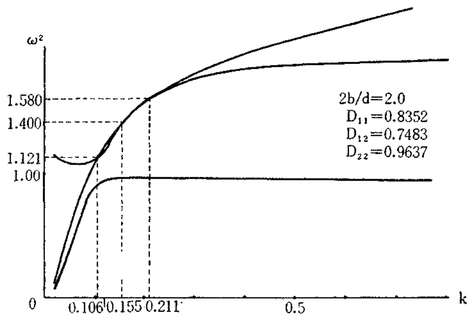


Fig. 4.3 Case 4, 60% Loaded.

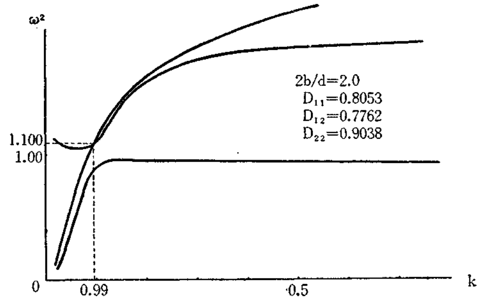


Fig. 4.4 Case 4, 50% Loaded.

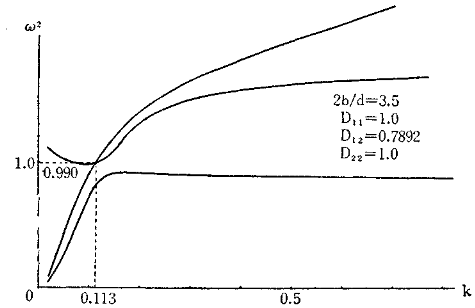


Fig. 4.5 Case 8, Fully Loaded.

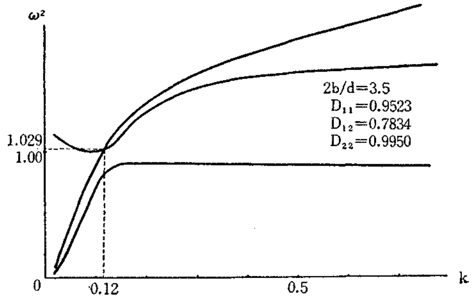


Fig. 4.6 Case 8, 80% Loaded

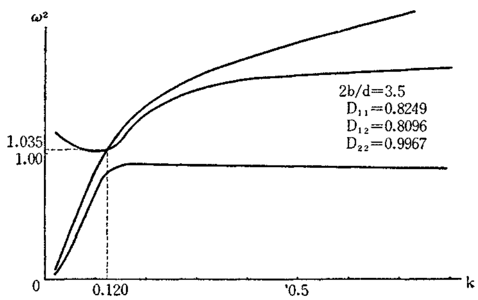


Fig. 4.7 Case 8, 60% Loaded.

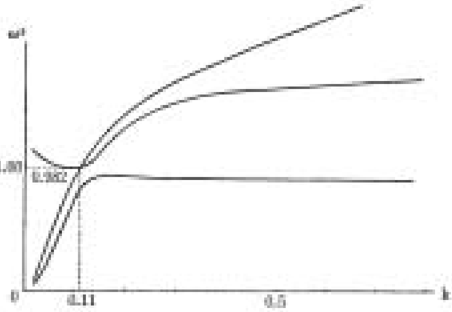


Fig. 4.8 Case 8, 50% Loaded.

3. Consideration.

From the curves indicated in Fig. 4, the spectrum given by eq. (9) hardly varies for various values of b/d to be an almost straight-line except the region where k is less than 0.3, while the spectrum given by eq. (10) varies conspiciously depending on the parameter k.

By the order estimation, the eq. (9) is reduced to a more simple form, because the mass density of air ρ, which is equal to 0.125×10^{-3} ton-sec²/m⁴, is so small that $S=\dfrac{sb}{mk^2}$ is negligible in the region where k is comparatively large. Thus eq. (9) becomes approximately as,

$$\omega^4-\omega^2(\omega_1^2+\omega_2^2)+\omega_1^2\omega_2^2$$
$$=(\omega^2-\omega_1^2)(\omega^2-\omega_2^2)=0$$

that is to say, eq. (9) indicates that k–ω^2 curves become asymptotically parallel to the k-axis in the spectrum diagram, for which the two corresponding frequency parameters approach to the frequencies of vertical and torsional vibrations, respectively, for the large values of k.

Two kinds of curves corresponding to eq's (9) and (10) seem to intersect at several points, with which the parameter k and squared flutter frequency ω^2 are determined (Table 4).

* Effect of Structural factor

Inspecting the variation of flutter speed according to various values of the structural factor $2b/d$, the critical velocity increases remarkably until the ratio $2b/d$ reaches to 3.0.

So far as the structural dimensions used here is concerned it may be said that the geometric shape factor $2b/d$ should be chosen to exceed 1.5, since, in our calculation the critical velocity of $2b/d$ between 1.5 and 2.5 decreases remarkably depending on the loaded length (Fig. 5).

Table 4

Case	$2b/d$	Load	k	ω	V_{cr} (m/sec)
1	1.00	a	0.116	1.100	112.45
		b	0.100	1.074	127.35
		c	0.089	1.058	140.97
		d	0.072	1.051	173.10
2	1.50	a	0.337	1.319	54.61
		b	0.320	1.314	57.50
		c	0.101	1.077	148.79
		d	0.091	1.056	161.92
3	1.75	a	0.342	1.332	56.71
		b	0.323	1.327	59.82
		c	0.222 (0.160, 0.105)	1.281 (1.208, 1.077)	84.02 (109.94, 149.35)
		d	0.095	1.049	160.78
4	2.00	a	0.360	1.334	57.17
		b	0.335	1.332	59.64
		c	0.211 (0.155, 0.106)	1.257 (1.183, 1.058)	89.36 (114.48, 149.77)
		d	0.100	1.049	157.35
5	2.25	a	0.325	1.321	62.29
		b	0.312	1.315	64.59
		c	0.105	1.044	152.37
		d	0.100	1.032	156.62
6	2.50	a	0.295	1.298	68.51
		b	0.282	1.290	71.23
		c	0.106	1.037	149.51
		d	0.103	1.025	154.85
7	3.00	a	0.125	1.059	133.65
		b	0.123	1.046	135.30
		c	0.110	1.015	146.82
		d	0.104	1.008	153.75
8	3.50	a	0.120	1.017	136.67
		b	0.120	1.014	136.67
		c	0.113	0.995	141.99
		d	0.110	0.991	145.28
9	4.00	a	0.122	0.997	133.98
		b	0.121	0.995	133.79
		c	0.115	0.982	138.93
		d	0.112	0.976	141.79

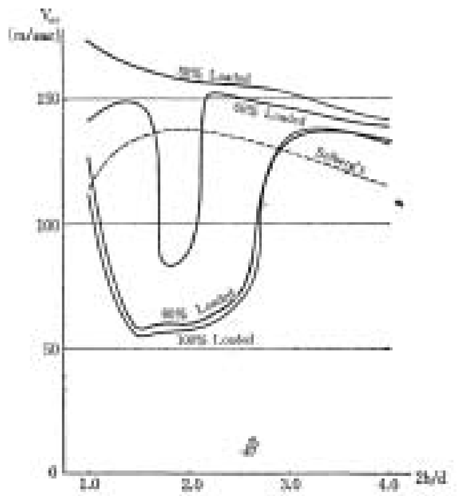


Fig. 5 Change of Critical Wind Velocity relating the Structural Factor b/d.

* The Effect of Variation of Wind loading Length in the spanwise direction (Fig. 6).

The particular attention should be paied on the lowest value of kinds of flutter velocities which takes a more important role in the structural point of views.

The acting length of wind on a suspension bridge has not so large effect on the flutter speed. Especially for the large ratio of b/d, that is, the flat floor, the difference of flutter speeds between the case of fully loaded and the case of half loaded along the spanwise is less than 7%. Moreover it should be noted that there are some cases in which the flutter speeds under fully loaded condition do not reach to the minimum. This seems to be an effect of higher order terms of the free vibrational modes. If we take into account the existence of side spans the effect of higher order terms becomes rather small in fundamental mode.

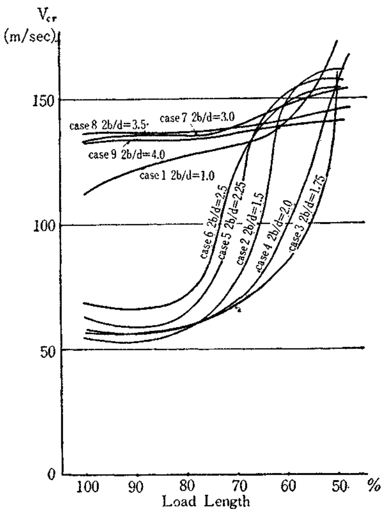


Fig. 6 Change of Critical Wind Verocity relating the Load Condition.

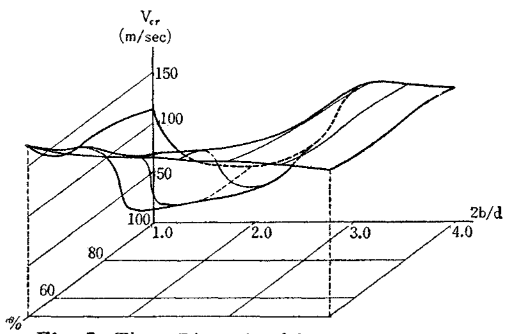


Fig. 7 Three Dimentional Expression of V_{cr}, b/d and Load Length.

Fig. 7 indicates a three dimensional relation, of V_{cr}, b/d and load condition, from which it is considered that the most profitable shape be given as a stationary point on this surface in the diagram.

An empirical form of expressions of critical wind velocity for a bridge being based on flutter theory is proposed by A. Selberg[4]. In consequence with deformation of flutter equation, he demonstrates the effect of various structural parts to improve aerodynamic stability of a suspension bridge, since the flutter velocity V_f has no direct bearing to the actual bridge sections. However, it may be used as a reference value for comparison of different wind tunnel tests. He introduces the factor K as follows,

$$V_{cr}=KV_F \quad\cdots\cdots(14)$$

and determine the K-values from experiments of section models. His investigation suggests to give the bridge cross-section such a form and such dimensions that the factor in eq. (14) will be a maximum.

Furthermore an empirical formula for V_F is given as

$$V_F \quad 0.44\, b\sqrt{(\omega_T{}^2-\omega_r{}^2)\frac{\sqrt{\nu}}{\mu}} \quad\cdots\cdots(15)$$

where

$$\nu=8\left(\frac{r}{b}\right)^2, \quad \mu=\frac{\pi\rho\, b^2}{4\,W}$$

Thus, from eq. (15), it is noticed that the flutter velocity V_F is apparently proportional to the bridge width, and further that the most effective measure is to increase ω_T to increase the aerodynamic stability.

Substituting the same dimensions of our calculation to the equation (15), the critical wind velocity obtained is indicated in Fig. 5.

Fig. 8 indicates the variation of critical wind velocity for the suspension bridge considered here with the various combinations of deflectional and torsional circular frequencies. In this diagram, the relationship is given as a spatial curve which is projected to the V_{cr}-ω_1 plane. This signifies that the critical velocity depend in complicate fashion on the circular frequencies. In this case the value of V_{cr} seems to reach the locally maximum near $\omega_1=0.94$ throughout the four kinds of loading conditions and this accidentally corresponds to the case 5 ($2\ b/d \doteqdot 2.00$) in Table 3.

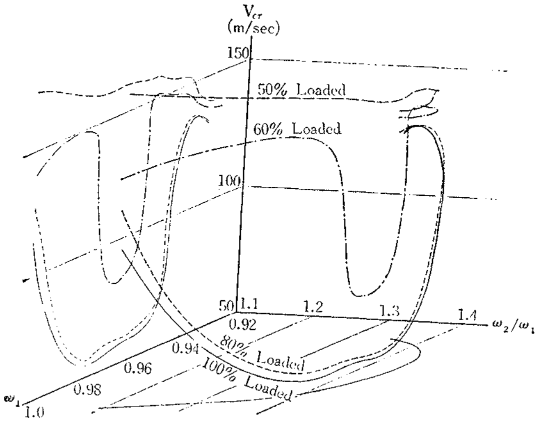


Fig. 8 The Variation of Critical Wind Velocity relating the Circular Frequency of Free Vibration.

3. Conclusion.

In the previous paragraphs, we introduced the effect of spanwise-distributed wind and the geometric sharp b/d into the problem of aerodynamic stability of suspension bridges for the purpose to improve the F. Bleich's flutter theory.

Based on result of our calculations we may conclude that the geometric factor $2\,b/d$ should exceed 3.0 for such structural scale of bridge as considered here.

In practical design, it is obvious that we must take into an account of the structural strength of the bridge at first, and then above condition for geometric shape should be satisfied in the view of aerodynamic stability.

In spite of the facts that some uncertain matters still remains to be discussed such as the application of thin airfoil theory to the decision of external forces acting on floor system and the fundamental equation of torsional vibrations, a criterion for the aerodynamic problems of suspension bridge can be founded analytically according to the theoretical considerations presented in this paper.

Acknowledgement.

The authors wish to express their graditute for financial surpports of Scientific Research Funds by the Ministry of Education which makes this investigation possible.

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(*Received* 12*th July*, 1965)