DIGITAL COMPUTER SIMULATION OF TURBULENT PHENOMENA

*By Mikio Hino**

ABSTRACT

In almost all cases, phenomena we usually treat accompany with them a property of random variation. At the same time a majority of them is possesed of a characteristic of continuity. A simple way of simulation of random phenomena frequently used is a method of random walk or more generally the Monte Carlo method. In order to fulfill the second characteristic of continuity, the author proposes a modified method of simulation consisting essentially of the weighted sum of several groups of random number series, each of which has the different time length of duration and the R.M.S. intensity. Practical procedures are described to put the technique into operation by a digital computer. Mathematical representations of the correlation and spectrum functions for the proposed model are derived.

Statistical analyses by the method of Tukey and Blackman of a thus composed random phenomenon are given to show the correctness and availability of the technique.

INTRODUCTION

Almost all time-dependent phenomena we treat accompany more or less with random oscillations around their mean values. One of the most familiar phenomena experienced is breathing of a gusty wind. Ocean waves which look like as if they are composed of a single harmonic component have a broad-band frequency spectrum. Vibrations of vehicles, buildings, aeroplanes and so on are in almost cases analysed as random processes to be discussed in terms of the frequency density spectra.

Recourses are often made to a simulation of the phenomenon when the governing factors are so complicated to allow for analytical procedures. One way of the numerical simulation is known as the Monte Carlo method (the random walk simulation). Essentially it is applied to stochastic processes. But it has been extended to the non-stochastic problems of decisive solution such as the definite integration, the boundary value problems of the Laplace and Poisson equations, the integral and differential equations and so on.

Another important phase of random phenomena in nature is a characteristic of continuity. Except for a few cases such as shock and radioactive radiations, fluctuations occur never abruptly but continuously. For example, let us follow a small particle with no gravitational effect discharged into turbulent atmosphere. It moves downwinds with about mean wind speed. At the same time, it fluctuates for- or backward, up- or downwards and left or right. However, it cannot change its motion abruptly. It will, for instance, move upwards gradually repeating small up and down fluctuations irregularly. The very property of continuity or smoothness cannot be simulated by the mere "random walk model".

Thus it is natural to consider that random motions are composed of several classes of random processes each of which has different characteristic time of duration and the root mean square intensity.

In what follows, a new technique is described in detail. The fundamental idea together with preliminary examples have been given in reference 1). Mathematical expressions of the correlation and spectrum for the proposed simulation technique are derived. Results of statistical analysis by the method of Tukey and Blackman are represented to show the availability of the technique.

SIMULATION BY RANDOM NUMBER SERIES[1]

a) Random phenomena : For the understanding of any random phenomenon, it is a usual practice to make recourse to the Fourier analysis,

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thus obtaining a frequency density spectrum $S(\omega)$, contribution from a frequency ω to total energy of oscillation. That is to say a random phenomenon is considered to be an aggregation of harmonic oscillations ranging continuously. Therefore, it may be one way of simulation to express the phenomenon as summation of finite numbers of harmonics with differing frequencies and phases. However, the method cannot succeed in giving a continuous spectrum. It is superior to replace a harmonic composing a random oscillation with a step function of random amplitude but of the same corresponing frequency. The method to be proposed is more conveniently describable in terms of a turbulence theory as follows. The mathematical derivation of statistical characteristics will be given later.

b) Turbulon theory : Turbulent flow fields are composed of infinite classes of eddies (nominated by E. Inoue as Turbulons or *Ranshi*), superposed on the mean flow field. The idea was first given by C.F. von Weizsäcker[2] and further put forward by E. Inoue[3]. Turbulon of any rank is characterized both geometrically and dynamically by its scale Λ_n and its velocity V_n.

The fluctuating velocity at a point is thus given by

$$v' = V_0 + V_1 + V_2 + \cdots\cdots + V_\infty \quad \cdots\cdots\cdots (1)$$

where suffix 0 refers to the largest eddy, suffix ∞ to the smallest one. Probability distribution of velocity of any rank of turbulon is a Gaussian with R.M.S. value V_n.

The rate of energy dissipation of the turbulon of n-th rank per unit time and unit mass, ε, is related to these characteristics, assuming the so-called Stokes' resistance law, as

$$\varepsilon \sim \frac{V_n^3}{\Lambda_n} \quad \cdots\cdots (2)$$

The energy of a turbulon should be transferred descendingly, at the equal rate irrespective of ranks, to ones of higher ranks (smaller turbulons), finally being dissipated in the smallest tuabulon into heat by the molcular viscosity. The hypothesis leads to the well-known Kolmogoroff's negative five-thirds power law of the turbulon spectrum

$$F(k) \sim \varepsilon^{2/3} k^{-5/3} \quad \cdots\cdots (3)$$

where k denotes the turbulon wave number corresponding to $1/\Lambda$.

On the other hand, the life time of turbulon τ, defined as the time necessary to consume its own energy V^2 at the rate ε is expressed,

$$\tau_n \sim V_n^2/\varepsilon \sim \Lambda_n/V_n \quad \cdots\cdots (4\,a)$$

The other charcteristic time, the passage time, is defined as the time necessary to pass through a point

$$T_n = \frac{\Lambda_n}{U} \sim \frac{V_n^3}{\varepsilon U} \quad \cdots\cdots (4\,b)$$

Therefore, the R.M.S. velocity of the n-th turbulon is propotional to the one-second power of the life time, and also to the one-third power of the passage time

$$\frac{V_n}{V_0} = \left(\frac{\tau_n}{\tau_0}\right)^{1/2} \quad \cdots\cdots (5\,a)$$

$$= \left(\frac{T_n}{T_0}\right)^{1/3} \quad \cdots\cdots (5\,b)$$

Also, the turbulent diffusion coefficient (kinematic viscosity coefficient) is shown to be

$$K \sim \varepsilon^{1/3}\Lambda^{4/3} = V_n^{4/3}/\varepsilon \quad \cdots\cdots (6)$$

While, the Lagrangian spectrum is given by Inoue as

$$F(f_*) \sim \varepsilon^{2/3} f_*^{-2}.$$

c) Turbulon model : Let R_n be a random number of normal distribution with standard deviation $\sigma = 1$ and $A_n R_n = V_n$ be a characteristic velocity of the n-th rank turbulon at time t. From the foregoing descriptions, the turbulent fluctuating velocity v is expressed

$$v = \sum_{n=0}^{\infty} A_n R_n \quad \cdots\cdots (7)$$

Two models of the turbulon theory schematically visualized are illustrated in Figs. 1 a and 1 b, where circles mean the generation of (pseudo) random numbers, lines solid or dotted correspond to the continuous existance of the random numbers. The life times of turbulon are prescribed in our model by the geometric series in the model I or the algebric series in the model II. In order to prevent turbulons from simultaneous generation and disappearance which give rise to an abrupt change in v lags are given between them as shown in the figures. The coefficients in eq. (7) should be, from eqs. (5 a) and (5 b),

$$\frac{A_n}{A_0} = \left(\frac{\tau_n}{\tau_0}\right)^{1/2} \quad \text{[Lagrangian behavior]} \cdots (8\,a)$$

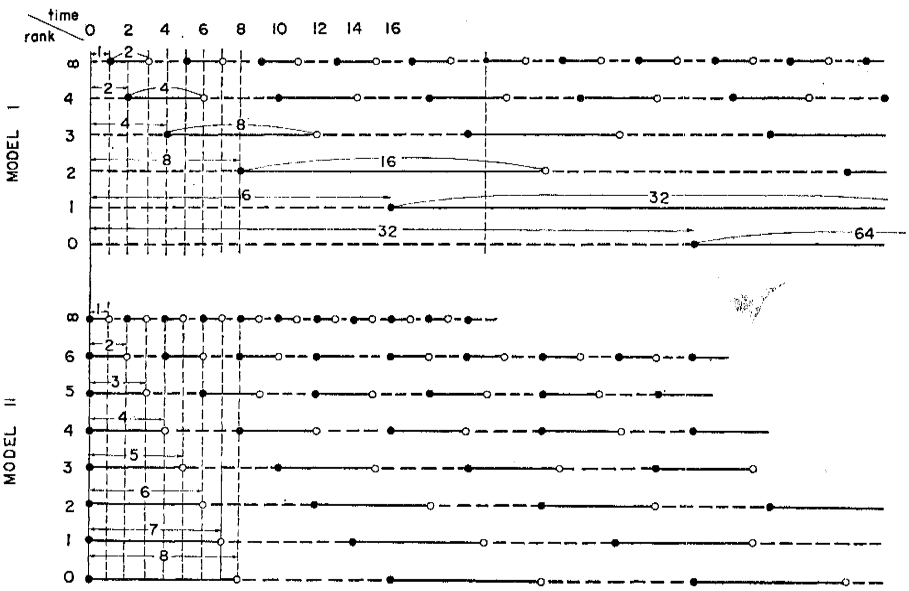

Fig. 1

or

$$\frac{\Lambda_n}{\Lambda_0} = \left(\frac{T_n}{T_0}\right)^{1/3} \text{[Eulerian behavior]} \cdots\cdots (8\,\mathrm{b})$$

Although pure random-numbers may be obtained by recording number of occurrences of physical phenomena such as cosmic ray and γ-ray radiation (physical random number), we can generate pseudo-random numbers by some algebric procedures, for instance the mid-square method and congruential method, thus enabling the use of high speed electronic digital computers. Pseudo-random numbers thus generated are usually of uniform distribution. From the centre limit theorem, random numbers with Gaussian distribution may be approximated by making the mean of several random numbers of uniform distribution.

Let $R_n(j)$ be a random number to be used after weighting by A_n as the j-th member of the turbulon of rank n. At time t, j is given for models I and II, respectively, by

$$j = \left[2 + \frac{1}{\tau_n}(t - \tau_{n-1})\right] \cdots\cdots\cdots\cdots\cdots\cdots (9\,\mathrm{a})$$

$$j = [1 + (t/\tau_n)] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9\,\mathrm{b})$$

where brackets mean the integer part. Whenever j of any rank is increased, a random number from an elementary random number series produced successively by digital computer should replace the older random number of that rank. Therefore it is sufficient practically to produce only one elementary series of random numbers.

d) Mathematical expression : First, we will consider a simple random-step function which corresponds the turbulon of any rank. Correlation function $R(\tau)$ should be of course unity at $\tau = 0$, and should decrease linearly to zero at $\tau = T_n$, thus we have

$$R(\tau) = \begin{cases} 1 - \dfrac{\tau}{T_n} & (0 \leq \tau \leq T_n) \\ 0 & (\tau > T_n) \end{cases} \cdots\cdots (10)$$

Also, the spectrum function determined as the Fourier transform is

$$F(f) = 4\int_0^\infty R(\tau)\cos 2\pi f\tau \, d\tau$$

$$= \frac{1}{(\pi f)^2 T_n}\left\{1 - \cos 2\pi f T_n\right\} \cdots (11)$$

The turbulon model, complex random-step function, is described as eq.(12),

$$X = X_0 + X_1 + X_2 + \cdots\cdots + X_\infty \quad \cdots\cdots\cdots (12)$$

where X_n means a simple random-step function of the life or passage time T_n.

Since different terms of right side of eq. (12) have no correlations each other, the correlation $R(\tau)$ is derived as

$$R(\tau) = \frac{\bar{X}_0^2}{\bar{X}^2}\left[\left(1 - \frac{\tau}{T_0}\right) + \left(\frac{\bar{X}_1^2}{\bar{X}_0^2}\right)\left(1 - \frac{\tau}{T_1}\right) + \cdots \right.$$

$$\left. \cdots + \left(\frac{\bar{X}_n^2}{\bar{X}_0^2}\right)\left(1 - \frac{\tau}{T_n}\right)\right] \cdots\cdots\cdots\cdots\cdots (13)$$

where $T_{n+1} < \tau \leq T_n$ and $\bar{X}^2 = \sum_{n=0}^{n}(\bar{X}_n^2)$.

Put

$$\left(\frac{\bar{X}_n}{\bar{X}_0}\right)^2 = \left(\frac{T_n}{T_0}\right)^p \quad \cdots\cdots\cdots\cdots\cdots\cdots(14)$$

then eq. (13) reduces with an infinitisimally small difference of the life or passage times between turbulons of neighbouring ranks

$$R(\tau) = \frac{F(T_0)}{\bar{X}^2}\left[\int_\tau^{T_0}\left(\frac{T}{T_0}\right)^p\left(1-\frac{\tau}{T}\right)dT\right]$$
$$= \frac{F(T_0)T_0}{\bar{X}^2}\left[\frac{1}{p+1}\left\{1-\left(\frac{\tau}{T_0}\right)^{p+1}\right\} - \frac{\tau}{T_0 p}\left\{1-\left(\frac{\tau}{T_0}\right)^p\right\}\right] \quad \cdots\cdots\cdots(15)$$

where $F(T_0)dT = \bar{X}_0^2$ means a contribution to the R.M.S. intensity from the largest turbulon. Considering a property of $R(\tau)$ that $R(0)=1$, we have

$$F(T_0)T_0/\bar{X}^2 = p+1,$$

and thus

$$R(\tau)\begin{cases} = 1-\left(\dfrac{\tau}{T_0}\right)^{p+1} - \dfrac{p+1}{p} \times \dfrac{\tau}{T_0}\cdot\left\{1-\left(\dfrac{\tau}{T_0}\right)^p\right\} & (0 \leq \tau \leq T_0) \\ = 0 & (T_0 < \tau) \end{cases} \quad \cdots\cdots\cdots(16)$$

Finally, we obtain the spectrum function by the Fourier transformation of eqs. (13) and (16), for a finite number of ranks as,

$$F(f) = \sum_{n=0}^{n}\frac{1}{T_n(\pi f)^2}(1-\cos 2\pi f T_n)\frac{\Sigma(T_n)^p}{(T_n)^p} \quad \cdots\cdots(17)$$

and, for infinite number of ranks with an infinitisimal difference of the life or passage times between neibouring turbulons.

$$F(f) = 4T_0\left[\left(\frac{\sin 2\pi fT_0}{2\pi fT_0}\right) + \frac{1}{p}\int_0^1 t^{p+1}\cos(2\pi fT_0 t)dt - \frac{p+1}{p}\int_0^1 t\cos(2\pi fT_0 t)dt\right]$$
$$= 4T_0\left[\frac{p+1}{2p}\frac{\sin^2\pi fT_0}{(\pi fT_0)^2} - \frac{1}{2p}\frac{\sin(2\pi fT_0)}{\pi fT_0} + \frac{1}{p}\int_0^1 t^{p+1}\cos(2\pi fT_0 t)dt\right] \quad \cdots\cdots(18)$$

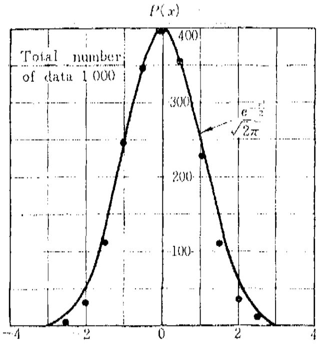

Fig. 2

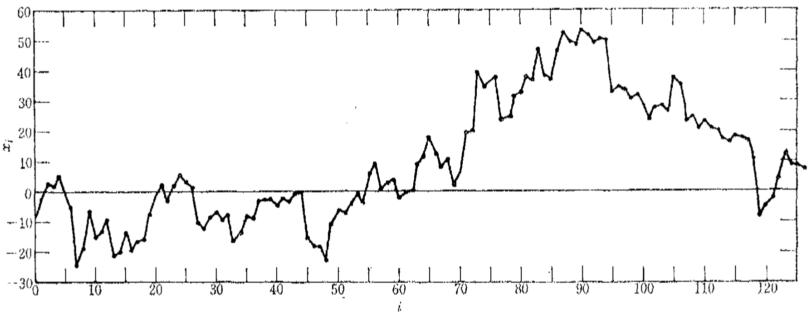

Fig. 3 (a)

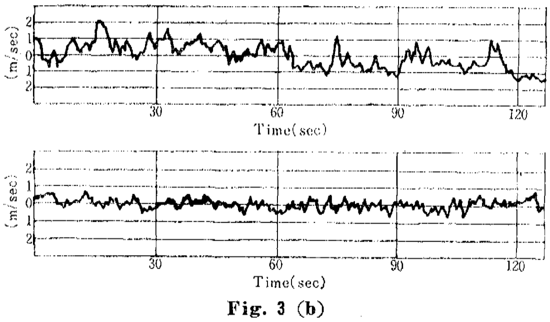

Fig. 3 (b)

Furthermore, when $2\pi fT$ is smaller than unity, the last term in eq. (18) reduces to

$$I = \int_0^1 t^{p+1}\cos 2\pi fT_0 t\,dt$$
$$= \frac{\Gamma(1)}{2\left(\frac{p}{2}+1\right)}\sum_0^\infty \frac{(-1)^n\left(\frac{p}{2}+1\right)(2\pi fT_0)^{2n}}{\left(\frac{p}{2}+n+1\right)\Gamma(2n+1)} \quad \cdots\cdots\cdots(19)$$

e) Statistical Analysis of Random Number

Series

i) Random Number Produced by Digital Computer :

In our study, random numbers with uniform distribution between 0 and 1 are generated by the conguential method by the electronic digital computer IBM 7090. Means of them yield random number of the Gaussian distribution,

$$\frac{U_1+U_2+\cdots\cdots+U_n-(n/2)}{\sqrt{n/2}}$$

where n is twenty in a subroutine used.

Probability distribution of random number thus produced is examined, results are shown in Fig. 2.

Fig. 3 (a) is an example of plots of turbulent fluctuation simulated. By way of comparison, Fig. 3 (b) shows samples from a record of natural wind fluctuations.

ii) Data Processing : Generally we can only record averaged or equally spaced discrete samples of finite length from infinite time series. In order to reduce rational and significant results on statistical analysis, there are several formulas expressing explicit requests derived from the point of view of communications engineering[4]. The procedures are known as the Blackman and Tukey method.

If random variables are read every uniformly spaced time intervals Δt, the maximum frequency calculated by the Fourier transform of correlation is $f_N=1/2\Delta t$. This is nominated as the folding frequency. However, we can use only first 2/3 part of the spectrum obtained. Accordingly, if we need to cover frequencies up to some f_{max}, then the time interval Δt is given by

$$\Delta t=\frac{1}{3f_{max}}=\frac{1}{2f_N} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(20)$$

On the other hand, the resolution in cps (the frequency separation between adjacent estimates) is given as

$$W=\frac{1}{m\Delta t} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(21)$$

in which m is number of the largest lag for correlation calculation.

The relationship among the duration of record T or the total number of data $N=T/\Delta t$, the resolution W and the fractional accuracy of estimation required (in X db) is represented as follows

$$T(\text{in sec})=\left(\frac{1}{2}+\frac{200}{(X:90\text{ per cent range in db})^2}+\frac{\text{pieces}}{3}\right)\Big/W\cdots\cdots\cdots\cdots\cdots(22)$$

Accordingly, there is a conflicting request between the resolution and the accuracy of estimation, increase in resolution, i.e. decrease in W, resulting in reduction in fractional accuracy or stability of analysis. In our study, these characteristic values are determined as follows,

$\Delta t=10^{-2}$, $f_{max}=33.3$ cps, $m=20$ and 100,
$W=1$ cps and 5 cps, $X=0.8$ db and 4 db,
$T=50$ sec, $N=5{,}000$.

Spectra of random number series are obtained by the following procedures. Let $x_0, x_1, x_2, \cdots, x_n$ be the time series and let Δt be the time interval between adjacent values. First prewhitening is performed by

$$\tilde{x}_i=x_i-0.6\,x_{i-1}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(23)$$

Next, compute mean lagged products

$$C_r'=\frac{1}{N-r}\sum_1^{N-r}\tilde{x}_i\tilde{x}_{i+r}-\left(\frac{1}{n}\sum_1^N\tilde{x}_i\right)^2 \quad\cdots\cdots\cdots\cdots(24)$$

We calculate the finite cosine series transform,

$$V_r=\left[C_0'+2\sum_{q=1}^{m-1}C_q'\cos\frac{qr\pi}{m}+C_m'\cdot\cos r\pi\right] \quad\cdots\cdots\cdots\cdots(25)$$

and the result of hanning

$$\left.\begin{aligned}U_0&=\frac{1}{2}(V_0+V_1)\\U_r&=\frac{1}{4}V_{r-1}+\frac{1}{2}V_r+\frac{1}{4}V_{r+1},\ (1\le r\le m-1)\\U_m&=\frac{1}{2}V_{m-1}+\frac{1}{2}V_m\end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(26)$$

Smoothed estimates of the power density is given by correcting both prewhitening and correction for the mean

$$\left.\begin{aligned}P_0&=\frac{N}{N-m}\,\frac{1}{1.36-1.20\cos\frac{2\pi}{6m}}U_0\\P_r&=\frac{1}{1.36-1.20\cos\frac{2r\pi}{2m}}U_r,\quad(1\le r\le m-1)\\P_m&=\frac{1}{1.36-1.20\cos\left(1-\frac{1}{6m}\right)2\pi}U_m\end{aligned}\right\} \quad\cdots\cdots\cdots\cdots\cdots\cdots(27)$$

where P_r is the power density spectrum corresponding frequency $f=r/2\,m\,\Delta t$.

iii) Comparisons of Numerical Calculation with the Turbulon Theory and the Mathematical Expressions.

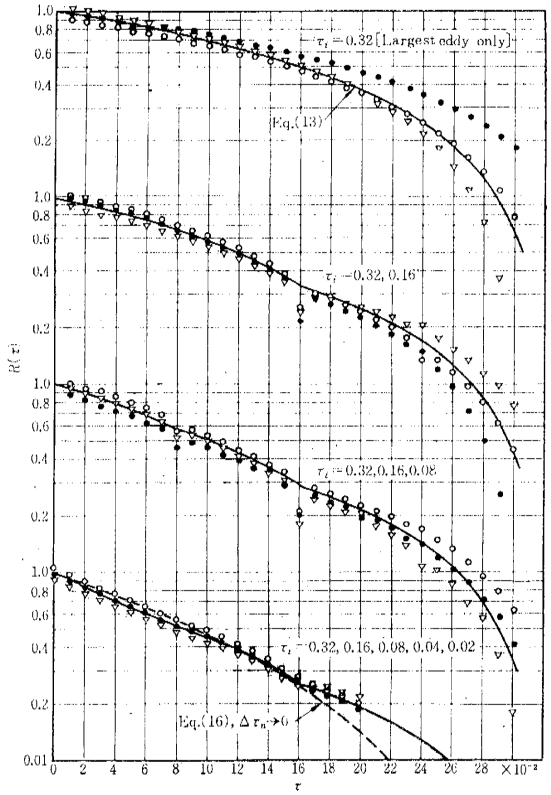

Fig. 4

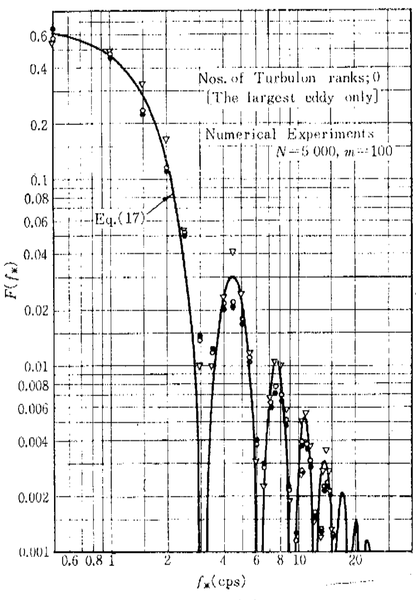

Fig. 5 (a)

The finer the differences between the life or passage times of random number series, the better the simulation of turbulent phenomena. In this respect, the Model II is preferable to the Model I. On the other hand, the frequency band covered by the models should preferably be wider with less computation times, thus the Model I in which the life or passage times increase as geometric series being superior to the

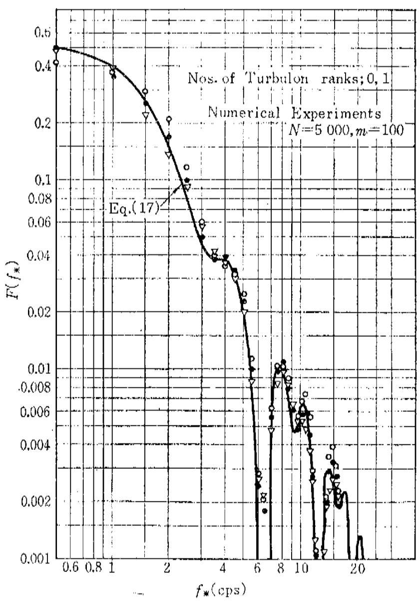

Fig. 5 (b)

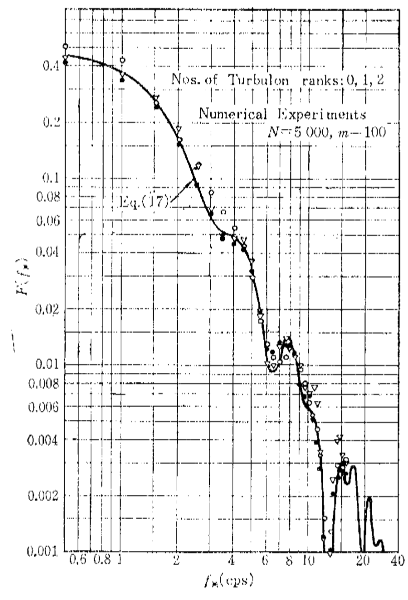

Fig. 5 (c)

Model II. Therefore, the Model I is firstly tested to proceed, if necessary, to the Model II.

Statistical analyses of the Model I for the Lagragian behaviour are conducted successively increasing the number of ranks which compose the Model. Total numbers of data are chosen 5,000, as mentioned before, the largest of life time of eddies being fixed as $\tau_0 = 32\times10^{-2}$sec for convenience of data reduction. Figs. 4 a through 4 d represent comparisons of numerical results on the correlation function $R(\tau)$ with the mathematical expressions (13) and (14) where $p=1$.

Likewise Figs. 5 a through 5 d (for $m=100$)

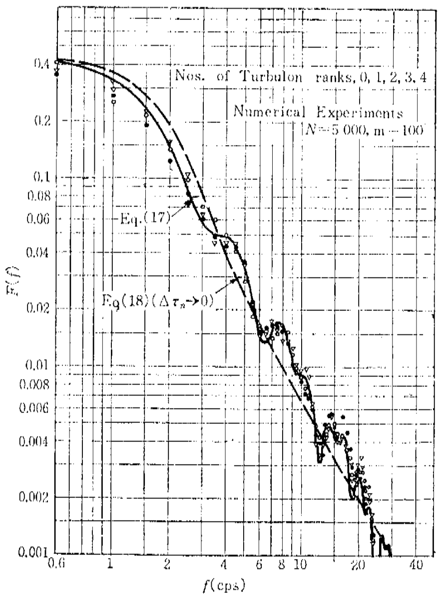

Fig. 5 (d)

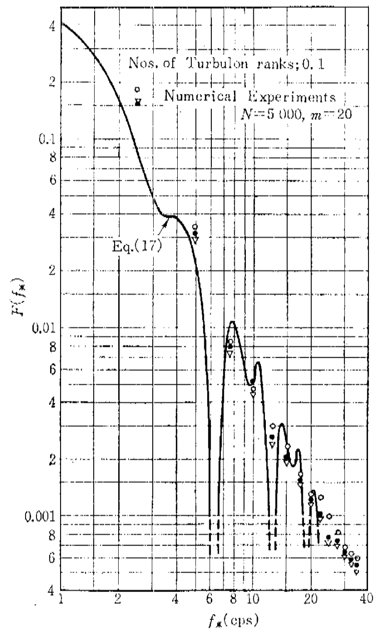

Fig. 6 (b)

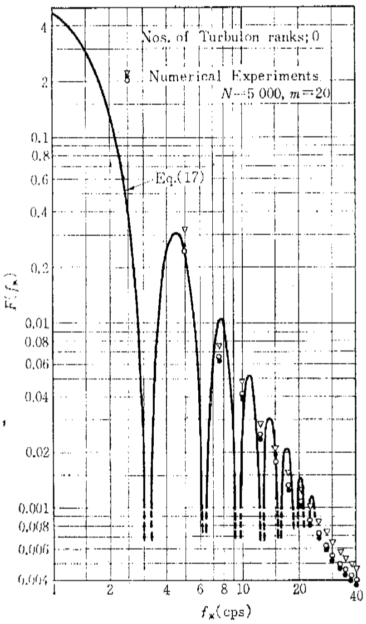

Fig. 6 (a)

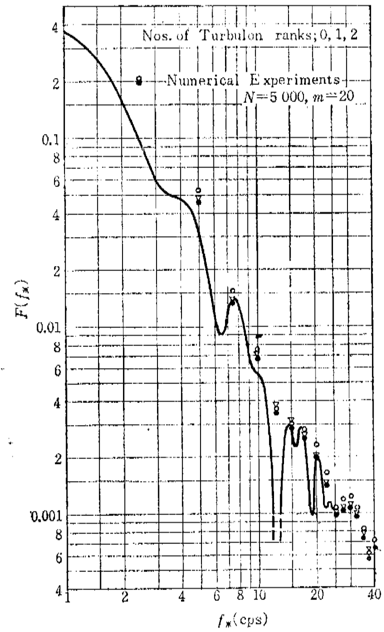

Fig. 6 (c)

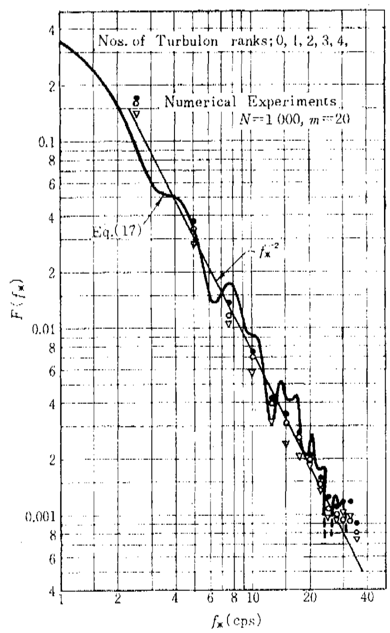

Fig. 6 (d)

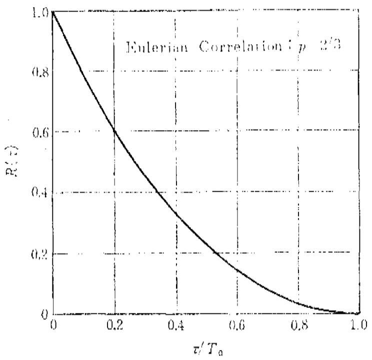

Fig. 6 (e)

and Figs. 6 a–6 d (for $m=20$) are given for comparisons between numerical experiments on the spectra and the equation (17).

Because of reduced resolution ($m=20$), the plots are flattened in Figs. 6 a–d.

These graphs Figs. 4,5 and 6 show clearly that correlations and spectra of turbulent fluctuations simulated by the turbulon model agree considerably well with the mathematical derivations. By the way, Figs. 7 and 8 are results for other runs, including Model II. These figures prove also coincidence between the simplified turbulon model and the theoretical prediction given by Inoue[3] including the so-called Kolmogorov's negative five thirds law.

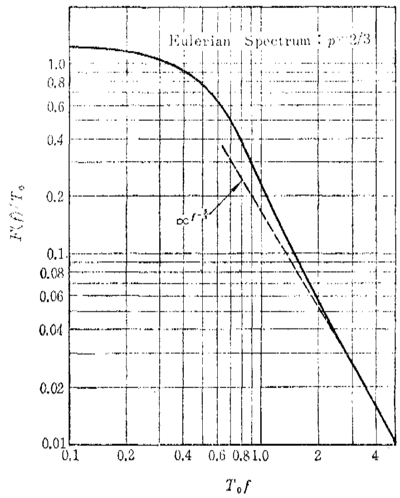

Fig. 6 (f)

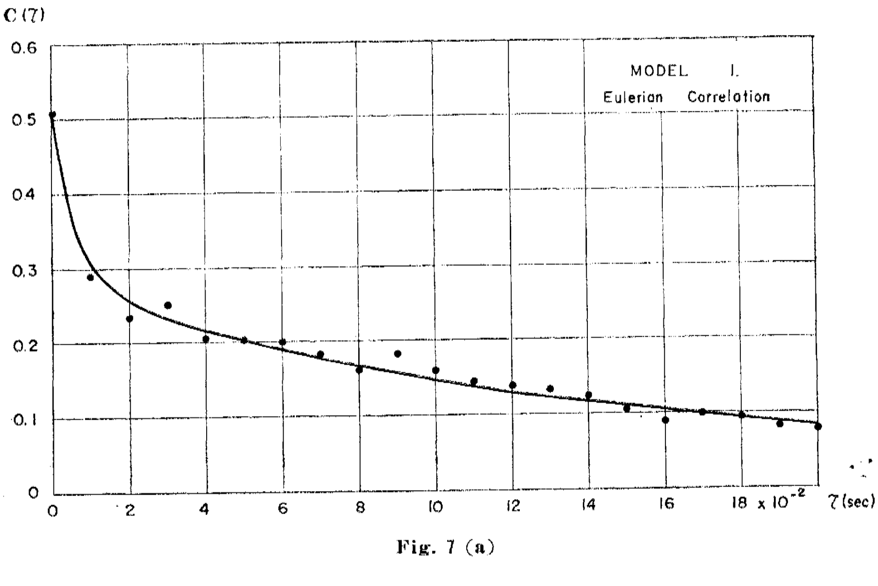

Fig. 7 (a)

With sufficiently small differences between life times of neighbouring ranks, the correlations and spectra approach to eqs. (16) and (18), respectively. Comparing Figs. 5 d and 6 d and considering the fact that usually number of samples are at the most 1,000 with less resolusivility from stability requirement, we can conclude that the turbulon model thus rather simplified composed of 5 or 6 ranks are sufficient enough for simulation of turbulent fields. Physical interpretation of this conclusion is given as that a random step function with a definite duration time τ_n covers the frequency band between 0 to $1/2\tau_n$ cps. Thus relatively small groups of them connect sufficiently wide frequency region with a smooth curve.

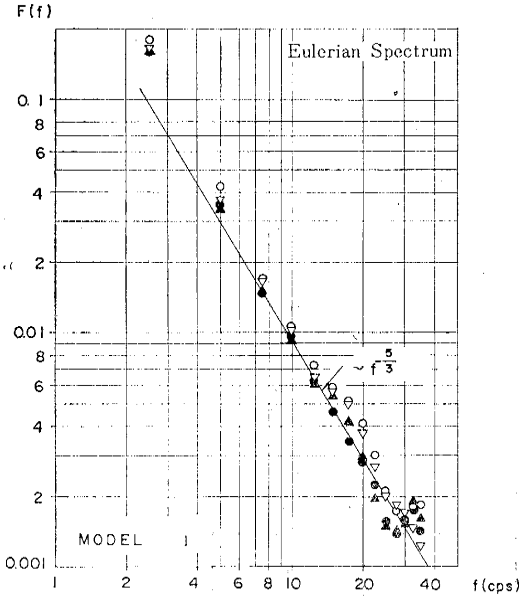

Fig. 7 (b)

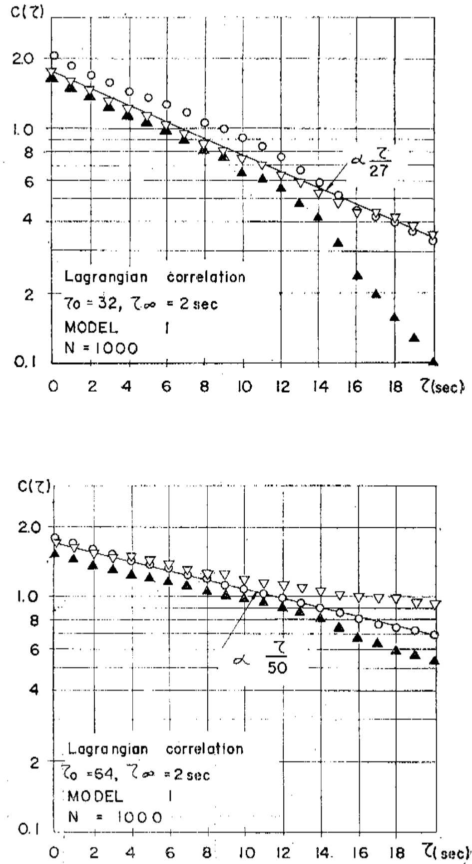

Fig. 7 (c)

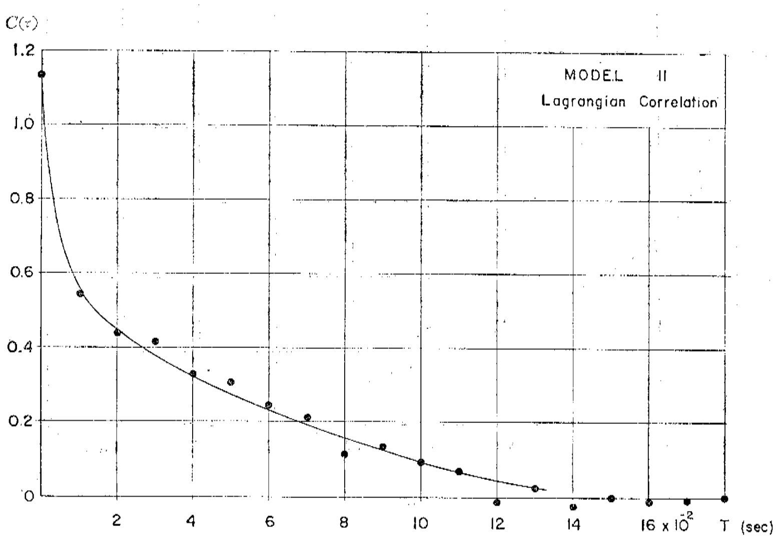

Fig. 8 (a)

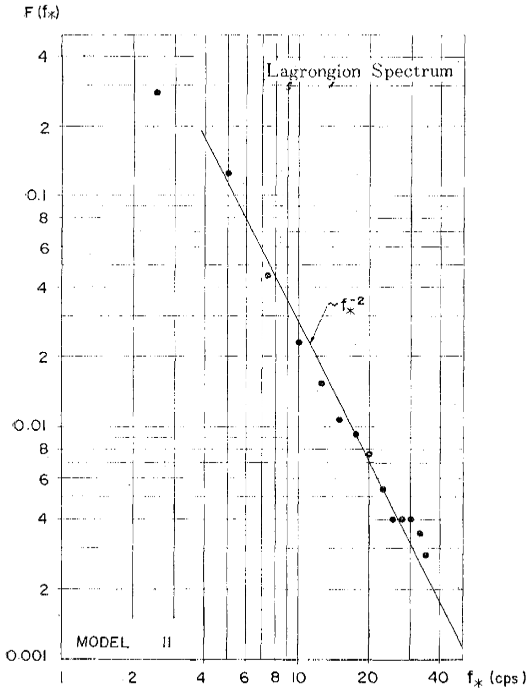

Fig. 8 (b)

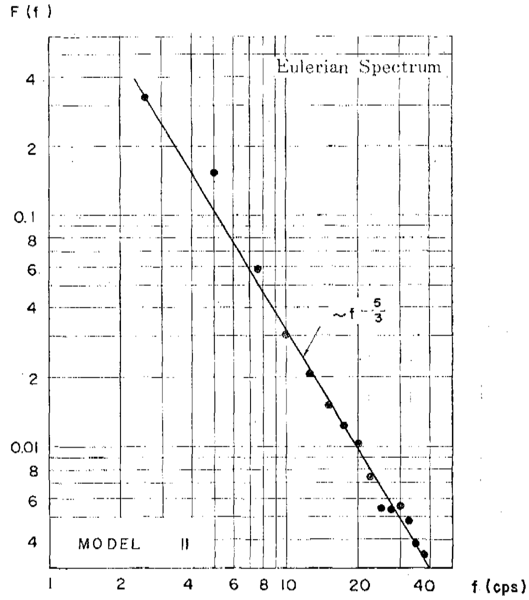

Fig. 8 (d)

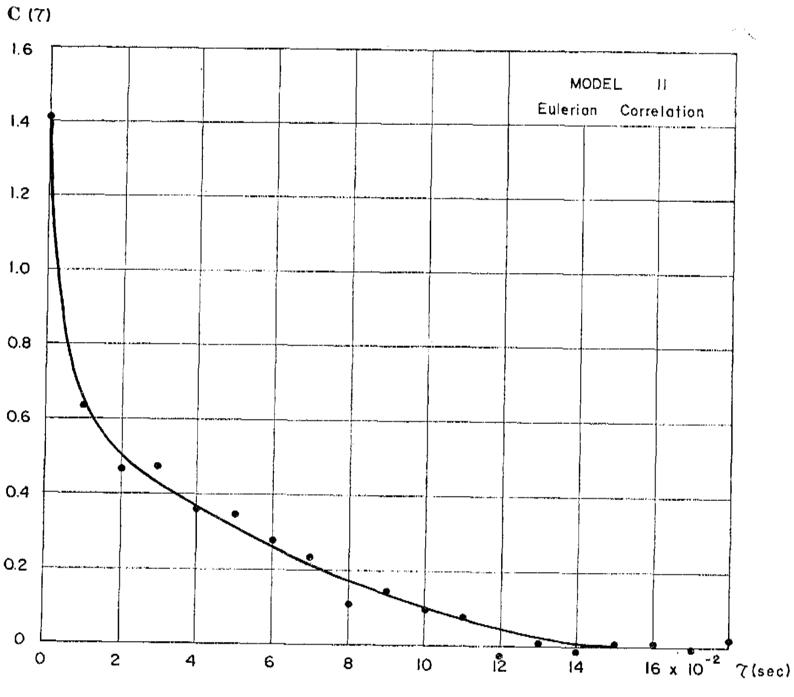

Fig. 8 (c)

The simulation technique presented herein may be applied to various problems. Some of examples are available elsewhere.[5,6] The readers are refered to these articles.

CONCLUSIONS

A technique to simulate a random phenomenon has been proposed based on the turbulon theory. Both numerical and mathematical analyses of the random phenomena thus simulated have been shown.

It should be emphasized that the method of simulation is applicable not only to fluid dynamic problems[5,6] but also to mechanical vibrations as well as other random phenomena. Furthermore, the method is believed to be superior to the simple Monte Carlo method in the point that it simulates not only the random characteristics but also the property of smoothness.

The author acknowledges great thanks to Mrs. Kazuko Hino and Miss S. Nakase for their valuable assistances in digital computer programming.

REFERENCES

1) Hino, M. : An attempt to simulate turbulent phenomena by the Monte Carlo method. (in Japanese), Preprint of the 19th annual conference of Jap. Soc. Civil Engrs. Part II, II-27 Apr. (1964).
2) Weizsäcker, C.F. von : (introduced by G.K. Batchelor) Nature, Vol. 158, (1946).
3) Inoue, E. : On the structure of wind near the ground, Bulletin of Nat. Inst. Agricaltural Science, Ser. A, No. 2, (1952).
4) Blackman, R.B. and Tukey, J.W. : The measurement of power spectra from the point of view of communications engineering, Dover Publications, Inc.
5) Hino, M. and Hino, K. : Response characteristics of Tokyo Bay to incident long waves, (in Japanese), Proc. 11th Conference on Coastal Engineering, Japan, J.S.C.E., (1964).
6) Hino, M. : Some numerical experiments on turbulent diffusion by Monte Carlo technique, Proc. the 9th Conference on Hydraulic Research, JSCE, Feb. (1965).
6′) Hino, M. : Digital Computer Simulation of Turbulent Diffusion, Proc. 11th Congress of IAHR, Sep. (1965)

(Received Nov. 11, 1964)

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昭和40年11月15日印刷
昭和40年11月20日発行
土木学会論文集 第123号　定価150円（〒20円）

編集兼発行者　東京都新宿区四谷一丁目　社団法人 土木学会　羽田　巌
印刷者　東京都港区赤坂溜池5　株式会社 技報堂　大沼正吉

発行所　社団法人 土木学会　振替東京16828番
東京都新宿郵便局区内　新宿区四谷一丁目　電話（351）代表5138番

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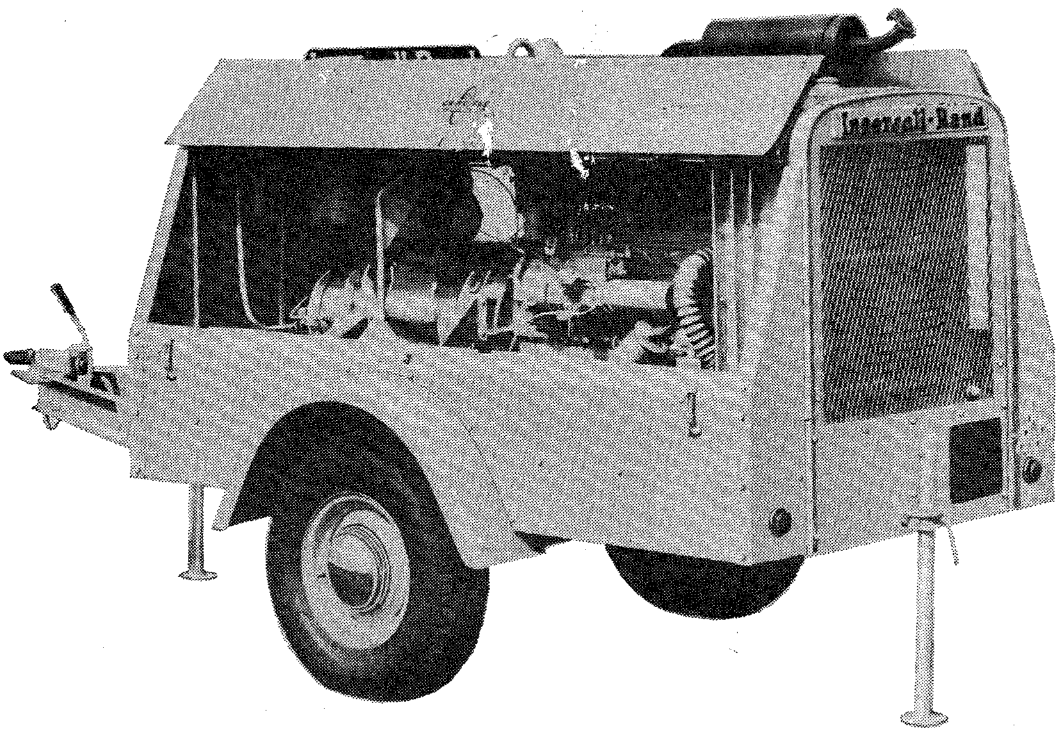

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昭和三十七年五月二十八日 第三種郵便物認可
昭和四十年十一月十五日印刷
昭和四十年十一月二十日発行（毎月一回二十日発行）
土木学会論文集 第一二三号
定価百五十円