EFFECT OF RATE OF LOADING ON THE MODULUS OF DEFORMATION OF MATERIALS EXHIBITING VISCOELASTIC BEHAVIORS.

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Synopsis.

Physical interpretation is given based on the linear viscoelastic theory to the wellknown fact that the modulus of deformation of some materials is greater for the rapid application of load than for slow loading. The principle for this understanding leads to the determination of the viscoelastic constants in accordance with the time of loading employed in the test.

The constants thus determined are used to describe the creep or relaxation curve as a means of representing these physical characteristics. Constitutive equations between stress and strain at any specific time of loading are developed that can be applied to analyse the test result obtained at a constant rate of stressing or at a constant rate of straining. Since the rapid transient test can be performed at considerably small time of loading, it may be justified to replace the high frequency vibration test by the transient testing method. The principle of choosing simplified model is suggested to make it easy to use it in the stress analysis problems. A comprehensive amount of the transient test data for concrete and soil having been reported in many papers was brought together to analyse them by the use of the proposed method. The results of the analysis are presented in the form of creep curve.

Introduction.

In the past research works studying the stress-strain characteristics, it has been pointed out that the stress-strain curve for some materials obtained in the rapid application of load exhibits different feature from the one for the same materials obtained in the slow increment of load application. Stress-strain curve in the slow loading shows in general a deviation from linearity, the deformation tending to increase markedly prior to ultimate failure, as the load is increased. Contrary to this tendency, the stress-strain relation remains linear in the rapid loading up to such high level of load intensity where yielding would take place if the load is applied slowly.

The greater the rate of loading, the steeper the curve in the stress-strain diagram. This phenomenon shows the fact that the mechanical properties of the material depend appreciably upon the time of loading or the rate of loading. Not only the stress-strain relation but also the strength of the material is generally influenced by the time of loading. Generally speaking, the material resists greater for the rapid application of load than for slow application of load. These two effects, that is, the effect on the strength and that on the stress-strain characteristics, are cus tomarily called "time effect, rate effect or speed effect."

Actually, a great majority of tests have been worked out by application, on test specimens, of linearly increasing load or restraint to obtain knowledge as to the response characteristics of materials to dynamical application of load. However, no reasonable means of analytical treatment have been proposed to give better understanding for the rate effect which were given experimental evidence generally.

The cause to which the appearance of the rate effect can be ascribed is thought to be the presence of anelastic properties of materials. Among those theories which can illustrate the anelastic behaviors of materials, the one based on the viscoelastic point of view can be considered most promising and successful for our purpose. Thus, the viscoelastic theory developed extensively so far was adoped to illustrate the

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rate effects that appear in the transient testing. Our effort to relate the transient test result with the conventional viscoelastic theory results inevitably in evaluation of the viscoelastic constants of the materials. It can therefore be said conversely that the transient test result as interpreted from viscoelasticity is useful for determining viscoelastic constants around the time of loading at which test is made. The transient testing principle seems to open a new way for evaluating viscoelastic constants.

In determining the viscoelastic constants, the ways of attack undertaken in the past can be classified into two major groups according to testing principles. They are; (a) the measurement of the transient phenomena caused by the sudden application of a constant load or a constant deformation, (b) the measurement of the steady-state behavior under alternating load or deformation. The creep test and the relaxation test fall in the category of the former methods. The latter includes the methods called vibration test. Which method should be used for specification of viscoelastic properties of a material is a problem to be determined in accordance with the time of loading around which the existing structural material may be exerted by external load. In principle, creep or relaxation method is suitable for obtaining viscoelastic properties for times of loading or straining of more than a few seconds. The steady-state vibration method involves measurements with higher frequencies, corresponding to times of loading or straining smaller than a few seconds. The first method is suitable for the study of the processes with long times of loading or straining, whereas the second one for those with short times of loading. The two methods therefore complete themselves mutually. With the combined use of these two methods, complete information of the properties of a material can be obtained for wide range of time of loading covering every possible cases encountered in the engineering circumstances. From a survey of many of papers published so far, it is seen that the treatment has been restricted principally to these two methods of specification.

In addition to these methods, the use of the transient testing principle is certainly a powerful means to map out a picture of the viscoelastic behavior which is available for relatively short times of loading or straining.

In what follows, an attempt will be made to relate the transient test result with the nature of viscoelaitic materials. As a result, the transient testing principle is justified that leads to development of a useful testing method which is applicable at the short time of loading. Establishment of the formulas throwing an physical insight into the phenomena of rate effect is not only meaningful in itself, but also contributes to the development of a new principle which enables the material properties at the short time of loading to be assessed quite easily.

I. GENERAL FORMULAS.

There have been proposed several ways of approach for formulation of linear viscoelastic law, although they are equivalent to each other in its background. Among them, the most useful approach is the one which dupricates material properties by a model consisting of some suitable combination of springs and dashpots.[1] Other mathematical challenges were offered in rather elegant form for the phenomenological behaviors of materials[2),3),4),5)]. Interrelation between these two approaches was also established through the concept of retardation or relaxation spectra. In the consideration which follows, deduction of necessary formulas will be made with recource to the model simulation method. As usual, every type of deformation will be formulated in terms of shear deformation, but analogous relations exist for bulk compression, simple tension, etc.

(1) Generalized Voigt model.

A Voigt element consists of a spring and a dashpot connected in parallel. This element expresses the simplest form of retarded elastic behavior. The strain associated with this element follows the differential equation,

$$\sigma = \mu\epsilon + \mu' \frac{d\epsilon}{dt} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where, σ is relevant stress component, ϵ strain component, and μ is spring constant, μ' constant of dashpot.

Fig. 1 shows the generalized Voigt model

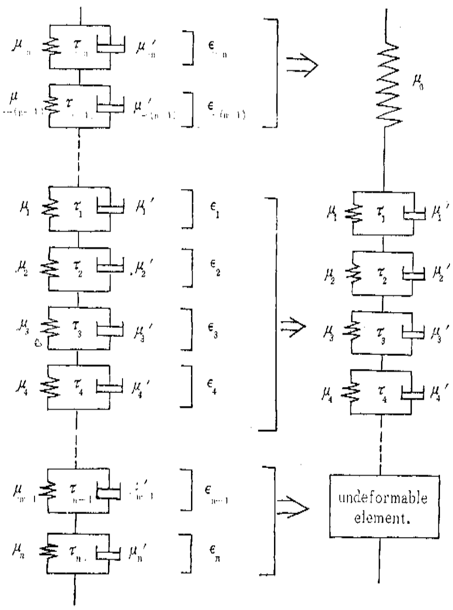

Fig. 1 Generalized Voigt Model.

consisting of a large number of Voigt elements all connected in series. The total strain ϵ is made up of the contribution from each element. Denoting the strain in i-th element by ϵ_i, the whole system should obey a set of n-differential equations,

$$\left.\begin{aligned}\sigma&=\mu_i\epsilon_i+\mu_i'\frac{d\epsilon_i}{dt}\\ \epsilon&=\sum_{i=-n}^{n}\epsilon_i\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

$$(i=-n,-(n-1),\cdots1,2,3,\cdots n)$$

If the viscoelastic properties of a material are specified by this generalized Voigt model, it is quite easy to compute the extension response to any simple stress sequence, that is, the response of each Voigt element can be calculated separately and the individual contribution to the strain is added together.

A constant force imposed upon at time zero and left unchanged thereafter causes a typical strain response generally called "creep function". Integration of eqs. (2) under the zero initial condition yields[2];

$$\epsilon=\left[\sum_{i=-n}^{n}\frac{1}{\mu_i}(1-e^{-t/\tau_i})\right]\sigma\cdots\cdots\cdots\cdots(3)$$

where $\tau_i=\dfrac{\mu_i'}{\mu_i}$ is retardation time of the i-th Voigt element. The ratio ϵ/σ is generally called "creep function" with discrete spectrum or simply "creep function". τ_i means the time taken to reach a proportion $1-e^{-1}$ of the final value of $1/\mu_i$.

Next, consideration will be made for the generalized Voigt model which is subject to a sinusoidally oscillating force. Let the model be subject to an alternating force $\sigma(\omega)e^{j\omega t}$ of radian frequency ω. After sufficient time has elapsed for the effect of the initial condition to be neglected, the strain will also be of radian frequency of ω. If this strain response is designated by $\epsilon(\omega)e^{j\omega t}$, it follows that;

$$\epsilon(\omega)=\left[\sum_{i=-n}^{n}\frac{1}{\mu_i+j\,\omega\mu_i'}\right]\sigma(\omega).\quad\cdots(4)$$

The ratio ϵ/σ is known as "complex compliance".

Next, let us consider the behavior of the generalized Voigt body to which a linearly increasing load of the form $\sigma=\sigma_0 t$ is applied. Integration of eqs. (2) with the zero initial condition gives;

$$\epsilon=\left[\sum_{i=-n}^{n}\frac{1}{\mu_i}\left\{1-\frac{\tau_i}{t}(1-e^{-t/\tau_i})\right\}\right]\sigma\qquad\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

where $\sigma=\sigma_0 t$.

The ratio ϵ/σ may be called "transient creep function with discrete spectrum". If the series of n-Voigt elements is replaced by a continuous destribution of spring constant $1/\mu(\tau)$ covering a whole range of τ, eq.(5) becomes,

$$\epsilon=\left[\int_0^\infty\frac{1}{\mu(\tau)}\left\{1-\frac{\tau}{t}(1-e^{-t/\tau})\right\}d\tau\right]\sigma\qquad\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

The ratio in eq.(6) may be called "transient creep function with continuous spectrum". The stress-strain relation of the form of eq.(5) plays a key role in interterpreting the result of transient test in connextion with the viscoelastic low.

(2) Generalized Maxwell model.

The combination of a spring and a dashpot connected in series is known as Maxwell element. In Fig. 2 is shown the generalized Maxwell model consisting of a large number of Maxwell elements all linked in parallel. In this model, the strains associated with all elements are equal, but the total stress is divided among the elements. The stress component on the i-th Maxwell element is related to the common st-

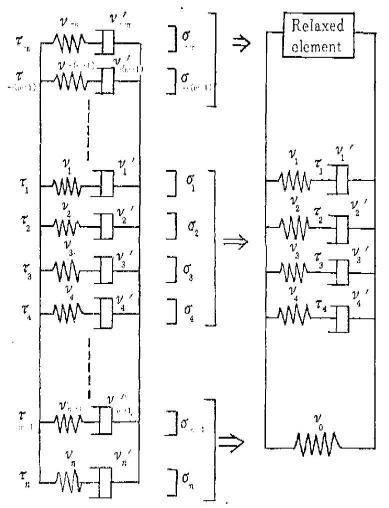

Fig. 2 Generalized Maxwell Model.

rain by the differential equation,

$$\left.\begin{aligned}\frac{d\epsilon}{dt}&=\frac{1}{\nu_i}\frac{d\sigma_i}{dt}+\frac{\sigma_i}{\nu_i'}\\ \sigma&=\sum_{i=-n}^{n}\sigma_i\end{aligned}\right\}\cdots\cdots\cdots(7)$$

$$(i=-n,-(n-1),\cdots1,2,3,\cdots n)$$

where ν_i denotes the spring constant, and ν_i' is the constant of spring. While the Voigt specication allows a simple predition of the strain as a function of time for a known stress, the maxwell model is designed to allow an equally simple prediction of the stress as a function of time when a known strain sequence is imposed upon the material.

The induced stress for a given strain can be computed for individual Maxwell element separately and the result added together. The relaxation of stress at constant strain is a particularly simple case. Integrating the eqs. (7) with the initial condition $\epsilon=0$ at time $t=0$, we obtain[2),

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i e^{-t/\tau_i}\right]\epsilon,\cdots\cdots\cdots(8)$$

where $\tau_i=\nu_i'/\nu_i$ denotes relaxation time of i-th element. The expression in the blacket of eq. (8) is called "relaxation function with discrete spectrum" or more briefly "relaxation function."

Next, consideration will be made for the generalized Maxwell model exerted by a sinusoidally oscillating strain. When an alternating strain $\epsilon(\omega)e^{j\omega t}$ of radian frequency ω is imposed upon the generalized Maxwell body, the resulting steady-state force will be also of a function of radian frequency ω. Inserting the force of the form $\sigma(\omega)e^{j\omega t}$ in eqs.(7), together with $\epsilon(\omega)e^{j\omega t}$, a stress-strain relation that is valid for vibration problems can be derived as follows.

$$\epsilon(\omega)=\left[\sum_{i=-n}^{n}\left(\frac{1}{\nu_i}+\frac{1}{j\omega\nu_i'}\right)\right]\sigma(\omega)\cdots\cdots\cdots(9)$$

The ratio σ/ϵ in the above relation is known as complex modulus.

Next, let us consider the stress-strain relation which is applicable to the generalized Maxwell body subjected to a linearly increasing strain $\epsilon_0 t$. If eqs.(7) are integrated under the condition of zero strain at time $t=0$, the resulting stress component is expressed by,

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i\left\{\frac{\tau_i}{t}(1-e^{-t/\tau_i})\right\}\right]\epsilon\ ,\cdots(10)$$

where $\epsilon=\epsilon_0 t$.

Here the definition of "transient creep function with discrete spectrum" may be given to the ratio σ/ϵ in eq.(10). If the number of single Maxwell element involved is increased infinitely and the spring constant is expressed by a continuous distribution function $\nu(\tau)$, eq.(10) can be reduced to,

$$\sigma=\left[\int_0^\infty\nu(\tau)\left\{\frac{\tau}{t}(1-e^{-t/\tau})\right\}d\tau\right]\epsilon.\quad(11)$$

Thus, we obtained a new function which may be termed "transient relaxation function with continuous spectrum". The stress-strain law as given by eq.(10) will be shown to play the key role in treating the transient test data which are obtained by the application of constant rate of deformation.

We have deduced the stress-strain relations which are applicable to the various types of loading or straining. *Here it has to be noticed that in the case of stress controlling test, the generalized Voigt model is well suited to specify material properties, whereas in the case of deformation controlling test, favorable use is made of the generalized Maxwell model.* Either method contains an equivalent physical meanings and the transformation from the one specification to the other can be attained in principle. It seems to be interesting to overlook three types of stress-strain relations for the generalized Voigt and Maxwell models to see their mutual relationships. The stress-strain relations in a viscoelastic material are expressed; for stress application,

$$\epsilon=\left[\sum_{i=-n}^{n}\frac{1}{\mu_i}(1-e^{-t/\tau_i})\right]\sigma$$

$$\cdots\cdots\text{constant stress}\cdots\cdots\cdots\cdots\cdots\cdots\cdots(12\text{-}1)$$

$$\epsilon=\left[\sum_{i=-n}^{n}{}'\frac{1}{\mu_i}\left\{1-\frac{\tau_i}{t}(1-e^{-t/\tau_i})\right\}\right]\sigma\cdots\cdots$$

$$\text{constant rate of stress}\cdots\cdots(12\text{-}2)$$

$$\epsilon=\left[\sum_{i=-n}^{n}\left\{\frac{1}{\mu_i}\frac{1}{1+\omega^2\tau_i^2}-j\frac{1}{\mu_i}\frac{\omega\tau_i}{1+\omega^2\tau_i^2}\right\}\right]\sigma$$

$$\cdots\cdots\text{alternating stress,}\cdots\cdots\cdots\cdots\cdots(12\text{-}3)$$

and for strain application,

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i e^{-t/\tau_i}\right]\epsilon\cdots\text{constant strain}\cdots(13\text{-}1)$$

$$\sigma=\left[\sum_{i=-n}^{n}\nu_i\left\{\frac{\tau_i}{t}(1-e^{-t/\tau_i})\right\}\right]\epsilon$$

$$\cdots\cdots\text{constant rate of strain}\cdots\cdots\cdots(13\text{-}2)$$

$$\sigma=\left[\sum_{i=-n}^{n}\left\{\nu_i\frac{\omega^2\tau_i^2}{1+\omega^2\tau_i^2}+j\,\nu_i\frac{\omega\tau_i}{1+\omega^2\tau_i^2}\right\}\right]\epsilon$$

$$\cdots\cdots\text{alternating strain}\cdots\cdots\cdots\cdots\cdots(13\text{-}3)$$

II. METHOD OF OBTAINING CREEP CURVE FROM THE RESULTS OF TRANSIENT TESTS.

In the preceding section, new formulas of stress-strain relation which can be used successfully in the transient loading condition were proposed. Such kinds of materials that are subject to change, depending upon the time of loading, in the shape of stress-strain diagram can be represented analytically by the use of these formulas to specify their mechanical properties. Experimental work of transient testing is usually performed by imposing proportionally increasing load or strain on the test specimen and recording the resulting strain or stress, respectively. Conventional creep test can be made by reading the strain produced in the specimen under sustained load. Although the situation in the static tests is different from the one in the transient test, there must exist an unique physical interconnection between the results of test obtained from two methods on the same specimen.

The stress-strain relations of the forms (12-1) and (12-2), or(13-1) and (13-2) expressed in terms of the same viscoelastic constants for two types of loading can be considered to give the physical interrelation between these two types of loading condition. The same statement can be made also for the interrelations between constant loading, transient loading, and alternate loading conditions. We will however exclude herein the consideration for alternate loading condition, because it has been discussed exhaustively elsewhere.

Different types of stress-strain relation expressed in terms of the same viscoelastic constants suggest that it is possible to obtain these physical constants from either relation. The transient test results may be transformed into the static test results formally and vice versa. Since it seems convenient to represent material properties in the form of creep or relaxation curve, it poses problems of considerable interest to demonstrate the results of transient test in the form of creep or relaxation curve. The creep or relaxation curve constructed for short times of loading by the use of transient test results must be the extension, to the range of short times of loading, of the creep curve obtained in the static test.

Two creep or relaxation curves, one from static test and the other from transient test, may be joined together around the time of loading at which static and transient tests are distinguished.

In the discussion which follows, a detailed description of the method of depicting creep or relaxation curve from the results of transient test will be given. First, consideration will be restricted to the construction of a creep curve.

In the model representation of generalized Voigt type, all the fractions of strain responses are distributed over whole range of time scale with corresponding retardation time ranging from zero to infinity. Therefore, when attention is drawn to a certain small interval of time in question, there must exist predominant contributive fractions of strain response which plays primary role in characterizing the sharpe of creep curve at that time of loading. If the time of loading under consideration is shifted, the succeeding predominant part of retardation spectra comes to play a essential role. Thus, if a result of transient test at a time of loading is given, its analysis determines the predominant Voigt element around that time of loading. In the same way, if another result of transient test on the same material at a different time of loading is given, the most contributive Voigt element at the corresponding time of loading is obtained and the creep curve is depicted around that time of loading. Thus, the domain of the creep curve can be extended to include the information of the creep curve in the larger range of time of loading than the one detemined by a single transient test. In this way, if the results of a series of transient tests are given, the creep curve with more accuracy can be drawn over the wider range of time of loading of practical importance. This procedure will be explained in detail henceforth. Let us consider the generalized Voigt model depicted in Fig. 1. In this model, let the τ_i be arranged such that,

$$\left.\begin{array}{l}\tau_{-n}<\tau_{-(n-1)}<\cdots\cdots<\tau_1<\tau_2<\tau_3<\\ \quad\tau_4<\cdots\cdots<\tau_{n-1}<\tau_n\\ \text{and}\\ \mu_{-n}<\mu_{-(n-1)}<\cdots\cdots<\mu_1<\mu_2<\mu_3\\ \quad<\mu_4<\cdots\cdots<\mu_{n-1}<\mu_n\end{array}\right\}\cdots\cdots(14)$$

The stress-strain relation available for the linearly increasing load application is given by eq. (12-2).

If, for example, four sets of transient test data performed at the times of loading t_1, t_2, t_3 and t_4 are given, the relaxation times τ_1, τ_2, τ_3 and τ_4 can be taken to correspond to the values t_1, t_2, t_3 and t_4.

Hence, the values $t/\tau_{-i}(i=1,2,\cdots n)$ can be considered to be infinite and t/τ_i $(i=5,6,\cdots n)$ to be infinitely small. Since

$$\left.\begin{array}{l}\lim\left\{1-\frac{\tau_{-i}}{t}(1-e^{-t/\tau_{-i}})\right\}=1\ \ (t/\tau_{-i}\to\infty)\\ \text{and}\\ \lim\left\{1-\frac{\tau_i}{t}(1-e^{-t/\tau_i})\right\}=0,\ \ (t/\tau_i\to 0)\end{array}\right\}\cdots(15)$$

eq. (12-2) can be reduced to the simpler form,

$$\epsilon=\left[\left(\frac{1}{\mu_{-n}}+\frac{1}{\mu_{-(n-1)}}+\cdots+\frac{1}{\mu_{-1}}\right)+\frac{1}{\mu_1}\left\{1-\frac{\tau_1}{t}(1-e^{-t/\tau_1})\right\}+\cdots\right.$$
$$\left.\cdots+\frac{1}{\mu_4}\left\{1-\frac{\tau_4}{t}(1-e^{-t/\tau_4})\right\}\right]\sigma.\ \ \cdots\cdots(16)$$

The first term in eq. (16) can be replaced by the expression σ/μ_0 briefly. Those portions of the generalized Voigt model which have the retardation times smaller time of loading in question can be thought to deform quickly before the actual time of loading t_1 elapses. Therefore, the elastic constant equal to μ_0 comes to be observed in reality.

Substituting,

$$\frac{1}{\mu_{-n}}+\frac{1}{\mu_{-(n-1)}}+\cdots+\frac{1}{\mu_{-1}}=\frac{1}{\mu_0}\cdots\cdots(17)$$

in eq. (16) it follows that;

$$\epsilon=\left[\frac{1}{\mu_0}+\frac{1}{\mu_1}\left\{1-\frac{\tau_1}{t}(1-e^{-t/\tau_1})\right\}+\cdots\right.$$
$$\left.\cdots+\frac{1}{\mu_4}\left\{1-\frac{\tau_4}{t}(1-e^{-t/\tau_4})\right\}\right]\sigma.\ \ \cdots\cdots(18)$$

The above equation is the one which is needed to represent the stress-strain curves obtained in the transient test. It is indeed possible to determine the viscoelastic constants in eq. (18) by fitting, in diagram, the relation (18) to the four stress-strain curves obtained from the transient test at four different times of loading.

Let it be supposed that the four stress-strain curves obtained at the times of loading t_1, t_2, t_3 and t_4 are given as shownin Fig. 3. When the

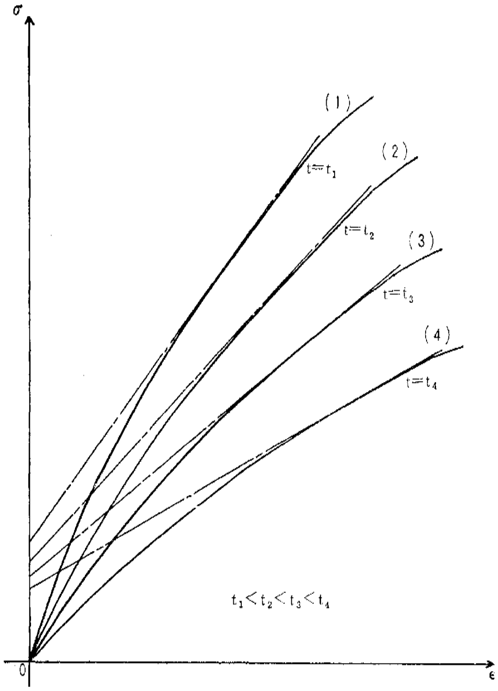

Fig. 3 Stress-strain curves obtained at four different times of loading.

time range around t_1 is of major concern, the values of t_1/τ_2, t_1/τ_3 and t_1/τ_4 are nearly equal to zero in eq.(18). The fractions of strains such as ϵ_2, ϵ_3 and ϵ_4 may be dropped off therefore from eq.(18) as is understood from eqs.(15). The resulting formula is then,

$$\epsilon=\left(\frac{1}{\mu_0}+\frac{1}{\mu_1}\right)\sigma-\frac{\tau_1\sigma_{01}}{\mu_1}(1-e^{-\sigma/\tau_1\sigma_{01}}) \quad \cdots\cdots\cdots(19)$$

where $\sigma=\sigma_{01}t$ is used to eliminate time factor t.

A characteristic curve of this equation is shown in Fig. 4 with solid line. The corresponding model is also indicated together with the curve. It can be said that, so long as the time of loading under consideration ranges around τ_1, the generalized Voigt model can be degenerated to the simplified form as shown in Fig. 4. Simplification of the complicated model is nothing but the proper choice of that Voigt element which contributes most sensitively to the formation of a stres-strain curve.

Comparison of the given curve(1) in Fig. 3 with that in Fig. 4 indicates that there is a close similarity in the shape of these two curves, except the portion where yielding seems to take place at the high level of stress intensity. Therefore, if attention is centered on the stage where no collapse takes place, the representation of the curve(1) by means of eq.(19) is successfully made by determining the viscoelastic constants appearing in eq.(19). It is seen in Fig. 4 that, as the applied load increased, the stress-strain curve approaches an asymptotic line which cuts σ-axis at the point $\frac{\tau_1\sigma_{01}}{\mu_1}\Big/\frac{1}{\mu_0}+\frac{1}{\mu_1}$ and has a slope whose tangent is equal to $1\Big/\frac{1}{\mu_0}+\frac{1}{\mu_1}$. Therefore, let the asymptotic line be drawn in Fig. 3 such that as long part of the straight line as possible may contact the stress-strain curve(1). This asymptotic line is drawn in Fig. 3 with dotted line. Reading of the slope of the straight line(1) and its intersecting point with σ-axis yields the values,

$$\frac{1}{1/\mu_0+1/\mu_1} \quad \text{and} \quad \frac{\tau_1\sigma_{01}/\mu_1}{1/\mu_0+1/\mu_1}. \quad \cdots\cdots\cdots\cdots(20)$$

From these, it is possible to determine the value τ_1/μ_1 for the given rate of loading σ_{01}.

Next, let us draw our attention to the stress-strain curve(2) in Fig. 3. Since this curve is obtained at the time of loading t_2, and the ratio t_2/τ_2 plays an important role to form the stress-strain curve, the third term in eq.(18) can no longer be neglected and the Voigt element having retardation time τ_2 have to be taken into account. In this case t_2/τ_1 is very large and the second term in eq.(18) is reduced to σ/μ_1. Hence, eq.(18) becomes,

$$\epsilon=\left(\frac{1}{\mu_0}+\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)\sigma-\frac{\tau_2\sigma_{02}}{\mu_2}(1-e^{-\sigma/\tau_2\sigma_{02}}). \quad \cdots\cdots\cdots\cdots\cdots\cdots(21)$$

The characteristic curve of this equation is shown in Fig. 5 with solid line. The curve(2)

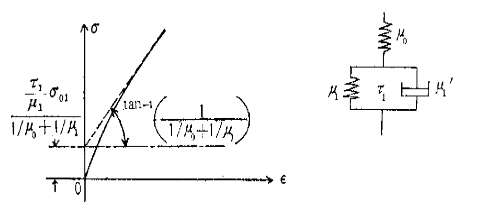

Fig. 4 Stress-strain curve at the time of loading t_1 around τ_1.

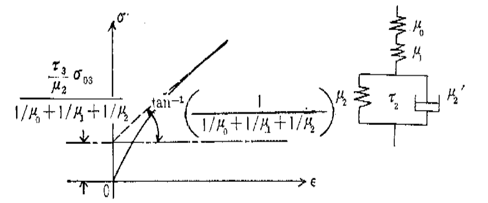

Fig. 5 Stress-strain curve at the time of loading t_2 around τ_2.

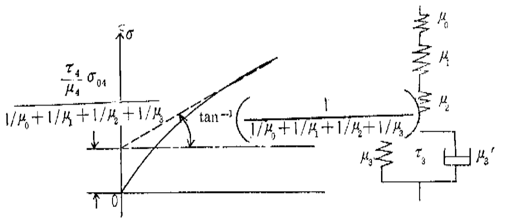

Fig. 6 Stress-strain curve at the time of loading t_3 around τ_3.

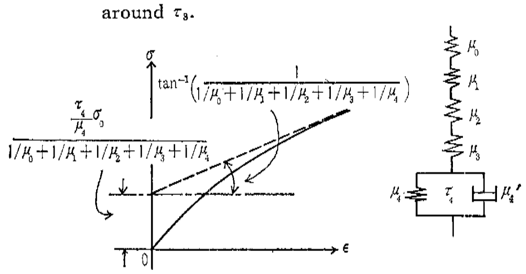

Fig. 7 Stress-strain curve at the time of loading t_4 around τ_4.

in Fig. 3 can be fitted to the one in Fig. 5. Reading of the slope of the straight line(2) and the point of its intersection with σ-axis gives,

$$\frac{1}{1/\mu_0+1/\mu_1+1/\mu_2} \text{ and } \frac{\tau_2\sigma_{02}/\mu_2}{1/\mu_0+1/\mu_1+1/\mu_2}. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(22)$$

With reference to the values in (20) obtained above, it is possible to determine the value τ_2 and μ_2 for the given value of loading rate σ_{02}.

The analysis of the stress-strain relation around the time of loading τ_3 is the next step to be followed. The contributive term including the effect of τ_3 in eq.(18) have to be retained for consideration at this stage. From the similar reasoning mentioned above, the stress-strain equation is from eq.(18)

$$\epsilon=\left(\frac{1}{\mu_0}+\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3}\right)\sigma-\frac{\tau_3\sigma_{03}}{\mu_3}(1-e^{-\sigma/\tau_3\sigma_{03}}) \qquad \cdots\cdots\cdots\cdots\cdots\cdots(23)$$

Determination of the viscoelastic constants can be made as previously by assuming that the curve(3) in Fig. 3 is represented by the above equation. Reading of the slope of the asymptotic line and the point of its intersection with σ-axis yields,

$$\frac{1}{1/\mu_0+1/\mu_1+1/\mu_2+1/\mu_3} \text{ and } \frac{\tau_3\sigma_{03}/\mu_3}{1/\mu_0+1/\mu_1+1/\mu_2+1/\mu_3}. \qquad \cdots\cdots\cdots\cdots\cdots(24)$$

With reference to the values in(22), μ_3 and τ_3 can be evaluated from(24). Finally, in the same way as above, we can determine the values μ_4 and τ_4 by fitting the curve(4) to the characteristic curve in Fig. (7). Similar step may be followed successively in the case where there are given more stress-strain curves at longer times of loading. In this way, it is possible to evaluate the viscoelastic constants successively. In interpretating the result, it is to be noted that the individual value μ_0, μ_1 and μ_1' remain undeterminated and the combined modulus $1\Big/\frac{1}{\mu_0}+\frac{1}{\mu_1}$ is only to be determined. This fact causes however no trouble in the stage of using these constants. A typical shape of the creep curve that is a strain response under sustained constant load is depicted in Fig. 8 by taking abscissa as time in logarithmic scale and ordinate as the magnitude of strain. The procedure of depicting this creep curve by the use of the data obtained above is as follows.

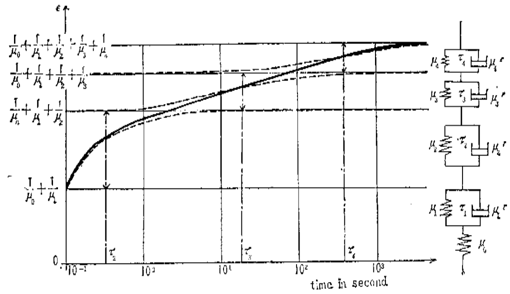

Fig. 8 A diagram illustrating the method of constructing a creep curve from the result of transient tests.

Firstly, the values,

$$\frac{1}{\mu_0}+\frac{1}{\mu_1},\quad \frac{1}{\mu_0}+\frac{1}{\mu_1}+\frac{1}{\mu_2},$$

$$\frac{1}{\mu_0}+\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3} \text{ and}$$

$$\frac{1}{\mu_0}+\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3}+\frac{1}{\mu_4}$$

are to be plotted in the ordinate to determine each strain of which each Voigt element takes its share.

After plotting the values of retardation time corresponding to each element, it is easy to draw creep curves in the individual Voigt element, as shown with dotted line in Fig. 8. The overall feature of the creep curve ranging from the order of τ_1 to that of τ_4 is able to be drawn by superposing all the strains resulting from each spectrum. The superposition can be done on the diagram. The resulting feature of the creep curve is shown with solid line in Fig. 8. In the right portion of Fig. 8, the corresponding Voigt elements are indicated.

In the example mentioned above, the results of transient test at four various times of loading were used to specify the visco-elastic properties. If use is made of a variety of transient tests at a number of times of loading, the more accuracy may be acquired in the shape of the creep curve. On the contrary, the result of fewer transient tests will give cruder approximation to the real shape of the creep curve.

We have described the creep curve by superposing all strains produced in the specific set of Voigt element selected from the generalized Voigt model. The choice of the specific set is to be made depending upon the time of loading employed in the transient test. If another set of time of loading is used in the transient test for the same material, a different set of viscoelastic constants would result to describe the same creep curve. Therefore it can be said that the way of selection of each contributive Voigt element can not give to rise to any difference in the shape of the resulting creep curve. Whatever Voigt element we may use, the resulting creep curve is to be the same, if the number of Voigt element used is assigned to deduce the result within the limit of definite accuracy.

III. METHOD OF OBTAINING RELAXATION CURVE FROM THE RESULTS OF TRANSIENT TESTS.

In the foregoing paragraphs, a principle to analyse the results of transient test performed at constant rate of loading was illustrated. The essential aspect of this method of analysis consists in the use of the generalized Voigt model corresponding to such type of loading as is applied at constant speed. Most of the conventional high speed test employs the loading apparatus by which constant speed of load can be produced. There are however some apparatus which can control the rate of deformation. In order to achieve similar analysis in the case of strain-rate control, it is indeed expedient to use the generalized Maxwell model. In this case, the viscoelastic constants determined from the transient test result are used to construct relaxation curve.

In the generalized Maxwell model, the imposed strain is common to all element and the stress is distributed to individual element in accordance with the corresponding relaxation time. The reduction in stress at a specific time of straining is therefore mostly attributed to a few contributive Maxwell elements predominant around that time of loading. Thus, if a rate of straining is given in a transient test, the feature of stress relaxation can be made known mainly by the behavior of the predominant element which reveals itself mostly around the given time of straining. Other elements far from that time of loading have no remarkable influence upon the stress reduction at this stage. If the given time of straining is shifted beyond the range of the time of straining employed previously, a succeeding Maxwell element with comparable relaxation time comes to reveal its influence upon the feature of stress relaxation. If similar step is followed several times, the characteristics of relaxation process over wide range of time of straining would be visualized.

Let us consider the generalized Maxwell system as shown in Fig. 2. In this system, let the relaxation time τ_i and spring constant ν_i be arranged so that,

$$\left.\begin{array}{l} \tau_{-n}<\tau_{-(n-1)}<\cdots<\tau_1<\tau_2<\tau_3<\tau_4< \\ \quad\cdots<\tau_{n-1}<\tau_n \\ \nu_{-n}<\nu_{-(n-1)}<\cdots<\nu_1<\nu_2<\nu_3<\nu_4< \\ \quad\cdots<\nu_{n-1}<\nu_n \end{array}\right\} \cdots(25)$$

When a linearly increasing strain $\epsilon=\epsilon_0 t$ is applied to this model, the equation relating strain with the induced stress is given by eq. (13-2). We will consider to analyse as an example the result of transient tests carried out at four different time of straining t_1, t_2, t_3 and t_4. If the relaxation time τ_1, τ_2, τ_3 and τ_4 are specified so that they correspond to the times of straining t_1, t_2, t_3 and t_4 respectively, the non-dimensional values t/τ_{-i} $(i=1,2,\cdots n)$ become infinite and t/τ_i $(i=5,6,\cdots n)$ infinitely small. Since,

$$\left.\begin{array}{ll} \lim \dfrac{\tau_{-i}}{t}\left(1-e^{-t/\tau_{-i}}\right)=0 & (t/\tau_{-i}\rightarrow\infty) \\ \lim \dfrac{\tau_i}{t}\left(1-e^{-t/\tau_i}\right)=1 & (t/\tau_i\rightarrow 0) \end{array}\right\} \cdots(26)$$

eq. (13-2) is simplified as follows,

$$\sigma=\left[\nu_1\frac{\tau_1}{t}\left(1-e^{-t/\tau_1}\right)+\cdots+\nu_4\frac{\tau_4}{t}\left(1-e^{-t/\tau_4}\right)\right.$$
$$\left.+\nu_5+\nu_6+\cdots+\nu_n\right]\epsilon \quad \cdots\cdots\cdots(27)$$

By putting,

$$\nu_5+\nu_6+\cdots+\nu_n=\nu_0$$

it followes that,

$$\sigma=\left[\nu_0+\nu_1\frac{\tau_1}{t}\left(1-e^{-t/\tau_1}\right)+\right.$$
$$\left.\cdots+\nu_4\frac{\tau_4}{t}\left(1-e^{-t/\tau_4}\right)\right]\epsilon \cdots\cdots\cdots (28)$$

The equation derived above can be considered to be the stress-strain relation in the case where the strain is applied at four different rate. Let us suppose that a set of four stress-strain curve is given as shown in Fig. 9, which are

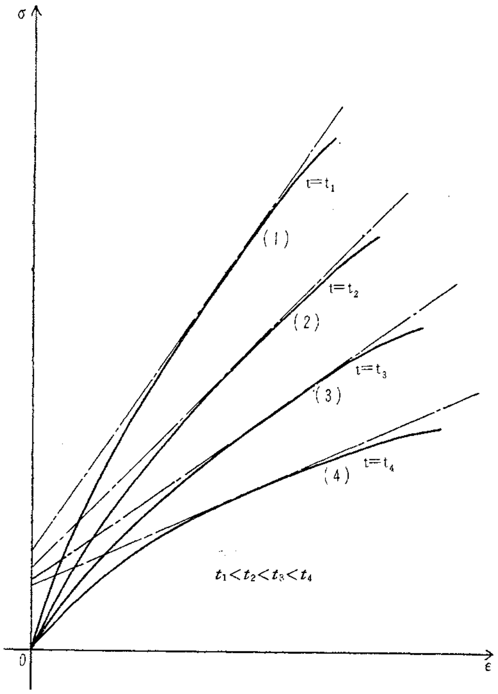

Fig. 9 Stress-strain curves obtained at four different times of straining.

obtained at times of straining t_1, t_2, t_3 and t_4. We will consider the curve (4) firstly. Since the time of straining t_4 employed in obtaining the curve (4) is comparable with relaxation time τ_4, the quantities t_4/τ_1, t_4/τ_2 and t_4/τ_3 can be supposed to be large. Therefore the terms in eq. (28) involving τ_1, τ_2 and τ_3 may be dropped off as apprantly seen from (26).

Then we have,

$$\sigma=\nu_0\epsilon+\epsilon_{04}\nu_4\tau_4(1-e^{-\epsilon/\tau_4\epsilon_{04}}) \cdots\cdots\cdots\cdots (29)$$

where $\epsilon=\epsilon_{04}t$ is used to eliminate time parameter t. The curve characterizing this relation is shown in Fig. 10. By drawing an asymptotic line and reading the slope and the point of intersection with σ–axis, the following values are determined for the given rate of straining ϵ_{04}.

$$\nu_4\tau_4 \text{ and } \nu_0. \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

If the curve (3) in Fig. 9 is considered that was obtained at the time of straining t_3, the quantities t_3/τ_1 and t_3/τ_2 are supposed to be large, whereas the quantity t_3/τ_4 is rather small, compared with t_3/τ_3. Hence the equation (28) takes the form,

$$\sigma=(\nu_0+\nu_4)\epsilon+\nu_3\epsilon_{03}\tau_3(1-e^{-\epsilon/\tau_3\epsilon_{03}}). \cdots\cdots (31)$$

The characteristic curve of eq. (31) is presented in Fig. 11. By fitting the curve (3) in Fig. 9 to the curve in Fig. 11, the following quantities can be evaluated for the known value of ϵ_{03},

$$\nu_3\tau_3 \text{ and } \nu_0+\nu_4. \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (32)$$

When the time of straining is increased up to t_2 in the next transient test, the value t_2/τ_1 is rather large in comparison with t_2/τ_2. Accordingly, the stress-strain equation is given with good approximation by the formula,

$$\sigma=(\nu_0+\nu_3+\nu_4)\epsilon+\nu_2\epsilon_{02}\tau_2(1-e^{-\epsilon/\tau_2\epsilon_{02}}). \cdots (33)$$

The characteristic plot of this curve may have the shape as presented in Fig. 12. By

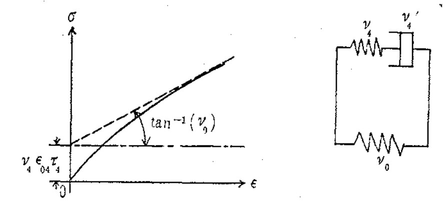

Fig. 10 Stress-strain curve around the time of straining τ_4.

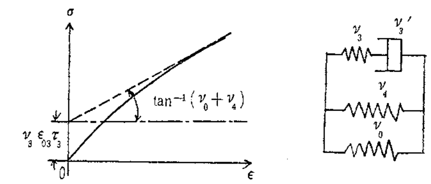

Fig. 11 Stress-strain curve around the time of straining τ_3.

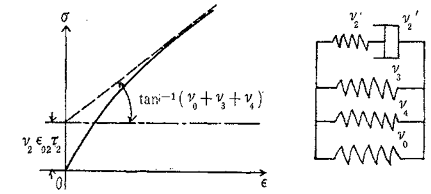

Fig. 12 Stress-strain curve around the time of straining τ_2.

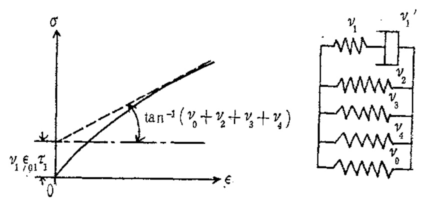

Fig. 13 Stress-strain curve around the time of straining τ_1.

fitting the factual curve (2) to the theoretical curve in Fig. 12, it is easy to obtain the quantities,

$$\nu_2\tau_2 \text{ and } \nu_0+\nu_3+\nu_4. \cdots\cdots\cdots\cdots\cdots\cdots\cdots (34)$$

Lastly, the stress-strain curve obtained at the fastest time of straining t_1 is fitted to the curve in Fig. 13 which is characterized by,

$$\sigma=(\nu_0+\nu_2+\nu_3+\nu_4)\epsilon+\nu_1\epsilon_{01}\tau_1(1-e^{-\epsilon/\tau_1\epsilon_{01}}). \cdots\cdots\cdots\cdots\cdots\cdots (35)$$

From the above equation, the following values are evaluated,

$$\nu_1\tau_1 \text{ and } \nu_0+\nu_2+\nu_3+\nu_4 \cdots\cdots\cdots\cdots\cdots\cdots (36)$$

With the use of a set of values in (30), (32), (34) and (36), it is possible to determine the values ν_0, ν_2, ν_3, ν_4 and τ_2, τ_3, τ_4, the values ν_1 and τ_1 being left undetermined. To determine ν_1 and τ_1, an additional result of transient test with the time of straining shorter than t_1 is required. Once all constants are thus evaluated, it is quite easy to construct a relaxation curve with the use of these visco-elastic constants. Diagram shown in Fig. 14 is helpful to under-

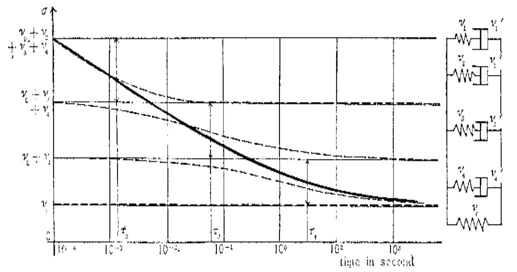

Fig. 14 A diagram illustrating the method of constructing a creep curve from the result of transient tests.

stand the method of constructing a relaxation curve. The procedure to be followed is firstly to plot the quantities ν_0, $\nu_0+\nu_4$, $\nu_0+\nu_3+\nu_4$ and $\nu_0+\nu_2+\nu_3+\nu_4$ on the σ-axis. By this procedure, individual quantitiy of stress to be relaxed by the corresponding Maxwell element is determined.

The next step is to assign the points of τ_2, τ_3 and τ_4 on the logarithmic scale in the abscissa. By drawing a set of relaxation curves which correspond to each Maxwell element and by superposing these relaxation curves, the complete configuration of relaxation curve can be obtained which covers the whole range of time of straining under consideration. In Fig. 14, relaxation curves corresponding to each Maxwell element are depicted with dotted lines and the superposed curve with solid line.

Finally, it should be noted that the same relaxation curve can be obtained by different sets of transient tests. In fact, if another set of transient test is performed at different times of straining from the previous test, each Maxwell element constructed from the latter may be of different shape. Nevertheless, the superposition of all stress components for which each Maxwell element is responsible will produce the same configuration of the relaxation curve as is found in the previous set of tests. In obtaining the relaxation curve in the specified range of time interval, it is clearly understood that the more closely the time of straining is spaced in the time scale, the more accuracy is expected in the resulting relaxation curve.

IV. SIMPLE MODELS USABLE IN THE PRACTICAL PROBLEMS.

It is much cumbersome for practical utility to take into account a number of elements in the generalized Voigt or Maxwell model. The choice of as small number of element as possible should be made for easier analytical treatment within the limit of required accuracy. Generally, the best choice is the one that satisfies the requirement that the selected model enables explicit solutions for the given problem to be found and further that it best fits the viscoelastic properties of actual material in the time interval under consideration. It is generally true that this choice is a simple model consisting of at most three elements. If the duration of applied load or strain in a considered problem is limited within the time interval $t_0 \leqq t \leqq t_1$, the contribution from the models distributed outside this time range can be made so small as to be negligible. Then the stress analysis resulting from the simple model is made equal to that for the same problem for any other viscoelastic material that has equal constants in the time range $t_0 \leqq t \leqq t_1$. The fact that the viscoelastic constants may be different outside the considered interval is of no significance.

In describing a creep or relaxation curve over the broad range of period the number of fractional elements to be superposed and their location, in the time scale of the retardation or relaxation time, could be arbitrary and are to be determined depending upon the required accuracy of the result. Such procedure followed in making up creep or relaxation curve suggests conversely the possibility of dividing the entire response into arbitrary number of components with arbitrary retardation or relaxation time. This division should be made according to the range of time appropriate to the practical application. It is impossible in discussing model fitting to lay down general rules valid in all cases. It is

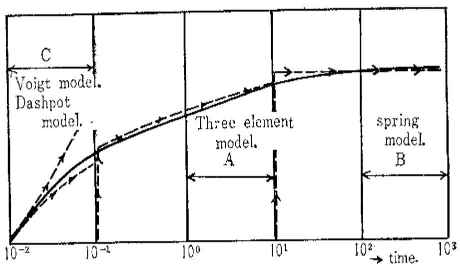

Fig. 15-1 Typical creep curve from which simple models are taken out.

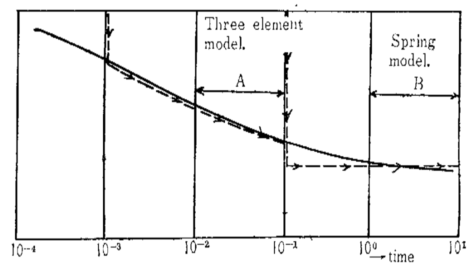

Fig. 15-2 Typical relaxation curve from which simple models are taken out.

only possible nevertheless to state guiding principles and the approximate ways in which simple models are made up. In what follows, the practical methods to derive some of simple models will be illustrated.

(1) Method of choosing simple models from a given creep curve.

The most preferable approximation can be made by the use of a three element model as shown in Fig. 16 or more rougly by a Maxwell model. In some range of time where the creep curve has particular simple shape, approximation is possible by the use of single spring.

In general, the plot on semi-logarithmic graph

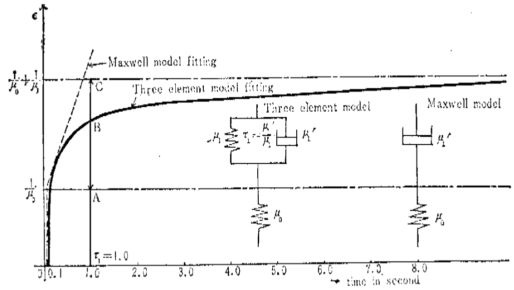

Fig. 16 Diagram illustrating the method of choosing simple models from creep curve.

is adopted as shown in Fig. 15 to express the creep properties so that a wide range of time may be covered in a limited space of paper. Hence in using this curve it is convenient to reproduce on the natural time plot that portion of the creep curve whose range of time is of current concern.

◦ Three element model.

We consider to make up a three element model from the creep curve given on Fig. 15 around the time of loading ranging from 1.0 sec. to 10 sec. This range is denoted by A in Fig. 15-1. A tentative method is illustrated as follows. The strain reached at 0.1 sec. (one-tenth of the smallest time under consideration), may be taken as the instantaneous strain component to determine the value of $1/\mu_0$. The largest strain attained within the limited time interval $1.0 \leq t \leq 10$ sec. may be taken as the ultimate strain to evaluate the value $1/\mu_0+1/\mu_1$. Once the initial unretarded spring constant and the final combined spring constant are determined and the curve on the semi-logarithmic plot is reproduced in the natural time scale plot, it is easy to evaluate the retardation time within the range of time concerned. In this fashion, it is generally possible to represent the behavior of a material with good accuracy by the use of the three element model within the specified range of time.

◦ Maxwell model.

The rougher approximation can be attained by the use of single Maxwell model. If the formul for strain response in the three element model is expanded in ascending series of t/τ, it follows that,

$$\epsilon=\left[\frac{1}{\mu_0}+\frac{1}{\mu_1}(1-e^{-t/\tau})\right]\sigma=\left[\frac{1}{\mu_0}+\frac{t}{\mu_1{}'}+\cdots\right]\sigma \quad \cdots\cdots(37)$$

This expansion is correctly applicable only when $t/\tau<1$. Hence it should be noticed that the use of Maxwell element as an approximation to three element model has to be restricted to the time range where the time of loading is less than τ, that is, $t<\tau$. The same viscoelastic constants as determined in making up a three element model can stand for those in Maxwell model.

◦ **Spring model.**

More use of a single spring may be permitted in that portion of the given creep curve where change in strain response is so small as to enable the creep curve to be regarded as flat straight line.

Let us consider the same creep curve as considered above. In that curve, the domain denoted by B may be replaced by a straight line as drawn with dotted line. The ray in the line indicates the route along which strain is assumed to go on. The elastic constant μ is determined straightforwards by reading the ordinate of the curve.

◦ **Voigt model.**

Approximation by means of single Voigt model may be allowed in such case in which the instantaneous component of strain response is particularly small compared with subsequent viscoelastic flow.

The zone in the creep curve which is simulated by Voigt model is indicated by C in Fig. 15-1. Since occurance of such zone seems rare in the time scale encountered in practice, its use may be limitted.

◦ **Dashpot model.**

When the rate of flow is predominant, compared with the instantaneous quantity of strain, it may be permitted to simulate the creep curve by a single dashpot. In Fig. 15-1, the domain designated by C typifies the strain response to be represented by a dashpot.

The viscous constant μ' can be evaluated by reading the slope of the creep curve that is depicted on the natural time plot.

(2) Method of choosing simple models from the given relaxation curve.

The method of choosing simple models from the given relaxation curve is much similar to the one followed in the case of creep curve.

The procedure is illustrated in Fig. 17. For example, we consider to take out a three element model valid for the time interval ranging from 0.01 sec. to 0.1 sec. from the given relaxation curve shown in Fig. 15-2. If the plot on the logarithmic time scale is transformed on the one in the arithmetic time scale so that the transformed plot may cover the relaxation curve within the time range $0.01<t<0.1$ sec. the relaxation curve shown in Fig. 17 is obtained.

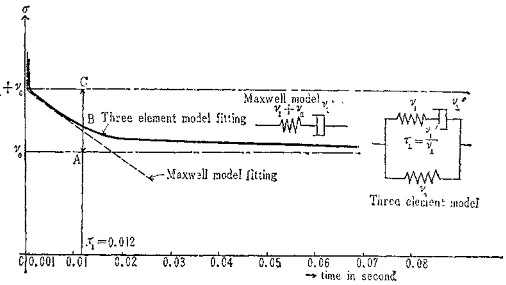

Fig. 17 Diagram illustrating the method of choosing simple models from a given relaxation curve.

As before the starting point of relaxation may be taken as the time 10^{-3} sec. (one-tenth of the shortest time of straining under consideration). The stress at the time 10^{-1} sec. may be taken as the one at infinite time.

The corresponding relaxation time τ_1 is easily determined from the reproduced relaxation curve.

The use of Maxwell model can be made instead of three element model. Expanding the formula for three element model in ascending power series of t/τ_1, we obtain,

$$\sigma=(\nu_0+\nu_1 e^{-t/\tau_1})\epsilon\doteqdot\left[(\nu_0+\nu_1)-\frac{t}{\nu_1{}'}+\cdots\right]\epsilon \quad \cdots\cdots(38)$$

Since the expansion is applicable only for $t/\tau_1<1$, it can be said that the use of Maxwell model is limited to such case where the time of straining is shorter than τ_1.

Approximation by a single spring may be

allowed in such case in which the relaxation curve is flat straight line in the time range under consideration.

V. EXAMPLES OF CREEP CURVES OF SOME ENGINEERING MATERIALS.

In the preceeding discussion, the principle of understanding transient stress-strain relation was illustrated based upon the linear viscoelastic theory in parallel with the method of its application. In fact, there have been conducted a great majority of transient tests on steel, concrete, soil, asphalt and so on, by many investigators. It was thought to be of great interest to consider these results in the light of the principle proposed above and to reproduce them in the form of creep curve or relaxation curve. Here the consideration will be confined to the test results on concrete and soils. The consideration for other engineering materials may be made in the similar manner.

(1) Concrete.

One of the earliest investigation as to the rate effect on concrete was worked out by Abrams[6]. The succeeding comprehensive investigations on the dynamical properties of plane concrete were carried out by Evans[7], Jones and Richart[8], and Watstein[9].

These investigators studied the dynamical properties of concrete as distinguished from the ones in the static condition.

The results of these tests all indicated the increases in the compressive strength and elastic constants with increase in the speed of loading or straining.

Quite resently, an extensive amount of transient tests was conducted by T. Hatano[10], and Takeda and Tachikawa[11] which aim at obtaining some information as to the effect of testing speed on the mechanical properties of concrete. The results are most suitably applied to the method stated above.

The procedure as exposed above was followed to evaluate the viscoelastic constants and the creep curves were drawn by the use of these constants. The creep curves obtained in this way are shown in Fig. 18 to 20.

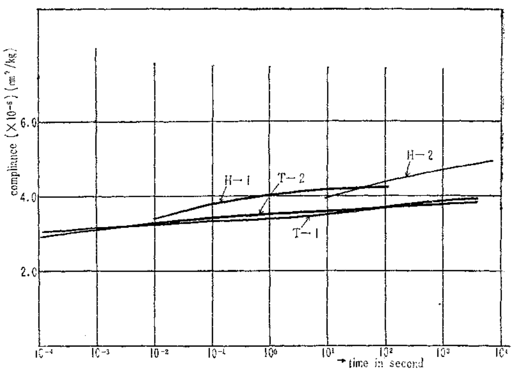

Fig. 18 Creep curve of concreete (Group I) $w/c=0.37\sim0.40$.

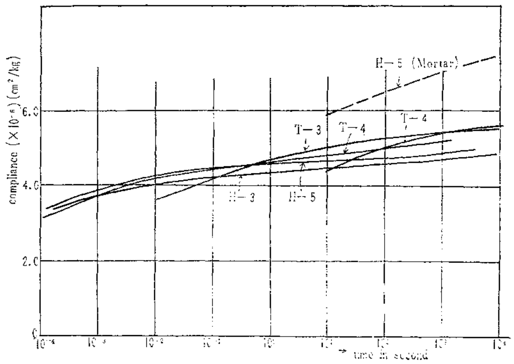

Fig. 19 Creep curve of concrete (Group II) $w/c=0.50\sim0.55$.

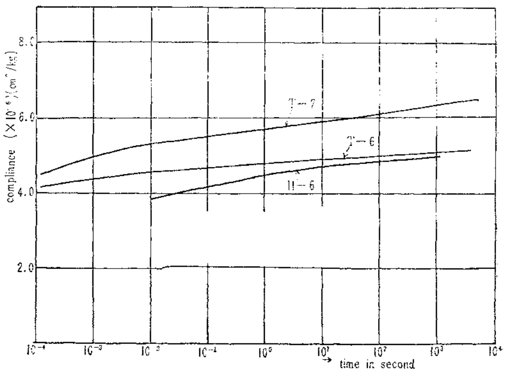

Fig. 20 Creep curve of concrete (Group III) $w/c=0.65\sim0.70$.

The test results from Hatano are designated by the notations H-1, H-2, ⋯ and those from Takeda and Tachikawa by the notations T-1, T-2, ⋯. The results of tests from these two sources can be classified into two groups in accordance with the water-cement ratio employed in the tests. In the group I, test specimens of relatively low water-cement ratio of

0.37 to 0.4 are included. Fig. 18 shows the creep curves pertaining to group I. The specimens of intermediate water-cement ratio ranging from 0.5 to 0.55 fall within the group II.

The creep curve pertaining to the group II are presented in Fig. 19. The specimen H-5 of the group II is made of mortar mixture lacking aggregate of large grain size. The group III composed of the specimens of relatively high value of water-cement content ranging from 0.65 to 0.7. In the Figs. 18 to 20, it is to be noticed that the ordinate represents the compliance. The reciprocal of compliance is equivalent to the nature of Young's modulus. A survey of these creep curve indicates that the value of compliance increases with increase in the water-cement ratio. This is consistent with the well-known fact that the modulus of elasticity increases with increasing water-cement ratio.

(2) Soil.

A considerable amount of investigations worked out by Casagrande and Shannon[12] seems to be the first on record of this kind of investigation, including a variety of soils from plastic clay to sand. Valuable physical interpretation was given by Seed and Lundgren to the behavior of sands during the rapid loading.

A comprehensive study on the effect of time of loading was also made by Whitman[14),15] on many kinds of soils ranging from very soft clay to coarse sand. Quite recently, F. Kawakami[16] presented the results of transient tests on the compacted soils obtained at the successive stages of time of loading. From the survey of these investigations, it is felt that the accurate information concerning the mechanical properties of soils is considerably difficult to be made known in its detail and much remains to be done at the present phase of investigation.

Nevertheless, rough consistent conclusions deducable from the knowledge of the past studies indicate an increase in the deformation modulus as well as the compressive strength in the transient test over those in the slow test. Although many involved factors such as pore pressure, void ratio and so on lead to the difficultly to specify straightforwards the viscoelastic properties of soils, it is planned herein to reproduce some of the tests data presented by some authors by means of the method proposed above.

The results for sandy soils are to be excluded from the current discussion, since it seems unreasonable to represent the mechanical properties of granular medium by means of the viscoelastic theory. Test data selected for the present discussion are of clay from Casagrande and Shannon and of clay from Kawakami.

The stress-strain curves obtained at various times of loading were analysed by using the proposed method and the viscoelastic properties

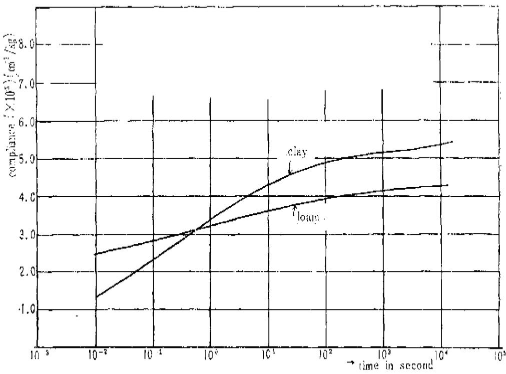

Fig. 21 Creep curve of clayey soils (From Kawakami)
clay : w=20%, s=72% loam : w=20%, s=73%

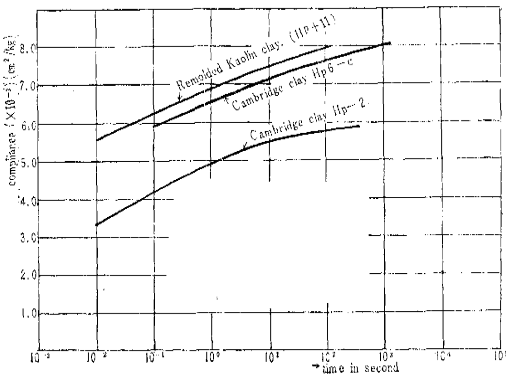

Fig. 22 Creep curve of clayey soils (From Casagrande and Shannon)
Remolded Kaolin clay : w=40~50%
Combridge clay HP-6 : w=30~51%
Combridge clay HP-2 : w=33~39%

determined from this analysis are demonstrated in Fig. 21 through Fig. 22 in the form of creep curve. The creep curve in Fig. 21 refers to the results from Kawakami, and those in Fig. 22 refers to the data from Casagrande and Shannon.

The original data are those which were obtained by uni-axial compression test, hence the reciprocal of compliance being equivalent to Young's modulus. Since the water contents employed in Casagrande-Shanno's tests are somewhat greater than those in Kawakami's tests, the compliances obtained are greater for the former than for the latter. The increasing tendency of the compliances with time as shown in these figures agrees well with the fact that the secant modulus of soils increases with increasing rate of loading.

NOTATION

ϵ : relevant strain component.

ϵ_0 : rate of strain.

ϵ_{0i} : rate of strain.

j : unit of imaginary number. $(=\sqrt{-1})$

t : time.

σ : relevant stress component.

σ_0 : rate of stress.

σ_{0i} : rate of stress.

ν_i : spring constant related to Maxwell element.

ν_i' : dashpot constant related to Maxwell element.

τ_i : relaxation time, or retardation time.

μ_i : spring constant related to Voigt element.

μ_i' : dashpot constant related to Voigt element.

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