

# FUNDAMENTAL STUDY ON THE POLLUTION CHARACTERISTICS OF GROUND WATER\*

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## Abstract

As a result of recent development of industries in Japan, pollution of ground water by industrial wastes is becoming very serious problem. However, we still have less knowledge about the pollution mechanisms of ground water.

As one of the fundamental studies, the author discussed the movement of pollutants in ground water through porous media, and then introduced theories to estimate the variation of concentration of pollutants in ground water. Furthermore, the author ascertained the validity of these theories by laboratory experiments using a filter column.

## 1. Introduction

Generally, the pollutants which have entered into ground water do not travel with the flowing liquid by the hindrance effects of the medium layer, such as the adsorption capacity, for example, and so these pollutants ordinarily appear a little later than the wet front of the liquid which contains them. Therefore, in order to discuss the pollution problem of ground water, we have to make clear the hindrance effects of the medium layer in the first place. In the following discussion, the author classified the hindrance effects into three cases and studied them separately.

(1) Dissolved Substances : The movement of dissolved substances in ground water is greatly affected by the adsorption capacity of the medium layer and is much slower than the movement of flowing liquid.

(2) Suspended Substances : These substan-

ces are hindered, for example, by the mechanical straining action of the interstices between the soil grains and cannot travel with the flowing liquid. Furthermore, the hydraulic factors such as velocity and loss of head of flowing liquid are greatly influenced as a result of the stoppage of the interstices. The velocity of the flowing liquid decreases and the loss of head increases. In most cases membrane of the suspended substances, which is formed close to the source of pollution, becomes an important factor. The author will call it the filtering membrane for convenience.

(3) Chlorine Ion : Approximately speaking, the movement of this ion in ground water is hardly affected by the adsorption capacity or the mechanical straining action of the medium layer and so moves with the flowing liquid. However, we have to take into account other factors. Some of these are the density current caused by the density difference between fresh water and saline water, and the diffusion current resulting from concentration gradient of chlorine ion.

In this paper, the author analysed the change of concentration of the dissolved substance and the suspended substance in ground water, theoretically, using the B.E.T.'s Adsorption Theory<sup>1)</sup> and the concept of accumulation rate based on the mechanical straining action of the interstices in the soil medium, respectively. Furthermore, the theories were verified qualitatively by the laboratory experiments using a filter column. The experiments were carried out by feeding the  $\text{CoCl}_2$  solution, the liquid suspended with kaolin and the diluted liquid of nightsoil.

## 2. Theoretical Consideration

### (1) Dissolved Substances :

Generally, the soil grains have more or less adsorption capacity and the movement of dissol-

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ved substances through the soil media is hindered.

Basing upon such a standpoint, Kaufman and others<sup>2)</sup> presented a theoretical rule governing the concentration of the substances dissolved in ground water. However, from the point of the kinetics of reaction velocity their theory was devoid of definite explanation about the time during which the adsorption capacity of the soil media reached its saturation value.

On the other hand Iwai and others<sup>3)</sup> analysed the change of concentration of dissolved substances in ground water using "The Langmuir mono-molecular layer adsorption theory" under an assumed ideal condition such that the waste liquid, containing only one dissolved substance, flows one-dimensionally, through a homogeneous porous soil media. However, the Langmuir's theory includes an assumption which states that energy of adsorption is the same level anywhere on the adsorbent, and is only applicable to the case such that the molecules comparatively ready to be adsorbed are adsorbed on the flat metallic surface. We can easily imagine that the multi-molecular layer adsorption is more ready to occur in the soil media.

From the above considerations the author introduced an equation of adsorption rate using "The B.E.T.'s multi-molecular layer adsorption theory" and analysed the behaviour of dissolved substances in ground water, theoretically. In this case, though the n-th multi-molecular layer adsorption has to be considered in general, the author assumed the di-molecular layer adsorption, in order to simplify the procedure of analysis.

From the B.E.T.'s theory, the rates of increase of the substance adsorbed in a unit volume of soil medium as mono-molecular layer and di-molecular layer, are expressed by the following equations, respectively.

$$\begin{aligned} \frac{\partial q_1}{\partial t} = & \text{(the rate of desorption from di-molecular layer)} \\ & + \text{(the rate of adsorption to bare layer)} \\ & - \text{(the rate of adsorption to mono-molecular layer)} \\ & - \text{(the rate of desorption from mono-molecular layer)} \end{aligned}$$

$$= b_2 \varphi \theta_2 + a_1 \varphi \theta_0 Cu - a_2 \varphi \theta_1 Cu - b_1 \varphi \theta_1 \dots \dots \dots (1)$$

$$\begin{aligned} \frac{\partial q_2}{\partial t} = & \text{(the rate of adsorption to mono-molecular layer)} \\ & - \text{(the rate of desorption from di-molecular layer)} \\ & = a_2 Cu \varphi \theta_1 - b_2 \varphi \theta_2 \dots \dots \dots (2) \end{aligned}$$

Here,

- C = concentration of the adsorbable substance contained in a unit volume of liquid
- q<sub>1</sub> = mass of the adsorbable substance adsorbed as mono-molecular layer in a unit volume of soil medium
- q<sub>2</sub> = mass of the adsorbable substance adsorbed as di-molecular layer in a unit volume of soil medium
- φ = a total number of adsorption beds contained in a unit volume of soil
- θ<sub>0</sub> = a ratio of beds where the adsorption has not been completed, to total number of the beds
- θ<sub>1</sub> = a ratio of beds where the adsorption has been completed as mono-molecular layer, to total number of the beds
- θ<sub>2</sub> = a ratio of beds where the adsorption has been completed as di-molecular layer, to total number of the beds
- u = linear velocity of the flowing liquid
- t = time
- a<sub>1</sub>, a<sub>2</sub>, b<sub>1</sub>, b<sub>2</sub> = constants.

At an equilibrium state of adsorption,  $\frac{\partial q_1}{\partial t}$  and  $\frac{\partial q_2}{\partial t}$  can be considered to be zero. Hence, from equations (1) and (2), the constants b<sub>1</sub> and b<sub>2</sub> are expressed as follows :

$$b_1 = a_1 \cdot C \cdot u \cdot \frac{\theta_{0c}}{\theta_{1c}}, \quad b_2 = a_2 \cdot C \cdot u \cdot \frac{\theta_{1c}}{\theta_{2c}} \dots \dots \dots (3)$$

Here, θ<sub>0c</sub>, θ<sub>1c</sub> and θ<sub>2c</sub> are values at an equilibrium state of θ<sub>0</sub>, θ<sub>1</sub> and θ<sub>2</sub>, respectively.

Applying these expressions of b<sub>1</sub> and b<sub>2</sub> to the equations (1) and (2), and considering that φθ<sub>1</sub> = q<sub>1</sub>, φθ<sub>2</sub> = q<sub>2</sub>, φθ<sub>0</sub> = φ(1 - θ<sub>1</sub> - θ<sub>2</sub>) = φ - q<sub>1</sub> - q<sub>2</sub>.

we obtain the following differential equation.

$$\frac{\partial q}{\partial t} = KuC\{S + f(x, t) - q\} \dots\dots\dots (4)$$

in which,

$$f(x, t) = -\frac{\beta}{K^2} \cdot \frac{1}{Cu} \cdot \frac{\partial}{\partial t} \left( \frac{1}{Cu} \cdot \frac{\partial q}{\partial t} \right)$$

$q = q_1 + 2q_2 =$  mass of the adsorbable substance stored in a unit volume of soil medium

$$K = \frac{\alpha(1 + \eta_2 + \eta_1\eta_2)K'}{\alpha + 1 + \eta_1 + \alpha\eta_2}$$

$$S = \frac{\varphi(2 + \eta_2)}{1 + \eta_2 + \eta_1\eta_2}$$

$$\beta = \frac{\alpha(1 + \eta_2 + \eta_1\eta_2)}{(\alpha + 1 + \eta_1 + \alpha\eta_2)^2}$$

$$K' = a_1, \alpha = \frac{a_2}{a_1}, \eta_1 = \frac{\theta_{0c}}{\theta_{1c}}, \eta_2 = \frac{\theta_{1c}}{\theta_{2c}}$$

Taking a small portion of soil, to which distance from the point that a pollutant has entered into ground water is  $x$ , the rate of concentration changing of the adsorbable substance contained in the liquid is quantitatively related, according to the Law of Conservation of Mass, to the accumulation of the substance adsorbed in the soil portion.

$$\lambda \frac{\partial C}{\partial t} + \frac{\partial q}{\partial t} = -\frac{\partial(uC)}{\partial x} \dots\dots\dots (5)$$

where,

$\lambda =$  porosity of the soil media.

The equation (5) is identical with the one introduced by Iwai and others<sup>3)</sup>.

From an idea similar to Iwai and others', the boundary and the initial conditions of the equation (5) may be taken as expressed by equation (6), which mean that at the point where  $x$  is zero  $C$  must be an original concentration of the adsorbable substance in the liquid,  $C_0$ , and that at any distance of  $x$ ,  $q$  should be zero so far as the wet front does not reach there.

$$\left. \begin{aligned} C(x=0, t) &= C_0, \\ q\left(x, t = \frac{\lambda x}{\bar{u}}\right) &= 0, \quad 0 \leq t \leq \frac{\lambda x}{\bar{u}} \end{aligned} \right\} \dots\dots\dots (6)$$

Here,  $\bar{u}$  is a mean value of  $u$  from  $t=0$  to  $t=t$ , and  $C_0$  is assumed to be constant.

Solving the simultaneous differential equations of (4) and (5) under the boundary and the initial conditions of (6),  $C$  and  $q$  should be obtained as a functional form of  $x$  and  $t$ .

However, due to complication of the procedure, the author solved these equations approxi-

mately as follows :

First of all, the author obtained the solutions of  $C$  and  $q$  for an assumption that  $f(x, t)$  in equation (4) is zero. Using these values of  $C$  and  $q$ , the functional form of  $f(x, t)$  can be determined explicitly. From this procedure, the following approximate solutions are obtained :

$$\left. \begin{aligned} \frac{C}{C_0} &= \frac{\eta(l, y)}{C_0 K \int_0^y \eta(l, y) dy + e^{KSi}} \\ q &= C_0 \cdot \frac{K \int_0^y \left\{ S + \frac{\beta S e^{KSi}}{e^{KSi} - 1 + e^{KC_0 y}} \right\} \eta(l, y) dy}{C_0 K \int_0^y \eta(l, y) dy + e^{KSi}} \end{aligned} \right\} \dots\dots\dots (7)$$

in which,

$$\eta(l, y) = e^{C_0 K y} \left( 1 + \frac{e^{KSi} - 1}{e^{C_0 K y}} \right)^{-\beta}$$

$$y = \int_0^t u dt - \lambda l$$

$$l = x$$

(2) **Suspended Substances :**

The movement of suspended substances in ground water may be hindered by the following actions :

- (a) Mechanical straining action of the interstices between soil grains.
- (b) Physical attraction between soil grains and suspended substances.
- (c) Inertia force induced by the microscopical changing of flow direction of the flowing liquid.
- (d) Sedimentation of suspended substances within the interstices between soil grains.

Furthermore, as a result of stoppage of the interstices between soil grains by the stored substances, the hydraulic factors such as velocity and loss of head of flowing liquid are greatly affected.

Generally, the stoppage of the interstices are remarkable in the vicinity of the inflow point of the substances to ground water and "the filtering membrane" is gradually developed there. This membrane, which is composed of soil grains and suspended substances, has great hindrance capacity because of its minute openings of the interstices and exerts much influence upon the behaviour of suspended substances in

ground water. After the first arrival of suspended substances the concentration in ground water will continue to rise to a maximum, after which it will fall as the substances will accumulate in the straining sites, suggesting that the accumulating substances enhance the straining removal<sup>4)</sup>. Krone and others<sup>4)</sup> have expressed this change of the concentration in ground water by a logarithmic function of the distance. However, the change of concentration has to be expressed by a functional form of not only distance but also time, and so their proposal having no consideration about the change of concentration with time cannot be accepted satisfactorily.

In order to simplify the procedure of analysis, the author considers an ideal condition that ground water which is containing only one suspended substance flows one dimensionally through porous media.

Taking a small portion of soil, to which distance from the point that a suspended substance has entered into ground water is  $x$ , the changing rate of concentration of the substance is related, according to the Law of Conservation of Mass, to the rate of accumulation of the substance stored in the soil portion.

$$\frac{\partial}{\partial t}(N + \lambda M) = - \frac{\partial(uM)}{\partial x} \dots\dots\dots(8)$$

in which,

- $M$  = mass of suspended substance contained in a unit volume of ground water
- $N$  = mass of suspended substance stored in a unit volume of soil media.

In this case, porosity,  $\lambda$ , and linear velocity of the flowing liquid,  $u$ , are not constant and are expressed by the functions of distance  $x$  and time  $t$ . However,  $\lambda$  and  $u$  can be related to each other by a continuous equation concerning volume of the flowing liquid.

$$\frac{\partial \lambda}{\partial t} = - \frac{\partial u}{\partial x} \dots\dots\dots(9)$$

Hence, from equations (8) and (9), the following differential equation can be obtained.

$$\frac{\partial N}{\partial t} + \lambda \frac{\partial M}{\partial t} = -u \cdot \frac{\partial M}{\partial x} \dots\dots\dots(10)$$

Corresponding to the equation of adsorbent

rate in case of dissolved substance, the author introduces an idea of the rate equation in case of suspended substance similarly.

The equation of rate, which the author will call "the equation of accumulation rate" for convenience, is defined as mass of suspended substance stored in a unit volume of soil media in a unit time. Here, it has been assumed that the hindrance action of soil media for suspended substance contained in ground water is mainly carried out by the mechanical straining between the interstices of soil grains. Furthermore, the author assumes that the equation of accumulation rate is expressed by the product of mass of the substance flowing into a small portion of soil in a unit time and a number of the interstices between soil grains existing in that portion at that time, and that suspended substance which was previously captured never drops off.

From these assumption the equation of accumulation rate can be expressed as follows :

$$\frac{\partial N}{\partial t} = A \varphi(1 - \theta) \cdot M \cdot u \dots\dots\dots(11)$$

where,

$$A = \text{constant.}$$

The symbols  $\varphi$  and  $\theta$  represent the total number of the interstices in a unit volume of soil media, and a ratio of the interstices, where the stoppage has been completed, to the total, respectively. Here, a number of the interstices means a number of openings in a cross section of soil media to the direction of flow and size of the openings is assumed to be constant in a small distance  $dx$ .

However, the suspended grains in ground water do not fit exactly to the openings of soil medium in general. In almost all cases suspended substance cannot close up the openings without some newly produced openings of smaller size and so  $\varphi$  cannot be considered to be a constant value.

Assuming that after suspended substance of mass  $N$  has been accumulated in a unit volume of soil media the interstices of a number  $\epsilon N$  are newly produced in it,  $\varphi$  can be expressed by the following equation :

$$\varphi = \varphi_0 + \epsilon N \dots\dots\dots(12)$$

Here,  $\varphi_0$  represents the number of the interstices which have initially existed in a unit volume of soil media and though the value of  $\epsilon$  changes depending upon size and shape for both the interstices and the suspended grains. It can be considered to be constant from a statistical viewpoint in case of the suspended grains of comparatively uniform size.

Therefore, combining the equations (11) and (12), and considering that  $\varphi\theta$  equals to  $N$ , the following rate equation of accumulation can be obtained.

$$\frac{\partial N}{\partial t} = A\{\varphi_0 - (1 - \epsilon)N\}Mu \dots\dots\dots (13)$$

However, as the number and size of the interstices may variously change depending on time and location, it is generally difficult to represent the rate of accumulation by one equation only.

On account of very small velocity of ground water, "the tunnelling effect", which means that suspended substance suddenly breaks into soil media through the openings of the filtering membrane broken by hydraulic force, can be neglected and in many cases soil medium zone can be distinguished from filtering membrane zone both in permeability and porosity. Therefore, in order to simplify the procedure of analysis, the author divides soil media into a filtering membrane zone and a soil medium zone, and applies the equations of accumulation rate having different constants to these zones, respectively.

(a) A filtering membrane zone

The continuous equation :

$$\lambda \frac{\partial M}{\partial t} + u \frac{\partial M}{\partial x} = - \frac{\partial N}{\partial t} \dots\dots\dots (14)$$

The equation of accumulation rate :

$$\frac{\partial N}{\partial t} = A\{\varphi_0 - (1 - \epsilon)N\}Mu \dots\dots\dots (15)$$

The boundary and the initial conditions may be expressed by equations (16), which mean that, at the point where  $x$  is zero,  $M$  must be an original concentration of suspended substan-

ce in the liquid,  $M_0$ , and that, at any distance of  $x$ ,  $N$  should be zero so far as the wet front does not reach there.

$$\left. \begin{aligned} M(x=0, t) &= M_0, \\ N\left(x, t = \frac{\int_0^x \lambda dx}{\bar{u}}\right) &= 0, \quad 0 \leq t \leq \frac{\int_0^x \lambda dx}{\bar{u}} \end{aligned} \right\} \dots\dots\dots (16)$$

(b) A soil medium zone

The continuous equation in this zone is identical with the equation (14), but the equation of accumulation rate is expressed by the following equation having constants,  $A', \varphi_0', \epsilon'$ .

$$\frac{\partial N}{\partial t} = A'\{\varphi_0' - (1 - \epsilon')N\}Mu \dots\dots\dots (17)$$

Assuming that at the starting point of the soil medium zone,  $x = \delta$  (the symbol  $\delta$  represents the thickness of the filtering membrane zone and is assumed to be approximately constant),  $M$  equals to  $M_\delta(t)$ , and that  $N$  should be zero so far as the wet front does not reach there, the boundary and the initial conditions can be expressed as follows :

$$\left. \begin{aligned} M(x=\delta, t) &= M_\delta(t), \\ N\left(x, t = \frac{\int_0^x \lambda dx}{\bar{u}}\right) &= 0, \quad 0 \leq t \leq \frac{\int_0^x \lambda dx}{\bar{u}} \end{aligned} \right\} \dots\dots\dots (18)$$

In order to obtain the solutions of  $M$  and  $N$  from the above equations, the differential equations (14) and (15) must be solved under the conditions (16).

The solutions are,

$$M = \frac{M_0 e^{M_0 A(1-\epsilon)(V - \int_0^x \lambda dx)}}{e^{A\varphi_0 x} - 1 + e^{M_0 A(1-\epsilon)(V - \int_0^x \lambda dx)}} \dots\dots (19)$$

$$N = \frac{\varphi_0}{1 - \epsilon} \cdot \left[ \frac{e^{M_0 A(1-\epsilon)(V - \int_0^x \lambda dx)} - 1}{e^{A\varphi_0 x} - 1 + e^{M_0 A(1-\epsilon)(V - \int_0^x \lambda dx)}} \right] \dots\dots\dots (20)$$

Therefore, solving the differential equations (14) and (17) under the boundary and initial conditions of equations (18) after the function of  $M_\delta(t)$  in the equations (18) has been determined from the value of  $M$  in the equation (19) where  $x$  equals to  $\delta$ , the solutions  $M$  and  $N$  can be obtained as a function of  $x$  and  $t$ .

$$\frac{M}{M_0} = \frac{e^{M_0 P_3 (V - \int_0^x \lambda dx)}}{[e^{P_4} - 1 + e^{M_0 P_3 (V - \int_0^x \lambda dx)}]} \times \frac{[e^{P_4} - 1 + e^{M_0 P_3 (V - \int_0^x \lambda dx)}]^{P_1/P_3}}{[\{e^{P_4} - 1 + e^{M_0 P_3 (V - \int_0^x \lambda dx)}\}^{P_1/P_3} + e^{-P_3} \{e^{P_2(x-\delta)} - 1\}]} \dots\dots\dots (21)$$

$$N = \frac{P_2}{P_1} \cdot \frac{[ \{ e^{P_4} - 1 + e^{M_0 P_3 (V - \int_0^x \lambda dx)} \}^{P_1/P_3} - e^{P_1 P_4} ]}{[ \{ e^{P_4} - 1 + e^{M_0 P_3 (V - \int_0^x \lambda dx)} \}^{P_1/P_3} + e^{P_1 P_4} \{ e^{P_2(x-\delta)} - 1 \} ]} \dots\dots\dots (22)$$

in which,

$$V = \int_0^t u dt, \quad P_1 = A'(1 - \epsilon'), \quad P_2 = A' \phi_0',$$

$$P_3 = A(1 - \epsilon), \quad P_4 = A \phi_0 \delta.$$

In the process obtaining the above solutions,  $\lambda$  has been assumed to be a known function. However,  $\lambda$  is practically related to  $N$  by the following equation.

$$\lambda = \lambda_0 - \frac{N}{m} \dots\dots\dots (23)$$

where,

$\lambda_0$  = initial porosity of the soil media,  
 $m$  = density of suspended substance.

Therefore, in order to find out the most possible form of function  $\lambda$ , the approximate method for calculation must be introduced. The procedures are as follows;

- (i) As the first approximation, the values of  $N$  are calculated from the equation (20) under the assumption that  $\lambda$  equals to  $\lambda_0$ .
- (ii) Using the values of  $N$  which have been obtained by the procedure (i), the values of  $\lambda$  can be determined from the equation (23).
- (iii) From these values of  $\lambda$  more accurate values of  $N$  can be calculated by the equation (20).
- (iv) The above procedures must be repeated till the values of  $\lambda$  are fixed.

Finally, the method of determining the constants  $P_1, P_2, P_3, P_4$  and  $\delta$  in the equations (21) and (22) should be explained for practical use.

While the filtering membrane has not fully developed yet,  $\delta$  in the equation (21) may be considered to be approximately zero and changes of concentration of suspended substance in flowing liquid can be supposed to depend mainly on the hindering action in the soil medium zone. Then, the equation (21) reduces to the following equation.

$$\frac{M}{M_0} = \frac{e^{M_0 P_1 (V - \int_0^x \lambda dx)}}{e^{P_2 x} - 1 + e^{M_0 P_1 (V - \int_0^x \lambda dx)}} \dots\dots\dots (24)$$

Expressing by a logarithmic form, we get,

$$\log_{10} \frac{M}{M_0 - M} = 0.434 M_0 P_1 \left( V - \int_0^x \lambda dx \right) - \log_{10} (e^{P_2 x} - 1) \dots\dots\dots (25)$$

Plotting the values of  $\left( V - \int_0^x \lambda dx \right)$  on the arithmetic scale and the values of  $\left( \frac{M}{M_0 - M} \right)$  on the logarithmic scale, the experimental data for small value of  $t$  will be expressed by a straight line. Hence, from the value of  $\left( \frac{M}{M_0 - M} \right)$  where  $\left( V - \int_0^x \lambda dx \right)$  equals to zero,  $B_1'$  and the slope, of the straight line,  $R'$ , the constants  $P_1$  and  $P_2$  can be determined as follows ;

$$\left. \begin{aligned} P_1 &= \frac{R'}{0.434 M_0} \\ P_2 &= \frac{1}{0.434 x} \cdot \log_{10} \left( 1 + \frac{1}{B_1'} \right) \end{aligned} \right\} \dots\dots\dots (26)$$

In contrast to this, after a certain time has elapsed and the filtering membrane has well developed, the hindering action in the filtering membrane zone plays an important role in the concentration change of suspended substance in flowing liquid. Compared with the hindering action in this zone, the hindering action in the soil medium zone can be neglected and suspended substance which has passed through the filtering membrane zone can be supposed to travel through the soil medium zone without serious loss of suspended substance. However, it must be noticed in this assumption that for large distance  $x$  the hindering action in the soil medium zone cannot be often neglected. In case where the above assumption is satisfied, the concentration of suspended substance in flowing liquid will be approximately expressed by  $M_s(t)$ , which represents the concentration of suspended substance in liquid at the end of the filtering membrane zone, but the time scale calculated in terms of cumulative discharge of ground water through a unit cross sectional area will be delayed the value of  $\int_0^x \lambda dx$ . Basing upon this, the constants  $P_3$  and  $P_4$  are determined as follows ;

From the equation (19), we get,

$$M_{\delta}(t) = \frac{M_0 e^{M_0 A(1-\varepsilon)(V - \int_0^{\delta} \lambda dx)}}{e^{A\varphi_0 \delta} - 1 + e^{M_0 A(1-\varepsilon)(V - \int_0^{\delta} \lambda dx)}} \dots\dots\dots (27)$$

Substituting  $V$  in the equation (27) for  $(V - \int_{\delta}^x \lambda dx)$ ,

$$M = \frac{M_0 e^{M_0 A(1-\varepsilon)(V - \int_{\delta}^x \lambda dx - \int_0^{\delta} \lambda dx)}}{e^{A\varphi_0 \delta} - 1 + e^{M_0 A(1-\varepsilon)(V - \int_{\delta}^x \lambda dx - \int_0^{\delta} \lambda dx)}} \\ = \frac{M_0 e^{M_0 A(1-\varepsilon)(V - \int_0^x \lambda dx)}}{e^{A\varphi_0 \delta} - 1 + e^{M_0 A(1-\varepsilon)(V - \int_0^x \lambda dx)}} \dots\dots\dots (28)$$

Expressing by a logarithmic form, we get,

$$\log_{10} \frac{M}{M_0 - M} = 0.434 M_0 P_3 \left( V - \int_0^x \lambda dx \right) - \log_{10} (e^{P_4} - 1) \dots\dots\dots (29)$$

in which,

$$P_3 = A(1-\varepsilon), P_4 = A \varphi_0 \delta$$

Therefore, from the value of  $\left(\frac{M}{M_0 - M}\right)$  where  $\left(V - \int_0^x \lambda dx\right)$  equals to zero,  $B$ , and the slope of the straight line,  $R$ , which will be approximately obtained from the experimental data plotted on the semi-logarithmic paper for the large value of  $t$ , the constants  $P_3$  and  $P_4$  can be expressed by the following equations.

$$\left. \begin{aligned} P_3 &= \frac{R}{0.434 M_0} \\ P_4 &= \frac{1}{0.434} \log_{10} \left( 1 + \frac{1}{B} \right) \end{aligned} \right\} \dots\dots\dots (30)$$

It is practically very difficult to measure the thickness of the filtering membrane zone,  $\delta$ . However, in discussing the behaviour of suspended substance in ground water, the distance from the incoming point of the substance to the point in question,  $x$ , is generally very large compared with the value of  $\delta$ , and so the approximation neglecting  $\delta$  in contrast to  $x$  will have no objection from the practical point of view.

### 3. Experimental Studies

In order to examine the adequacy of the theories introduced here, experiments using a filter column were carried out.

#### (1) The procedure of the experiments:

The experimental apparatus is shown in Fig. 1. The filter column was filled with rapid filter sand which was used at the Keage Water Purification Plant, in Kyoto City. The filtration

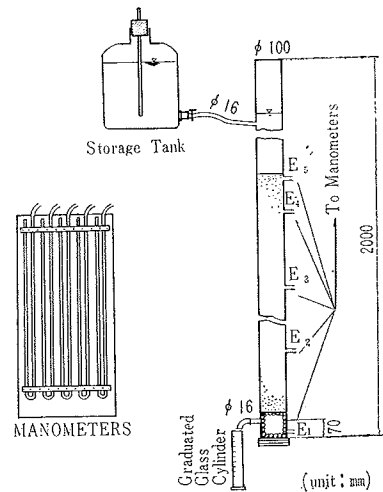


Fig. 1 Experimental Apparatus.

experiments were carried out under a condition such that the depth of water column above the sand surface was constant during each experiment.

The filtering velocity was calculated from the amount of filtrate flowed out from the bottom of the column and the loss of head in the sand column was measured by the manometers of U-shaped glass pipe. Furthermore, the concentration of a substance in the filtrate was determined.

#### (a) Filtration experiment with $\text{CoCl}_2$ solution :

The filtration experiment using  $\text{CoCl}_2$  solution was performed in order to examine the validity of the theory for dissolved substances.

The conditions in this experiment were as follows :

Property of media (sand)	specific gravity=2.57
	effective size=0.42 mm
	uniformity coefficient=1.57
Porosity	43.6%
Depth of the filter layer	99 cm
Liquid depth above the surface of sand layer	67 cm
Initial concentration of $\text{Co}^{++}$	286 ppm

After the filtering velocity was determined from the amount of the filtrate, the ionic concentration of  $\text{Co}^{++}$  in it was measured. This was carried out by the colorimetric method with 0.5% solution of Nitroso-R salts.

#### (b) Filtration experiments with kaolin or diluted liquid of nightsoil :

In order to examine the validity of the theory for suspended substances, filtration experiments with kaolin, which has negligible solubility to water, were carried out. Moreover, an experiment on diluted liquid of nightsoil was performed.

The conditions in these experiments were as follows :

(i) Experiments with kaolin

	Run No. 1	Run No. 2
Property of kaolin		
specific gravity	2.61	2.61
effective size	0.0042 mm	0.0042 mm
uniformity coefficient	1.38	1.38
Property of media (sand)		
specific gravity	2.57	2.60
effective size	0.42 mm	0.57 mm
uniformity coefficient	1.57	1.33
Porosity	45.2%	43.7%
Depth of the filter layer	97 cm	99 cm
Liquid depth above the surface of sand layer	69 cm	69 cm
Initial concentration*	899 ppm	943 ppm
Positions of manometers**		
1)	4 cm	4 cm
2)	34 cm	34 cm
3)		66 cm
4)	96 cm	96 cm
5)	107 cm	107 cm

\* measured as mass of suspended solid.

\*\* represented by the distance from the bottom of the column.

(ii) Experiment with diluted liquid of nightsoil

Property of media (sand)	specific gravity=2.57 effective size=0.42 mm uniformity coefficient=1.57
Porosity	45.3%
Depth of the filter layer	97 cm
Liquid depth above the surface of sand layer	69 cm
Initial concentration	57.8 ppm (C.O.D.) <sub>l</sub>
Positions of manometers	the distance of 4 cm, 34 cm, 74 cm, 92 cm, and 103 cm from the bottom of the column.

The nightsoil which was used here had been very finely pulverized by steam injection.

#### 4. Experimental Results

##### (1) Filtration experiment with $\text{CoCl}_2$ solution :

The data obtained by observing changes of

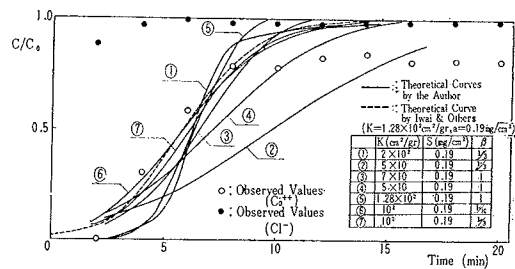


Fig. 2 Changes of Ionic Concentration of  $\text{Co}^{++}$  and  $\text{Cl}^-$  in the Filtrate with Time.

ionic concentration of  $\text{Co}^{++}$  and  $\text{Cl}^-$  in the filtrate with time are shown in Fig. 2, in which the ratio of concentration in the filtrate,  $C$ , to the initial concentration,  $C_0$ ,  $C/C_0$ , and time,  $t$ , are taken on the ordinate and abscissa axes respectively.

As  $\text{Cl}^-$  would approximately move with the flowing liquid, as stated in Article 1, it is supposed that liquid which has contained  $\text{Co}^{++}$  flows out from the bottom of the column just when  $\text{Cl}^-$  appears in the filtrate. On the other hand  $\text{Co}^{++}$  would travel more slowly compared with the flowing liquid, because of the adsorption capacity of the medium layer.

The filtering velocity in this case was constant at the level of 0.3 cm/sec.

In order to ascertain the adaptability of the theory introduced at the preceding article, the observed data of concentration of  $\text{Co}^{++}$  in the filtrate were compared with the theoretical results computed by the equations (7) in Fig. 2, in which theoretical curves for various values of  $K$ ,  $S$  and  $\beta$  are shown on account of the difficulty of determining the values of the constants. Moreover, the theoretical curve computed from the equation introduced by Iwai and others<sup>3)</sup> is shown in Fig. 2 for comparison.

As shown in Fig. 2, the theoretical curves qualitatively agree with the experimental results. However, the theoretical equation based on the B.E.T.'s theory is very difficult in determining the values of constants and also the numerical calculation of the equation is much complicated. Therefore, it is necessary to make a further study of the determination of the constants, in addition to the theory more applicable to the practical problem quantitatively.



(2) Filtration experiments with kaolin or diluted liquid of nightsoil :

Figs. 3 and 4 show the filtering velocities in Run No. 1 and Run No. 2 of the experiments in which kaolin is used as a suspended substance, respectively.

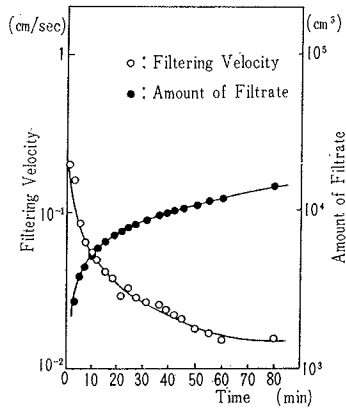


Fig. 3 Changes of Filtering Velocity and Amount of Filtrate with Time (Run No. 1, Experiment with Kaolin under Constant Head)

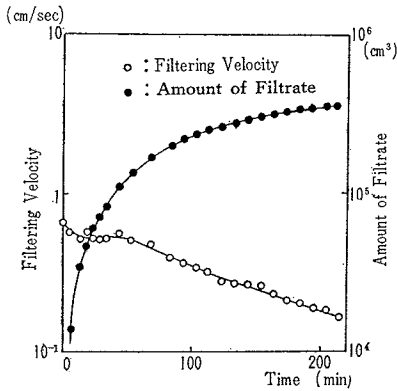


Fig. 4 Changes of Filtering Velocity and Amount of Filtrate with Time (Run No. 2, Experiment with Kaolin under Constant Head)

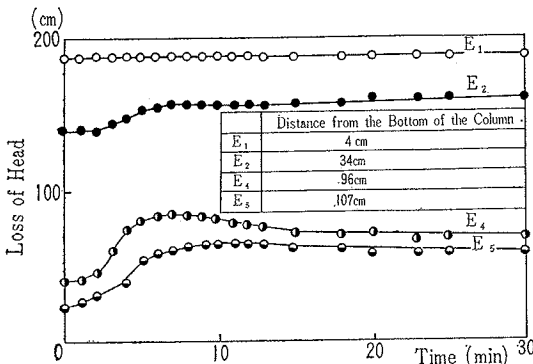


Fig. 5 Changes of Loss of Head with Time (Run No. 1, Experiment with Kaolin under Constant Head)

All of the filtering velocity curves in these experiments gradually decrease on account of the accumulation of kaolin particles in the interstices between sand grains.

Furthermore, changes of loss of head with time in Run No. 1 and Run No. 2 are shown in Figs. 5 and 6, respectively, indicating that they considerably fluctuate with time, too.

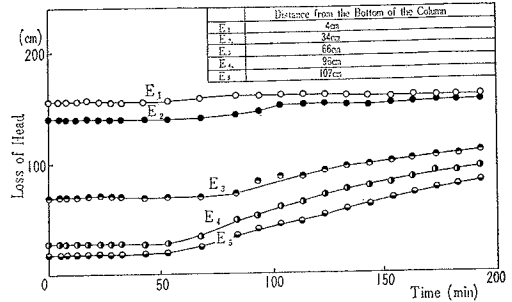


Fig. 6 Changes of Loss of Head with Time (Run No. 2, Experiment with Kaolin under Constant Head)

Figs. 7 and 8 show changes of concentration of kaolin in the filtrate in Run No. 1 and Run No. 2, respectively, in which the ratio of concentration in the filtrate,  $M$ , to the initial concentration,  $M_0$ ,  $M/M_0$  is taken on the axis of ordinate and time,  $t$ , on the axis of abscissa.

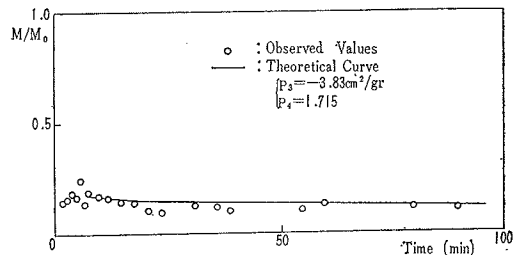


Fig. 7 Changes of Concentration of Kaolin in the Filtrate with Time (Run No. 1, Experiment with Kaolin under Constant Head)

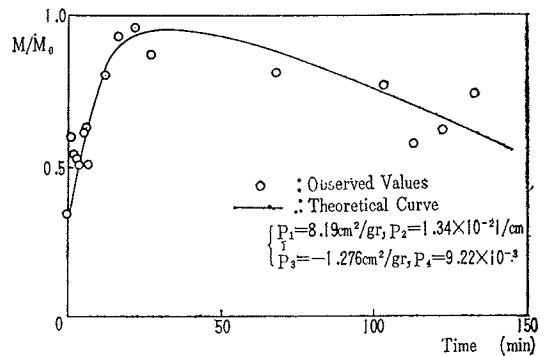


Fig. 8 Changes of Concentration of Kaolin in the Filtrate with Time (Run No. 2, Experiment with Kaolin under Constant Head)

It can be qualitatively understood from these figures that the concentration increases at first but begins to decrease because of the great hindrance capacity provided by the development of the filtering membrane zone. Therefore, the development of the filtering membrane zone is proved to be a great factor for the variation of concentration of suspended substances in flowing liquid.

The observed values of concentration of kaolin in the filtrate were compared with the theoretical results computed by the equation (21), in order to verify the applicability of the theory introduced by the author, and it turned out that the theoretical curves satisfactorily coincide with the observed values.

Finally, the similar considerations were performed on the experiment with diluted nightsoil.

Figs. 9 and 10 show changes of the filtering

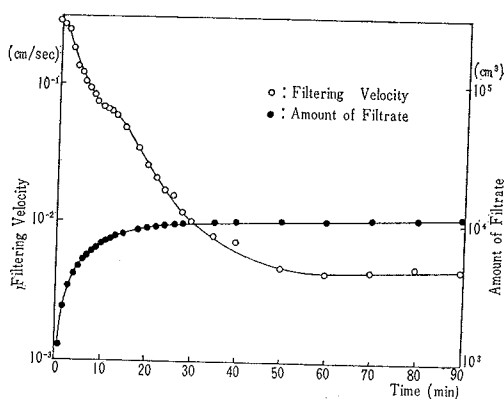


Fig. 9 Changes of Filtering velocity and amount of Filtrate with Time (Experiment with Diluted Liquid of Nightsoil under Constant Head)

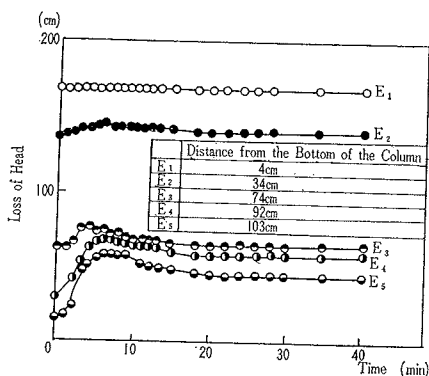


Fig. 10 Changes of Loss of Head with Time (Experiment with Diluted Liquid of Nightsoil under Constant Head)

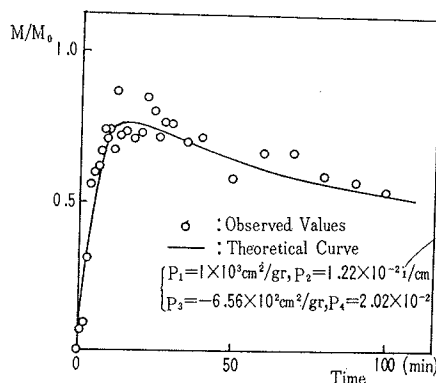


Fig. 11 Changes of C.O.D. in the Filtrate with Time (Experiment with Diluted Liquid of Nightsoil under Constant Head)

velocity and the loss of head with time, respectively. These figures qualitatively express the same variation as those with kaolin.

The observed values of changes of C.O.D. in the filtrate with time, which are shown in Fig. 11, indicate the behaviour similar to the observed values obtained in case of the experiments with kaolin. Furthermore, applying the theoretical equation for suspended substances to this case equally, the observed values satisfactorily agreed with the theoretical results computed by the equation (21).

### 5. Summary and Conclusions

As one of the fundamental studies, the author discussed the movement of pollutants in ground water through porous media, on the basis of an ideal condition that liquid, containing only one substance, is flowing one dimensionally through a homogeneous porous media of soil, and then introduced theories to estimate the variation of concentration of dissolved substances or suspended substances in ground water. Furthermore, laboratory experiments using a filter column were carried out to prove their validity.

#### (1) Dissolved Substances :

After obtaining the equation of adsorption rate for di-molecular layer adsorption from the B.E.T.'s multi-molecular layer adsorption theory, the author introduced a theoretical equation to express the behaviour of dissolved substances in ground water.

In order to examine the applicability of the

theory, the author carried out filtration experiments using a filter column with  $\text{CoCl}_2$  solution and obtained the satisfactory coincidence between the theoretical values and the observed values, qualitatively.

## (2) Suspended Substances :

The author introduced a equation of accumulation rate, which was defined as mass of suspended substances stored in a unit volume of soil media in a unit time, from an assumption that the hindering action of soil media on suspended substances would be produced by the mechanical straining action of the interstices between soil grains

The result of the author's theoretical analysis, equation (21), was verified to be qualitatively correct by the filtration experiments with kaolin.

Furthermore, the author ascertained that the theoretical equation for suspended substances could be satisfactorily applicable to the case of nightsoil, too.

Generally, in addition to the mechanical straining action of the interstices between soil grains, the behaviour of nightsoil in ground water is affected by many complicated factors such as the ion exchange capacity of soil media to Calcium ion, the bio-chemical removal and so on. Therefore, the behaviour of nightsoil in ground water should be practically very complicated.

However, considering that the observed values of the experiment on nightsoil show the tendency similar to the experimental data with kaolin and satisfactorily coincide with the theoretical results computed by the equation (21), it might give less error in the application

of the theoretical equation for suspended substances to a practical problem for nightsoil, too.

Above discussion is the basic study on an ideal case. However, the behaviour of pollutants in ground water is naturally more complicated, being affected by many factors such as soil formation, characteristics of soil, the valence of dissolved ions, pH of solution, the interference of coexisting electrolytes, the desorption of the substances which has been adsorbed in soil media, and so on.

Furthermore, though the author has separately discussed the behaviour of dissolved substances and suspended substances in order to simplify the analysis of the problem, both substances usually coexist in a natural case. Therefore, the interference between these substances must be considered.

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## References

- 1) S. Brunauer, P.H. Emmett, and E. Teller : Adsorption of Gases in Multimolecular Layers, Jour. Amer. Chem. Soc., 60, p. 309, (1938)
- 2) W.J. Kaufman, R.G. Orcutt and G. Kline : Underground Movement of Radioactive Wastes, Progress Report 1, Sanitary Engineering Research Laboratory, University of California, Sept. (1955)
- 3) S. Iwai, T. Goda, K. Koyama, and Y. Inoue : On the Removal of Dissolved Substances by Sand Layer, Jour. Japan Soc. Civil Engrs., Vol. 40, p. 664 (1955)
- 4) R.B. Krone, G.T. Orlob, and Carl Hodgkinson : Movement of Coliform Bacteria through Porous Media, Sewage and Industrial Wastes, Vol. 30, No. 1, Jan. 1958, pp. 1-13

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