STUDIES ON PHOTOGRAMMETRIC TECHNIQUES FOR PRECISE MEASUREMENT AND THEIR APPLICATION TO THE INDUSTRIAL FIELD

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Abstract

Research work in photogrammetry at the Institute of Industrial Science, Tokyo University, has been focussed on the application of terrestrial photogrammetry since 1949. At first, this research was applied mainly to the compilation of topographic maps for dam construction. Recently, photogrammetric techniques for use in industrial engineering fields were developed using short range terrestrial photography.

Among these developments are the applications of photogrammetry to the technique for drawing the body lines of automobiles, and to precision checking of hydraulic turbines.

This paper deals with the techniques, procedures, and results of short range terrestrial photogrammetry, and also includes as practical examples the results of the two above mentioned problems.

1. Introduction

Since 1959, The University of Tokyo has been engaged in projects relating to the application of photogrammetric techniques to engineering problems. Among these are the applications of photogrammetry to automobile models for obtaining the body line drawing, and to checking the complicated surface of hydraulic turbine runners. This paper deals with the photogrammetric techniques applied to the above mentioned problems.

1.1 Object of Automobile Measurements

The first step in the styling of an automobile is to make a clay model (which is improved upon) from which the body line drawings for the ultimate design are produced. But the classical method of producing the line drawings from the clay model is very laborious and time-consuming. Photogrammetry provides an alternative method which is much less laborious and is superior to the classical method in the following respects :

1) The automobile drawing can be produced in very short time with the required accuracy.

2) Accordingly, the body design programs and the other projects preparatory to the production of the automobile can start at the same time when the body lines have been drawn.

3) The drawing lines, such as the window border of the automobile, which is difficult to draw by the conventional method because of its three-dimentional aspects, can be easily and quickly obtained.

4) As the condition of the object at the time of photographing can be reproduced at any time later by using a plotting instrument, the results of plotting and measurement can be checked by inserting the photographic plates into the projector holder of the plotter.

5) As a result of the time saving nature of photogrammetric work, the jig facilities that were used to make the clay models are made avaliable for the next job.

6) The results of the photogrammetric measurements are given in section lines along three axes. These contours are produced by plotting a series of points along the contour with the plotting instrument and connecting these pointing by a smooth curve.

The co-ordinates of these points are also recorded in type on a sheet of paper by the EK 3 automatic co-ordinate printer attached to the plotting instrument. (Fig. 48)

1.2 Object of Hydrauric Turbine Measurement

Modern hydralic turbines may be divided into

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three types; the Pelton wheel, the Francis turbine and the Kaplan turbine. The type is selected according to operating head and other conditions. The form of the hydraulic turbine blades that is best suited for efficiency is generally determined by a theoretical calculation and an efficiency test of a model.

There is a runner in the center of a casing of the vertical shaft type Francis turbine, and guide vanes surround the outer periphery of the runner. The supply of water to the runner depends upon the opening of the guide vanes.

As the runner is generally made from cast iron, the angular direction and pitch of the blades are often incorrect at some parts. Moreover, the surface of runner blades consists of a complicated three-dimentional curve. Often this surface is cast with some deviations from the original design, and these cannot be easily checked and rectified.

The problem was to measure the complicated surface of the runner blades photogrammetrically, and to compare it with the original design figure. This request came from the Chairman of the Technical Research Committee of Hydraulic Turbines, Mr. T. Takagi, Chief of the Technical Department of Tokyo Electric Power Co. Ltd.

Formerly, the form of the surface of the runner blades was checked by measuring directly with a rule, but it is difficult to determine the correct fundamental plane for measurement. Therefore, Photogrammetric methods can be conveniently used for checking the surface form of the complicated shape, and also has the following advantages :

1) The condition of the object can be reproduced at any time later by using the plotting instrument as referred to earlier under 1.1, 4).

2) A uniform degree of accuracy is ensured in the measurement over the entire surface of the object, regardless of the complexities and magnitude of the exposed surface.

3) We can easily obtain longitudinal sections parallel to the outlet end of the guide blades, and circular sections at any radial distance from the centre of turbine.

4) The results of photogrammetric measurements are shown in two ways; plotted directly as contours, and also as typed co-ordinates at certain points along the fixed cross section lines on the object, as mentioned under 1.1, 6).

2. Principles of Photogrammetry

The principles of photogrammetry are based upon our optical function of stereoscopic vision by which it is possible to realize and measure the size and distance of objects in space.

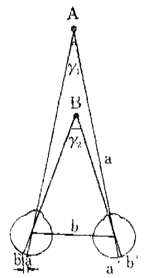


Fig. 1 Principle of optical function of eyes.

In Fig. 1, the convergence of the axes of the eyes when looking at points A, B is γ_1, γ_2 respectively, and $(\gamma_1-\gamma_2)$ is the fundamental factor that makes us perceive the distance between two points. These images, which are projected with the sharpest vision on the retina of our eyes, can be replaced by the photographic images which are taken of the same viewed objects by two cameras set at a certain distance from each other. In Fig. 2, the stereoscopic model

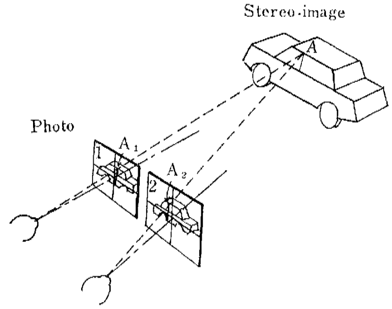


Fig. 2 Reproduction of stereoscopic vision with a pair of photographs.

can be reproduced as an intersection point A of the same photographic images A_1 and A_2. Fig. 3 shows the relationship between two camera positions and an automobile model in the case of terrestrial photogrammetry.

When drawing with the plotting instrument,

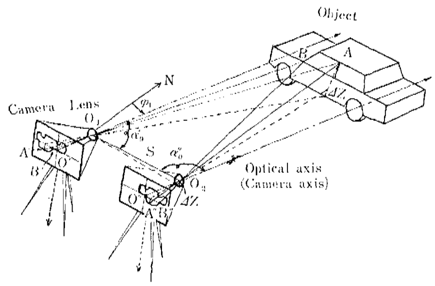


Fig. 3 Principle of terrestrial photogrammetry.

it is necessary to know the values of the inner and outer orientation. As the photography is done on solid ground or on a specially designed platform, the outer orientation of the camera can be determined very accurately.

The inner orientation of the cameras—the principal point, and principal distance—is known from the camera caribration; the outer orientation—the three coordinates x, y, z of each exposure station, the two angles ϕ and ω which determine the direction of the camera axis, and the angle κ which determines the location of the pencil of rays around the camera axis—is determined in connection with the photography.

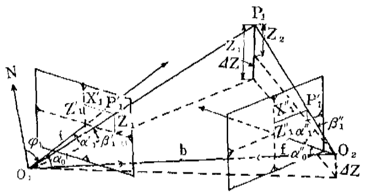


Fig. 4 Reconstruction of the object point P_1 by intersection of rays. (From Schwidefsky)

Fig. 4 shows the general principles for the determination of the coordinates of the point P_1. The rays towards P_1 from the two stations can be reconstructed from the image co-ordinates of the point P_1' and P_1''. From the left picture, we obtain the angles α_1 and β_1 which determine the ray O_1 P_1' from the co-ordinate plane as :

$$\tan\alpha' = \frac{X_1'}{c} \qquad \tan\beta' = \frac{Z_1' \cos\alpha'}{c}$$

In the same manner the corresponding ray from the right picture can be determined. As the direction angles of the camera axis are known from the photography, the direction angles of the rays O_1P_1 and O_2P_1 can be determined. The co-ordinates of P_1 can be determined by an intersection from the known points O_1 and O_2.

We can also determine the elevations of P_1 in order to check the measurements and computations. If there is a discrepancy between the elevations calculated for P_1, there are some errors in the measurements or computations. The control points can be used to find the discrepancy and to find the right orientation of the photographic plates as correctly as possible. Thus, the image reproduction in the plotting machine can be orientated by the intersection method numerically, optically and mechanically. In case of the automobile and the hydralic turbine, as the object was inmovable during the time it was being photographed, a low sensitive and specially fine grained negative plate was used.

One must pay special attention to the selection of photographing stations as they affect the direction errors in the section lines of the drawing. The direction of the optical axis is selected so as to be parallel to the centre line of the automobile in the case of side photography, and to be normal to the centre line in the case of front and rear photography. In case of the hydraulic turbine, the optical axis must be kept normal to the plane of the outer periphery of the runner. At each camera point on the photographing base line, the camera axis was normal to the line and was maintained in the horizontal, i.e., stereoscopic photography with parallel camera directions, perpendicular to the base.

Fig. 5 shows a section through the camera axis in the case of usual photography. The left

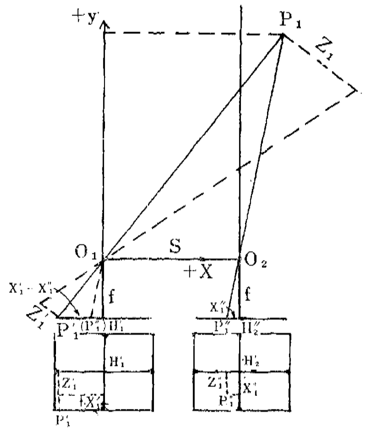


Fig. 5 Co-ordinate determination according to the normal stereophotogrammetric case. (From Hallert)

camera station O_1 is selected as the origin of the co-ordinate system and its three co-ordinates are introduced with the positive X-axis directed toward O_2, the positive Y-axis to the left camera direction, and the Z-axis positive upwards, perpendicular to the XY-plane. The image of the point P_1 is obtained on the left and right plate with the image co-ordinates X_1', Z_1' and X_1'', Z_1'' in the co-ordinate system, the origin of which is placed in the principal point of the left camera. The photographing base length is S and the focal length of the cameras, with no distortion lenses, is equal to f, and it is assumed that $O_1(P_1')$ is parallel to O_2P_1''. It is easily derived from the similarity of the triangles $\Delta O_1O_2P_1$, and $\Delta O_1P_1'(P_1')$ that :

$$\frac{y}{S}=\frac{f}{X_1'-X_1''}$$

$X_1'-X_1''=p$, the horizontal parallax, and substituting this expression in the above equation, one finds

$$y=\frac{S\cdot f}{p} \quad\cdots\cdots(1)$$

and in the same manner, one obtains the following expressions :

$$X=\frac{S}{p}X_1',\ Z_1=\frac{S}{p}Z_1',\ Z_2=\frac{S}{p}Z_1''$$

The point elevations may accordingly be determined in two different ways. A discrepancy in the elevation determination is known as vertical parallax. Normally in terrestrial photogrammetry, the accuracy of the measurements is so high that such vertical parallaxes can be neglected. Defferentiation of equation (1) yields

$$\frac{dy}{y}=\frac{dS}{S}+\frac{df}{f}-\frac{dp}{p} \quad\cdots\cdots(2)$$

dS, df and dp are assumed to be small errors in the base S, the camera focal length f, and the parallax p respectively.

If each term is treated separately ; one sees that ;

$$dy_s=\frac{y\cdot dS}{S}=\frac{f\cdot dS}{p} \quad\cdots\cdots(3)$$

$$dy_c=\frac{y\cdot df}{f}=\frac{S\cdot df}{p} \quad\cdots\cdots(4)$$

$$dy_p=\frac{y\cdot dp}{p}=\frac{y^2\cdot dp}{S\cdot f} \quad\cdots\cdots(5)$$

dS as well as df can be made so small by accurate measurement that they are of little significance in comparison to the influence of dp, which causes an error in y proportional to y^2.

If dp is assumed to be equal to 0.02 mm and $S=1.2$ m and $f=200$ mm, one obtains ;

$$dy=\frac{y^2\times 0.02}{1200\times 200}$$

For $y=5$ m, $dy=2.08$ mm : for $y=10$ m, $dy=8.32$ mm.

dp is due to a number of error sources, e.g., errors in the inner and outer orientation, the resolving power, the observational errors, etc.

If eq. (5) is written

$$\frac{dy}{y}=\frac{y\cdot dp}{S\cdot f}$$

One has the connection between the relative accuracy of y and the quotient S/y, which is called the base ratio.

If $dp=0.01$ mm and $f=200$ mm and

$$\frac{dy}{y}\leqq\frac{1}{1000},$$

the following is derived from the above expression ;

$$\frac{S}{y}\geqq\frac{1}{10}$$

that means, the base under the given conditions must not be less than one tenth of the maximum point distance, if the relative distance error is not to exceed 1/1000. On the other hand, a maximum base can also be determined with respect to the largest suitable parallax, which normally must not exceed 50 mm with respect to the stereoscopic vision and the measuring instrument,

From the eq. (1)

$$y=\frac{S\cdot f}{p}$$

is derived, using the above indicated $f=200$ mm, we get

$$\frac{S}{y}\leqq\frac{1}{4}.$$

3. Instruments

3.1 Phototheodolite

The basic requirements for the photographic apparatus of terrestrial photogrammetry are constant inner orientation of the surveying camera and the possibility of readily ascertaining the elements of outer orientation.

In the case of the automobile and hydraulic turbine, the objects have to be photographed

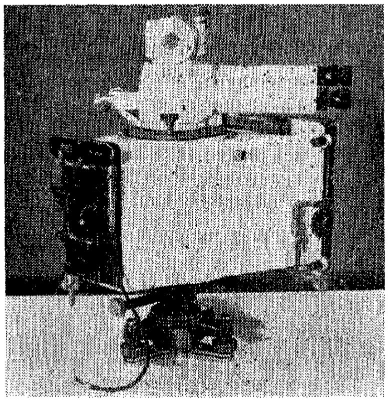


Fig. 6 C 3 B Phototheodolite.

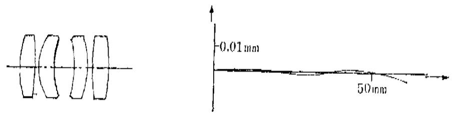


Fig. 7 Lens section and distortion of the Zeiss Orthoprotar (From Gruber)

from a close distance. Taking all these factors into consideration, a C 3 B Phototheodolite was used (Fig. 6,7)

This consists of a camera, and a theodolite to determine directions. This camera has three lens systems, separated from each other with 35 mm clearance vertically. The lens holder is 13 by 18 cm. The Orthoprotar normal angle lens with yellow filter has a fixed relative aperture 1 : 25. (Fig. 6)

The camera constant which allows photographing over an infinite range, is 193.30 mm, but it was extended to 200.30 mm for short range photography. (Fig 11)

Numerous experiment runs were performed to study the depth of focus which might result from shifting the plate, and various degrees of image sharpness were examined with respect to the distance of the object.

In the C 3 B Phototheodolite, the angle between the direction line of the theodolite and the camera axis may be adjusted to a right angle or to fixed angles with left and right horizontal angular displacement of 35 g.

But from the nature of the measurement for the automobile and turbine the angle selected was a right angle.

3.2 T 2 Theodolite and Ni 2 Level

As mentioned above, the direction of the base line for photographing is important and

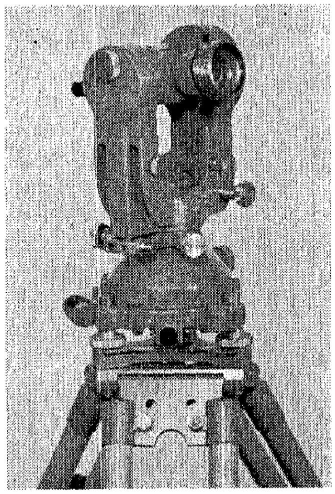


Fig. 8 T 2 Theodolite.

must be determined by precise measurement. For this purpose, a T 2 theodolite (Fig. 8) was used. Especially for setting the clay model of the automobile, two or three theodolites were used. For checking the elevation, and to know the difference of elevation of photographing stations, a Ni 2 level (Fig. 9) was used.

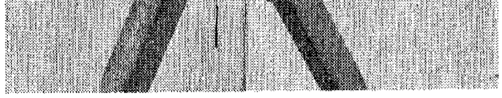


Fig. 9 Ni 2 Level.

3.3 Plotting Instrument : Wild A 7 Autograph

The A 7 Autograph is a plotting instrument with first order accuracy and with the mechanical projection principle which is used in both aerial and terrestrial photogrammetry. (Fig. 10) The space rods, for guiding the measuring mark which has a diameter of 4/100 mm, determine the direction of the model. This measuring mark can be moved horizontally to the left and to the right of the base direction by the X handle, in a direction at right angles to the base by the Y handle, and also vertically in the direction of Z by a foot disc.

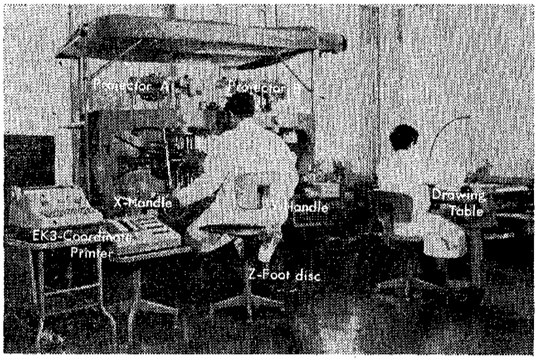


Fig. 10 Plotting Machine A 7 autograph.

These three movements can be measured, and recorded in type by an EK 3 co-ordinate counter with a co-ordinate printer. Furthermore, the X and Y movements can be transmitted by a transmission gear to a drawing table on which a plan of the spatial model can be mapped in a desired scale. By fixing the height movement of the foot disc, the object surface can be traced moving the measuring mark by the X and Y handles. The movement of the drawing pencil in this case follows a curved line on the surface of the object. We can also draw the figure of X–Z section by interchanging the motion between the Y and Z axis, and the Y–Z axis data can also be typed by the co-ordinate printer.

These contours are produced by making a series of points along the contour in the plotting instrument and connecting these points by a smooth curve, and sometimes these data are typed as co-ordinates and are punched on tape for calculation and plotting automatically in a electronic machine.

4. Problems of Photogrammetry applied to the Engineering field

At first terresterial photogrammetry was applied mainly to making topographic maps and measuring the excavation volume for dam construction purposes where access was difficult.
In these cases, the object is located at an appreciable distance from the camera stations, and the camera focal length allows photography focused at infinity, and the mapping scale extends from 1/200 to 1/1000. However, such precise measurements as required for the body line drawing of the automobile and contouring the hydraulic turbine, which have been carried out by the authors, were an unprecidented trial in Japan for which the object had to be photographed from a close distance and contoured at a large scale of between full size, that is 1/1, and 1/5. To fill these requirements, it was necessary to tackle and solve the special problems concerned which differ from the procedure for making the topographic maps for the dam construction mentioned above.

These special problems are as follows :

(1) Rearrangement of camera principal distance for the short range photography.

(2) Relationship between the object and the photographing stations :

- (a) Relationship between the photographing distance and the photograph ing base length.
- (b) Requisite number of photographing stations, and the spatial position of the photographing axes in relation to the object.

(3) Method of reflection to guard against glitter on the object, so that the object's surface is not obscured.

(4) Resonable shutter speed in photographing.

(5) Conditions of lighting for the object, and the development of the plates.

(6) Setting of the control points, and their number.

(7) Mapping and measuring procedures.

These fundamental experiments mentioned above have been under progress since 1959.

5. Fundamental Experiments

5.1 Rearrangement of the Camera Principal Distance for the Short Range Photography

The principal distance of the camera must be extended for the short range photography to get a clear image on the plate. For this purpose two small metal bars with a square cross section were attached to the rear side of camera box (see Fig. 11, 12). These were of two kinds with a width of 5 mm and 7 mm. The principal distance was measured in two ways, one by using the optical bench at the laboratory, and

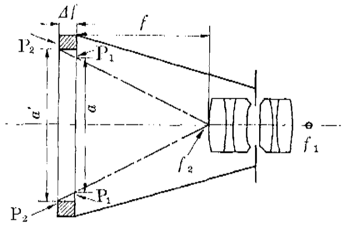


Fig. 11 Camera section of C 3 B Phototheodolite Two small metal bar attachments for short range photogrammetry (width Δf=5 mm & 7 mm, in Figure)

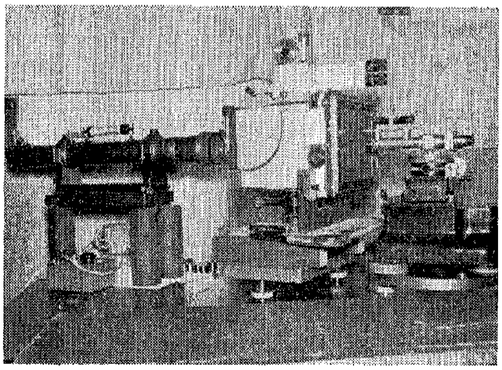


Fig. 12 Equipment for measuring the principal distance.

the other by measuring the distance of a measuring mark on the negative plates with and without attaching the metal bars.

5.2 Experiments on the Plate Characteristic

In photogrammetry, the photographic plates are the basis of the subsequent plotting. It is therefore indispensable to know the characteristics of the photographic plates, in order to have good reproduction images. In the case of the automobile and the hydraulic turbine, artificial illumination was used as it was important to maintain a uniform brightness on the surface of the object, that is to say, the distribution of the reflection brightness should be reproduced with a good image density which lies on the straight line part of the photographic emulsion's "Characteristic Curve" for the whole surface area of the object.

As terrestrial photography is taken from solid ground or specially designed platforms, superfine grained emulsion can be used regardless of the shutter speed, but the sensitivity and the grain of the emulsion are of opposite characteristics, i.e., the highly sensitive layers have a coarser grain than the less sensitive ones.

The resolving power of the emulsion is also of great importance for survey photographs. High resolution enables us to reproduce fine details clearly and also affects the apparent sharpness of the image. Therefore, one must first find the relationship between the sensitivity and the grain of the emulsion for a certain plate. The next problem is to clarify the characteristics of the resolving power of the plate and lens. Two plates were tested, Prochrome Plate and SG Plate of Sakure Co Ltd., which are popularly used in terrestrial photogrammetry in Japan. The graph of Fig. 13 shows the sensitivity and spectrum of these plates.

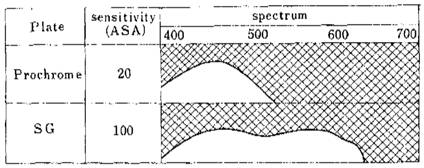


Fig. 13 Sensitivity and spectrum of plate for terrestrial photogrammetry.

5.2.1 Characteristic Curves

The density of the negative is, as mentioned above, only approximately in direct proportion to the illumination. The relationship between the density and the illumination is given in the so-called "Characteristic Curve" from which the plates' behaviour may be seen.

On the abscissa, the illumination is entered in logarithmic scale, while the ordinate is the standard for the density. The Characteristic Curves were calculated according to the specifications of N.S.G. (Nippon Shasin Gakkai method for sensitometry). The chief use of the characteristic curve is to determine the sensitivity or speed of the plates, the contrast, the latitude of exposure, and the manner in which tones will be reproduced. The light source which is adopted internationally is a tungsten filament electric lamp operated at a colour temperature of 2360° K., and screened with a daylight filter (Davis-Gibson). (Fig. 14, 15). A typical intensity scale sensitometer is a step tablet or optical wedge

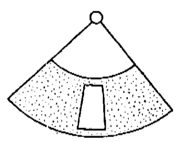


Fig. 14 Pendulum shutter.

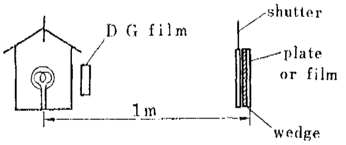


Fig. 15 Equipment for making optical wedge. N.S.G. (Nippon Shasin Gakkai method)

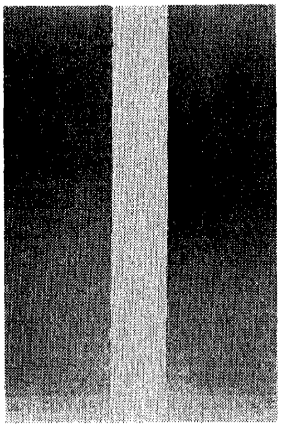


Fig. 16 Optical wedge.

consisting of a series of areas having precisely known light transmissions (Fig. 16). Development of the exposed strips was carried out in Konidol Fine Developer (fine grained) and Konidol D.P. (standard) for the time of 6,8,10 and 12 minutes at a standard temperature 20°C ±0.5°C.

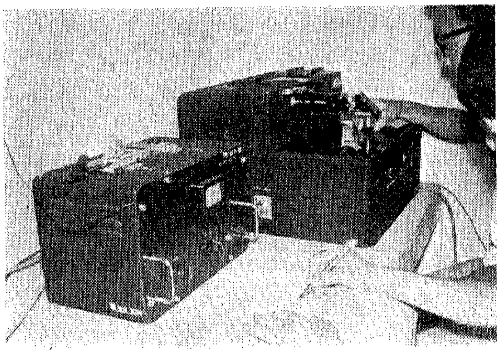


Fig. 17 Automatic Balancing Densitometer. (Narumi Co. Ltd.)

Density is measured in a Automatic Balancing Densitometer made by Narumi Co. Ltd. (Fig. 17). Characteristic Curves are drawn by plotting the densities against the logarithms of the exposure. The straight middle line corresponds to correct exposure. The inclination of this straight part of the curve is a measurement of the gradation of the negative. The inclination is expressed mathematically by the tangent of the inclination angle, and was called by Hurter and Driffield gamma (the Greek letter γ). The gamma curve is frequently used as a means of expressing contrast (Fig. 18,19,20 & 21).

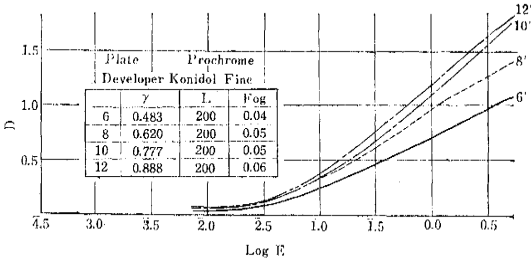


Fig. 18 Characteristic curve.

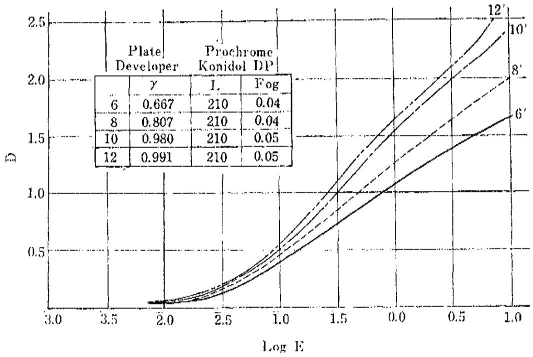


Fig. 19 Characteristic curve.

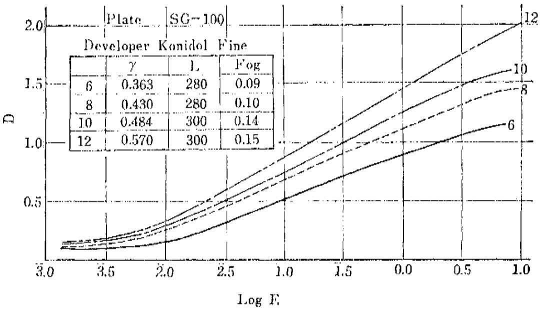


Fig. 20 Characteristic curve.

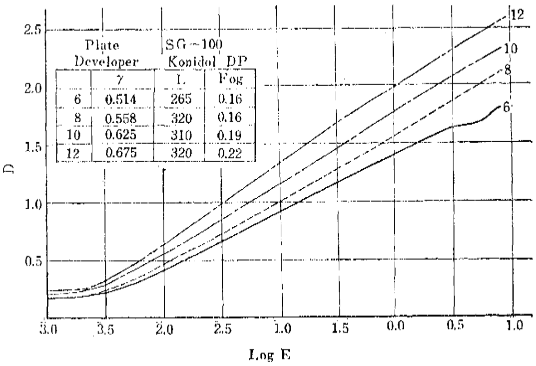


Fig. 21 Characteristic curve.

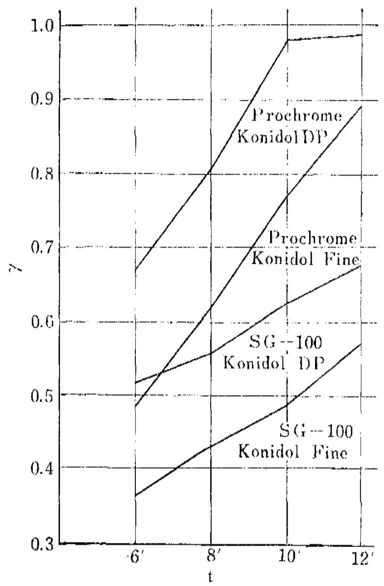


Fig. 22 t-γ curve.

From these experimental results, the t-γ curve (Fig. 22) was produced, and we can conclude as follows :

1) As the time of development increases, in the case of the same plate and developer, the density increases.

2) Prochrome plate shows generally bigger values than the SG-100 plate and has sharper contrast.

3) Both plates have a considerably wide range of latitude, but SG-100 plate has a wider latitude than Prochrome plate.

4) In the case of Prochrome, so little fog appears that it can be neglected, but SG-100, shows a remarkably noticiable fog when it is developed for more than ten minutes in the developer Konidol DP.

5) SG-100 shows a wide range in latitude in the underexposure region.

6) Generally it is prohibited to develop the SG-100 plate for more than ten minutes from the view point of fog, and it is better for Prochrome plate to have a developing time of 10 minutes for Konidol DP developer, and 12 minutes for Konidol Fine developer.

5.2.2 Resolving Power of Photographic Emulsion and Camera Lens

5.2.2.1 Resolving Power of Photographic Emulsion

Resolving power is defined by the ability to distinguish fine detail in the photograph. It is usually expressed as the number of lines per milimeter which can just be separately distinguished in the photograph. The following shows the resolving power measured by Koana type resolving power machine.

5.2.2.2 Resolving Power of Camera Lens

Resolving power of a lens is also defined as the number of lines per millimeter which can just be separately distinguished in the photographic image plane.

Resolving power of the ideal lens is approximately expressed as;

$$R=1/1.22\lambda F$$

where R = resolving power, λ = wave length, F = F-number of lens. In this equation, the F-number is generally inversely proportional to the resolving power, but in the case of a small F-number, the resolving power is almost governed by its opening aberration and its opening diffraction in the case of the largest F-number. Accordingly, the largest resolving power is usually in the range of 1:5.6–1:8. Fig. 23 shows

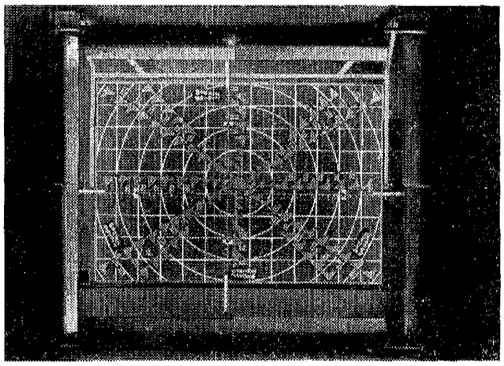


Fig. 23 Chart for resolving power measurement.

the chart for resolving power measurement. This chart is used to find the limitation of the smallest line spacing that may be separately distinguished in various types of pattern, each of which is shown in three lines with the same clearance. The following graph shows the results of measurements of the resolving power of the lens (Table 1). The standard specification which is authorized by the Association for Camera Testing shows 27 lines per one millimeters for 35 mm film and 16 line per one millimeter for 6×6 film as the standard.

Table 1

Plate	Resolving Power	
	Central Part of Photograph	Marginal Part of Photograph
SG Plate	19.2	25.8
Prochrome	26.1	29.0

In the case of terrestrial photogrammetry, where the plate format is 13×18 cm, it is easy to enlarge from 6 to 10 times the original negative size, judging from the resolving power of this lens. In fact, one must take into consideration the practical resolving power combined with the theoretical resolving power and the reproduction of the test images ; this combination is called the response function.

5.3 Limitation of Measurement Accuracy in One Plane

In the case of photogrammetry, the accuracy of measurements in the plane of the photographs is generally higher than measurements perpendicular to this plane. Fig. 24 shows a grid

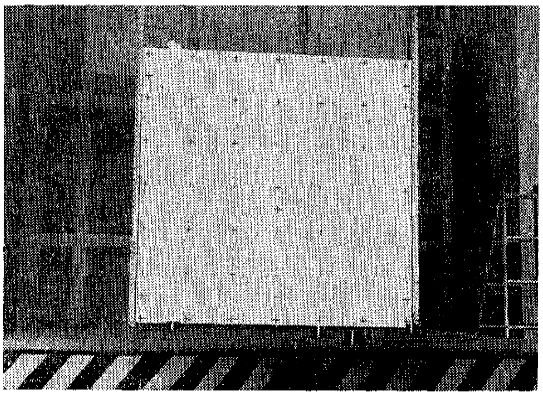


Fig. 24 Grid test board.

test board 2.4 m by 2.4 m with a grid interval of 40 cm drawn very accurately. The board was welded in place so as to occupy the same position as the object in the case of actual short range terrestrial photography. The base line was set parallel to the plane surface of the board which was placed at a distance of 6.7 m from camera station. The grid was plotted with an A 7 Autograph. Each co-ordinate of grid intersections were typed out five times by an EK 3 automatic co-ordinate printer and were compared with the direct measurement value. Fig. 25 shows the vectors of the discrepances between the photogrammetric measurements and

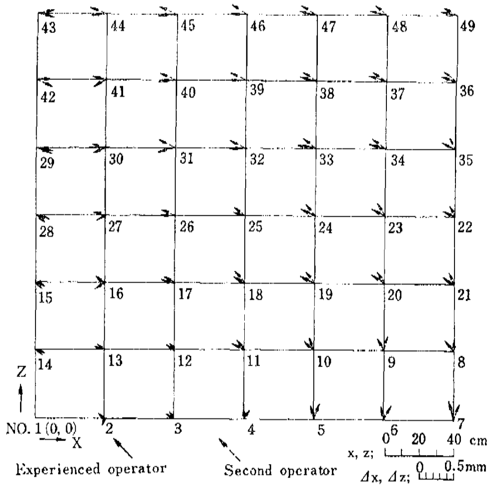


Fig. 25 Vector of discrepancy in a plane. (Grid test)

the direct measurements. The photogrammetric measurements were made by two operators, one of whom was more experienced. The co-ordinates of each grid intersection were shown as the mean value of five readings. The mean square errors are as follows ;

1. Measurements of experienced operator :

$$\varepsilon_1 = \sqrt{\frac{\Sigma(\Delta x)^2}{47}} \fallingdotseq 0.16 \text{ mm for } \Delta x$$

$$\varepsilon_2 = \sqrt{\frac{\Sigma(\Delta z)^2}{47}} \fallingdotseq 0.07 \text{ mm for } \Delta z$$

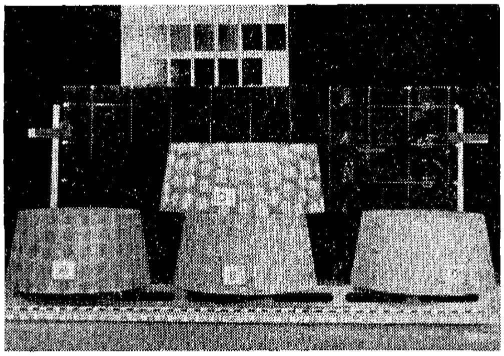
2. Measurements of second operator :

$$\varepsilon_1 = \sqrt{\frac{\Sigma(\Delta x)^2}{47}} \fallingdotseq 0.15 \text{ mm for } \Delta x$$

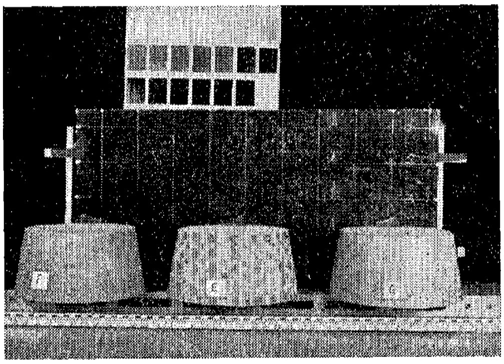
$$\varepsilon_2 = \sqrt{\frac{\Sigma(\Delta z)^2}{47}} \fallingdotseq 0.10 \text{ mm for } \Delta z$$

5.4 Fundamental Test to Select the Most Suitable Surface Finish of the Object

In photogrammetry, as mentioned above, it is of major importance to obtain the negative plate with a good image quality that can easily be measured in the plotting instrument. The photographs are taken at an indoor studio with uniform illumination, but the surface of object tends to glisten and to show bright spots of light reflection, or highlights. These highlights adversly affect the image quality. In order to test several surface finishes, as a protection against glistening and highlights on the object's surface, several kinds of test pieces were made having a predetermined shape measured by direct measurement. Fig. 26 and Fig. 27 show the test patterns painted; the former is for the

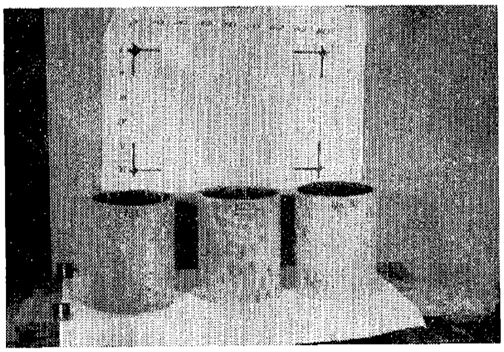


(a)

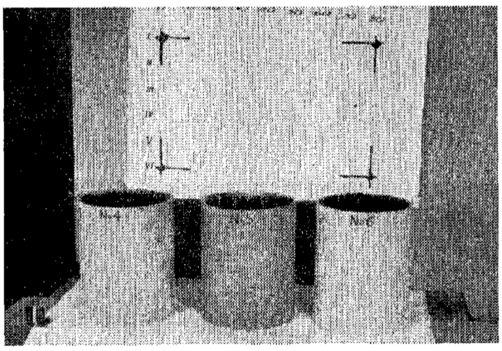


(b)

Fig. 26 Patterns of pointing test for automobile measurement.



(a)



(b)

Fig. 27 Patterns of painting test for hydraulic turbine measurement.

automobile measurement and the latter is for the hydraulic turbine measurement. Asa result of these tests, a painting technique for the model was specified as follows :

1) The thickness of the paint has to be thin enough so as not to have any effect on the photogrammetric measurements.

2) The paint should have a matt finish, and must not show irregular effects caused by the illumination, the photographing distance, or the position of camera lens in space.

3) The model should be painted in a light colour with random spots of a darker colour.

4) The painting should be in a pattern that aids the operator to make measurements in a plotting instrument.

5) The painting must be easily removed : for this reason it should be a water soluble paint.

In taking the photographs, the problem is the relationship between the automobile and the photopraphing stations. To photograph and measure rationally, one must consider the various factors which influence the photographing and measuring procedures and, moreover, their relationship to the measurement accuracy. Taking into account these considerations, the following experiments were performed in connection with the photography.

1) The effect of distance between the photographing stations and object.

2) The effect of base line length.

3) A comparison of photography from front and side stations.

4) The effect of changing the camera station's position along the base line.

5) The effect of different orientation marks.

6) The effect of personal ability and experience of the measurement techniques.

7) The effect of density differences of the photographic image.

5.4.1 Painting Tests for Automobile Measurement

5.4.1.1 Test Piece for Painting Tests

These conical test pieces of Hineko pine are shown in Fig. 26. Their shape was measured directly with a rule for comparison with the photogrammetric measurements. The seven classes of paint combinations of background colour and spot colour used are shown in the following table.

Painting type	Background colour	Spot colour
A	Whitish grey	black
B	a little darker gray than A	grey
C	the same as B	no spots for the left half. red for the right half.
D	a grey colour	light grey
E	the same as B	darker grey than B
F	darker grey than B	no spots
G	vanish with lacquer	no spots.

These test patterns were set before a grid board with 20 cm grid which was used for the orientation. The board surface direction was parallel to the photographing base line. Fig. 28 shows the relationship between the test patterns and the three photographing base lines, at a distance of 4.5 m, 5.5 m and 6.5 m respectively from the test pattern. Prochrome and SG-100 plates were used and were developed for 12 minutes in the Konidol fine developer and 10 minutes in the Konidol DP.

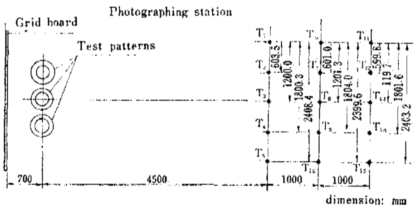


Fig. 28 A plan of test patterns and photographing stations.

5.4.1.2 Measuring Method

At the extreme end of the three cross sections on the surface of the test piece, cross marks were set to determine the measured section Photogrammetric measurements of depth at each 50 mm interval point, or each cross section, were compared with the direct measurement values. The mean value of five measurements was taken. (Fig. 29)

5.4.1.3 Comparison of Measurement results

Comparing photogrammetric data with the direct measurements, the following conclusions were arrived at :

(1) As all the test patterns have a circular section, the depth readings around the sides of the cylinders show a general tendency to larger errors with fairly large dispersion, than the

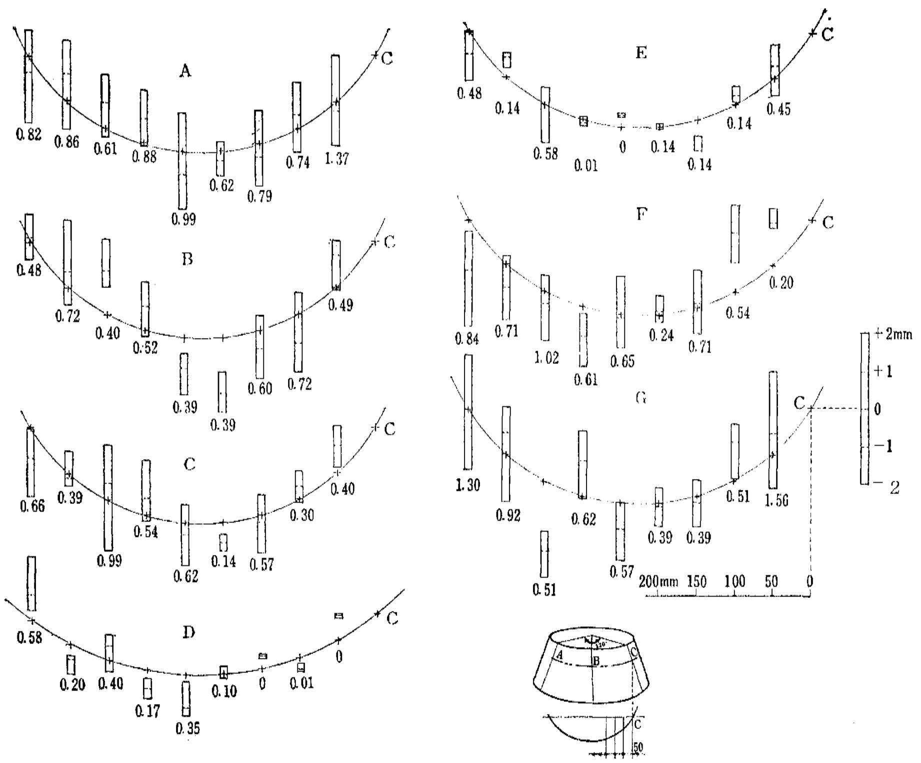


Fig. 29 Figures denote standard deviation.

readings of the cylinder fronts.

(2) The depth readings of test pieces D and E show the smallest dispersion of errors.

(3) The fact that test piece C shows greater dispersion of errors on that half without spots than on that half with red spots, shows that painting is an important factor.

5.4.2 Painting Test for Hydraulic Turbine Measurement

5.4.2.1 Test Pieces for Painting

Cast iron test pieces of the same material as the turbine were used, in the shape of a circular pipe with a diameter 160.02 mm and a height 400 mm each. Six classes of painting were selected in order to find the most suitable surface finish for the photogrammetric measurements. The results of the measurements were compared with the values of the direct measurements. Fig. 30 shows the relotionship between the camera stations and the test pieces in question.

5.4.2.2 Result of the Measurements of the Test Pieces

The results of the measurements and photographing of the test pieces were followed by the same procedures as described in article 5.4.1 and shown in Fig. 31. Two measurement cross section lines were chosen. As shown in the Fig. 31, one is in the lateral direction and the other is in the longtudinal direction. The dispersion errors of the measurements are shown in the form of a frequency graph. (Fig. 32) The ordinate is the frequency, and the abscissa is the error in millimeters.

5.4.2.3 Calculation of the Radius and Comparison with the Actual Radius

From the co-ordinates of the surface points along the section line, one can calculate the radial

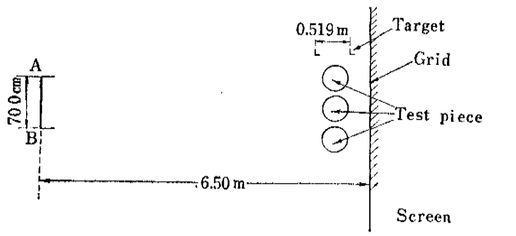


Fig. 30 Relationship between camera station and patterns of painting test. (for hydraulic turbine)

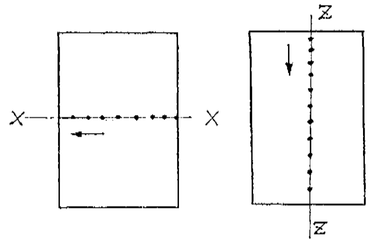


Fig. 31 Measurement points on the two section lines on test piece.

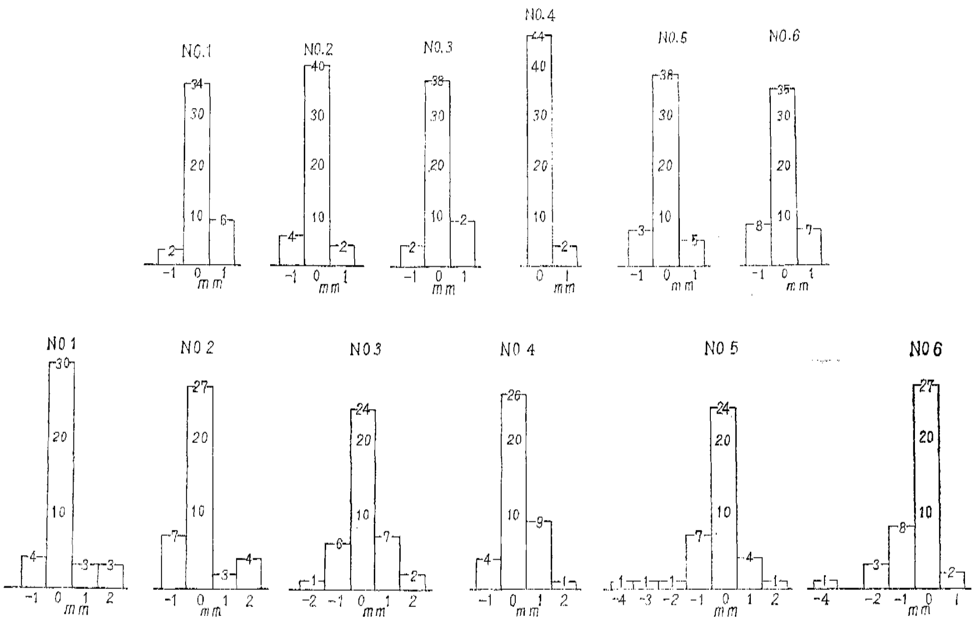


Fig. 32 Frequency graph of dispersion of errors of measurement.

lengths and compare them with the actual radial lengths as measured. The calculation of radial lengths was done using the method of least squares of Deming.

Table 2 shows the radio resulting from the calculation.

Table 2 Radio resulting from calculation

Test piece	Calculated radius from photogrammetric measurement	Calculated value (Δr_i) subtracted from design value ($r=160.02$ mm)
No. 1	$r_1=162.52$ mm	$\Delta r_1=+2.50$ mm
No. 2	$r_2=164.27$ mm	$\Delta r_2=+4.25$ mm
No. 3	$r_3=160.33$ mm	$\Delta r_3=+0.31$ mm
No. 4	$r_4=161.99$ mm	$\Delta r_4=+1.97$ mm
No. 5	$r_5=161.90$ mm	$\Delta r_5=+1.88$ mm
No. 6	$r_6=161.31$ mm	$\Delta r_6=+1.29$ mm

These results show that the measurements depend upon the kind of surface finish, and that No. 3 pattern is superior to the rest. In the practical case, vanish and lacquer was used.

5.5 Fundamental Experiments for Photographing the Automobile

The drawing line of the automobile body consists of three lines parallel to the three coordinate axes; they are called T.L. (vertical section normal to the body centre of automobile), W.L. (horizontal cross section), and B.L. (vertical section parallel to the body centre of automobile).

5.5.1 Outline of Experiments

The result of the painting experiments showed that two types, D and E, were superior, especially type E. Type E was used on the wooden master of the "Colt" automobile, the surface of which was finished to ± 0.4 mm accuracy. The rear part of the model, which is one of the difficult parts of the model to measure by the conventional method, was selected for a trial test using the photogrammetric techniques, for comparison with the results by the conventional method. Orientation marks were placed at fixed points on the cast iron jig holding the clay model of the automobile. Small pins and cross tapes were used. Fig. 33 shows the relationship between the wooden master model and the photographing stations. Plotting was carried out in a series of points along the body line contours as already described (1.1.6) Fig. 34 shows the wooden master metal.

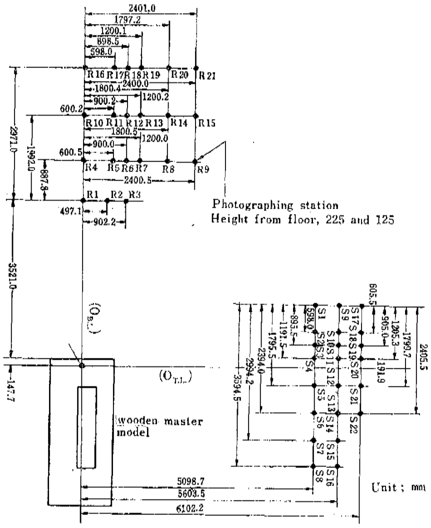


Fig. 33 Relationship between wooden master model and photographing station.

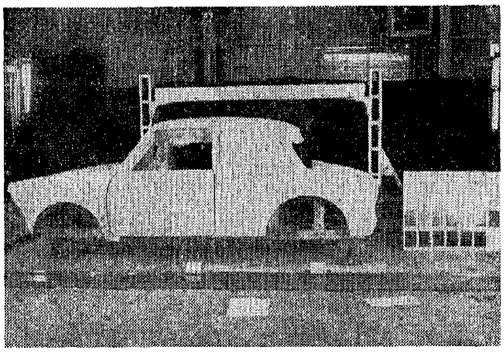


Fig. 34 Wooden master model.

5.5.2 Details of Experiments

(1) To study the effect on the accuracy of the photographing distance, photographs were taken from R_4-R_5 base line, R_{10}-R_{11} base line, and R_{10}-R_{17} base line (see Fig. 33), which are located at distances of 440.9 cm, 551.3 cm and 649.2 cm respectively, and a comparison of the results was made.

(2) To find the most suitable base ratio, two kinds of base were selected, i.e., 600 mm and 900 mm. These base ratios are both within the above mentioned theoretical limits.

(3) To compare the accuracy of side photography and rear photography, measurements were made on three pairs of photographs, R_{10}-R_{11}, R_{14}-R_{15}, and S_{17}-S_{19}.

(4) To compare the photographs taken from the photographic station R_{10}-R_{11} with R_{14}-R_{15}, which are on base line R, and the photographs

taken from the photographic station S_{21}-S_{22} with S_{20}-S_{22} on base line S, photographs which were taken from different heights were used. The effect of the direction and height of the photographic stations on the accuracy was studied.

(5) Two types of orientation marking, one being a small pin, other crossing tapes, were attached to the body surface for comparson.

(6) The photographic density has an important effect on the measurements. Three kinds of plates, such as the plates developed for 8 minutes, with exposure times of 2.5 sec., 5 sec., and the plates developed for 7 minutes with an exposure time of 10 sec., were used. The developer was konidol D.P. at a temperature of 20°C.

(7) Three men were selected to examine the effect of the technique and experience of the operators of the plotting instrument on the accuracy of the results.

operator A : excellent man with experience of five years.

operator B : man with experience of one year.

operator C : beginner with experience of two months.

(8) The place of the image on the plate is also a very important factor closely related to the accuracy. Fig. 35 shows the position of the images of the orientation points on the plate that was taken from a distance of 610.22 cm, side photography.

The values of the photogrammetric measurements were compared with the direct measurement values at each orientation point.

5.5.3 Study of the Results of Measurements and their Comparison

(1) Effect of photographing distance (see Fig. 36).

(a) The result of the measurements with a photographing distance of 649.2 cm is superior

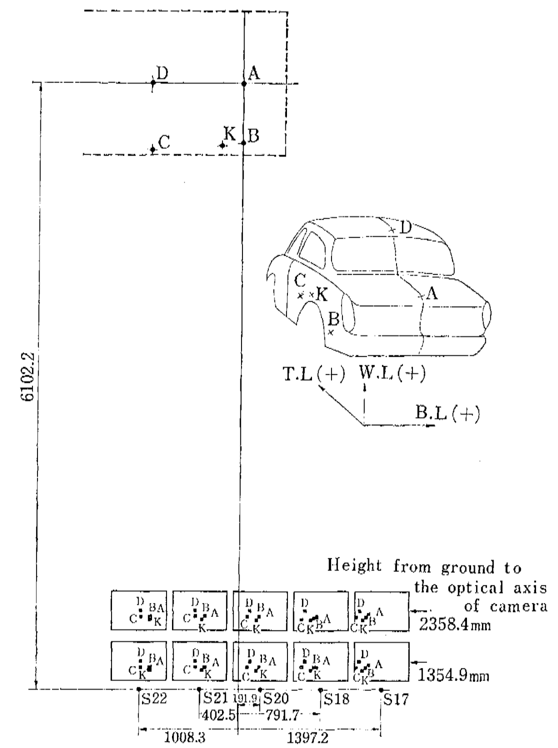


Fig. 35 Relationship between image positions of the orientation points on the plate and the photographing stations.

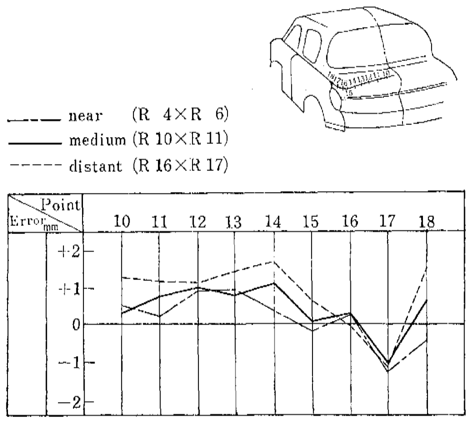


Fig. 36 (a) Comparison of errors in measurement using photographs from three different station.

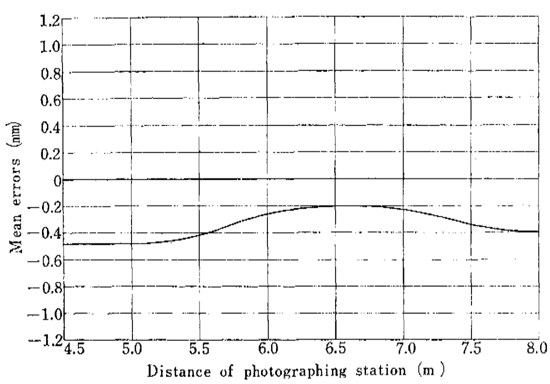


Fig. 36 (b) Graph of mean square errors versus distance of photographing station from object.

to the other two distances : the distance from 630 cm to 690 cm is shown to be excellent.

(b) The limiting short range focusing distance of this camera with attachment is 450 cm and measurement errors within the distance of 450 cm become unacceptably large.

(c) The standard deviation of the mean measurement values are as follows :

1. Photographing distance 440.9 cm :
   The maximum value 0.86 mm~The minimum value 0.20 mm.
2. Photographing distance 551.3 cm :
   The maximum value 0.78 mm~The minimum value 0.10 mm.
3. Photographing distance 649.2 cm :
   The maximum value 0.81 mm~The minimum value 0.10 mm.

(2) Effect of photographing base length (see Fig. 37).

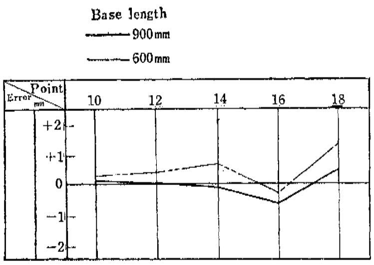


Fig. 37 Comparison of errors in measurement using two different base lengths.

Measurement with the base length of 900 mm is superior to the one with the base length of 600 mm. From these results, it appers that the longer the base length, the higher the accuracy, as far as the limitations of the theoretical base ratio, as mentioned above, are observed.

(3) Comparison of measurement accuracy with side and rear photography (Fig. 38).

(a) Depth measurement

For the T.L. measurement, side photography plates show the best measurement result, and for the B.L. rear photography is better.

(4) Effect of the photographing stations on the same base line (Fig. 39).

(a) One can easily measure the images that are photographed around the centre of the plate. The farther the images are from the centre of the plate, the less the accuracy

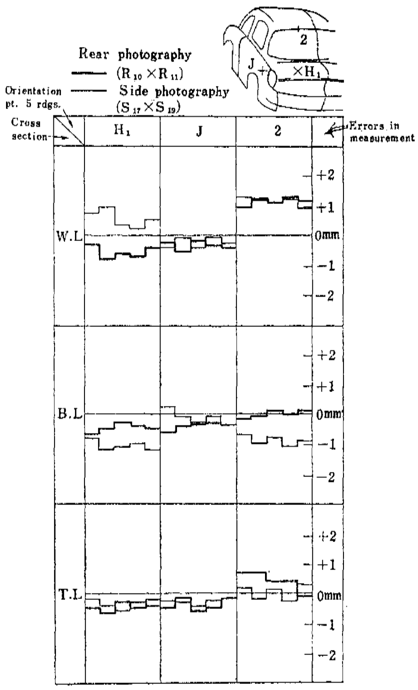


Fig. 38 Comparison between front and side photography.

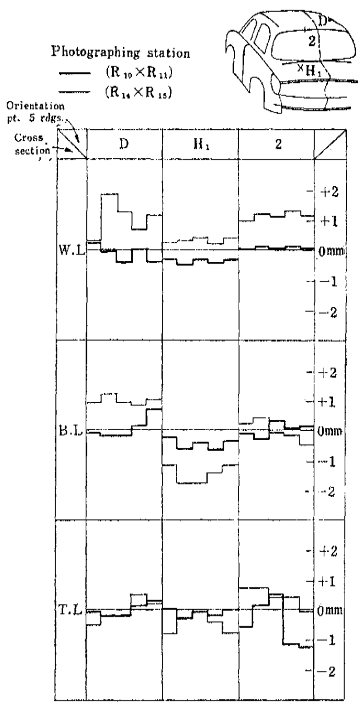


Fig. 39 Comparison between two photographing stations along the same base line--on car centreline, and to one sid cof car centreline.

becomes. The accuracy of the measurements made using the plates taken from the platform, was less than that with the plates taken from the ground, except for several pairs of plates with some accidental errors.

(b) Measurement which is caused by the height difference of camera (Fig. 40) depends

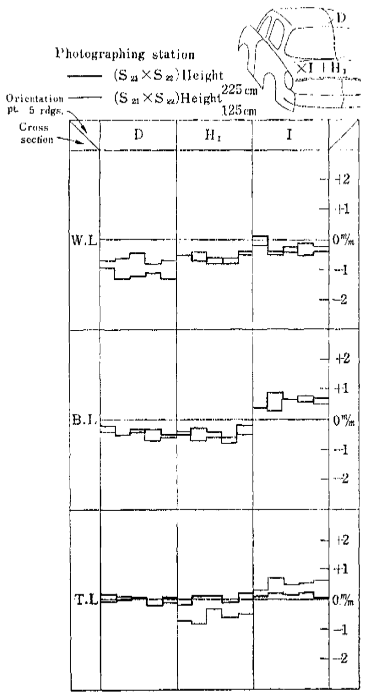


Fig. 40 Comparison between heights of photographing position.

on the position of the object in space, the inclination of the camera angle and the surface of the object. Generally, the greater part of the boot and roof is shown with good accuracy using photographs taken from the ground.

(5) Effect of the marking as orientation points (Fig. 41).

The tape markings show superior results for measurement; the accuracy of the markings

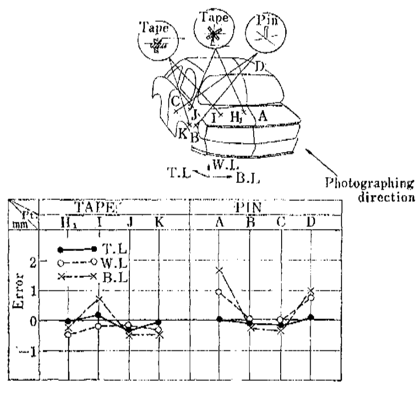


Fig. 41 Comparison between types of orientation point.

depend on the contrast behind them, and is sometimes good, but sometimes bad results occur. In both cases, one must take care with the angle at which the marking is seen from the photographing stations.

(6) Effect of density difference (Fig. 42).

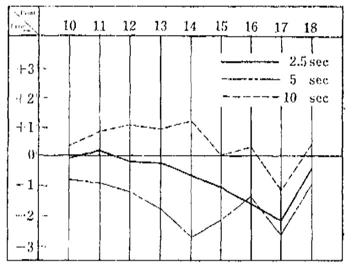


Fig. 42 Comparison between difference of photographic densities.

The plate with the exposure time of 2.5 sec is superior, but in the case of the plate with the exposure time of 10 sec, the plate improved with shorter developing times.

(7) Effect of the operator (Fig. 43).

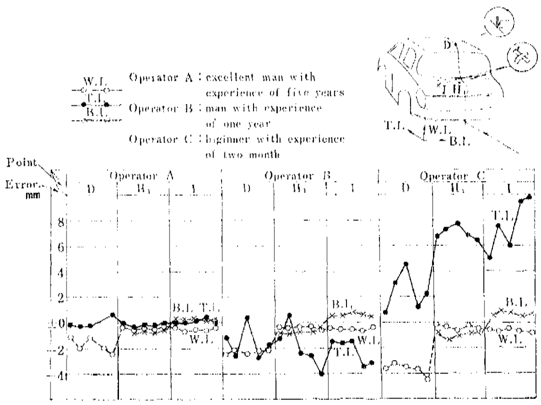


Fig. 43 Comparison between results obtained by different operators.

The results of the measurements show clearly that operator technique and experience are important considerations. They show that operator trainning for more than one year is essential for presision measurements.

(8) Effect of image position on the plate. (Fig. 44).

This is a very important factor for precision measurements, especially when choosing the orientation points. Fig. 44 shows that good results are obtained when the orientation points are placed with a reasonable arrangement over the whole plate, and not restricted to one part.

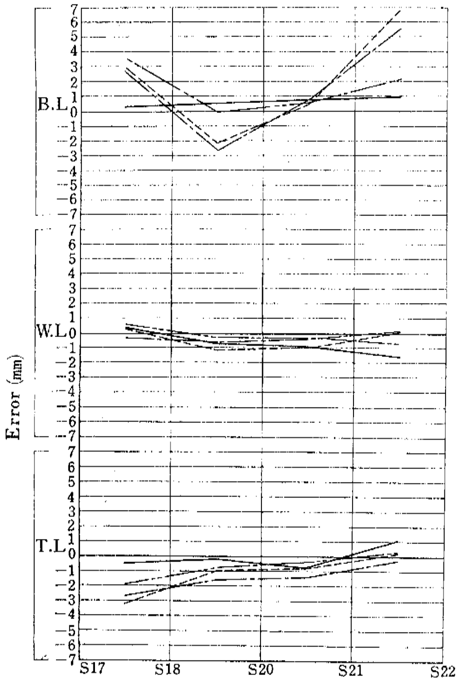


Fig. 44 Relationship between image position on the plate and accuracy of measurement. (Starting from Point A)

The most important problem is the starting point for the measurements, and for this reason, the starting point governs the accuracy of all the points.

6. Practical Example

This practical application of photogrammetry to the body line drawing of automobile was undertaken following the request of the Nagoya Car Works of the Shin Mitsubishi Heavy Industries to conduct experiments during the development stage of their Colt automobile series, in particular, the recent Colt 1000. The co-operation received from Mitsubishi during this period was an important factor for the successful completion of this application of photogrammetry to automobile engineering.

6.1 Practical Example of the Body Line Drawing of Automobiles

The result of the various kinds of fundamental experiments were taken into consideration, together with all the factors mentioned above, for the practical case of drawing the body lines for an automobile. Such research work must be considered to be one the essential procedures during the design of a new automobile.

6.1.1 The Progress of the Practical Examples

The first project of this nature was under taken in July 1961, when the body lines of the "Colt" were drawn, the predecessor of the present "Colt 600".

This was followed by two projects covering some fundamental problems, and in December 1962, by the fourth & fifth projects to draw the body lines of the "Colt 600 Van". Photography was from a concrete platform in the first project, but it was found to have large shrinkage movements with changes in temperature. For the second project, the body drawings of a clay model were made at a scale of 1 : 3.

This work proved that the photography for the body line drawings of an automobile could be taken indoors. As the fifth project, a practical model measurement was undertaken following the techniques established from the previous four projects, as the first step in one part of the production process for a new automobile. To date, several kinds of drawings for

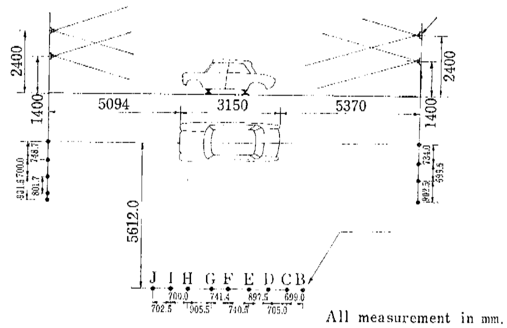


Fig. 45 Relationship between photographing stations and clay model—"Colt 1000".

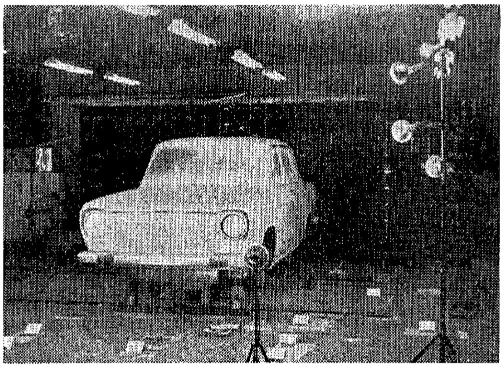


Fig. 46 Clay model—"Colt 1000".

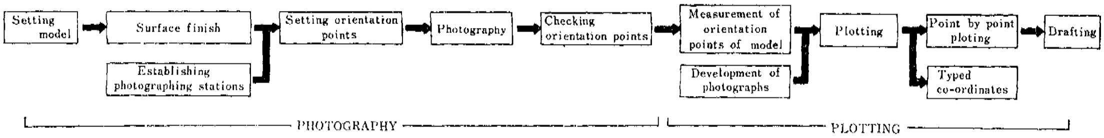


Fig. 47 Schematic diagram of procedure for producing body line drawing by photogrammetric techniques.

automobile production have been completed in this Research Institute. Another practical example is the body line drawing of the "Colt 1000" automobile clay model which formed part of the recent work of the Research Institute.

6.1.2 Example of Body Line Drawing of "Colt 1000" Clay Model

Fig. 45 shows the relationship between the photographing stations and the clay model. Fig. 46 shows a photograph of "Colt 1000" clay model, and the photographing room.

The clay model must be kept in the correct position along the three axes, T.L., W.L., and B.L., as above mentioned. Among them, the fundamental line must be determined along each axis. The jig equipment arranged in the automobile design room was used to set the model with precision, as well as for its normal purpose. At the same time, the jig can be used as a convenient location for the orientation points, saving time and ensuring accuracy. Painting the model and setting the photographing stations follow after setting the model as shown in Fig. 47. Photography was taken from three sides, but in the fourth test, the model was set on a rotating table so that the photographs could all be taken from the same base line.

Fifteen pairs of photographs were taken, but the number need not be so large, and ten pairs are sufficient. The orientation points must be protected from disturbances and movement until the photography is finished.

After taking the photographs, the orientation points must be checked to see whether they were moved or not. The photographed plates were developed in a dark room which was next to the photographing room, and the

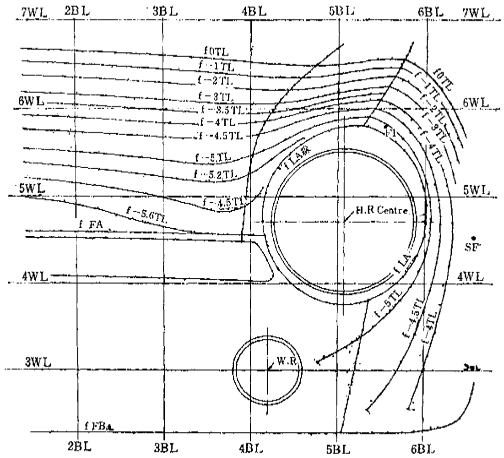
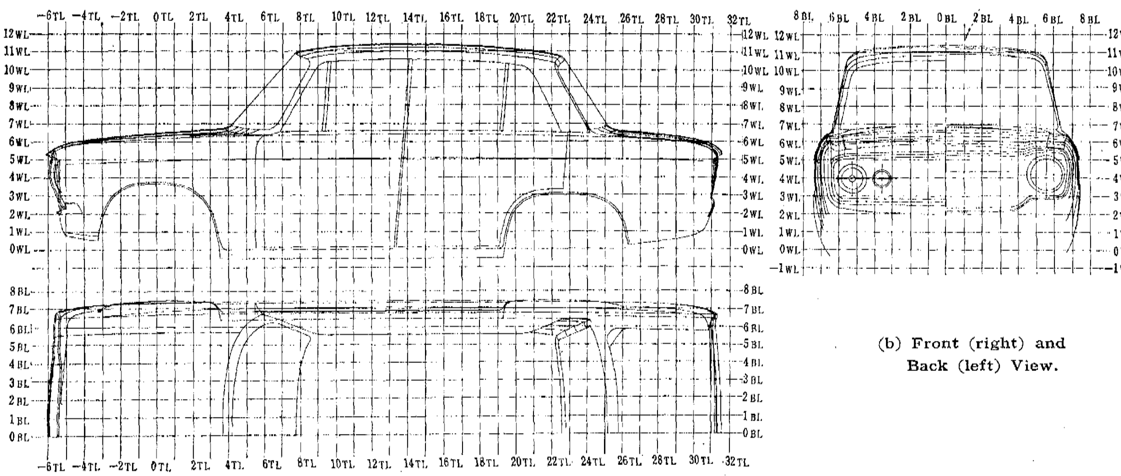


Fig. 48 Example of plotting contours point by point--detail around left headlight.

(a) Side View



(b) Front (right) and Back (left) View.

(c) Plan

Fig. 49 Drawing of the body line of "Colt 1000" result was checked. All the plotting was done with care by plotting a series of points along the contours of the automobile body. An experienced operator can plot twenty or thirty points a minute. Some A.K. Kent paper, with aluminuim inserted, was used for drawing to avoid effects of temperature change. Fig. 48 shows the drawing of a part around the right headlight of the bonnet cover. Fig. 49 shows the drawing of the body line of "Colt 1000."

6.1.3 Accuracy and Application of Results

In the normal case of short range photogrammetry, the range of errors is within ±0.5 mm. But under the same conditions, some differences in the accuracy are found due to the variable nature of the surface finish of the model, and, therefore, one must choose suitable photographic plates for the project in question dependent on the many factors mentioned above. The following figure shows the relationship between the distance of photographing and their measurement accuracy. In this case two kinds of base length were used. To determine the necessary base length and accuracy, this diagram can be used very conveiniently.

The time needed for plotting is a very important factor. Fig. 51 shows the time for each step procedure of the photogrammetric measurement. Except for the experimental step, when these techniques have been established and the preparations prior to photography have been completed, the time for plotting can be less than that shown by up to one third.

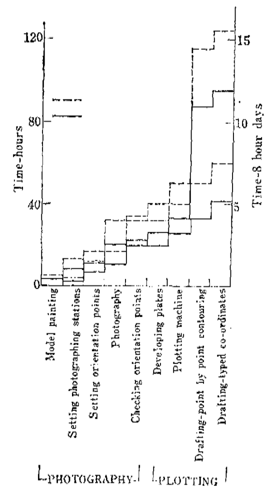
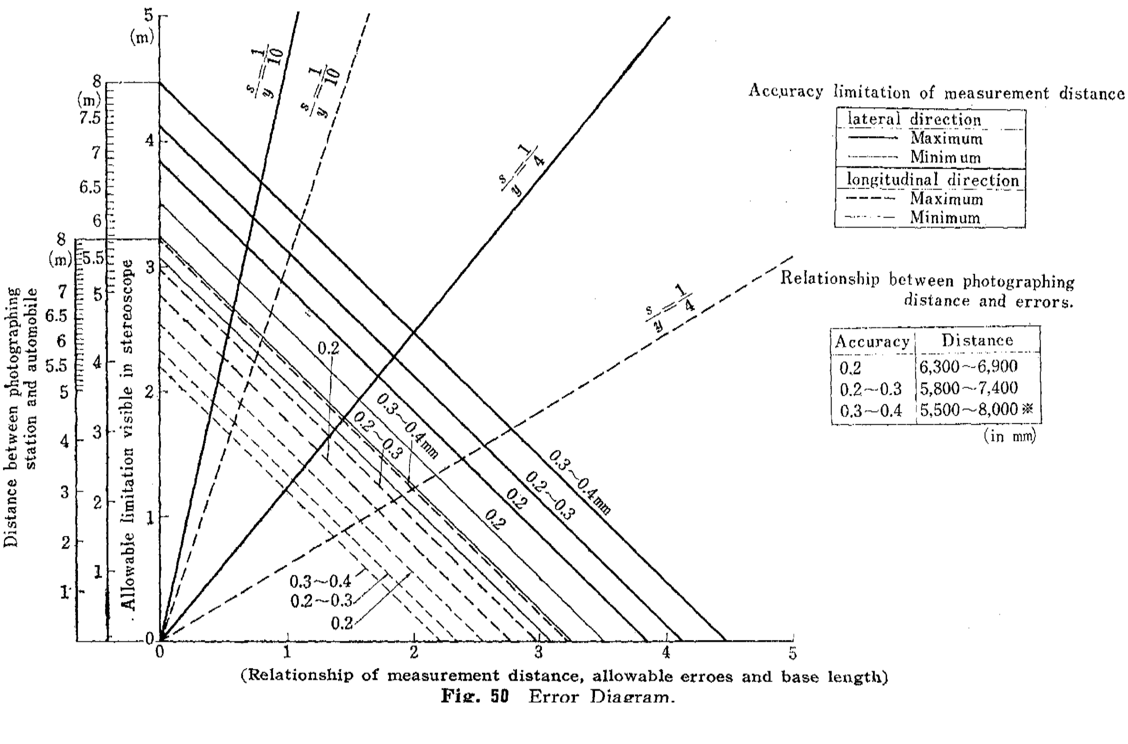


Fig. 51 Time consumed for each step in procedure.

6.2 Practical Example of the Hydraulic Turbine

6.2.1 Setting of Photographing Stations and Photography

As shown in Figs. 52 and 53, the turbine shaft was set in the horizontal and the base direction of photographing was set parallel to the



Accuracy	Distance
0.2	6,300~6,900
0.2~0.3	5,800~7,400
0.3~0.4	5,500~8,000 ※

(in mm)

(Relationship of measurement distance, allowable erroes and base length)

Fig. 50 Error Diagram.

surface of the runner band. Fig. 54 shows the object and five photographing stations on the same line. Four pairs of plates were used to measure the surface of the runner vane. The extreme end points of the base were used to measure the dead area between the runner blades. As orientation points, four points on the surface of the runner band, four points at the front on the floor, and four points on the

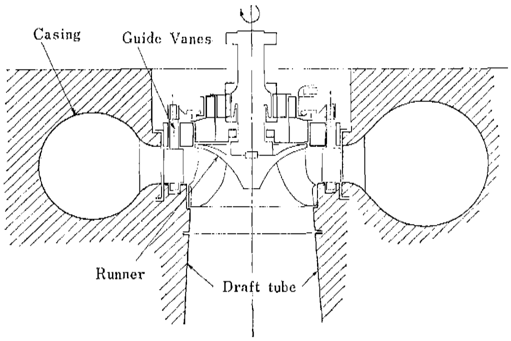


Fig. 52 Cross section of Turbine Runner and Casing.

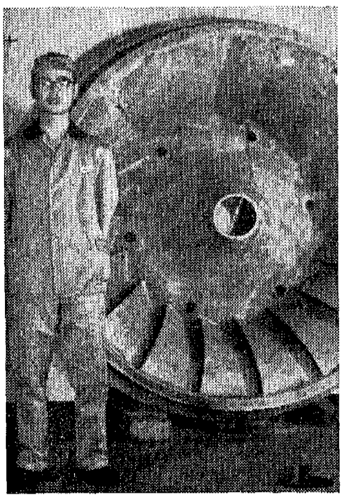


Fig. 53 (a) General view of Turbine Runner.

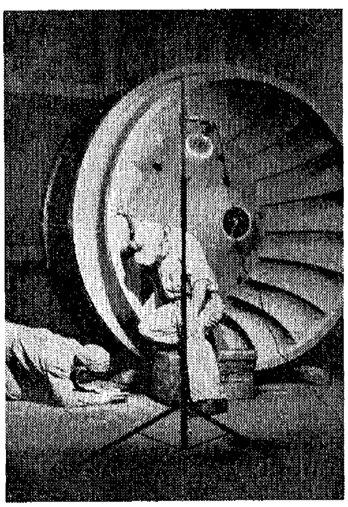


Fig. 53 (b) Setting orientation points on runner band.

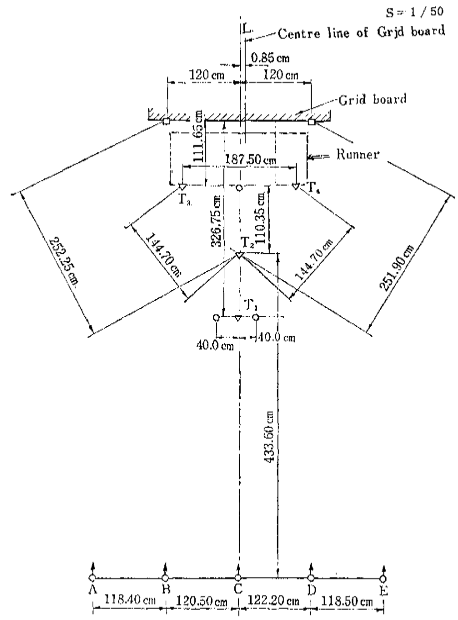


Fig. 54 Relationship between Tubine Runner and photographing stations. Grid board and orientation points (T) shown.

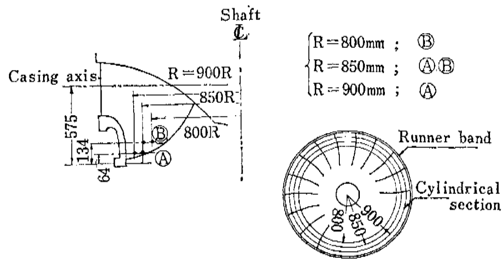


Fig. 55 (a) Diagram showingl ocations of vertical and cylindrical sections.

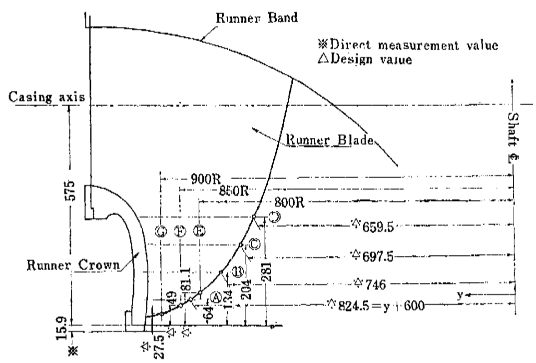


Fig. 55 (b) Radial section of Runner blade showing details lacations of vertical and cylindrical sections for photogrammetric measurements.

grid board were selected. Eight electric bulbs of 500 W were used for illumination.

6.2.2 Measurements and Results

Plotting with the A 7 Autograph was done on an instrument scale of 1 : 15 and a mapping scale of 1 : 2.5. The range of depth for plotting was from 2.10 m to 7.27 m in this case. (Fig. 55)

The resulting drawings from the plotting are as follows;

1. The front of runner on a scale of 1 : 2.5.
2. Cross sections parallel to the base line. (Fig. 56) The four sections are A, B, C and D in Fig. 55.
3. Circular cross sections with a diameter of 800 mm, 850 mm and 900 mm from the centre of the vertical shaft (Fig. 57). The three sections are E, F and G in Fig. 55. In plotting the points with an interval of 8 mm, each point on the circle was predetermined on the drawing on a scale of 1:2.5, and the co-ordinates of each point were typed with an EK 3 co-ordinate printer.

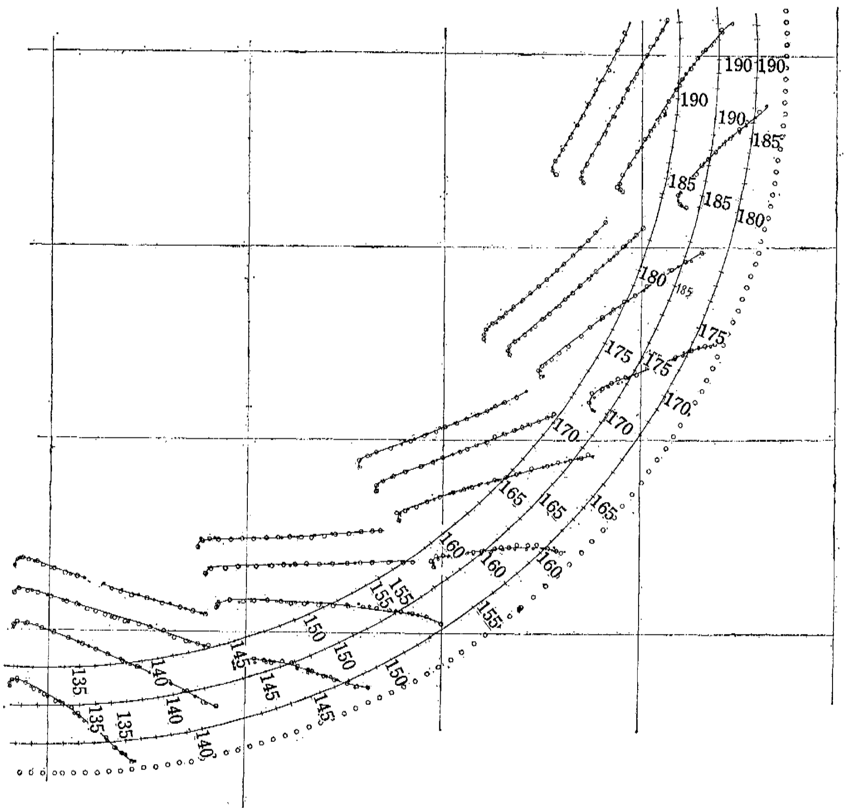


Fig. 56 Vertical cross section of Runner showing blades (see Fig. 55)

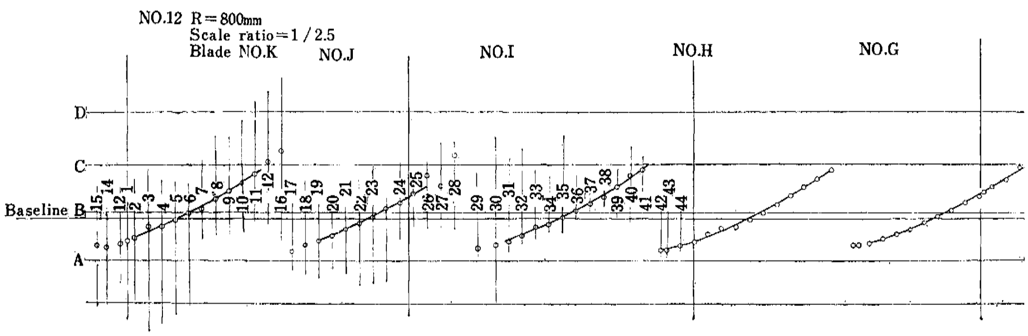


Fig. 57 Clylindrical section showing blades *K* to *G*.

6.2.3 Accuracy of the Measurements

As the method of checking the measurement accuracy, co-ordinates of 375 points on the interior rim of the surface of the runner band were measured, and the computed radial length from the 375 points' co-ordinates were compared with radial values of the runner band, which is finished to within a fine tolerance.

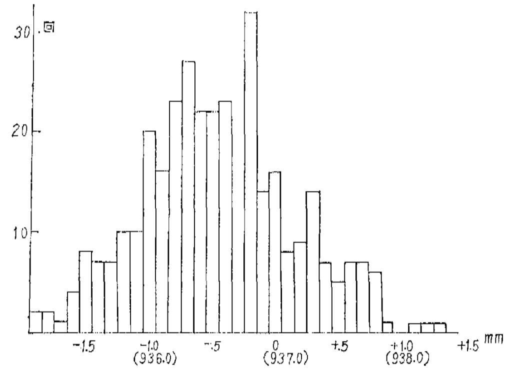


Fig. 58

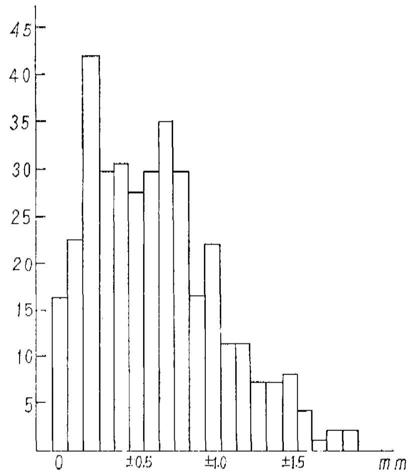


Fig. 59

Fig. 58 and Fig. 59 show frequency distributions and the cumulative frequency distribution of these point measurements. The mean square value, which was compared with the designed value of radial lengths, was 0.57 mm. Fig. 60 shows the graph of the radial errors of photogrammetric measurements, arranged in numerical order.

6.2.4 Discussion and Application of Results

The Hydraulic Technical Committee members discussed the data gained by the photogrammetric method and compared it with the design data.

Their conclusions were as follows :

1) The form of the guide blades of the runner were completed fairly well in accordance with the design value, but in some places an inequality of the clearance between the guide vanes was found.

2) The cross section of each runner blade was projected from the circular section, and the angle of intersection with the A plane, which is normal to the runner axis, was measured. This angle was measured for all planes, A to R, 18 in total. The dispersion of differences between θ and the design θ_0, ($\Delta\theta = \theta - \theta_0$) in Fig. 61, are shown in Fig 62. The maximum difference is less than 1°.

3) The place of measurement of the pitch error of the runner blade is shown in Fig. 63. The chord length of each runner vane was

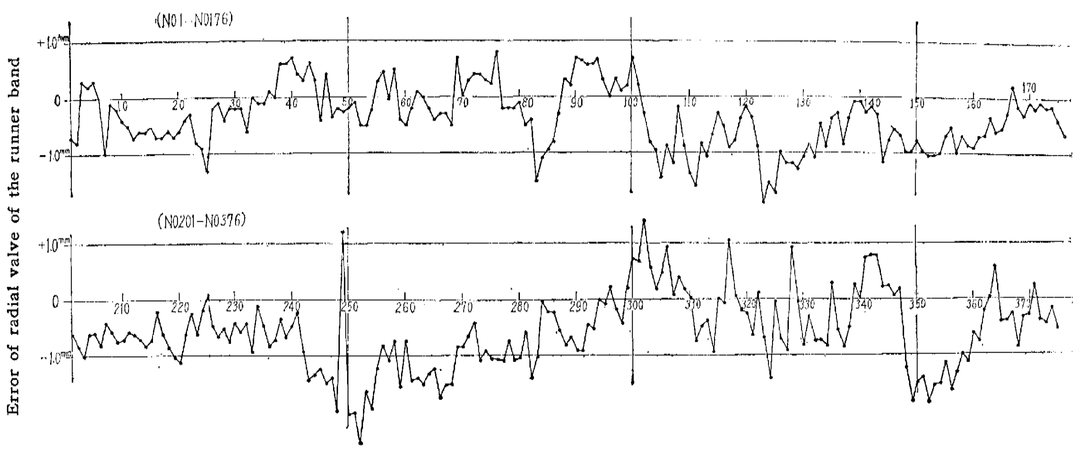


Fig. 60 Cylindrical Test Piece—measurements of diameter after painting.

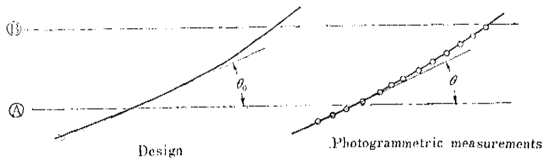


Fig. 61 Blade inclinations at point A.

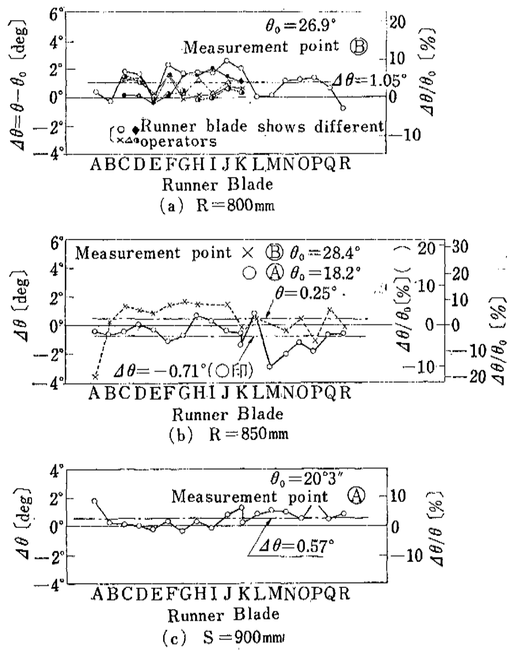


Fig. 62 Errors in blade inclination.

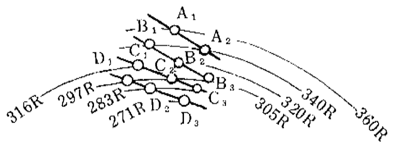


Fig. 63 Diagram of measurement points for errors in pitch.

measured from Fig. 55.

Mean chord length

$$t_m = 2\,r\sin(\pi/18) = 0.347296\,r$$

Pitch error

$$\varepsilon = \frac{t - t_m}{t_m} \times 100\%$$

Mean square error

$$\bar{\varepsilon} = \sqrt{\sum_1^{18} \varepsilon^2/18}$$

The result of these calculations is shown in Fig. 64 and 65. An inequality of the pitch was found at some places between the guide vanes from the results of the photogrammetric measurements, but almost all the errors are less than 1%, which is small enough to be neglected.

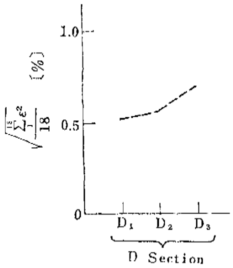


Fig. 64 Mean square errors of photogrammetric measurements of a pitch.

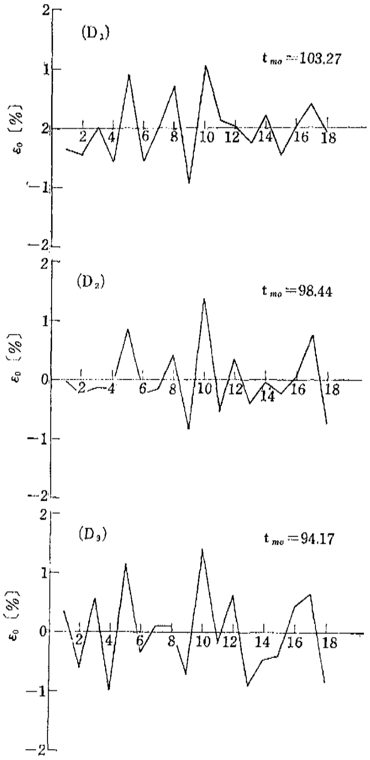


Fig. 65 Pitch errors of Photogrammetric measurements.

7. Summary

Short range photogrammetric methods have played an important role in the field of industrial engineering in obtaining the body line drawings from a clay model of a new automobile, and for checking the form of an hydraulic turbine. Moreover, there are many various problems which can be solved in the future by these techniques with advantages in accuracy and efficiency.

As these techniques have many advantages over the more complicated and laborious conventional methods, they can be applied not only to the limited field of automobile engineering, but also to the benefit of a wide range of industry. Now experiments are under way to measure the deformation of a model on a scale of 1 : 30 of the National Indoor Stadium to be used in Tokyo for the Olympic Games this year.

Also, beginning in 1964, measurements will be made of artificial snow avalanches using photographs taken by a pair of synchronized wild P. 30 phototheodolites.

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