SIMPLIFIED CALCULATIONS FOR CABLE TENSION IN SUSPENSION BRIDGE

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Introduction

In order to obtain the numerical values of suspension bridges, whenever they may be the design calculations or only a trial one, cable tension must need troublesome numeric calculs concerning every statical value and every loading condition. When the horizontal component of cable tension H of a single suspended span is correctly determined or given in the form easily calculated, it is easier to make the elastic equations for more complicated type of suspension bridges such as continuous one.

This brief report deals with the horizontal component of the cable tension in the following form;

$$H_r = H_d + H$$

$$H = \frac{1}{1+\mathfrak{N}} \cdot \frac{l}{8f}\left[\int p(x)k(x)dx + \mathfrak{A}\Delta L - \mathfrak{B}T\right] \qquad \cdots\cdots (1)$$

where, $k(x)$ is a function of the co-ordinate x at which unit concentrated load acts on the stiffning girder. This is otherwise than proportional to the influence line of the cable tension, and also defined so that the mean value of the function is equal to unit. As the shape of the function $k(x)$ is determined from the deflection curve when the uniform distributed load acts along the suspended girder, so $k(x)$ concerns also the parameter $cl = l\sqrt{H_r/EJ}$ and boundary conditions of stiffning girder. Two cases of girder conditions, simply supported and clamped at both ends, are calculated and shown in Table 1 Fig. 2. In practical meaning, the shape of $k(x)$ is almost similar to the parabolic, and especially in case of simply supported suspended girder, $k(x)$ shows few difference even when the parameter cl varies from zero to infinitive.

$\mathfrak{N}$ of denominator is a characteristic value concerning the bending rigidity of girder, cable dimensions, parameter N and so. It changes from zero to infinitive when stiffness of girder varies from zero to infinitive. Considering a suspension bridge of very low stiffness and a uniform distributed load p on whole span length, the cable tension is equel to $pl^2/8f$. Multiplier $1/(1+\mathfrak{N})$ means such a stiffness ratio in the suspension bridge comparing the deflection theory to the elastic theory.

ΔL is a small amount of distance change between both tops of towers where the cable rests. It occurrs when the towers are flexible or during the period of construction work, cable saddles may be moved longitudinally.

"T" denotes temperature increase of the cable from ordinary.

A parameter N is a dimensionless value and

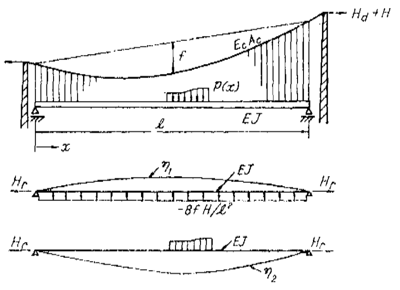

Fig. 1

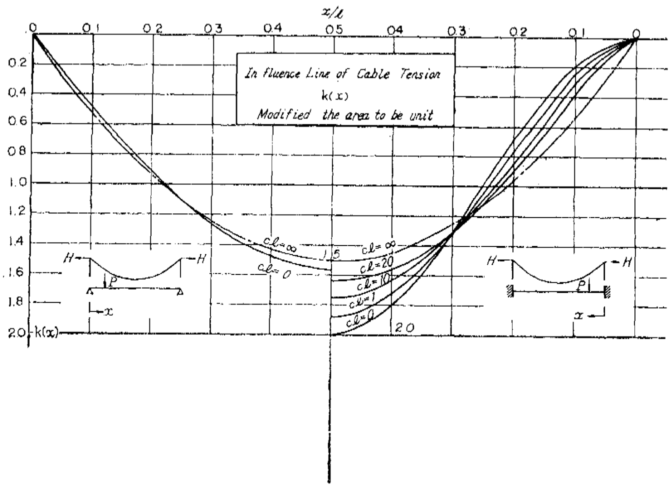

Fig. 2

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defined also like $k(x)$ as a function of cl only. When a suspended girder deforms under the uniform distributed load p deflection curve of girder is given as;

$$\eta = p\cdot\frac{l^4}{EJ}\cdot N\cdot\frac{\mathfrak{N}}{1+\mathfrak{N}}\cdot k(x)\cdots\cdots\cdots\cdots\cdots(2)$$

N values is also tabled and shown in Table 2 and Fig. 3. Constants $\mathfrak{N}$, $\mathfrak{A}$ and $\mathfrak{B}$ of equation (1) contain the parameter N in the dividend form $(1/N)$, and do not include any other variables upon cl, in other words, equation (1) is so simplified that it saves the time for try-and-error calculs of suspension bridges excluding complicated treatment of hyperbolic functions and only referencing the numeric tables.

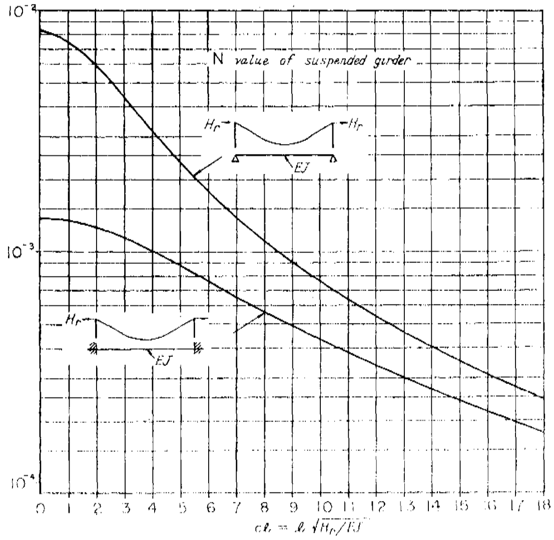

Fig. 3

As is understood from Figs. 2/3, $k(x)$ is, practically, fewly affected upon the value of cl, and N can be approximately given as a linear function or at most a secondary function of cl for the proposed calculs, equation (1) is given as the arithmetic function, and does not include no more hyperbolic or exponential functions. These simplification has the merit to save the complicated numeric calculs and to shorten the programm treatment when the digital computation is used.

After every suspended span is simplified, it is easier to make the elastic equations of continuous suspention bridge under complicated loading conditions, arbitral temperature change, even when considering the tower stiffness which cause that every span has a different horizontal component of the cable tension.

In this report, the author deals with the typical type of suspension bridge, that is, the cable takes a parabolic curve and stiffning girder has an equal bending rigidity. If the bending rigidity is not equal through the span and the cable takes a shape other than parabolic, idea of calculation is the same to get N and $k(x)$ from the deflection curve of such structure.

Evaluations of Deflection Theorem

The differential equation used in this chapter is based on the deflection theory of suspension bridge.

$$EJ\frac{d^4\eta}{dx^4}-(H_d+H)\frac{d^2\eta}{dx^2}+\frac{8f}{l^2}H=p(x)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

Let $H_r=H_d+H$ and $\eta=\eta_1+\eta_2$ be, and equation (1) shall be separated into two cases.

$$EJ\frac{d^4\eta_1}{dx^4}-H_r\frac{d^2\eta_1}{dx^2}=-\frac{8f}{l^2}H\cdots\cdots\cdots(4\text{-}1)$$

$$EJ\frac{d^4\eta_2}{dx^4}-H_r\frac{d^2\eta_2}{dx^2}=p(x)\cdots\cdots\cdots\cdots\cdots(4\text{-}2)$$

These equations are the same with that of a straight beam which has the bending stiffness EJ and is expanded by axial tensile force H_r at both ends. External loads are the uniform distributed load $-8fH/l^2$ and the actual load $p(x)$, respectively. With the boundary conditions of stiffning girder, η_1 and η_2 can be solved when arbitral external force acts if necessary, but from the point of view to the following explanation, we show the two special solution of η_1 only.

i) Simply supported at both ends of stiffning girder.

$$\eta_1=\left(-\frac{8f}{l^2}H\right)\cdot\frac{l^4}{EJ}\cdot\frac{1}{c^4l^4}\left[\frac{c^2x(l-x)}{2}-\tanh\frac{cl}{2}\sinh cx+\cosh cx-1\right]$$

ii) Clamped at both ends of stiffning girder.

$$\eta_1=\left(-\frac{8f}{l^2}H\right)\cdot\frac{l^4}{EJ}\cdot\frac{1}{c^4l^4}\left[\frac{c^2x(l-x)}{2}-\frac{cl}{2}\sinh cx+\frac{cl}{2}\coth\frac{cl}{2}(\cosh cx-1)\right]$$

where

$c=\sqrt{H_r/EJ}$

Increase or decrease of cable tension H from the state, the external force off, is given by the conditions, that total elongation of cable length such as by the deformation of girder and by the distance change between tops of towers, is nothing but the elongation due to elasticity of cable and to the temperature change.

Difference of cable length due to the deformation of girder is the geometrical one. For η_1 and η_2 we get

$$\Delta L_1 = \frac{8f}{l^2}\int_0^l \eta_1 dx$$

$$\Delta L_2 = \frac{8f}{l^2}\int_0^l \eta_2 dx$$

To get the integral of η_2, we consider the unit concentrated load $P=1$ acting at $x=\xi$, then

$$\Delta L_2 = \frac{8f}{l^2}\int_0^l \eta_2\, dx = -\frac{1}{H}\cdot\eta_1(\xi)$$

Let ΔL_3 be the horizontal distance change between both tops of towers on which the cable rests. ΔL_4 shall be the elongation of total cable length of supposed span by temperature increase.

$$\Delta L_4 = \alpha L_t T$$

Finally let ΔL_5 be the total elastic elongation of cable by H.

$$\Delta L_5 = \frac{L_s}{E_c A_c}\cdot H$$

Equilibrium equation for H is then induced as follows when unit concentrated load $p=1$ works at $x=\xi$,

$$\Delta L_1 + \Delta L_2 + \Delta L_3 = \Delta L_4 + \Delta L_5$$

$$\therefore\quad H = \frac{-\dfrac{1}{H}\eta_1(\xi) + \Delta L_3 - \alpha TL_t}{\dfrac{L_s}{E_c A_c} - \dfrac{8f}{l^2}\cdot\dfrac{1}{H}\displaystyle\int_0^l \eta_1 dx} \cdots\cdots\cdots (5)$$

Now that the equation (5) is defined, successive simplification is considered introducing the following dimensionless function $k(x)$

$$k(x) = \frac{l}{\displaystyle\int_0^l \eta_1 dx}\cdot\eta_1(x)$$

i) simply supported at both ends of stiffning girder.

$$k(x) = \frac{1}{N_1}\cdot\frac{1}{c^4 l^4}\left[\frac{c^2 x(l-x)}{2} - \tanh\frac{cl}{2}\sinh cx + \cosh cx - 1\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots (6\text{-}1)$$

$$N_1 = \frac{1}{c^5 l^5}\left[-cl + \frac{c^3 l^3}{12} + 2\tanh\frac{cl}{2}\right] \cdots (7\text{-}1)$$

ii) Clamped at both ends of stiffning girder.

$$k(x) = \frac{1}{N_2}\cdot\frac{1}{c^4 l^4}\left[\frac{c^2 x(l-x)}{2} - \frac{cl}{2}\sinh cx + \frac{cl}{2}\coth\frac{cl}{2}(\cosh cx - 1)\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots (6\text{-}2)$$

$$N_2 = \frac{1}{c^5 l^5}\left[cl + \frac{c^3 l^3}{12} - \frac{c^2 l^2}{2}\coth\frac{cl}{2}\right] \cdots (7\text{-}2)$$

Conclusively, equation (5) is induced to the following form with the arbitral external force $p(x)$

$$H = \frac{1}{1 + \dfrac{l^2}{64f^2}\cdot\dfrac{EJ}{E_c A_c l^2}\cdot\dfrac{L_s}{l}\cdot\dfrac{1}{N}}\cdot\frac{l}{8f} \cdot\left[\int_0^l p(x)k(x)dx + \frac{l}{8f}\cdot\frac{EJ}{l^2}\cdot\frac{1}{lN} \cdot(\Delta L_3 - \alpha TL_t)\right] \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

N and $k(x)$ are tabled in Tables 1 and 2, and also shown graphically in Figs. 2 and 3 against the parameter cl comparing the two cases of boundary conditions in suspended girder.

Generally, the suspended structure shows the non-linear behavior of stress and strain against the working external force. A typical example is the equation (8) which includes in the right terms N and $k(x)$, those are functions of cl

Table 1 N value

cl	N1	N2	cl	N1	N2
0	0.0083333	0.0013888	10.25	0.0007202	0.0004194
0.25	0.0082809	0.0013868	10.50	0.0006892	0.0004061
0.50	0.0081277	0.0013806	10.75	0.0006601	0.0003934
0.75	0.0078845	0.0013705	11.00	0.0006328	0.0003813
1.00	0.0075676	0.0013566	11.25	0.0006071	0.0003696
1.25	0.0071958	0.0013391	11.50	0.0005828	0.0003585
1.50	0.0067882	0.0013184	11.75	0.0005600	0.0003478
1.75	0.0063623	0.0012948	12.00	0.0005385	0.0003375
2.00	0.0059329	0.0012686	12.25	0.0005181	0.0003277
2.25	0.0055115	0.0012402	12.50	0.0004989	0.0003182
2.50	0.0051061	0.0012101	12.75	0.0004807	0.0003092
2.75	0.0047224	0.0011785	13.00	0.0004634	0.0003005
3.00	0.0043633	0.0011458	13.25	0.0004471	0.0002921
3.25	0.0040303	0.0011124	13.50	0.0004316	0.0002841
3.50	0.0037235	0.0010785	13.75	0.0004168	0.0002764
3.75	0.0034421	0.0010445	14.00	0.0004028	0.0002689
4.00	0.0031849	0.0010105	14.25	0.0003895	0.0002618
4.25	0.0029503	0.0009768	14.50	0.0003768	0.0002549
4.50	0.0027366	0.0009436	14.75	0.0003647	0.0002483
4.75	0.0025419	0.0009109	15.00	0.0003532	0.0002419
5.00	0.0023647	0.0008790	15.25	0.0003422	0.0002358
5.25	0.0022033	0.0008479	15.50	0.0003317	0.0002299
5.50	0.0020561	0.0008177	15.75	0.0003217	0.0002242
5.75	0.0019218	0.0007884	16.00	0.0003121	0.0002187
6.00	0.0017991	0.0007601	16.25	0.0003030	0.0002134
6.25	0.0016868	0.0007327	16.50	0.0002942	0.0002082
6.50	0.0015840	0.0007064	16.75	0.0002858	0.0002033
6.75	0.0014896	0.0006811	17.00	0.0002777	0.0001985
7.00	0.0014029	0.0006567	17.25	0.0002700	0.0001939
7.25	0.0013231	0.0006334	17.50	0.0002626	0.0001894
7.50	0.0012496	0.0006110	17.75	0.0002555	0.0001851
7.75	0.0011817	0.0005895	18.00	0.0002487	0.0001809
8.00	0.0011189	0.0005690	18.25	0.0002421	0.0001769
8.25	0.0010608	0.0005493	18.50	0.0002358	0.0001730
8.50	0.0010068	0.0005304	18.75	0.0002298	0.0001692
8.75	0.0009568	0.0005124	19.00	0.0002239	0.0001656
9.00	0.0009102	0.0004951	19.25	0.0002183	0.0001620
9.25	0.0008668	0.0004786	19.50	0.0002129	0.0001586
9.50	0.0008264	0.0004628	19.75	0.0002077	0.0001553
9.75	0.0007886	0.0004477	20.00	0.0002027	0.0001520
10.00	0.0007533	0.0004332			

Table 2 $k(x)$

	x/l	$cl=0$	$cl=1$	$cl=2$	$cl=3$	$cl=4$	$cl=5$	$cl=6$	$cl=7$	$cl=8$	$cl=9$	$cl=10$
$K_1(x)$	0.05		0.2490567	0.2498513	0.2510783	0.2526203	0.2543555	0.2561770	0.2580026	0.2597758	0.2614616	0.2630419
	0.10		0.4909576	0.4922750	0.4943027	0.4968384	0.4996727	0.5026236	0.5055525	0.5083657	0.5110076	0.5134507
	0.15		0.7192859	0.7207357	0.7229581	0.7257198	0.7287815	0.7319368	0.7350311	0.7379629	0.7406748	0.7431420
	0.20		0.9284172	0.9296127	0.9314357	0.9336836	0.9361501	0.9386597	0.9410839	0.9433413	0.9453896	0.9472143
	0.25		1.1135050	1.1141432	1.1151076	1.1162782	1.1175379	1.1187875	1.1199578	1.1210085	1.1219221	1.1226975
	0.30		1.2704689	1.2703756	1.2702198	1.2700028	1.2697284	1.2694033	1.2690367	1.2686393	1.2682218	1.2677946
	0.35		1.3959841	1.3951289	1.3938126	1.3921669	1.3903279	1.3884146	1.3865177	1.3846988	1.3829947	1.3814236
	0.40		1.4874733	1.4859617	1.4836489	1.4807821	1.4776145	1.4743629	1.4711880	1.4654372	1.4654372	1.4629407
	0.45		1.5431001	1.5411473	1.5381659	1.5344826	1.5304302	1.5262916	1.5222746	1.5150701	1.5150701	1.5119764
	0.50		1.5617645	1.5596564	1.5564401	1.5524703	1.5481080	1.5436596	1.5393493	1.5316408	1.5316408	1.5283411
	x/l	$cl=11$	$cl=12$	$cl=13$	$cl=14$	$cl=15$	$cl=16$	$cl=17$	$cl=18$	$cl=19$	$cl=20$	$cl=$
	0.05	0.2645095	0.2658644	0.2671112	0.2682563	0.2693075	0.2702727	0.2711594	0.2719750	0.2727260	0.2734186	0.285
	0.10	0.5156870	0.5177206	0.5195623	0.5212264	0.5227286	0.5240845	0.5253089	0.5264157	0.5274172	0.5283248	0.540
	0.15	0.7453616	0.7473435	0.7491052	0.7506668	0.7520495	0.7532734	0.7543574	0.7553185	0.7561718	0.7569307	0.765
	0.20	0.9488196	0.9502197	0.9514341	0.9524840	0.9533901	0.9541718	0.9548464	0.9554294	0.9559339	0.9563715	0.960
	0.25	1.1233431	1.1238727	1.1243016	1.1246453	1.1249178	1.1251318	1.1252980	1.1254255	1.1255218	1.1255931	1.125
	0.30	1.2673665	1.2669448	1.2665352	1.2661417	1.2657671	1.2654131	1.2650803	1.2647688	1.2644783	1.2642080	1.260
	0.35	1.3799906	1.3786932	1.3775239	1.3764730	1.3755296	1.3746832	1.3739234	1.3732408	1.3726267	1.3720734	1.365
	0.40	1.4607042	1.4587145	1.4569514	1.4553919	1.4540131	1.4527932	1.4517124	1.4507527	1.4498988	1.4491369	1.440
	0.45	1.5092252	1.5067952	1.5046570	1.5027787	1.5011285	1.4996772	1.4983984	1.4972688	1.4962683	1.4953794	1.485
	0.50	1.5254130	1.5228322	1.5205662	1.5185794	1.5168372	1.5153077	1.5139621	1.5127754	1.5117256	1.5107941	1.5
$K_2(x)$	x/l	$cl=0$	$cl=1$	$cl=2$	$cl=3$	$cl=4$	$cl=5$	$cl=6$	$cl=7$	$cl=8$	$cl=9$	$cl=10$
	0.05		0.0680619	0.0691642	0.0709345	0.0732848	0.0761126	0.0793131	0.0827890	0.0864556	0.0902434	0.0940970
	0.10		0.2440010	0.2469378	0.2516240	0.2577869	0.2651101	0.2732744	0.2819876	0.2910012	0.3001162	0.3091809
	0.15		0.4890892	0.4931890	0.4996913	0.5081656	0.5181174	0.5290539	0.5405325	0.5521866	0.5637332	0.5749675
	0.20		0.7693797	0.7734024	0.7797427	0.7879298	0.7974280	0.8077116	0.8183183	0.8288769	0.8391132	0.8488421
	0.25		1.0556214	1.0583328	1.0625702	1.0679726	1.0741347	1.0806663	1.0872343	1.0935824	1.0995339	1.1049823
	0.30		1.3231862	1.3237131	1.3244932	1.3254040	1.3263138	1.3271043	1.3276857	1.3280016	1.3280279	1.3277665
	0.35		1.5520073	1.5500114	1.5468248	1.5426317	1.5376475	1.5320920	1.5261687	1.5200535	1.5138893	1.5077865
	0.40		1.7265290	1.7222350	1.7154499	1.7066551	1.6963989	1.6852212	1.6735992	1.6619188	1.6504681	1.6394462
	0.45		1.8356677	1.8297808	1.8205081	1.8085438	1.7946742	1.7796664	1.7641887	1.7487718	1.7338014	1.7195328
	0.50		1.8727843	1.8663293	1.8561703	1.8430789	1.8279275	1.8115647	1.7947276	1.7779983	1.7617967	1.7463972
	x/l	$cl=11$	$cl=12$	$cl=13$	$cl=14$	$cl=15$	$cl=16$	$cl=17$	$cl=18$	$cl=19$	$cl=20$	$cl=$
	0.05	0.0979738	0.1018415	0.1056765	0.1094614	0.1131837	0.1168348	0.1204086	0.1239014	0.1273107	0.1306353	0.285
	0.10	0.3180845	0.3267498	0.3351256	0.3431802	0.3508966	0.3582680	0.3652951	0.3719837	0.3783429	0.3843843	0.540
	0.15	0.5857512	0.5959989	0.6056650	0.6147327	0.6232052	0.6310990	0.6384391	0.6452553	0.6515801	0.6574467	0.765
	0.20	0.8579517	0.8663867	0.8741328	0.8812030	0.8876284	0.8934502	0.8987145	0.9034693	0.9077617	0.9116366	0.960
	0.25	1.1098775	1.1142104	1.1179998	1.1212815	1.1241005	1.1265051	1.1285439	1.1302628	1.1317041	1.1329061	1.125
	0.30	1.3272378	1.3264738	1.3255116	1.3243897	1.3231449	1.3218103	1.3204148	1.3189831	1.3175352	1.3160875	1.260
	0.35	1.5018267	1.4960670	1.4905446	1.4852819	1.4802894	1.4755693	1.4711176	1.4669263	1.4629846	1.4592797	1.365
	0.40	1.6289774	1.6191291	1.6099268	1.6013676	1.5934301	1.5860815	1.5792834	1.5729948	1.5671750	1.5617842	1.440
	0.45	1.7061147	1.6936161	1.6820494	1.6713901	1.6615913	1.6525939	1.6443338	1.6367465	1.6297697	1.6233452	1.485
	0.50	1.7319563	1.7185425	1.7061629	1.6947846	1.6843509	1.6747931	1.6660377	1.6580116	1.6506450	1.6438723	1.5

namely the functions of H itself. Numerical treatment must therefore search the just value by try-and-error method. Practically, however, after tabulating N and $k(x)$, it is easier to find the correct value of H without complicated calculation of hyperbolic functions and within simple interpolation of already calculated tables.

Computation of H is not so difficult when more progressive calculations are needed for complicated suspended structure which is a combination of two or more simple spans. If digital computers are available for design calculation, it would be desirable to shorten the programm step to use more simplified equations. For such purposes, it would be better to adopt the approximate simple functions of cl instead of exponential function for the value of N and $k(x)$.

Numerical Examples and Applications

Example 1

$l=1000$ m, $f=100$ m, $EJ=30\times10^7$t-m^2, $E_cA_c=10^7$t, $H_d=2\times10^4$t, $L_s=1500$ m, $L_t=1200$ m, $\alpha=1.2\times10^{-5}$

$$H=1.25\times\frac{1}{1+0.7\times10^{-4}/N}\left[\int k(x)p(x)\,dx+\frac{0.375}{N}(\varDelta L_3-1.44\times10^{-2}T)\right]$$

First approximate value $c^2l^2=H_d\cdot l^2/EJ=67$, then $cl=8.2$; from Table 1, we get $N_1=1.07\times10^{-3}$

$$H=1.25\times\frac{1}{1.065}\left[\int p(x)k(x)\,dx+350(\varDelta L_4-1.44\times10^{-2}T)\right]$$

when uniform distributed live loads $p=2$ t/m act on whole span and temperature drops 10°C degrees.

$$H=1.25\times\frac{1}{1.065}(1000\times2+350\times0.144)$$

$$=2\,410\text{ t}$$

More correct value of H is given in repeating again the above calculation with newly assumed value of $c^2l^2=(H_d+H)l^2/EJ$.

Example 2

When the cable of a simple supported symmetric suspension bridge is now fixed tightly at center of span with stiffning girder and large longitudinal force P acts along the bridge, the force will cause the cable tension $H_d+(P/2)$ in left half span and $H_d-(P/2)$ in righ thalf, also stiffning girder will move longitudinally about ΔL.

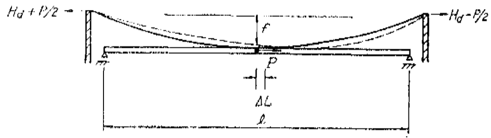

Fig. 4

To solve this problem, imagin two separate half spans, left half span expands and right half shortens. Using also the same data of suspension structure as example 1 and considering the span length $l'=l/2$ and $f'=f/4$ for half span we get from equation (5).

$$\frac{P}{2}=2.5\cdot\frac{1}{1+1.13\times(10^{-3}/N)}\cdot\frac{3}{N}\Delta L_3$$

N value of first approximation is 3.09×10^{-3} using $c^2l^2=16.7$ then

$$\frac{P}{2}=2.5\cdot\frac{1}{1+0.366}\cdot\frac{3\times10^3}{3.09}\Delta L\doteq1.8\times10^3\Delta L$$

This result means that longitudinal force $P\doteq3\,600$ t causes longitudinally 1 m displacement in stiffning girder.

Maximum deflection at a quarter point of whole span is then

$$\eta_{\max}=\frac{8f}{l^2}H\cdot\frac{(l/2)^4}{EJ}\cdot N\cdot k\doteq1.08\text{ m}$$

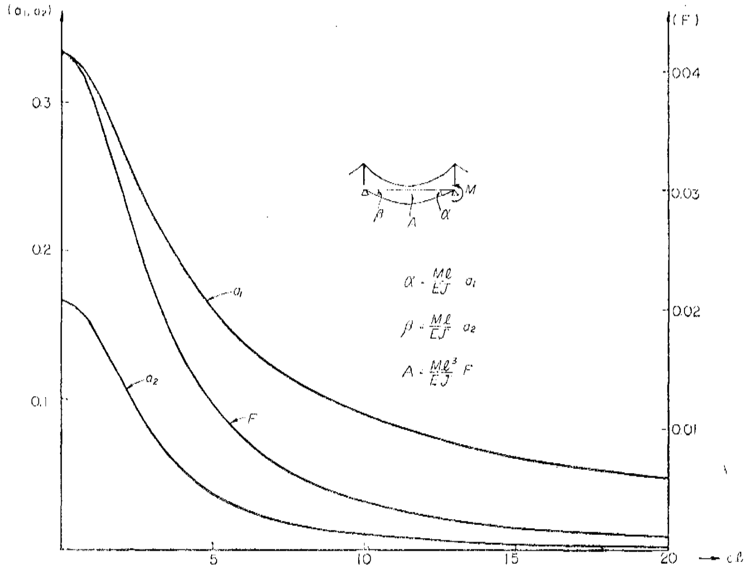

Fig. 5

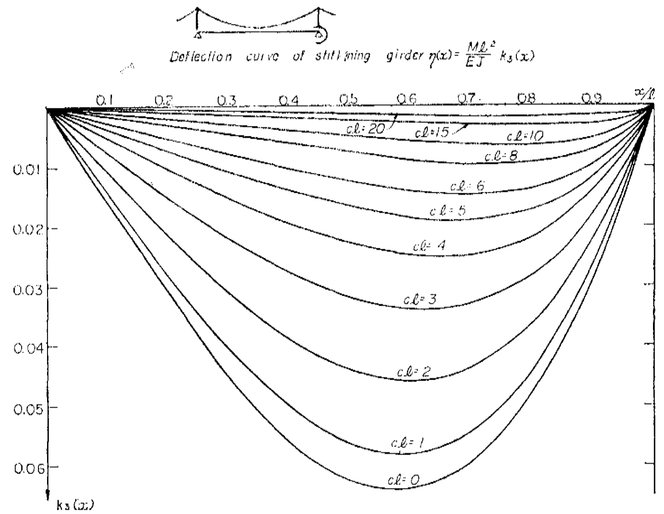

Fig. 6

Deflection and Displacement under Bending Moments at Supports

Preliminary calculations to reach a continuous suspension bridge we must deal with the effect of end moment working at either or both terminals of stiffning girder of each suspended span.

To take an approach, some fundamental values and equations are to be solved and also tabled in case when a simply supported suspension structure under a unit bending moment M at a end of stiffning girder at $x=l$. Symbols and functions are as follows;

deflection curve

$$\eta(x)=\frac{Ml^2}{EJ}\left[\frac{1}{c^2l^2}\left(\frac{x}{l}-\frac{\sinh cx}{\sinh cl}\right)\right]=\frac{Ml^2}{EJ}k_3(x)\cdots\cdots(9)$$

area of deflection curve

$$A=\int_0^l\eta\,dx=\frac{Ml^3}{EJ}\cdot\left[\frac{1}{c^3l^3}\left(\frac{cl}{2}-\tanh\frac{cl}{2}\right)\right]$$

Table 3

cl	a_1	a_2	F	K	cl	a_1	a_2	F	K
0.25	0.33195	0.16545	0.041407	5.00021	10.25	0.08804	0.00951	0.00383	5.31827
0.50	0.32790	0.16193	0.040650	5.00148	10.50	0.08616	0.00906	0.00367	5.32654
0.75	0.32146	0.15634	0.039449	5.00332	10.75	0.08436	0.00864	0.00352	5.33466
1.00	0.31303	0.14908	0.037882	5.00589	11.00	0.08264	0.00826	0.00338	5.34264
1.25	0.30308	0.14059	0.036044	5.00915	11.25	0.08098	0.00789	0.00324	5.35048
1.50	0.29208	0.13134	0.034029	5.01310	11.50	0.07939	0.00755	0.00312	5.35817
1.75	0.28048	0.12174	0.031924	5.01769	11.75	0.07786	0.00724	0.00300	5.36573
2.00	0.26865	0.11213	0.029800	5.02290	12.00	0.07638	0.00694	0.00289	5.37314
2.25	0.25686	0.10279	0.027715	5.02869	12.25	0.07360	0.00639	0.00268	5.38757
2.50	0.24542	0.09388	0.025709	5.03504	12.50	0.07496	0.00666	0.00278	5.38042
2.75	0.23438	0.08554	0.023809	5.04189	12.75	0.07227	0.00615	0.00259	5.39457
3.00	0.22389	0.07783	0.022031	5.04921	13.00	0.07100	0.00591	0.00250	5.40145
3.25	0.21394	0.07077	0.020381	5.05696	13.25	0.06977	0.00569	0.00241	5.40820
3.50	0.20460	0.06436	0.018860	5.06509	13.50	0.06858	0.00548	0.00233	5.41482
3.75	0.19585	0.05856	0.017464	5.07357	13.75	0.06743	0.00528	0.00225	5.42132
4.00	0.18766	0.05333	0.016187	5.08236	14.00	0.06632	0.00510	0.00218	5.42769
4.25	0.18002	0.04864	0.015021	5.09141	14.25	0.06525	0.00492	0.00211	5.43394
4.50	0.17289	0.04444	0.013958	5.10068	14.50	0.06420	0.00475	0.00205	5.44008
4.75	0.16623	0.04067	0.012989	5.11015	14.75	0.06320	0.00459	0.00198	5.44609
5.00	0.16001	0.03730	0.012107	5.11978	15.00	0.06222	0.00444	0.00192	5.45200
5.25	0.15420	0.03428	0.011302	5.12952	15.25	0.06127	0.00429	0.00186	5.45779
5.50	0.14876	0.03157	0.010567	5.13936	15.50	0.06035	0.00416	0.00181	5.46348
5.75	0.14367	0.02913	0.009896	5.14927	15.75	0.05946	0.00403	0.00175	5.46906
6.00	0.13889	0.02695	0.009282	5.15921	16.00	0.05859	0.00390	0.00170	5.47454
6.25	0.13440	0.02498	0.008719	5.16917	16.25	0.05775	0.00378	0.00166	5.47991
6.50	0.13017	0.02320	0.008203	5.17912	16.50	0.05693	0.00367	0.00161	5.48519
6.75	0.12620	0.02160	0.007730	5.18905	16.75	0.05613	0.00356	0.00156	5.49037
7.00	0.12244	0.02014	0.007293	5.19893	17.00	0.05536	0.00346	0.00152	5.49545
7.25	0.11890	0.01882	0.006892	5.20875	17.25	0.05461	0.00336	0.00148	5.50044
7.50	0.11555	0.01763	0.006521	5.21850	17.50	0.05387	0.00326	0.00144	5.50535
7.75	0.11238	0.01653	0.006178	5.22816	17.75	0.05316	0.00317	0.00140	5.51016
8.00	0.10937	0.01554	0.005860	5.23772	18.00	0.05246	0.00308	0.00137	5.51489
8.25	0.10651	0.01462	0.005566	5.24718	18.25	0.05179	0.00300	0.00133	5.51953
8.50	0.10380	0.01379	0.005292	5.25652	18.50	0.05113	0.00292	0.00130	5.52409
8.75	0.10122	0.01302	0.005038	5.26574	18.75	0.05048	0.00284	0.00127	5.52857
9.00	0.09876	0.01231	0.004801	5.27483	19.00	0.04986	0.00277	0.00123	5.53298
9.25	0.09642	0.01166	0.004580	5.28380	19.25	0.04924	0.00269	0.00120	5.53730
9.50	0.09418	0.01106	0.004373	5.29262	19.50	0.04865	0.00262	0.00118	5.54156
9.75	0.09204	0.01050	0.004180	5.30131	19.75	0.04806	0.00256	0.00115	5.54573
10.00	0.09000	0.00999	0.004000	5.30986	20.00	0.04750	0.00249	0.00112	5.54984

Table 4 $k_3\ (x)$

cl	$x/l=0.1$	$x/l=0.2$	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.014766	0.028679	0.040878	0.050483	0.056590	0.058259	0.054507	0.044294	0.026518
2	0.011121	0.021686	0.031115	0.038782	0.043993	0.045952	0.043736	0.036251	0.022195
3	0.007733	0.015160	0.021947	0.027702	0.031939	0.034034	0.033170	0.028261	0.017855
4	0.005309	0.010466	0.015292	0.019559	0.022943	0.024981	0.024988	0.021954	0.014372
5	0.003719	0.007366	0.010852	0.014044	0.016738	0.018599	0.019082	0.017289	0.011740
6	0.002690	0.005347	0.07928	0.010358	0.012509	0.014148	0.014853	0.013856	0.09755
7	0.002012	0.004010	0.05972	0.07858	0.09588	0.011004	0.011786	0.011294	0.08232
8	0.001553	0.003100	0.04630	0.06121	0.07526	0.08738	0.09520	0.09345	0.07041
9	0.001231	0.002460	0.03681	0.04882	0.06035	0.07070	0.07812	0.07835	0.06091
10	0.009989	0.001996	0.02990	0.03975	0.04932	0.05816	0.06502	0.06646	0.05321
11	0.000826	0.001651	0.02475	0.03294	0.04098	0.04857	0.05480	0.05695	0.04687
12	0.000694	0.001388	0.02081	0.02772	0.03455	0.04109	0.04671	0.04925	0.04158
13	0.000591	0.001183	0.01774	0.02364	0.02949	0.03517	0.04022	0.04294	0.03712
14	0.000510	0.001020	0.01530	0.02039	0.02546	0.03042	0.03494	0.03771	0.03333
15	0.000444	0.000888	0.01333	0.01777	0.02219	0.02655	0.03061	0.03334	0.30008
16	0.000390	0.000781	0.01171	0.01562	0.01951	0.02337	0.02702	0.02965	0.02726
17	0.000346	0.000692	0.01038	0.01383	0.01729	0.02072	0.02401	0.02652	0.02482
18	0.000308	0.000617	0.0925	0.01234	0.01542	0.01849	0.02146	0.02384	0.02267
19	0.000277	0.000554	0.0831	0.01108	0.01384	0.01660	0.01929	0.02154	0.02078
20	0.000249	0.004999	0.0749	0.0999	0.01249	0.01499	0.01743	0.01954	0.01911

$$= \frac{Ml^3}{EJ} \cdot F \cdots\cdots (10)$$

$$K = \frac{F}{N_1} \quad \text{where } N_1 \text{ is eq. (7-1)} \cdots\cdots (11)$$

deflection slope at $x=l$

$$\alpha = -\left(\frac{d\eta}{dx}\right)_{x=l} = \frac{Ml}{EJ}\left[\frac{1}{c^2l^2}\left(\frac{cl\cosh cl}{\sinh cl}-1\right)\right] = \frac{Ml}{EJ}a_1 \cdots\cdots (12)$$

deflection slope at $x=0$

$$\beta = \left(\frac{d\eta}{dx}\right)_{x=0} = \frac{Ml}{EJ}\left[\frac{1}{c^2l^2}\left(1-\frac{cl}{\sinh cl}\right)\right] = \frac{Ml}{EJ}\cdot a_2 \cdots\cdots (13)$$

Consequently, the same expression as eq. (5) or (8) for the cable tension H is introduced including the end moments as unknown forces.

$$H = \frac{1}{1+\frac{l^2}{64f^2}\cdot\frac{EJ}{E_cA_cl^2}\cdot\frac{L_s}{l}\cdot\frac{1}{N_1}} \cdot\frac{l}{8f}\left[\int_0^1 p(x)k_1(x)\,dx+\frac{l}{8f}\cdot\frac{EJ}{l^2}\cdot\frac{1}{lN_1}\cdot(\varDelta L_s-\alpha\, TL_t)+K(M_A+M_B)\right] \cdots\cdots (14)$$

where $k_1(x)$ and N_1 must be used the concerning values to the simply supported suspended structure.

References

1) Perry, D.J : An Influence Line Analysis for Suspension Bridges; Proc. A.S.C.E., Dec. 1954

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