VERTICAL STRESSES IN ELASTIC TWO-LAYER SYSTEMS

By Kenji Ishihara, C. E. Member*

Summary

Laboratory-scale experiments were performed of artificially-made twe-layer systems in order to examine the stress distribution and ascertain the validity of the elastic theory.

The materials employed were fine sand, gravel, oil clay, soil cement slab and asphalt slab. By the use of these materials, the task was undertaken of checking to what extent the actual stresses followed the stress pattern indicated by the theory. The results extracted from the experiments confirmed that in the elastic systems the actual load spreading ability of the upper layer obeyed the law of the elastic two-layer theory. However, in the systems in which either the upper layer or the lower layer or both layers were made up of the granular medium, representation by means of the elastic theory was found to be partially invalidated.

Stress concentration towards the center of the load occurred, as is generally the case in homogeneous ground of granular substance. A principle for basic understanding of this characteristic stress distribution was illustrated, based upon the concept proposed by Frölich.

Introduction.

The assessment of the efficiency -of a plate as means of transferring a relatively localized applied load to a underlying foundation poses a problem of considerable interest in several fields of civil engineering. The need for the solution of the problems is raised, for intance, in the design of footings for columns, in the design of highways, runways, taxiways and parking area and in the design of airport pavement. The design of the surface course on the slope of embankment and the determination of the load carrying capacity of a floating ice sheet involve also the problems of the same nature. The aspect common to all these practical problems is that the engineers are dealing basically with layered systems. Although multi-layer systems may be most representative of the actual conditions, two-layer system can approximately replace them, if necessary, hence the basic study being required to be made of two-layer system.

Although there have been many investigators such as D.M. Burmister$^{1)}$, L. Fox$^{2)}$, who contributed for the development of the elastic theories of two-layer system, little has been made of the experimental work concerning this item. Few data are available to confirm or reject any of the proposed theories for determining vertical stresses in the two-layer system. So far as the auther is aware, the test report by Sowers and Vesic$^{3)}$ seems to be one of the most reliable one. As the result of their test, it was found that a two-layer system having granular material as surface layer exhibits a pattern of stress distribution concentrating more towards the center of load than predicted from elastic theory. This result is ascertained basically also in our experiment.

The stresses at any point in a two-layer system consist of combinations of shear and normal stresses. Whether all the components of stress or only certain ones need be considered depends upon the criteria of the design of concerned structure. The eariest methods of design in pavement were mostly related to shear and shear failure in the subgrade. In this sense, the shear stresses were considered as a most governing factor. However recent studies of the pavement system indicated that deflection may be the better index to design ; and therefore the normal vertical stress is of greatest significance. In this paper only the vertical stresses are investigated.

1. Theoretical Stresses.

The theoretical stresses were computed by assuming that the interface of both layers is

---

* Dr. Eng. Assistant of Civil Engineering, University of Tokyo.

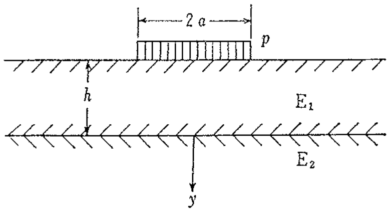

Fig. 1 Two-layer system.

frictionless, that is, free to move horizontally. This assumption is one of extreme cases, the other of which is that the interface constitutes a complete junction and no relative horizontal displacement is permitted along the interface of both layers. The actual condition is supposed however to be inbetween, hence, the frictionless assumption not being the cause of serious error in the result. The vertical stresses were computed for plane strain in order to be in conformity with the test condition. In calculating the stress, the variable parameters are the ratio of loading width to the depth of upper layer $S=a/h$ and the ratio of elasticity of the upper layer to that of lower layer, $C=E_1/E_2$. (see Fig. 1). The stresses were computed for S-value ranging from 0 to 2.0 and for C-value varying between 0 and 1 000. These ranges cover almost all the cases encountered in the tests and in the practise.

A part of the results of calculation was published elsewhere[4].

2. General Arrangement for Model Test.

(1) Box, loading equipment and loading plates.

The experiments were conducted in a large wooden box whose overall dimensions were 100 cm long, by 50 cm deep, by 28 cm wide. A sketch of general layout is shown in Fig. 2. A lever beam of a channel section was used to apply load to the loading plate placed on the model foundation. The load was transferred to the plate through a calirated proving ring. A steel roller was inserted between the contact planes of the lever and the top of proving ring, to avoid the tendency for horizontal force to be caused during settlement of the base plate. Model strip footings were made from 2.5 cm thick steel plate, 10 cm, 15 cm, and 20 cm wide, of common length of 25 cm. As the loading condition was considered to be identified with that of two-dimensional plane strain, the model footings were designed so that the width of strip load might be varied stepwise to investigate relative effect of width on the pattern of stress distribution in the soil. The allowable maximum load which the proving ring used can bear was 500 kg. The magnitude of applied load was inferred by reading the dial gage fitted to the ring with the aid of a calibrated characteristic curve. The dial gage used allowed a maximum stretch of 20 mm and permitted a minimum reading of 0.001 mm.

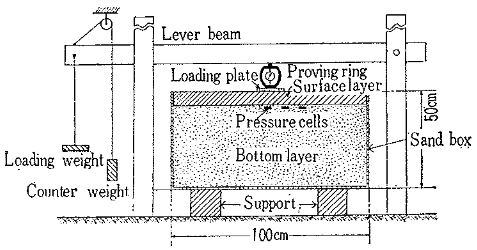

Fig. 2 Arrangement for tests.

(2) Measuring apparatus.

Simple diaphram type pressure cells, frequently used in this country, were employed to measure the vertical normal stresses in the interior of the two-layer systems. It consists of an elastic membrane, a thin disk of steel 4 cm in diameter which is fixed at its perimeter to a thicker circular base plate. The deflection at the center responding proportionally to the induced stress is transferred by a mechanism of lever system encased in the pressure cell box into a change in electric resistance. The acting pressure can be inferred by detecting the change in electric current that is caused by change in resistance through passage of Wheatstone bridge circuit. The ratio of cell diameter to its thickness was about 2.5. This ratio is not so large as to minimize the variable effects of soil arching over the deflecting diaphrams. The calibration diagram obtained by applying uniform fluid pressure to individual cell was taken as reference standard.

3. Materials Used for Two-Layer Components.

The two-layer materials tested were selected

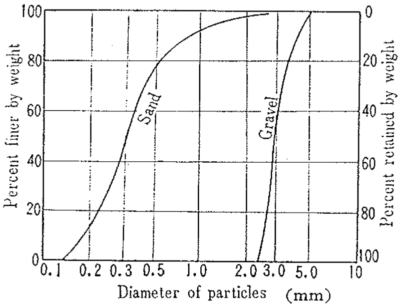

Fig. 3 Particle size distribution of sand and grvel.

so that it may cover the variety of natures of materials most frequently employed in the practice. The materials used for surface layer were soil cement slab and asphalt slab. Sand, gravel, and oil-mixed-clay were employed in the test as representatives of the bottom layer. Various combinations of these component materials served to form different types of two-layer systems that would be faced with in the practice.

(1) Sand and gravel.

Fine sand was taken from natural deposit and gravel of definite grain size range was obtained by sieving gravel-sand aggregate taken from river bed. The sand and the gravel used have specific gravities of 2.72 and 2.62 respectively. The gradation curves of them are presented in Fig. 3.

(2) Oil-clay.

The oil-clay which is a mixture of clay powder and machine oil was made up to provide us with a substitute of cohesive clay. It was impractical to utilize an ordinary soft clay to represent lower layer of cohesive properties, because there is a tendency for water content of clay to be changed during test period unless the temperature and humidity of the room are regulated adequately. The local change in water content causes unfavorable nonhomogeneity of material. Hence, an artificial clay termed oil-clay was made up to keep material condition constant during test period. As is known generally, the machine oil does not evaporate so rapidly as the water, and the clay mixed with oil does not tend to create an appreciable change in its physical characters in the test period continuing as long as several months. The process of manufacturing the oil-clay was as follows. The natural clay was dried in the air up to the water content of about 5% and it was crushed up by the roller or hammer to produce the powder of the clay. The fractional part which has the grain size smaller than 0.5 mm was selected by sieving the crushed clay. The powder clay thus obtained was mixed with machine oil at the mixing rate of 4 for clay to 1 for oil. The oil-clay appreared very cohesive and plastic and it was used to typify the poorer material.

(3) Soil cement slab.

Soil cement slabs of various component ratios and dimensions were fabricated to use it as representatives of the surface layer. The slabs were cured sufficiently long until stationary constant value was acquired in the elastic modulus. The dimensions of the slab were 90 cm long and 23 cm wide, the thickness being 5.0 cm, 7.5 cm and 10.0 cm. The length and the width were all equal, whereas the thickness was changed in three steps, in view of the size of the loading plate. Three kinds of mix proportion were employed to make up slabs with different elastic modulus. The slab with a mix proportion of 13% for cement, 20% for water and 67% for silty soil will be signified as C_{13}. The slab signified by C_{17} indicates that in which mix proportion is 17% for cement, 20% for water and 63% for silty clay. The slab having the greatest percentage of soil cement of 21% and the percentage fractions of 20% of water and 59% of silty clay will be termed C_{21}.

(4) Asphalt slab.

Asphalt slabs with various mix proportions and dimensions were fabricated to provide substitutes of the top layer of viscoelastic properties. The mix proportions were determined from the laboratory requirement concering compaction of asphalt aggregates. The mixed aggregates were heated at about 200°C in the oven and cooled in form boxes. The dimensions of the asphalt slabs were entirely equal to those of soil cement slab stated above. Three kinds of asphalt slab will be designated by A_8, A_{10}, and A_{14} in accordance with the quantity of asphalt contained in the mixed aggregates.

4. Physical Properties of Test Materials.

(1) Sand and gravel.

Among several factors which have influence on the physical properties of granular medium, the most significant in our investigation was the stress-deformation characteristics which plays the major role in determining the stress destribution in the interior of the two-layer system. Since two specifying constants are generally needed to represent the elastic property of a material, there arises a need for performing a couple of independent tests to determine the elastic constants. However, if a restrictive assumption concerning Poisson's ratio is imposed on the physical behavior of the material, there remains only one specifying constant to be made known. In accordance with the preceding theoretical treatment[5], the materials were considered, without unallowable loss of accuracy, to respond incompressibly. It is impossible to perform the conventional uniaxial compression test on the granular materials owing to the difficulty arising in forming test specimens of usual type. Thus, such simple test was run in which the plate resting upon the sand and gravel base was loaded vertically and the settlement of the plate was read as well as the intensity of applied load. The modulus of elasticity E was evaluated by the use of the following formula[4].

$$E=\frac{3h}{\pi}\frac{p}{v_0}\cdot\varphi \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

where,

$$\varphi=\int_0^\infty \frac{\mathrm{sh}^2 z}{\mathrm{ch}\,z\,\mathrm{sh}\,z+z}\,\frac{\sin\frac{a}{h}z}{z^2}\cdot dz$$

v_0 ; surface displacement.
p ; intensity of applied load.
h ; thickness of base materials.
a ; width of strip load.

The test apparatus used for this test was the same as the one employed in two-layer system test. The sand and gravel were packed in the box as dense as possible to enable the state of maximum density of the materials to occur whose condition was similar to the one encountered in the case of two-layer system test. The Young's modulus as evaluated by the formula (1) are summarized in Table 1.

Table 1. Young's moduli of test material in kg/cm²

Sand	Gravel	Oil clay	Soil cement			Asphalt		
S	G	C L	C_{13}	C_{17}	C_{21}	A_8	A_{10}	A_{14}
170	330	2.5	4 400	7 200	9 700	1 540	990	770

(2) Oil-clay.

The oil-clay used as a material of the base layer exhibited viscoelastic behavior. The creep test of plate load type which measures the displacement response following the sudden application of a constant load was run using the same setup as used in the test of gravel and sand. In considerating viscoelastic behavior, the factor of most significance is the time of loading or the time of straining in which the material of current concern is subjected to load. In short, mechanical properties exhibited by viscoelastic material are subject to change with change in the time interval of loading. Thus, the duration of loading time in which material properties itself are measured should be within the proper range of time of loading around which actual structure may be exerted by extermal disturbances. Since our experimental conditions concerning loading were almost same both for creep test and two-layer system test, the time of loading encountered in both tests was almost equal. From such consideration, the viscoelastic modulus determined from creep test was thought to be usuable directly to assess the theoretical stress distribution in two-layer system. In determining the viscoelastic constants, a model type to which the obtained creep curve is to be fitted must be postulated in advance. The rational and simple fitting can be made with the use of Maxwell model or three element model. Although three element model reveals more accurate representation than Maxwell model, the greater complexity is encountered for the former than for the latter in the course of analytical treatment. Thus, the fitting by means of Maxwell model was made on the creep curve to determine the viscoelastic constants appearing in Maxwell model. The relation between load intensity, surface displacement and time in the case of the plate loading test is given, for Maxwell model, by the following formula.

$$\frac{1}{E}+\frac{t}{E'}=\frac{\pi}{3h\varphi}\frac{v_0}{p} \quad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where, E ; spring constants corresponding to Young's modulus.

E' ; dashpot constant corresponding to Young's modulus.

By fitting the above equation to the creep curve obtained in the test, it was possible to determine the values of E and E'.

It was found theoretically elsewhere[5] that the state of stresses and displacements at the beginning of load application are governed solely by elastic components of two-layer substances in the case of Maxwell type representation. Viscous element comes to be mobilized only after a proper elapse of time. The effect of viscosity on the stress distribution could be put out of consideration, if attention is centered on the initial state of stress. Hence even if we are dealing with the materials of viscoelastic properties, it is possible to examine the validity of the elastic theory by comparing initial stress measured in viscoelastic system with the values evaluated theoretically by means of elastic theory. In evaluating theoretical stresses, the elasticity E of the materials corresponding to spring constant as represented by equation (2) should be used. From such consideration, merely the elastic properties of the asphalt are presented in Table 1.

(3) Soil cement slab.

A number of test specimens of soil cement 5 cm in diameter and 10 cm in length were made up, together with fabrication of soil cement slabs. Unconfined compression tests were run of all specimens in which stress-strain characteristics were determined. The Young's moduli thus determined are listed in Table 1.

(4) Asphalt slab.

Test specimens of 10 cm in diameter and 12 cm in height were made up in the mold, in parallel with fabrication of asphalt slabs. Since the physical properties of finished slabs were apt to change depending upon the degree of compaction in the process of forming slab, a great deal of precaution was arrested to keep the compaction in the mold in conformity with that in the slab form. Creep tests were performed on the cylindrical specimens thus fabricated. The period of time during which measurement of flow was made varied from about 3 sec. to 30 min. This range of time of loading was equal to that employed in the two-layer system test. By fitting the experimentally obtained creep curve to Maxwell model, viscoelastic constants were determined. In Table 1 are presented the elastic values that were the average of several creep curves obtained under different intensity of applied load.

5. Description of the Two-Layer System Test.

(1) Preparation for the test.

Three pressure cells were positioned at the junction face of both layers in the horizontal direction at the interval of 8 cm. Positioning of the presure cells in the granular media was conducted with great caution. The surface of the pressure cell tended to oblique and a great deal of attention was arrested to maintain the normal of the cell surface in the vertical direction. To accomplish this, temporary frame was set up during packing, so that cells were sited in their assigned positions. Sand and gravel were compacted so dense as to produce their maximum densities. When the sand and gravel were compacted up to that level where the pressure cells were to be located, the frame work was taken off. Three pressure cells were imbedded at the same depth to obtain the stress distribution along the horizontal plane of the fill. The location of the pressure cell installation was just at the interface of both layers as indicated in Fig. 2.

When oil-clay was used as the base material, a serious problem was encountered in installing pressure cells. As the weight of the pressure cells was greater than the specific weight of the oil-clay, there was a tendency for pressure cell to oblique and to sink downwards gradually by its own weight. To overcome this difficulty, a special frame work was constructed that seemed to function satisfactorily without disturbing seriously the surrounding soil conditions throughout test period. When the oil-clay was filled up to the assigned level where measurement was to be made, the frame equipped with pressure cells was inserted into the compacted mass of oil-clay. The frame appeared to function as if it were floating in heavy viscous fluid.

(2) Test procedure.

After all arrangement was completed, the measurement was started. Loading weights, each weighing 10 kg, were added step by step to the hanger equipped at the end of the lever beam. In each step of load increment, reading of the pressure indicator was repeated. After the load of about 200 kg was applied, the loading weights were taken off during which no measurement was made. After complection of this series of procedure, the loading plate was exchanged, and the same test was repeated the two-layer system being left in the same condition as it had been when the previous test was performed. The same test procedure was followed for the second loading plate and after that for the third one. The set of tests as mentioned above was repeated three times in which the base materials were refilled and recompacted. When the formed slab was used as the top layer, great caution was taken in producing uniform contact between the slab and the surface of the underlying base. When the creep type test was run on the viscoelastic two-layer systems, loading weights were applied suddenly to the lever beam and the change in stress with time was recorded for about 30 to 40 minutes. The induced stresses in the creep test were of course subjected to change with time. However, it was logically demonstrated[5] that, in the two-layer system of Maxwell type properties, the stress at the start of loading is simply determined by the elastic properties of two-layer materials, and the viscous component has no influence on the initial stress. Viscous element comes to play its role after a proper elapse of time. Thus, it can be said that the stress measured immediately after loading can be considered to be the stress which is determined by the elastic properties of two-layer substances with the use of the elastic theory. It was justified thus that the initial stress can be used to provide knowledge as to the experimental evidence for two-layer elastic theory. In this paper, only the stress obtained at initial time of loading is presented for discussion of two-layer elastic system.

6. Results of the Tests.

The data obtained for the vertical stresses are summarized in Fig. 4 through Fig. 9 as indevidual points on the graphs. They are shown as percentage stress of applied load intensity and as a function of distance from the

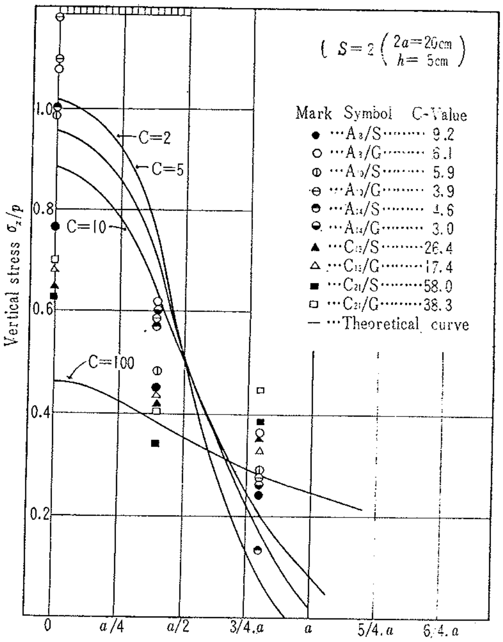

Fig. 4 Stress distribution.

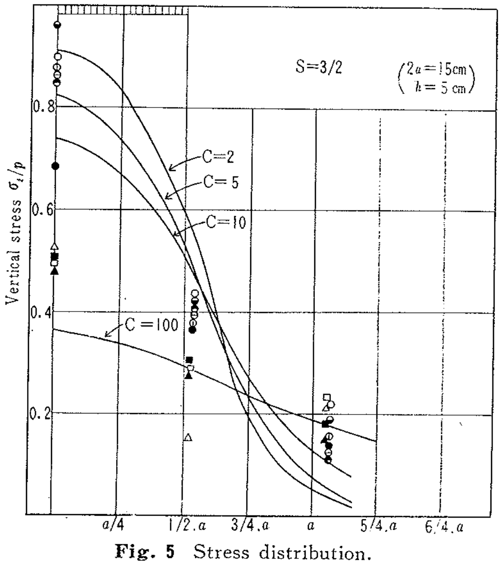

Fig. 5 Stress distribution.

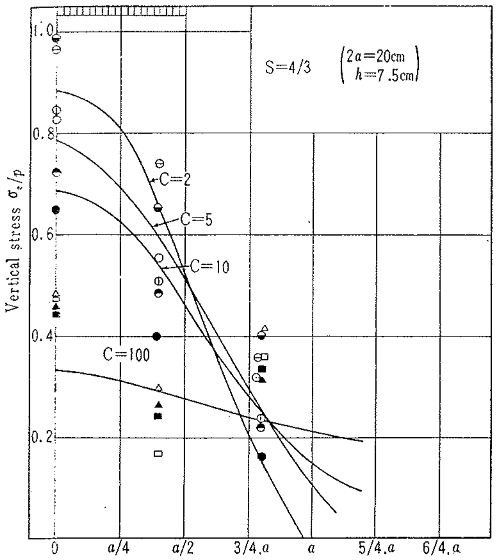

Fig. 6 Stress distribution.

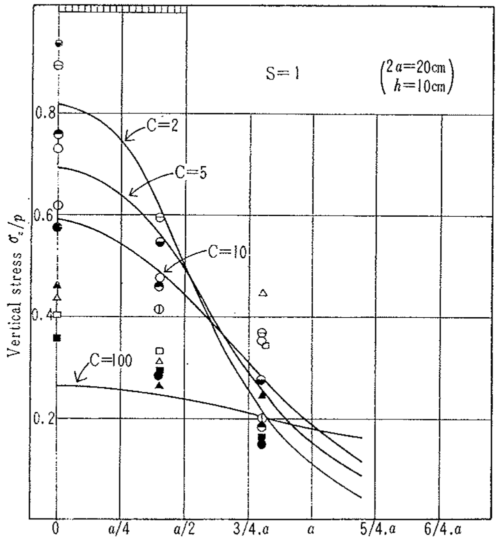

Fig. 7 Stress distriibution.

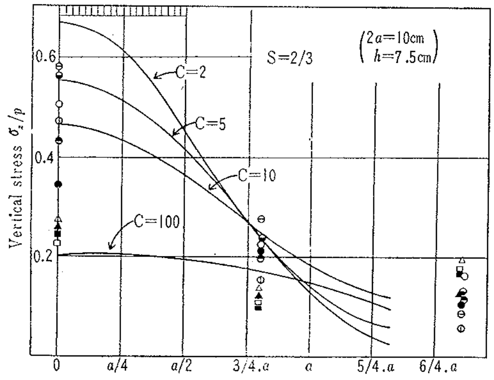

Fig. 8 Stress distribution.

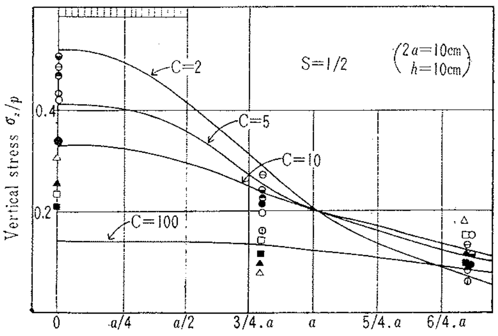

Fig. 9 Stress distribution.

center of the load. Since the pattern of stress distribution is governed theoretically by the elasticity ratio C of the upper layer to that of the lower layer, different marks are used in the graphs to distinguish between data obtained with different set of two-layer materials. Nearly all possible combination of materials were employed to provide wide range in the elasticity ratio of the two-layer systems. The elasticity ratio adopted varied between 3.0 and 58.0. The computed stresses were shown, together with the obtained data, to make it easy to compare between them. The computed values are portrayed with continuous curves for various values of elasticity ratio C, thereby the value of S being fixed. Fig. 4 refers to the stress distribution met in such condition where the ratio of loading width to the thickness of the upper layer is equal to 2.0. In Fig. 5 are presented the test data for $S=3/2$ which were obtained with the loading plate 15 cm wide and the slab of thickness of 5 cm. Figs. 6, 7, 8 and 9 refer to the cases of $S=4/3$, 1, 2/3 and 1/2, respectively. Another aspect of the test data is seen from Fig. 10 through Fig. 12. In these figures, the vertical stresses both computed and measured are presented as a function of elasticity ratio C. The

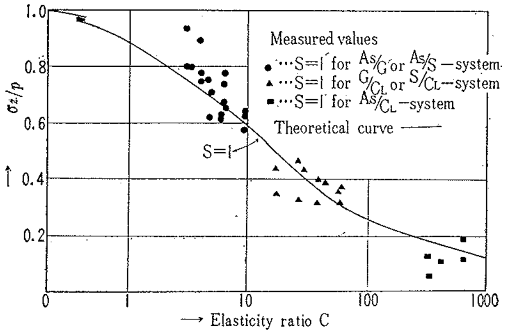

Fig. 10 Vertical stress as function of C.

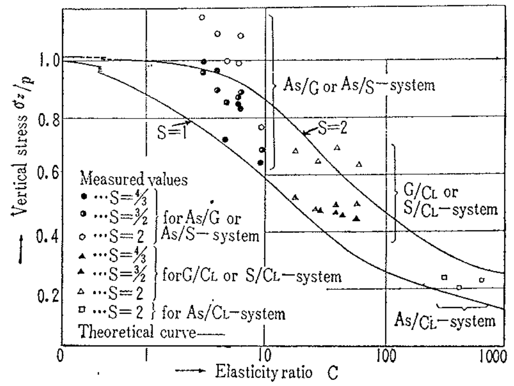

Fig. 11 Vertical stress as function of *C*.

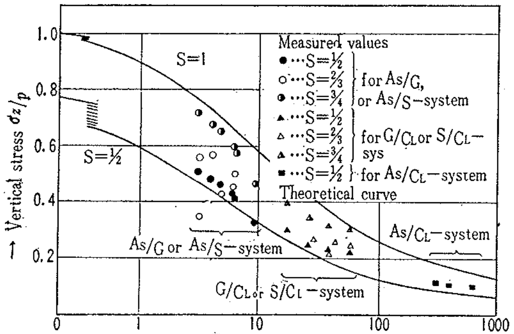

Fig. 12 Vertical strese as function of *C*.

data in these figures are the vertical stresses just below the center of load and are the same as those presented in Fig. 4 to Fig. 9. Additional data are also shown that were obtained by using asphalt slabs and oil clay. The combination of asphalt slab and oil clay served to present data for rather greater values of elasticity ratio ranging between 400 and 700. Theoretical curves depicted in these figs. are the same as the ones shown in Fig. 4 to Fig. 9.

7. Discussion of the Result.

The diagrams in Fig. 4 to Fig. 9 are presented in order to see the characteristic shape of stress distribution within two-layer systems. In general, the experimental results show good agreement with the theoretical values. However, the result obtained for the two-layer system comprising soil cement slab deviates slightly from the computed values. The values observed at the greatest distance from the center of load are somewhat greater than the theoretical values, overall feature of stress being of flatter shape than theory. The cause of this discrepancy is attributed to irregular contact between both layers that is encountered when the slab is placed on the underlying fill. It was difficult in the tests to produce uniform contact over the entire slab area because even slight unevenness of levelled fill could be responsible for producing uneven contact between the slab and the fill. However, the above disagree ment is not essential in making basic consideration for the characteristic nature of stress distribution because uneven contact of this kind may not be the case in the practice. From inspection of the entire diagrams, it is seen that the measured stresses at the center of load is markedly greater than those predicted from theory for the two-layer system comprising asphalt slab and granular masses. Moreover, percentage rise of measured values over the theoretical values is more remarkable for the larger values of *S*. For *S*-value below about 2/3, the pressure distribution is seen to be close to that given by elastic theory. On the other hand, no characteristic increase of this kind can be observed in the case of the systems consisting of soil cement and granular media. As is indicated in the legend of Fig. 4, the elasticity ratios of the surface layer to that of bottom layer are generally greater for C_L/S or C_L/G systems than for A_S/S or A_S/G systems. Hence it can be said that the discrepancy of experiment from theory depends upon the elasticity ratio *C* as well as the value of *S*. This distinctive tendency of stress concentration can be visualized more clearly from Fig. 10 to Fig. 12. In Fig. 10 to Fig. 12, additional data are plotted that were obtained from the two-layer systems composed of asphalt slab and oil clay. From these data, satisfactory agreement may be seen between theory and experiment although somewhat scattering. Hence, it can be concluded that the two-layer systems composed of elastic materials in both layers follows the law of stress distribution determined by the elastic theory. The load spreading efficiency of the surface layer as predicted by the theory could thus be substantiated by the experiment. Somewhat surprising result as stated above regarding the stress concentration in the system containing granular substance is analogous to that attained in the test by G.F. Sowers and A.B. Vesic[3]. They discussed the possible causes of stress concentration. The first factor that is undoubtedly responsible is that the materials used are incapable of withstanding much tension. Tensile cracks may develop in the two-layer system and destroy the continuity of the medium and invalidate the theory to some extent. Such cause for stress concentration always exists for the systems comprising granular mass either in the top layer or in the lower layer. The second controlling factor for stress concentration is that the elastic modulus increses with depth. In this case, the same effect is produced as that experienced in the two-layer system in which the elasticity of the upper layer substance is smaller than that of the lower layer. Increase in elasticity with depth thus offsets the gain in load spreading ability of the overlying layer. The latter factor appears however less responsible in our experiment, as compared with the former, because the depth of lower layer was not great enough to produce change in elasticity by the gravity force, as in the case of field practice. Thus, the cause of stress concentration could be attributed mainly to the first factor just mentioned above.

In the two-layer system comprising granular media, the mechanism of stress concentration beneath the load resembles that which is experienced in homogeneous granular materials such as sand or gravel, of which extensive discussion was made by Frölich[6]. However the degree to which the stress concentrates toward center of load depends upon the elasticity ratio C and S-value as noted above in the case of two-layer granular system. Moreover the dependence of stress concentration on these factors varies according that the granular material is used either in the surface layer or in the lower layer or in both layer.

Simple qualitative consideration will be made hereafter on the characteristic feature of the three types of the systems containing granular medium.

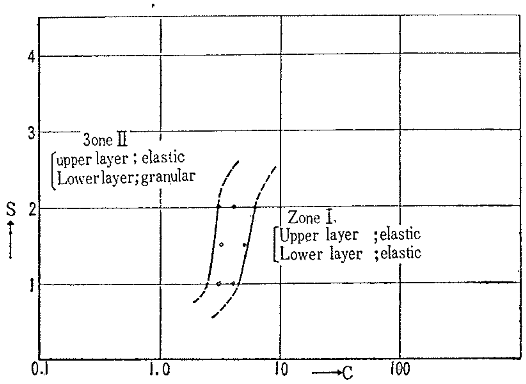

Fig. 13 Range of S and C within which elastic theory is applicable or invalidated.

(1) Granular medium is overlaid by elastic material.

When the elastic medium typified in our case by soil cement or asphalt rests over the granular material such as sand or gravel, the elastic two-layer theory does not hold valid within some range of C and S. The approximate range within which elastic theory is applicable is shown in Fig. 13 in terms of C and S. The points on the diagram indicate the values of C and S at which the measured stresses deviate by more than 15% over the computed values. The plot is made by using the data for A_S/G and As/S-systems shown in Fig. 10~12. The data obtained for large values of C seem not suitable for this kind of discussion, because the measured stresses are small in magnitude and may include the errors resulting from experimental techniques. More exhaustive work should be done, before this diagram is made applicable for various types of soils. In Fig. 13, Zone I indicates the one in which elastic theory is applicable. In the two-layer system belonging to Zone II, the elastic theory is no more usable and the system should be treated as the one comprising Frölich material in the under layer.

(2) Granular medium rests over elastic material.

In this case, the elastic theory is of no use for any value of C and S. When the depth of overlying granular medium is small, the load is transmitted immediately to the underlying elastic material without being widespread by the shear action of the overlying granular medium. Thus, the upper layer can be considered to be simply surcharge pressure and the two-layer system is reduced to the homogeneous ground that is subjected to the surcharge pressure plus the applied load. When the upper layer has the depth enough to develop shear action, the two-layer system should be treated as the one consisting of Frölich material in upper layer and elastic material in the lower layer.

(3) Granular media are deposited in both layers.

Although available data lack that may lead to a complete conclusion, it seems safe to consider both layers as consisting of Frölich materials. The elastic theory is undoubtedly of no use.

Conclusions

As the result of the experiments, the following conclusions were drawn.

1. In the two-layer system made up of elastic materials, the distribution of the measured stresses agrees well with those predicted from the elastic theory. The load spreading efficiency of the upper layer was substantiated by the results of experiments.
2. When the two-layer system contains granular material, either in the upper layer or in the lower or in both layers, the law governed by elasticity is destroyed and invalidated. Although alternative theory is not available at present, a principle for basic undersanding was presented qualitatively.

Acknowledgement.

In overall areas of this study, valuable advice of Prof.T. Mogami and great cooperation of Mr. Tsutomu Kimura were of great help, and here gratefully acknowledged.

References

1) Burmister, D.M. : The General theory of Stresses and Displacements in Layered Systems, Journal of Applied Physics. Vol. 16, pp. 296～302 (1945).

2) Fox, L. : Computation of Traffic Stresses in a Simple Road Structure, Proc. 2nd Interna. Conf. Soil Mech. & Found. Engrg, Vol. II, pp. 236～246 (1948).

3) Sowers, G.F. & Vesić A.B. : Stress Distribution beneath Pavements of Different Rigidities, Proc. 5th Interna. conf. Soil Mech. & Found. Engrg. Vol. II, pp. 237～331 (1961).

4) Ishihara, K. : Stress Distribution in Two-Layer Soil System. (in Japanese) Trans. Japan Soc. Civil Engineers, Vol. 68, pp. 32～40 (1960).

5) Ishihara, K. : The General Theory of Stresses and Displacements in Two-Layer Viscoelastic Systems, Soil and Foundation, Vol. II pp. 51～68 (1962).

6) Frölich, O.K. : Druckverteilung im Baugrunde, Berlin. (1934).

(Received July 5, 1963)