Higher-order FEM implementation of scissors structure

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1. Introduction

The temporary emergency bridge, "Mobile Bridge", is a structure that allows prompt recovery of a lifeline after natural disasters, in 2013, Ario[1] et al., have created an optimum deployable bridge inspired by origami skill. The one-unit scissors structure[2] in Fig.1 is a pre-assembled bridge with the main mechanism of the module structure of a pin-rigid body, in which the rigid body is connected by the pin, but currently there are various technical issues, such as buckling problems remained to be solved.

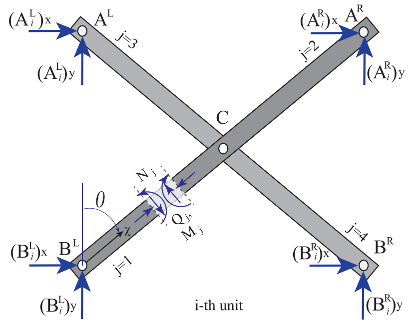

Fig. 1 The one-unit scissors structure model[2].

In terms of higher-order FEM[3], which can deal with more degrees of freedom beyond 1D analysis, it contributes to improve the accuracy on predicting the critical loads and buckling modes for intermediate length structures rather than using empirical transition equations.

This study aims to implement higher-order FEM into scissors finite elements for predicting the buckling phenomena of scissors structures. With these investigations, it can provide more accurate and efficient design for the structural components due to compressive stress.

2. Method

Differing from the standard FEM (h-version FEM) which only uses a piecewise linear approximation, the higher-order FEM (p-version FEM) derive the stiffness matrix based on hierarchical Legendre shape functions. From the pioneering work of Barna Szab and Ivo Babuka[4], the origins of h- and p-version FEM were refined for solving partial differential equations.

In this part, the procedure of applying higher-order FEM into scissors finite elements will be introduced, which mainly refers to the derivations of stiffness matrix and mass matrix for constructing the eigenvalue equation.

$$[k_m]\{a\} - \omega^2 [m_m]\{a\} = \{0\} \tag{1}$$

where $\{a\}$ represent amplitudes of the harmonic motion, ω is frequency and ψ is phase shift. The standard eigenvalue equation can be easily arranged following Eq.(1) Considering the simplified case for rod element[5], which is the simplest component of scissors finite elements, we arrange the following part as the derivations of element stiffness matrix and element mass matrix for rod element by using the set of Legendre shape functions $N_m(\xi)$.

3. Matrix derivation

For a reference element $x \in \{0, L\}$, the normalized element coordinate $\xi \in \{-1, 1\}$, the space of polynomial degree p defined on domain x and ξ can be denoted by

$$S^p(x) \quad S^p(\xi) \tag{2}$$

We define shape functions using Legendre polynomials for $t \in [-1, 1]$:

$$P_0 = 1, \quad P_1 = t, \tag{3}$$

$$P_{n+1}(t) = \frac{(2n+1)tP_n(t) - nP_{n-1}(t)}{n+1}, \quad n \geq 1. \tag{4}$$

Hierarchic shape functions on $\xi = [-1, 1]$ are functions $N_m(\xi) : \xi$ from domain of real numbers R, m from domain of natural numbers N defined using (4) shown in Fig.2:

$$N_1 = \frac{1}{2}(1-\xi), \quad N_2 = \frac{1}{2}(1+\xi), \tag{5}$$

$$N_m(\xi) = \sqrt{\frac{2m-3}{2}} \int_{-1}^{\xi} P_{m-2}(t)dt \quad m \geq 3. \tag{6}$$

Note that for all $N_m(\xi)$, $m \geq 3$ vanishes at the endpoints of x or ξ.

Keywords Hierarchical shape functions, Higher-order FEM, Scissors structure, Buckling analysis
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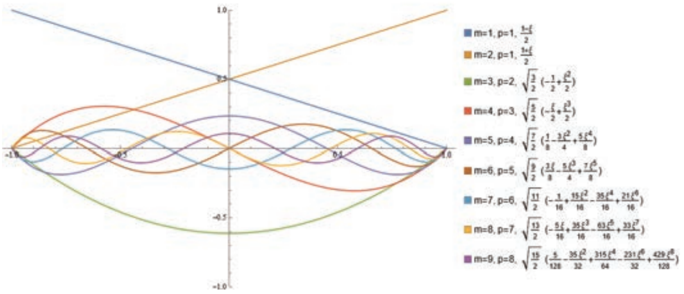

Fig. 2 The hierarchical Legendre shape functions $N_m(\xi)$, $m = 1, ..., 9$, $p = 1, ..., 8$, where p is the corresponding Legendre polynomial degree.

On evaluation of the integrals, the element stiffness matrix for rod element $K^r_{i,j}$ has obtained, where the notation r represents the rod element.

$$\begin{aligned} K^r_{i,j} &= \int_0^\ell EA \frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} dx \\ &= \frac{EA\ell}{2} \int_{-1}^{1} \frac{\partial N_i}{\partial \xi}\frac{\partial N_j}{\partial \xi} (\frac{2}{\ell})^2 d\xi \\ &= \frac{2EA}{\ell} \int_{-1}^{1} \frac{\partial N_i}{\partial \xi}\frac{\partial N_j}{\partial \xi} d\xi, \quad i, j = 0, 1, \cdots, m \end{aligned}$$

Following the similar procedure, the element mass matrix can be derived as,

$$\begin{aligned} M^r_{i,j} &= \int_0^\ell \rho A N_i N_j dx \\ &= \frac{\rho A \ell}{2} \int_{-1}^{1} N_i N_j d\xi, \quad i, j = 0, 1, \cdots, m \end{aligned}$$

4. Numerical results

A numerical benchmark problem[6] shows in Fig.3, where we assume the conditions $EA = L = 1$, $f(x) = -\sin(8x)$ and $F = 0$.

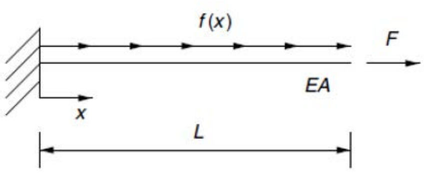

Fig. 3 The benchmark problem model: linear elastic rod[6].

Here, we apply the p-version FEM consists of nine Legendre shape functions $N_m(\xi), m = 1, 2, \cdots, 8$. Following the derived matrices and equations above, the approximate displacement function $u_{FE}(\xi)$ can be solved hierarchically. Here is the approximation expressed by the basis of Legendre shape functions.

$$u_{FE}(\xi) = N_1(\xi)u_1 + N_2(\xi)u_2 + \sum_{p=2}^{p_{max}} N_{p+1}(\xi)a_{p+1} \quad (7)$$

where $p_{max} = 8$, u_1 and u_2 denote the nodal displacements, whereas $a_3, \cdots, a_9$ are unknown coefficients determining the higher-order terms of the approximation $u_{FE}(\xi)$.

Next, the numerical results have shown in Fig.4, the FE displacement functions by p-version approximation with p =1, 4, and 8 have plotted together with the exact solution of the problem.

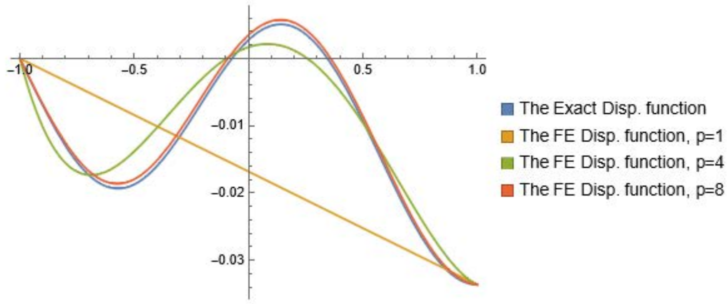

Fig. 4 The numerical results for exact solution $u_{EX}(x)$ and p-version approximation $u_{FE}(x)$.

5. Conclusion

In this study, we have compiled a part of implementations of higher-order FEM for the scissors finite elements which are used for discrete elements in FEM process, where p-version FEM uses hierarchical shape functions. Compared to traditional low-order FEM approaches, higher-order FEM provides more accuracy and efficient predictions for buckling analysis of scissors structures.

Moreover, since p-version FEM improves the accuracy of numerical results, the scissors components design will obtain refinements, and we proposed the new algorithm for scissors finite elements satisfying the buckling analysis. For further researches, we are looking forward to obtaining the matrix computation for beam elements and beam-cloumn elements, then making the comparisons with the reference numerical results for structural analysis of scissors structure[7], which has applied the traditional FEM.

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