GENERALIZED HIGH-ORDER NONLINEAR FRAME ELEMENTS

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1. INTRODUCTION

An important criterion not usually checked for structural analysis is the consistency between the equilibrium-compatibility and the interpolation functions used (shape functions), which produces a truncation error. This error is always present in low-order functions, e.g., in traditional structural analyses.

Some well-known plastic-length structural analysis software packages use the Direct Flexibility Method with models that cannot be used for high-order elements. Moreover, since no current studies have addressed high-order elements with nonlinear plastic length, the solution was to use a set of sigmoid functions to depict the nonlinear material behavior. Distinct node distributions for increasing the order were considered to generalize the solution and reduce the Runge phenomenon. A maximum of three additional internal nodes are usually enough for common structures, as was proved with diverse examples.

Compared with related *state-of-the-art* documents (Bai et al., 2020; Eisenträger et al., 2020; Kim et al., 2019; 2019; Sharifnia, 2022), the contributions exposed herein to those works are 1) a stiffness matrix formulation that can deal with nonlinear plastic length hinges on high-order elements, and 2) an effective manner to obtain the higher permissible error for p-adaptive (and h-adaptive) procedures. Moreover, a straightforward process to establish general high-order Timoshenko elements has been presented, and robust free software for dealing with the problem described has been developed.

2. PROBLEM FORMULATION

The solution posed here was to use the Hamilton principle (Finite Element formulation) from a generalized Timoshenko approach to establish the high-order shape functions and series of sigmoid functions to depict the nonlinear material behavior conserving the continuous displacement space. That is how the proposed stiffness matrix for nonlinear elements was obtained using $K^e = \sum_{r=1}^{\bar{n}-1} \int_0^L \boldsymbol{B}^T \boldsymbol{D_r} \boldsymbol{B} \partial x$, where $\boldsymbol{D_r}$ comes from using sigmoid functions $\sigma(\cdot)$. This generalized process allows separate beam nodes, Gauss integration points, and plastic length coordinates.

Furthermore, the high-order functions for bending and shear displacements for the stiffness and the mass matrix are $\underline{\boldsymbol{N_b}} = \underline{\boldsymbol{x}}^{\boldsymbol{T}}(\boldsymbol{C_b^{-1}})$ and $\underline{\boldsymbol{N_s}} = \underline{\boldsymbol{x}}^{\boldsymbol{T}}(\boldsymbol{C_s^{-1}})$ respectively, where C_b and C_s were found that can be gotten from Eq. (1).

$$\boldsymbol{C_{k_{i,j}}} = \begin{Bmatrix} \left(L_{distribution_i}\right)^{j-1} \\ (j-1)\left(L_{distribution_i}\right)^{j-2} \end{Bmatrix} + \begin{Bmatrix} -\emptyset_c L^2 \left(\dfrac{j^3 - 6j^2 + 11j - 6}{3}\right)\left(L_{distribution_i}\right)^{j-3} (0\ if\ k = s) \\ \emptyset_c L^2 \left(\dfrac{j^3 - 6j^2 + 11j - 6}{3}\right)\left(L_{distribution_i}\right)^{j-4} (0\ if\ k = b) \end{Bmatrix} \tag{1}$$

Where $\emptyset_c$ is the shear coefficient, i.e., $\emptyset_c = \frac{EI_y}{\kappa GAL^2}$, and L is the total element length.

Since increasing the function order slows the calculations, p-adaptive approaches can be employed. Those approaches consist of incrementing the shape function order only to the elements with a high truncated error. That means reducing the error e_m from subtracting the continuous equation of the element moments less the prescribed moments. Thus, the shape function order should be increased when ξ_{el} from Eq. (2) is greater than one.

$$\xi_{el} = \frac{\|e_m\|_{L_2}^2}{\bar{\eta}\left[\dfrac{\|\boldsymbol{d}^{xz}\|^2 + \|e_m\|^2}{number\ of\ elements}\right]^{\frac{1}{2}} \le \|e_m\|_{L_2}^2} \le 1 \tag{2}$$

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From Eq. (3), the problem of calculating the maximum permissible error $\bar{\eta}$ with iterations has been solved in different ways by diverse authors. Here it has been proposed to analyze $\bar{\eta}$ filtering the outliers using the Grubbs method $T_n(\cdot)$ with $\bar{\eta} \approx \max_{i\in n} T_n\left(\sum_{el=1}^{nel} \frac{e_{m_{el}}}{m_{y_{el}}}, significance\right)_{n\times 1}(i)$.

3. EXAMPLES

Various examples were developed, from straightforward 1-floor to medium-height buildings using energy dissipation devices. It has been found that the truncated and round-off errors follow the same pattern shown in Fig. 1. As a result, the appropriated internal nodes for p-adaptive methods will be near 2 (6th order functions). Finally, another important outcome is to show that the elements damaged after strong earthquakes can be different after using distinct shape orders, see Fig.2.

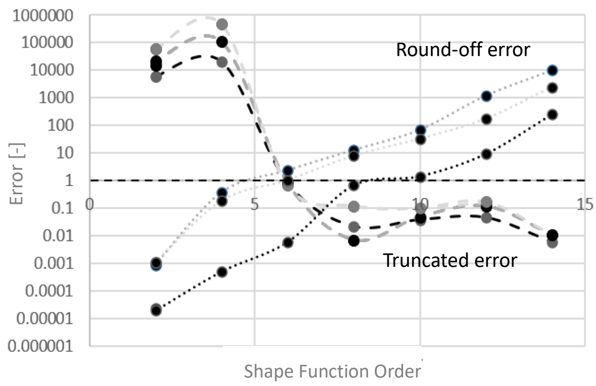

Fig. 1. Errors of various structures

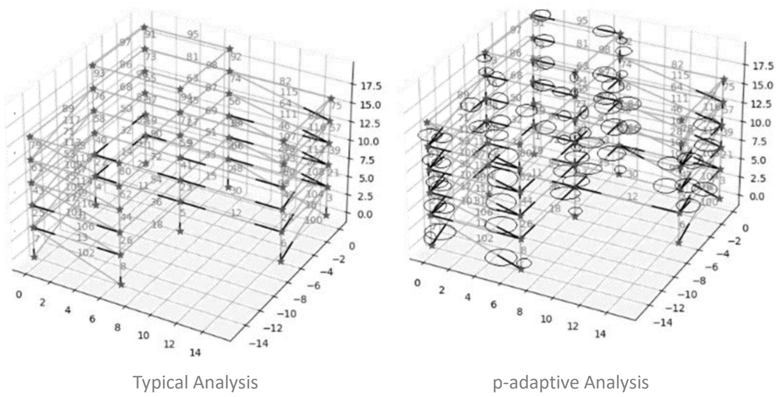

Fig. 2. Example of damage variations. Differences are enclosed

4. CONCLUSIONS

The distinct damage shown in the results indicates the impact of using different shape function order in elements. Regardless the response or damage is larger or smaller, it is important to highlight that the results are different, which encourages the use of appropriate function orders by implementing p-adaptive procedures.

In the examples, it was found that in all the cases, the optimum number of additional internal nodes is two or three, and less than two nodes produces large errors.

Moreover, the additional nodes can present important spurious high frequencies, which are usual problems in finite elements. Thus, different techniques have to be used, like the generalized alpha procedure, constant stiffness, Chebyshev nodes distribution, Ritz vector modal shapes, and the p-adaptive procedure.

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