Simplified Probable Maximum Loss Model from Generated Correlated Seismic Damaged Ratio

Tokyo City University Student Member ○Samantha Louise N. Jarder
Tokyo City University Regular Member Osamu Maruyama
De La Salle University Lessandro Estelito O. Garciano

1. INTRODUCTION

Losses due to hazards are inevitable. Estimating possible losses require numerous generations of random variables and possible combinations. Most studies focus on the independent impacts of a specific hazard to one specific site. But, seismic impacts are affected by various parameters. Lifeline systems are interconnected and affected by damage from its neighbors thus, correlation must be considered to account for uncertainty. A simplified model to obtain the correlated Probable Maximum Loss (PML) of a Pipeline system subjected to a seismic event is proposed. Data for the simulation and analysis were obtained from a previous study. The results show the cumulative histogram of the simulated PML show a lognormal trend. A model was formed and compared to the simulated results. This would aid in pre-disaster planning and would require only a few steps and time.

2. THEORY

2.1 Probabilistic Seismic Hazard Analysis

Earthquake hazards were obtained from a previous paper (Jarder et al, 2020). Historical seismic data of the area were the parameters for the analysis. Probabilistic Seismic Hazard Analysis (PSHA) was used to obtain the peak ground velocity, V, throughout the city (Kramer, 1996).

2.2 Damage Rate

Pipeline seismic damages can be estimated using the Isoyama et al (2000) method on estimating damages of pipelines per grid. Eq. 1 is the standard rate of damage.

$$R(V) = 3.11 \times 10^{-3} \times (V - 15)^{1.3} \tag{1}$$

2.3 Correlation, ρ

Using Normal Distribution for data generation is common and easy; but, generated multivariate normal data produces negative values when it is impossible for damages, thus, transforming to Poisson Distribution is needed. Yahan and Shmueli (2009) proposed a method to generate Multivariate Poisson Random data. Eqs. 2 and 3 shows the Cumulative Density Functions (CDFs) for Gaussian and Poisson, respectively. Where, u and λ are the mean for Gaussian and Poisson, respectively.

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du \tag{2}$$

$$\Xi(x) = \sum_{i=0}^{x} \frac{e^{-\lambda}\lambda^i}{i!} \tag{3}$$

The proposed method states to generate multivariate Gaussian random variables, using the correlated matrix in Eq. 4, where $\rho_{1,2}$ and $\rho_{2,1}$ are correlation coefficients of two sets of samples whose values ranges from -1 to 1. Once the number generations are done, calculate the Normal CDF as shown in Eq. 5. The values for $\rho_{1,2}$ and $\rho_{2,1}$ are equal.

$$\begin{bmatrix} 1 & \rho_{1,2} \\ \rho_{2,1} & 1 \end{bmatrix} \tag{4}$$

$$\Phi\left(X_i^N\right) \tag{5}$$

Assuming the Normal CDF is equal to the Poisson CDF, Eq. 6 shows how to obtain the Poisson Multivariate Random Number by obtaining the Poisson inverse CDF of Eq. 6 at the rate of λ. However, one of the limitations of using this method is that there is a discrepancy when the value of λ is less than 1. Thus, calibration is needed.

$$X_{Pois_i} = \Xi^{-1}\left(\Phi\left(X_{N_i}\right)\right) \tag{6}$$

To obtain the max and min uncorrected, the calculation is the same from Eqs. 5 to 6, however values of $\rho_{1,2}$ and $\rho_{2,1}$ should be 1 and -1 to get the ρ_{max} and ρ_{min}, respectively. Correction is needed, Eqs. 7, 8 and 9 show the calculations for the coefficients needed for Eq. 10, Corrected Poisson Correlation, ρ_{Pois}. Where, ρ_N is the Calculated Poisson Correlation.

$$a = -\frac{\rho_{max} \times \rho_{min}}{\rho_{max} - \rho_{min}} \tag{7}$$

$$b = \log\left(\frac{\rho_{max} + a}{a}\right) \tag{8}$$

$$c = -a \tag{9}$$

$$\rho_{Pois} = a \times e^{b\rho_N} + c \tag{10}$$

Once the ρ is corrected, the covariance per set can be obtained as shown in Eq. 11. Where, σ_x and σ_y are standard deviation of sets x and y.

$$cov(x, y) = \rho_{x,y} \cdot \left(\sigma_x \cdot \sigma_y\right) \tag{11}$$

Keywords: Probable Maximum Loss, Correlation, Seismic Damages
Contact address: 1-28-1 Tamazutsumi, Setagaya-ku, Tokyo, 158-8557, Japan, Tel: +81-3-5707-0104

2.4 Probable Maximum Loss

According to ASTM E2026-99, the PML can be obtained at 90% confidence of non-exceedance for the Scenario Upper Loss (SUL) of the CDF of the distribution. Eq. 12 was used to calculate the Physical Loss of the System (Maruyama, 2015). Where, X_{ij} is the damaged spot in one mesh area, Z_j is the total damage spots in the area; a_j is the loss; j represents a category; i is the value of each category.

$$W = a_1 \cdot Z^1 + a_2 \cdot Z^2 + \cdots + a_l \cdot Z^l = \sum_{j=1}^{l} \sum_{i=1}^{m} a_j \cdot X_i^j \quad (12)$$

Loss, in this paper, is the replacement cost, for a damage spot cost by the seismic event. The mean, m_j, and variance, v_j, of the physical loss was obtained using Eq. 13 and Eq. 14, respectively. The value of m_j is obtained in Eq. 13.

$$m_j = \bar{W}_j = \sum_{i=1}^{m} a \cdot r_i^j \quad (13)$$

$$v_j = \sigma_W^2 = \sum_{j=1}^{m} a^2 \cdot \sigma^2{}_{r_i}^{j} + \sum_{j}^{m} \sum_{\neq k}^{m} a_j a_k \cdot cov(r_i{}^j, r_i{}^k) \quad (14)$$

When the distribution fits the lognormal distribution, it is necessary to transform m_j and v_j to log mean, μ_{mj}, and log variance, $\sigma_{vj}{}^2$, respectively. Eqs. 15 and 16 show the parameters for the lognormal distribution based m_j and v_j.

$$\mu_{mj} = \ln\left(m_j / \sqrt{1 + v_j / m_j^2}\right) \quad (15)$$

$$\sigma_{vj}^2 = \ln\left(1 + v_j / m_j^2\right) \quad (16)$$

Eqs. 17 and 18 are used to obtain the Gaussian SUL and Lognormal SUL, respectively.

$$SUL_{GAU} = PML_{GAU90} = m_j + vjz_{90} \quad (17)$$

$$SUL_{LOG} = PML_{LOG90} = e^{\mu_{mj} + \sigma_{vj} z_{90}} \quad (18)$$

3. RESULTS

Data used was obtained from a previous study (Jarder, et at 2021). A total of 274 meshes were considered and each mesh has its own unique seismic damage rate, λ. Each mesh was simulated 100,000 to obtain the Empirical CDF. Sample data shows seismic level L0 (earthquake at 75 years return period).

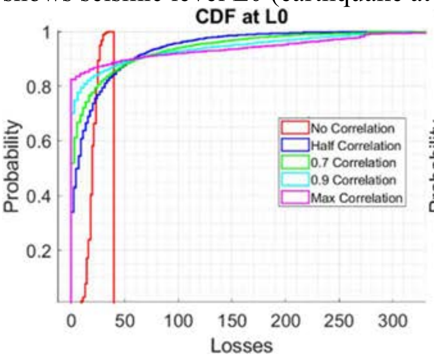

Fig. 1a CDF of Losses at L0

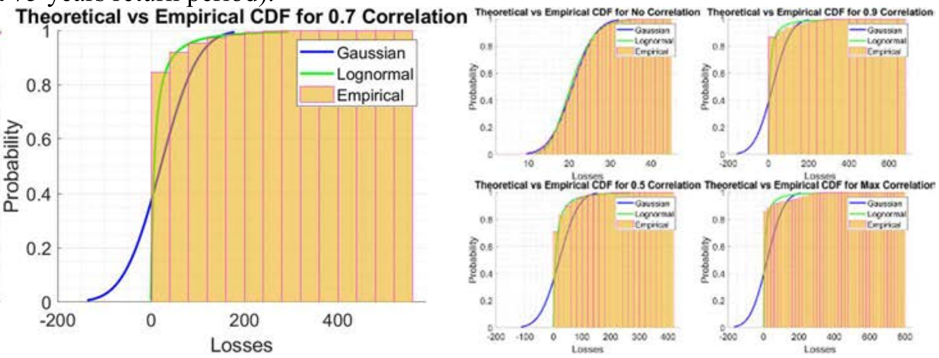

Fig. 1b Losses CDF Comparison of Gaussian, Lognormal and Empirical at L0

The CDF at different ρ cases is shown in Fig. 1a. It shows that PML increase as correlation increases. The Gaussian PML and Lognormal PML curves were compared to the simulated histograms as shown in Fig 1b. Fig1b (left), shows a sample comparison of the Gaussian, Lognormal and Empirical CDF at L0 with ρ=0.7, while Fig1b (right) shows the CDF at different ρ. It could be observed that only at No Correlation case is where all scenarios satisfy both the Gaussian and Lognormal PML curves, while the rest only satisfies and fit the Lognormal Curve. Including the correlation when estimating the Gaussian PML produces negative losses. Mathematically, negative losses do not exist.

4. CONCLUSION

The PML was obtained using the multivariate Gaussian to Poisson simulations. Including the concept of correlation when estimating the PML could cover the impacts of the surroundings and other uncertainties. The proposed Lognormal distribution model can be used to estimate the PML with minimal time, effort, steps and memory compared to the standard simulations for estimation.

REFERENCES

Kramer, S. L. (2016, January 21). Lateral Spreading. In Encyclopedia of Natural Hazards

Isoyama, R., Ishida, E., Yune, K., & Shirozu, T. (2000). Seismic Damage Estimation Procedure For Water Supply Pipelines. In 12th World Conference on Earthquake Engineering.

Jarder, S.L.N., Garciano, L.E.O. and Maruyama, O. (2021), "Probable maximum loss of a pipe network due to earthquakes: a case study in Iloilo city, Philippines", International Journal of Disaster Resilience in the Built Environment, Vol. 12 No. 2, pp. 223-237. https://doi.org/10.1108/IJDRBE-03-2020-0017

ASTM E2026-99 (1999), Standard Guide for the Estimation of Building Damageability in Earthquakes, ASTM International, West Conshohocken, PA

Yahav, I., & Shmueli, G. (2009). On Generating Multivariate Poisson Data in Management Science Applications. SSRN Electronic Journal. doi:10.2139/ssrn.1457347

Mayurama, O. (2015, October 15). Estimation of Probable Maximum Loss Index of Water Supply System. Geotechnical Safety and Risk V: Fifth International Symposium on Geotechnical Safety and Risk (ISGSR) 2015, 451-456. doi:10.3233/978-1-61499-580-7-451

Keywords: Probable Maximum Loss, Correlation, Seismic Damages
Contact address: 1-28-1 Tamazutsumi, Setagaya-ku, Tokyo, 158-8557, Japan, Tel: +81-3-5707-0104