

## CONJUGATE BEAM METHOD-BASED SETTLEMENT IDENTIFICATION OF UNDERGROUND STRUCTURES

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### 1. INTRODUCTION

During the period of rapid economic growth that began in the 1950s, Japan built many underground structures and infrastructure. For example, underground tunnels, sewers, culvert, etc. As underground structures gradually exceed their service life, the number of underground aging underground structures has increased dramatically, accompanied by damage and destruction. Among them, the road surface settlement caused by the structural deformation is an important reason for the road surface collapse. Therefore, it is very necessary to calculate the deformation of underground structures. At present, the calculation methods for underground structures mainly include the quadratic integration method and the conjugate beam method. Compared with the quadratic integration method, the conjugate beam method effectively avoids the deviation caused by multiple integration calculations. Therefore, this study will discuss calculating the deformation of underground structures through the conjugate beam method. And build a finite element model through finite element analysis to verify the reliability of the conjugate beam method in the analysis of underground structures.

### 2. METHODOLOGY

Underground structures such as tunnels and sewers are suitable for the conjugate beam method because they usually have the characteristics of large length and small deformation. What is more, according to material mechanics, changing the size and form of the load is equivalent to changing the bending moment distribution on the beam, that is, the curvature distribution. Based on the principle of conjugate beams, the curvature distribution of the actual beam is equivalent to the load distribution of the conjugate beam. In summary, it is reasonable to use the strain distribution to calculate the curvature distribution. In addition, the conjugate beam method has higher calculation accuracy for beams with larger lengths because it avoids the error caused by the quadratic integration.

Assuming that the structure in question satisfies Euler-Bernoulli beams, the small deformation and material properties are linear elastic materials. The structural strain is positive when the beam bottom is under tension, the structural deformation is positive when the displacement is downward, and the load on the conjugate beam is positive downward. The underground structure discussed at this time can be calculated as simply supported beams. From the principle of conjugate beams, it can be known that simply supported beams are their own conjugate beams suppose the length of the beam is  $L$ , and the beam is equally divided into  $n$  elements. After derivation, the bending moment  $M$  at the boundary between the  $p$  and  $p+1$  units can be obtained. At the same time, the corresponding point can be solved to obtain the deformation  $w_p$ . Deformation at point  $p$  is shown as follow Eq 1.

$$w_p = -\frac{L^2}{n^2} \left[ \frac{p}{n} \sum_{i=1}^n k_i \left( n - i + \frac{1}{2} \right) - \sum_{i=1}^p k_i \left( p - i + \frac{1}{2} \right) \right] \quad (1)$$

$w_p$ : Deformation at the joint between the  $P$  unit and the  $P+1$  unit.  $L$ : The length of the beam.

$k_i$ : Curvature at unit  $i$ .  $n$ : Total number of units

According to Eq 1, only the curvature can be obtained to obtain the deformation of the corresponding point of the structure. The curvature of the corresponding point of the structure can be obtained by formula 2. When the strain  $\varepsilon_1$ ,  $\varepsilon_2$  and the distance  $H$  between the two to the neutral axis are known, the curvature of the corresponding point  $k_{(i)}$  can be obtained. The relationship between curvature and strain is shown as Eq2.

$$k_{(i)} = \frac{\varepsilon_1 - \varepsilon_2}{H} \quad (2)$$

$\varepsilon_1, \varepsilon_2$ : Two strains at the same distance from the neutral axis.  $H$ : Distance between  $\varepsilon_1$  and  $\varepsilon_2$ .

The structural model is divided into 3 units, 4 units, 5 units, 6 units, 9 units, 10 units, 30 units, 60 units, and 100 units according to the length direction. Extract the corresponding strain data of each element in turn to calculate the curvature through Eq2, and then use Eq 1 to calculate the deformation of the structure under different unit divisions. The calculation results are compared with the finite element analysis results.

### 3. FINITE ELEMENT ANALYSIS

In this study, the finite element analysis software was used to construct the model and analyze it. The cross-sectional width of the model is 4 meters, the height is 3 meters, and the longitudinal length is 3 meters. Then ten such models are bound and constrained. The total number of nodes in the model is 441468, the total number of elements is 80400, the Young's modulus is 30GPa, and the Poisson's ratio is 0.18. There are two loadings to make the model deform. One is a uniform load of 0.1MPa on the upper surface of the model. Another is a concentrated load of 1000KN at the midpoint of the model. The model is deformed, and the strain values of the upper and lower surface elements of the model are extracted twice.

Keywords: Conjugate beam, Finite element analysis, Underground structure, Deformation

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## 4. DISCUSSION

### 4.1 LOADING METHOD

The result of finite element analysis is subtracted from the deformation result of each point on the structure calculated by the conjugate beam method. And divide the difference obtained by the finite element result as the deviation value. Take the case 9 units as an example, under the condition of applying a uniform load of 0.1MPa, the average error of deformation of each store on the conjugate beam method beam is 6.88%, and the average error of applying a concentrated load of 1000KN is 6.01%. It can prove that the error value of the conjugate beam method is unrelated to the loading mode of the load. The deformation results are shown in Fig-1 and Fig-2.

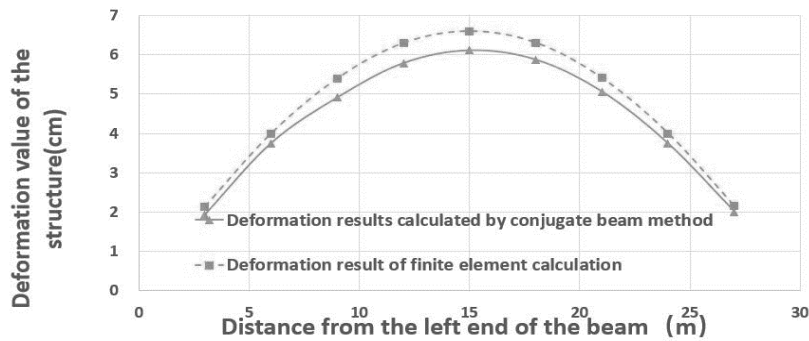


Fig-1 Structural deformation value calculation result (Uniform load)

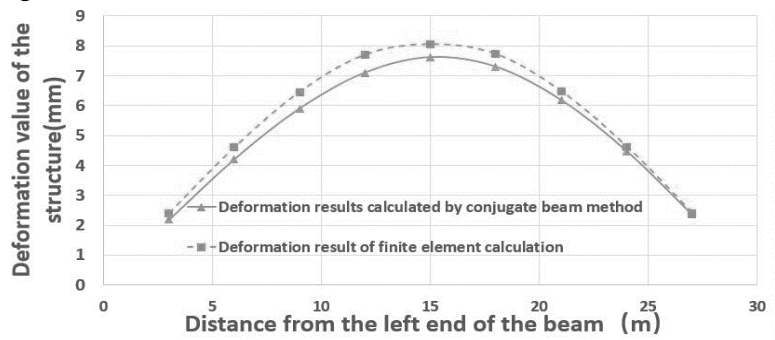


Fig-2 Structural deformation value calculation result (Concentrated load)

### 4.2 NUMBER OF UNITS

By calculating the structural deformation with different numbers of units. When the number of elements is 3, the calculation deviation of the conjugate beam method is 22.2%. The calculation deviation decreased with the increase of the amount of units. When the number of units is 100, the calculation deviation drops to 4.6%. And gradually tend to converge. It can be inferred from this that the calculation deviation of the conjugate beam method is only related to the number of element divisions. The deviation downward trend is shown in Fig-3.

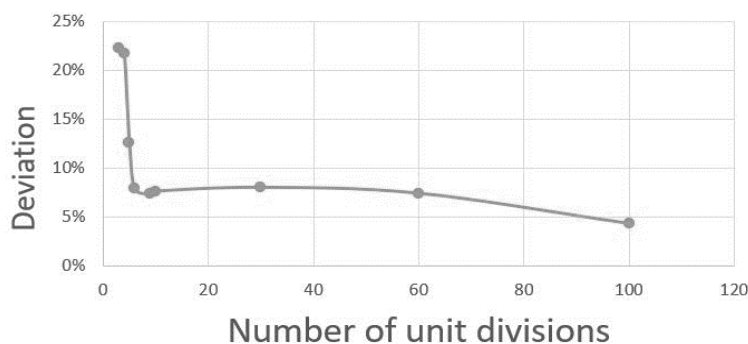


Fig-3 Deviation value of conjugate beam method calculation result.

## 5. CONCLUSION

1. This paper compares the deviation value of the conjugate beam method under concentrated load and uniform load. It can prove that the calculation accuracy of the conjugate beam method will not be interfered by the loading method when calculating the deformation of underground structures.

2. This paper calculates the deviation value of the conjugate beam method under different numbers of units. The deviation is 22.2% in 3 units. When units becomes 100, the deviation drops to 4.6%. It can be found that the deviation of the conjugate beam method decreases as the number of structural elements increases, and tends to converge. It can prove that the conjugate beam method is able to calculate the deformation of underground structures effectively.