FUNDAMENTAL CONSIDERATIONS ON HIGH VELOCITY OPEN CHANNEL FLOWS BELOW AN ABRUPT EXPANSION

Kyoto University Student Member ○Tha Pisey
Kyoto University Fellow Member Takashi Hosoda
University Sains Malaysia Non-Member Puay How Tion

1. INTRODUCTION

High velocity open channel flows in the downstream of an abrupt expansion illustrated in Fig. 1 are considered as one of the basic flows studied in Hydraulic Engineering. Puay and Hosoda [1] divided the flow into three regions based on the method of characteristics (MOC) as shown in Fig 2. But the analytical solution of λ_2 line staring at the point W in Fig 2 was not derived yet. In this study, the λ_2 line is calculated using the theoretical relations derived based on MOC. The results are verified in comparison to the simulated results obtained using the shallow water equations.

2. GOVERNING EQUATIONS AND BASIC RELATIONS

The shallow water equations denoted as Eq. (1) and (2) are used as the basic equations for analysis.

$$A_1\frac{\partial U}{\partial x}+A_2\frac{\partial U}{\partial z}=B \tag{1}$$

$$A_1=\begin{bmatrix}u & h & 0\\ g & u & 0\\ 0 & 0 & u\end{bmatrix}, A_2=\begin{bmatrix}w & 0 & h\\ 0 & w & 0\\ g & 0 & w\end{bmatrix}, U=\begin{bmatrix}h\\ u\\ w\end{bmatrix}, B=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix} \tag{2}$$

where u and w are the components of depth-averaged velocity vectors in x and z directions, respectively, h is the water depth, and g is the acceleration due to gravity.

The characteristic lines λ_1, λ_2, and λ_3 with its corresponding eigenvectors μ_1, μ_2, and μ_3 are derived as follows:

$$\lambda_1=\frac{dz}{dx}=\frac{w}{u},\ \lambda_2=\frac{dz}{dx}=\frac{uw+\sqrt{gh(u^2+w^2-gh)}}{u^2-gh},\ \lambda_3=\frac{dz}{dx}=\frac{uw-\sqrt{gh(u^2+w^2-gh)}}{u^2-gh} \tag{3}$$

$$\mu_1=\begin{bmatrix}1 & \frac{u}{g} & \frac{w}{g}\end{bmatrix},\ \mu_2=\begin{bmatrix}\frac{\sqrt{gh(u^2+w^2-gh)}}{uh} & -\frac{w}{u} & 1\end{bmatrix},\ \mu_3=\begin{bmatrix}-\frac{\sqrt{gh(u^2+w^2-gh)}}{uh} & -\frac{w}{u} & 1\end{bmatrix} \tag{4}$$

Multiplying Eq. (1) by eigenvectors, μ_1, μ_2 and μ_3 on the left side of Eq. (1), the following relations which are satisfied along the characteristic lines are derived as Eq. (5).

$$\lambda_1:\ \frac{\partial}{\partial x}\left(h+\frac{u^2+w^2}{2g}\right)+\frac{w}{u}\frac{\partial}{\partial z}\left(h+\frac{u^2+w^2}{2g}\right)=0,\ \lambda_2:\ \frac{\sqrt{gh(u^2+w^2-gh)}}{uh}\left(\frac{\partial h}{\partial x}+\lambda_2\frac{\partial h}{\partial z}\right)-\frac{w}{u}\left(\frac{\partial u}{\partial x}+\lambda_2\frac{\partial u}{\partial z}\right)-\left(\frac{\partial w}{\partial x}+\lambda_2\frac{\partial w}{\partial z}\right)=0 \tag{5}$$

Since the relation along λ_1 means the energy conservation along a stream line, the following equation is valid in the whole flow region.

$$h+\frac{V^2}{2g}=h_0+\frac{V_0^2}{2g}=H_0=\text{constant},\ V^2=u^2+w^2 \tag{6}$$

where h_0 and V_0 are the inlet depth and velocity, respectively.

The velocity components x and z directions are related to the angle ϕ between the stream line and z -axis.

$$u=V\sin\phi, w=V\cos\phi \tag{7}$$

Since the λ_3 -lines emancipating at $(0,b_0)$ are straight lines [1], the relation along λ_3 -line is given as Eq. (8).

$$\lambda_3:\ \frac{z-b_0}{x}=\frac{V^2\sin\phi\cos\phi-\sqrt{gh(V^2-gh)}}{V^2\sin^2\phi-gh}=F_3(h) \tag{8}$$

The relation along λ_2 -line is given as Eq. (9) (9)

$$\lambda_2:\ \frac{dz}{dx}=\frac{V^2\sin\phi\cos\phi+\sqrt{gh(V^2-gh)}}{V^2\sin^2\phi-gh}=F_2(h)$$

Using these two equations leads to the following relation which is satisfied along λ_2 -line starting at W.

$$\lambda_2:\ \frac{1}{F_2(h)-F_3(h)}\frac{dF_3}{dh}dh=\frac{dx}{x} \tag{10}$$

Eq. (9) can be integrated numerically for h ranging from h_0 to 0, resulting in the water depth distribution along λ_2 line starting at W.

Keywords: High velocity open channel flows, abrupt expansion, method of characteristics
Dept. of Urban Management., Kyoto Univ. (C1-3-265, Kyodai Katsura, Nishikyo-ku, Kyoto 615-8540, Tel. 075-383-3266)

© Japan Society of Civil Engineers

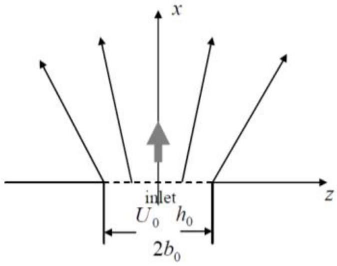

Fig. 1. Flow parameters used in the analysis

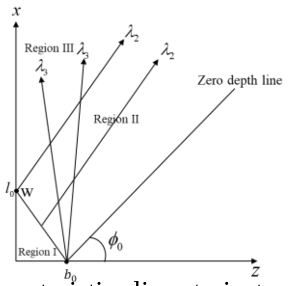

Fig. 2. Characteristics lines trajectories

3. NUMERICAL ANALYSIS

The basic equations in (x, z) system are transformed into the equations in (ξ, η) system defined as Eq. (11).

$$\xi = x - l_0, \quad \eta = \frac{z}{b(\xi)} \tag{11}$$

$$\begin{pmatrix} u & h & 0 \\ g & u & 0 \\ 0 & 0 & u \end{pmatrix} \begin{pmatrix} \dfrac{\partial h}{\partial \xi} \\ \dfrac{\partial u}{\partial \xi} \\ \dfrac{\partial w}{\partial \xi} \end{pmatrix} + \begin{pmatrix} -\dfrac{\eta}{b}\dfrac{db}{d\xi}u + \dfrac{w}{b} & -\dfrac{\eta}{b}\dfrac{db}{d\xi}h & \dfrac{h}{b} \\ -g\dfrac{\eta}{b}\dfrac{db}{d\xi} & -\dfrac{\eta}{b}\dfrac{db}{d\xi}u + \dfrac{w}{b} & 0 \\ \dfrac{g}{b} & 0 & -u\dfrac{\eta}{b}\dfrac{db}{d\xi} + \dfrac{w}{b} \end{pmatrix} \begin{pmatrix} \dfrac{\partial h}{\partial \eta} \\ \dfrac{\partial u}{\partial \eta} \\ \dfrac{\partial w}{\partial \eta} \end{pmatrix} = 0 \tag{12}$$

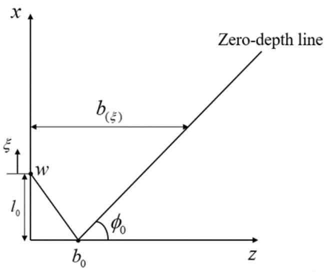

Fig. 3. The symbols used in coordinate transformation

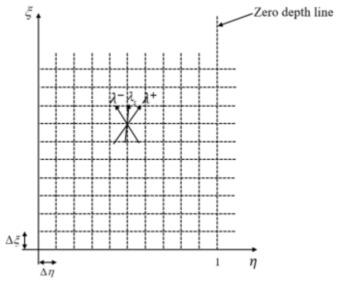

Fig. 4. Grid system in (ξ, η) coordinate system

Based on MOC, the equations which are satisfied along the characteristic lines are derived, and then are discretized into the finite difference equations with 1st-order upwind scheme. The simulation was conducted under the following conditions $h_0 = 0.125\,\mathrm{m}, b_0 = 0.25\,\mathrm{m}, Fr_0 = 4, \Delta\xi = 0.005$ m and $\Delta\eta = 0.01$.

4. NUMERICAL RESULTS

Fig. 5 shows the 3-D contour map of depth. Using the results, the calculated trace of λ_2 line and the depth distributions is compared to the numerical integration of Eq. (10) in Fig. 7 and Fig. 8.

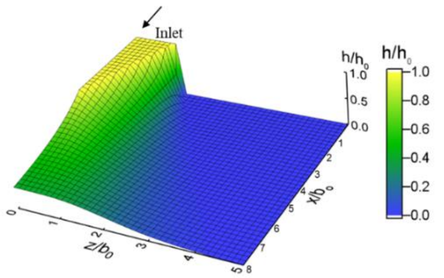

Fig. 5. Symmetrical half of the simulated flow contour

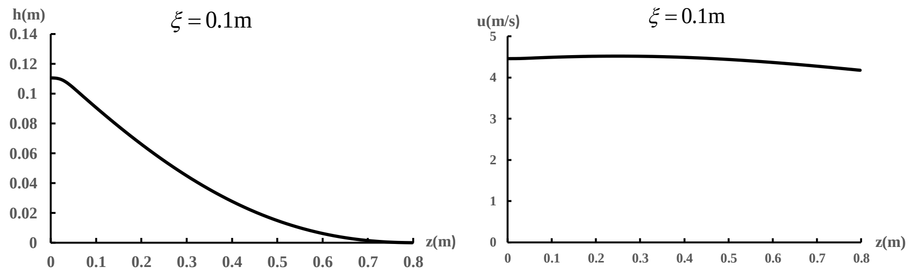

Fig. 6. Simulated depth and velocity distributions at $\xi = 0.1$m

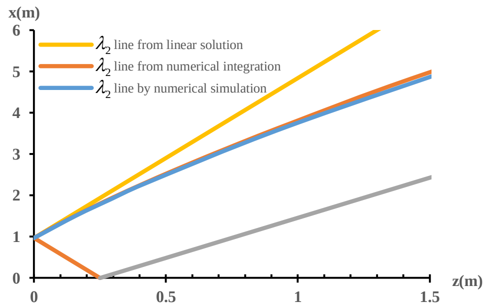

Fig. 7. The trace of λ_2 line by linear solution and numerical results

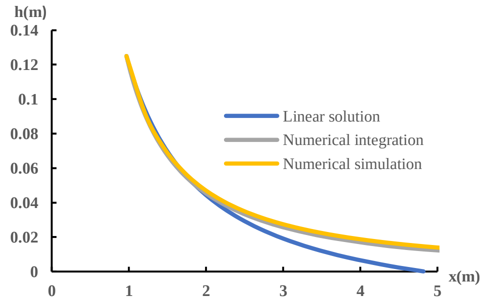

Fig. 8. Comparisons results of depth distribution on λ_2 line

REFERENCE

[1] Puay, H. T. & Hosoda, T. (2012), Journal of JSCE, Ser. A2, Vol. 68, No. 2, pp. I-539-546. (in Japanese)

© Japan Society of Civil Engineers