STRUCTURE PARAMTER ESTIMATION UNDER UNKNOWN SEISMIC EXCITATION WITH AN ADAPTIVE KALMAN FILTER

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Abstract: An adaptive extended Kalman filter (EKF) is proposed to conduct parameter estimation of civil structures under unknown seismic excitations in this paper. In order to avoid the tuning of process noise covariance matrix $\boldsymbol{Q}$ as normal EKF does, an offline noise estimation method is combined with the EKF method to adjust $\boldsymbol{Q}$ matrix automatically. The proposed method is numerically verified using a bridge pier model case showing high estimation accuracies.

1. INTRODUCTION

In the field of structural health monitoring (SHM), one appealing technique is the use of various nonlinear filters, e.g. EKF method. A number of works have been contributed to the topic numerically, e.g. Yang et al. (2006), and experimentally, e.g. Chatzi et al. (2015). While system inputs are assumed measured signals in the aforementioned studies, extensions of the filter methods considering unknown input were conduct by Lei et al. (2012). However, one common problem of the studies above is that the process noise covariance matrix $\boldsymbol{Q}$, which influences the performance of KF, is assumed to be known, but this is not the case in practice.

In this paper, an augmented state vector involving all system response, parameters and inputs is employed to conduct system parameter estimation considering unknown seismic input based on EKF method. An offline process noise estimation method is combined with the EKF to adjust the $\boldsymbol{Q}$ matrix automatically. The adaptive EKF is conduct in an iteration manner until parameter estimation results converge.

2. METHODOLOGY

Considering equation of motion (EOM) of a structure under seismic excitation in absolute coordinate as below

$$\boldsymbol{K}_{ss}\boldsymbol{x}_s + \boldsymbol{C}_{ss}\dot{\boldsymbol{x}}_s + \boldsymbol{M}_{ss}\ddot{\boldsymbol{x}}_s = -\boldsymbol{K}_{sg}\overline{\boldsymbol{x}}_g \tag{1}$$

in which subscript s and g stand for structural and ground DOFs; $\overline{\boldsymbol{x}}_g = \boldsymbol{x}_g + \beta\dot{\boldsymbol{x}}_g$ is regarded as equivalent displacement input to the system in which β is stiffness damping coefficient of Rayleigh damping, i.e. only stiffness damping is considered for supporting part of the system. The objective here is to estimate system parameters corresponding to $\boldsymbol{K}_{ss}$ and $\boldsymbol{C}_{ss}$ as well as unknown input $\overline{\boldsymbol{x}}_g$ based on some partially measured system responses, e.g. displacements, accelerations. The real earthquake displacement (or velocity, acceleration) can be solved by the differential equation of $\overline{\boldsymbol{x}}_g$ above. In order to estimate system parameters and inputs simultaneously, the parameter and input vector are both augmented into state vector as

$$\boldsymbol{X}_a = \left[\boldsymbol{x}(t)^T \quad \dot{\boldsymbol{x}}(t)^T \quad \ddot{\boldsymbol{x}}(t)^T \quad \boldsymbol{\theta}^T \quad \overline{\boldsymbol{x}}_g{}^T \right]^T \tag{2}$$

in which θ is a vector containing unknown parameters to be identified. By firstly designating an initial $\boldsymbol{Q}$ matrix, EKF is employed to estimate the state vector in each time instant, then the results can be 'smoothed' by extended Kalman smoother (EKS). Upon to this point, the $\boldsymbol{Q}$ matrix can be updated as

$$\hat{\boldsymbol{Q}} = 1/(N-1)\sum_{1}^{N-1}\boldsymbol{d}(k+1)\boldsymbol{d}(k+1)^T \tag{3}$$

$$\boldsymbol{d}(k+1) = \hat{\boldsymbol{X}}_a(k+1/N) - \boldsymbol{f}\left(\hat{\boldsymbol{X}}_a(k/N)\right) \tag{4}$$

in which $\boldsymbol{d}(k+1)$ is the so called innovation vector based on the smoothed state vector $\hat{\boldsymbol{X}}_a$; $\boldsymbol{f}$ is system equation in EKF; N is total number of time instants; k stands for a specific time instant.

The updated $\boldsymbol{Q}$ matrix is then regarded as known and input into next iteration computation. The estimation stops until parameter results converge to similar values in several consecutive iterations. It is worth noting here that $\boldsymbol{Q}$ matrix is only updated for state variables, i.e. x, $\dot{x}$ and $\ddot{x}$, and process noise corresponding to parameters and inputs are set as constant values as in Naets et al (2015).

3. NUMERICAL SIMULATION

A bridge pier model is simplified from a full scale bridge pier experiment, called C1-1 experiment, in E-defense shaking

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table laboratory as shown in figure below. The total length of the prototype pier is 7.5m, including 6m of pier itself and 1.5m of pier cap. Extra weights and steel deck supported by the pier represent inertial forces from superstructure. Here 0~6m part of pier is simplified as 3 same FEM beam elements with uniform cross section stiffness EI of 4.55×10^9 Nm^2 and linear density of 6.62×10^3kg/m; Rayleigh damping stiffness coefficient β=0.02 is considered here. The effect of superstructure weights is condensed into a concentration mass element with 2 DOFs added on the very top node. The mass matrix is calculated as [2.52×10^5, -3.48×10^5; -3.48×10^5, 5.63×10^5].

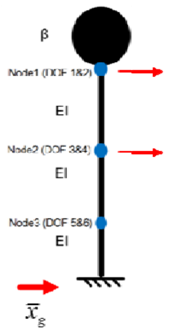


Fig. 1 simplified bridge pier model

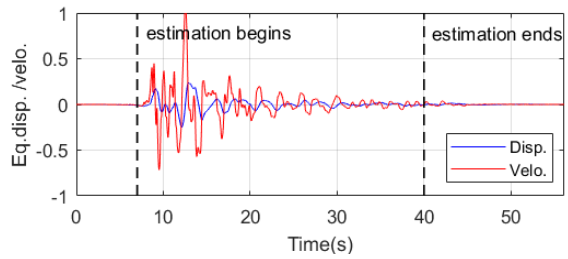
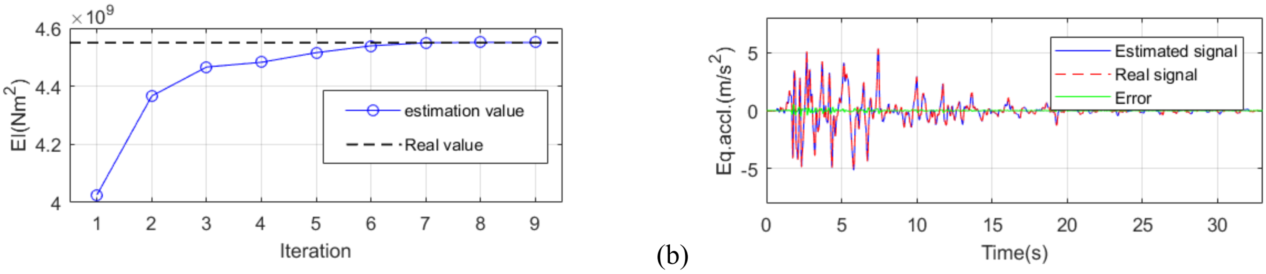


Fig. 2 Takatori earthquake displacement and velocity

The forward analysis is conducted using Generalized-α method and time sample equals to 0.005s. Takatori earthquake displacement and velocity shown in Fig. 2 are treated as excitations. White noise process whose root-mean-square (RMS) is 3% of signals' RMS are added to simulated accelerations. The noisy acceleration signals of NO.1 and 3 DOF as well as corresponding double integrated displacements are regarded as observations. For this simulation case, initial $\boldsymbol{Q}$ matrix is set as $\boldsymbol{Q}$=diag([$10^{-6}\boldsymbol{I}$, $10^{-4}\boldsymbol{I}$, $10^{-2}\boldsymbol{I}$,$\boldsymbol{0}$, $10^{-4}\boldsymbol{I}$]) while initial parameter values and error covariance matrix are $1.5\boldsymbol{\theta}_{real}$ and diag([$0.1\boldsymbol{\theta}_{real}$]^2) respectively.



(a) (b)

Fig. 3 (a) EI estimation results at each iteration; (b) comparison of estimated and real earthquake acceleration

In this case, 9 iterations are employed and EI value identification result at each iteration as well as final estimated earthquake acceleration signal are shown in Fig. 3. By using the proposed method, parameter converge to real value after 6 iterations with estimation error equals to about 0.1%. The estimation result of the 1st iteration shows large error due to inappropriate initial $\boldsymbol{Q}$ matrix applied. The earthquake acceleration is double derivative of estimated earthquake displacement which coincides with real signal pretty well. Both of the parameter and input estimation indicate the effectiveness of the proposed method.

4. CONCLUSION

An adaptive EKF method is proposed for structure parameter estimation under unknown seismic excitations. An offline noise estimation method is combined with EKF to adapt process noise covariance matrix automatically. The effectiveness of the proposed method is numerically verified by a bridge pier model case successfully. Further experiment validation is expected to clarify the performance of the method under practical conditions.

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