

# Experimental Analysis on Moving Vehicle Force Identification from Bridge Responses Using an Extended Kalman Filter

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## 1. INTRODUCTION

A vehicle passing across a bridge leads to bridge vibration. The effects from passing vehicle's load on bridge are divided into two categories: static effect and dynamic effect. The static effect usually indicates the pseudo-static responses of the bridge caused by the vehicle gross weight while the dynamic effect is related with factors including vehicle dynamic properties and bridge pavement roughness. In the literature, these two effects are usually treated as two different types of research, known as bridge weigh-in-motion (Wang et al, 2017) and moving load identification. In this paper, a method is proposed to identify vehicle's static weight and dynamic load time history as well as its transverse position only from responses recorded by portable accelerometers using an extended Kalman filter. The feasibility of the proposed method is proved through a field measurement.

## 2. EXTRACTION OF BRIDGE INCLINATION FROM 3-AXIS ACCELEROMETERS

Low-frequency signals, such as bridge displacement and girder strain, are important in the identification of vehicle's force on the bridge. However, measurement of bridge displacement is not always available while double integration of acceleration data loses the non-zero-mean signals, and the measurement of bridge girder strain has some specific on-site difficulties. Therefore, this paper tries to extract bridge inclination data as low-frequency components from 3-axis accelerometers (Nagayama and Zhang, 2016). The idea is shown in Fig. 1.

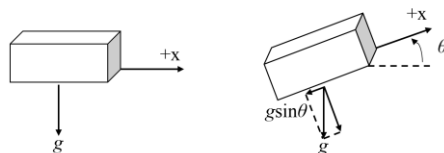


Fig. 1. Extraction of Bridge Inclination from 3-Axis Accelerometers

When the bridge starts to vibrate due to a passing vehicle, the signal in the +x direction is affected by the projection of gravitational acceleration. The inclination at the sensor point is thus obtained through Eq. (1).

$$\theta = \arcsin\left(\frac{a_x}{g}\right) \quad (1)$$

where  $a_x$  denotes the acceleration output from the sensor in +x direction,  $g$  is the gravitational acceleration, and  $\theta$  is the bridge inclination at the sensor location. This inclination data contains non-zero-mean characteristic of low-frequency bridge motion component mainly caused by pseudo-static effect induced by the weight of the moving vehicle and is adopted in the proposed method.

## 3. BASIC EQUATIONS IN VEHICLE-BRIDGE DYNAMICS

Using modal decomposition technique, the equation of motion of a bridge is decoupled as Eq. (2) for each mode.

$$\mathbf{M}_b \ddot{\mathbf{q}} + \mathbf{C}_b \dot{\mathbf{q}} + \mathbf{K}_b \mathbf{q} = \mathbf{F}_b \quad (2)$$

where  $\mathbf{M}_b$ ,  $\mathbf{C}_b$ , and  $\mathbf{K}_b$  are the diagonal modal mass, damping, and stiffness matrices, respectively,  $\mathbf{q}$  is a column vector containing the modal coordinates, and  $\mathbf{F}_{b,i}$  represents the  $i^{\text{th}}$  modal force with the form of Eq. (3).

$$\mathbf{F}_{b,i} = F_f \phi_i(x_f) + F_r \phi_i(x_r) \quad (3)$$

where  $\phi_i(x)$  indicates the  $i^{\text{th}}$  mode shape value at a distance  $x$  to one end of the bridge,  $x_f$  and  $x_r$  are the respective location on the bridge of the front and rear tire, and  $F_f$  and  $F_r$  are the front and rear tire forces, respectively.

In the most frequent case represented by a two-axle vehicle passing across the bridge, the wheelbase between front and rear tires of the vehicle is always much smaller than the bridge span, giving  $\phi_i(x_f)$  close to  $\phi_i(x_r)$ . This relation leads to a new expression of input modal force as Eq. (4).

$$\mathbf{F}_{b,i} \approx (F_f + F_r) \phi_i(x_v) \quad (4)$$

where  $x_v$  represents the equivalent point-load position of the vehicle on the bridge (e.g.,  $x_v = (x_f + x_r) / 2$ ). For the cases where vehicle wheelbase is considerably smaller than the bridge span, this assumption does not lead to large errors.

## 4. IMPLEMENTATION OF EXTENDED KALMAN FILTER

When using EKF to estimate system state, the parameters to be identified need to be included in the state vector. In the current problem of simultaneous identification of vehicle load and passing lane, the total force  $F_v = F_f + F_r$  and a parameter  $d$  indicating the transverse position of the vehicle are included in the state vector, as shown in Eq. (5).

$$\mathbf{X} = [\mathbf{q} \quad \dot{\mathbf{q}} \quad F_v \quad d]^T \quad (5)$$

The observation vector shown in Eq. (6). includes bridge vertical acceleration and bridge inclination extracted using the method of Section 2. Note that the inclination and vertical acceleration do not need to be measured at the same location.

$$\mathbf{Y}_k = [\ddot{u} \quad \theta]^T \quad (6)$$

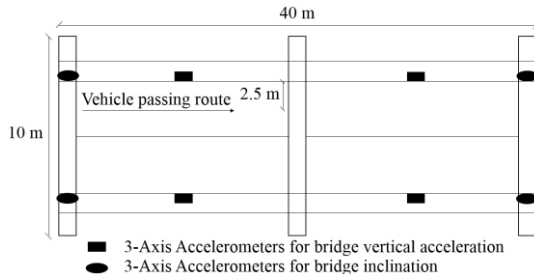
Keywords: Bridge monitoring, Bridge weigh-in-motion, Moving load identification, Extended Kalman filter.

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A function  $f(\cdot)$  is defined to describe the relation between  $\mathbf{X}_k$  and  $\mathbf{X}_{k+1}$  in the system state equation. Similarly, a function  $h(\cdot)$  is used for the relation between  $\mathbf{X}_k$  and  $\mathbf{Y}_k$ , known as observation equation. These functions depend on bridge dynamic properties as well as the vehicle's transverse position already included in system state vector shown in Eq. (5). Therefore, the functions need to be linearized through their Jacobian matrices.

## 5. EXPERIMENTAL VALIDATION

To validate the proposed method, an experimental field test was conducted at a 40-m long and 10-m wide two-span steel-girder bridge located in Yokohama, Japan. The natural frequencies, damping ratios, modal mass, and mode shapes are extracted using ambient vibration tests in advance. For the vehicle load identification problem, the measurement layout and the sensors installed on the bridge are shown in Fig. 2.



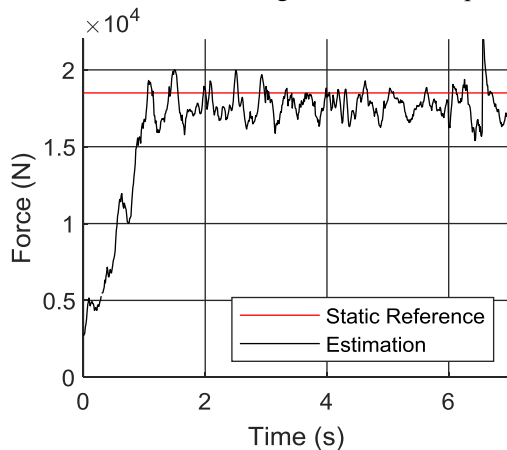
(a) Measurement Layout



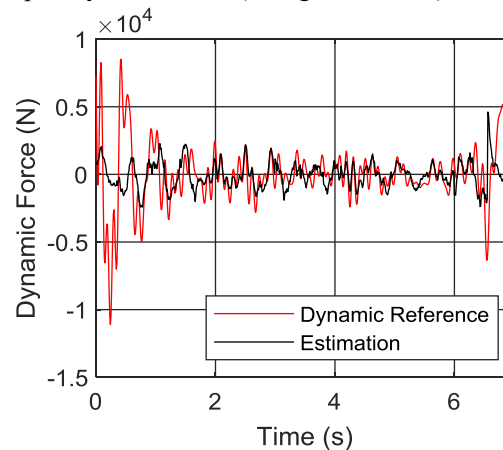
(b) Wireless 3-Axis Sensors (Sonas Corporation)

Fig. 2. Measurement Layout and Sensors

In the experiment, a test vehicle with a mass measured as 1850 kg was driven across the bridge three times at each speed for each direction. The driving speeds were chosen as 20, 30, 40, and 50 km/h. In total 24 driving tests were conducted. Following the process described above, the vehicle moving tire forces are estimated from bridge responses. One typical result with a driving speed of 20 km/h is shown in Fig. 3. Note that the dynamic reference shown in Fig. 3(b) is obtained through sensors on the vehicle using another technique developed by the authors (Wang et al, 2019).



(a) Total Force Estimation



(b) Dynamic Part Comparison

Fig. 3. Typical Results for Tests with Driving Speeds of 20 km/h

The static weight estimation shown in Fig. 3(a) is around 1700 kg with an error of 9.11 % by taking average value of the signal after 1 s. This estimated value was used as the initial value of another round of EKF to remove the effect from the converging process, and comparison of dynamic force is thus given in Fig. 3(b) by removing the static component. Large force values are observed at the beginning and the end of the reference signal due to the joint effect, which is not well captured in estimation due to the low values of the bridge mode shape values at those area. An error  $\varepsilon$  is defined by  $\|F_{ref} - F_{es}\| / F_{ref} \times 100\%$ , where subscripts 'ref' and 'es' indicates 'reference' and 'estimation'. By this definition, the error shown in Fig. 3(b) is 10.02 %.

## 6. CONCLUSIONS

In this paper, a moving force identification technique is proposed by implementing an Extended Kalman filter technique. The vehicle static weight and dynamic load time histories are identified from bridge vertical accelerations and bridge inclinations extracted from 3-axis wireless accelerometers. Experimental results show that the proposed algorithm successfully identifies the moving dynamic force with an error around 10 %.

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