

# APPLICATION OF MODIFIED LATIN HYPERCUBE SAMPLING METHOD FOR THE STUDY OF CEMENT-FLY ASH-SILICA FUME TERNARY PASTE SYSTEM

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## 1. Introduction

One general evolving tendency of modern concrete material is complex binder system. Fly ash is known as one of the most popular alternative binder materials, resulting in environmental and durable concrete. But it brings about lower strength development in the early ages. Silica fume, one highly reactive pozzolanic material, could complement the early age strength, inducing further particle packing. In such ternary blending system, the interactions between binder materials and water are rather complicated and cannot be described by simple linear relationships. With traditional concrete experiment design method, design points with equal intervals, it is hard to capture such intricate interactions. In this study, a modified Latin hypercube design is adopted to extract information to explore the ternary blending system.

## 2. Sampling and analytical methodology

There are three independent variables, water binder ratio (WB) 0.35-0.55, fly ash proportion (FA) 0-50%, silica fume proportion (SF) 0-20%, and one hidden variable: cement proportion (OPC) decided by FA and SF. Following the regular engineering routine, usually 3 equidistant levels per each variable were chosen, making 27 test points in total. On the other hand, to complete a thorough study (examining WB by 0.05, FA by 10%, SF by 5%) would create 150 test points, infeasible obviously.

Latin hypercube sampling (LHS), a space-filling sampling method popular in computational experiment practice, was applied here. When sampling inside a  $S$ -dimension ( $S$  independent variables) design space, the range of each variable is divided into  $N$  equally probable intervals. Combining the  $S$  one-dimension equal interval sampling, randomly, would produce  $N$   $S$ -dimension arrays, i.e., a specific LHS design [1]. Latin hypercube sampling requires the number of test runs predetermined at an arbitrary number, which breaks the limitation from traditional concrete sampling method. It provides better uniformity across the design space and excellent projection properties on each dimension. The number of test points for this study was set at 27. A measure of the uniformity, centered L2 discrepancy, was applied to compare random generated LHS designs. Consequently, the chosen LHS design gained preferable uniformity and representativeness in the design space.

Response surface methodology (RSM) consists a collection of statistical and mathematical techniques for analyzing responses influenced by several independent variables inside a given set of allowable ranges. By using the polynomial model in RSM, it promotes the investigation of the main effects and their interactions and gives a tool for optimizing operational variables. RSM is applied in this study for modeling paste performance.

## 3. Experiment procedure

The materials utilized were ordinary Portland cement, JIS type II fly ash, commercially available silica fume and tap water. Hardness test, which can be done with smaller specimens, reducing the labor and material, were adopted as the methodology to identify the strength development of the specimens. After demolding in the third day, the specimens were kept in standard water curing condition until testing age. Due to the high variation of hardness test, each specimen was tested for 25 times and took the average value.

## 4. Results and discussion

### 4.1. Correlation between variables

In Fig.1, it is shown that the three explanatory variables, WB, FA and SF, have random relationships and uniformly distributed across the design space due to the modified LHS. OPC is decided by FA and SF as a common mixture design. On account of the relative low usage of silica fume (0-20%), the correlation between OPC and FA had a strong linear relationship (correlation coefficient is smaller than -0.9).

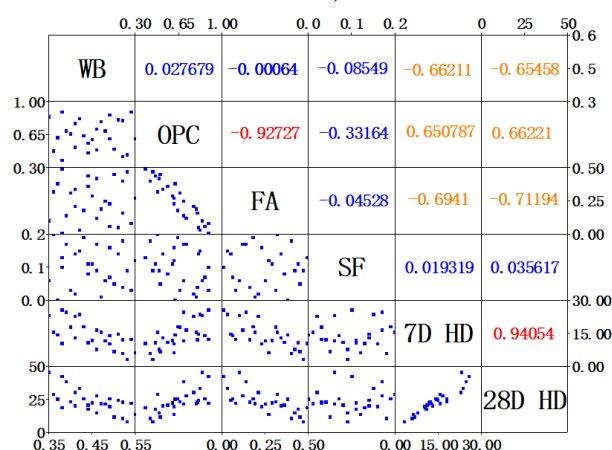


Figure 1 Correlation plots of variables

**Keywords:** Ternary paste system, Fly ash, Silica fume, Latin hypercube sampling, response surface methodology

7D HD and 28D HD stand for 7 days hardness and 28 days hardness value respectively. The hardness at different curing ages both indicated moderate correlation with WB, OPC and FA following the common concrete engineering experience. However, SF did not exhibit a clear effect on the strength development, which seems to be extremely unreal because silica fume is the essential ingredient of high/ultra-high strength concrete. From previous literature, concrete material incorporated with silica fume usually owned similar (usually slightly lower) strength at 7 days, and gave around 10-30% improvement at 28 days. It should be noted that in the aforementioned experiment, every single mix contained a complete ternary blending system, which secured the presence of fly ash, leading to direct decrease in strength. Besides, the incorporation of fly ash would reduce the proportion of cement in the meantime, further inhibiting the hydration process of silica fume. Because of the complicated interactions between various binder materials, the effect of silica fume was clouded in the statistical analysis.

#### 4.2. Comparing hardness models at different ages

Reduced quadratic models were chosen in RSM for modelling the hardness responses acquired at both 7 and 28 days. Ideal coefficient of determination, 0.9519 and 0.9674, were obtained, indicating precise approximation of the response surfaces. For visibility, some 2D contour plots were taken from the 3D models to observe the main effects and interactions. As denoted in Fig.2, silica fume showed trivial (almost straight vertical contour lines) influences on the hardness regardless of WB for each curing age when FA is 0. Moreover, silica fume had no significant effects constantly and it was independent from WB and FA treatment levels such as the discussion in 4.1.

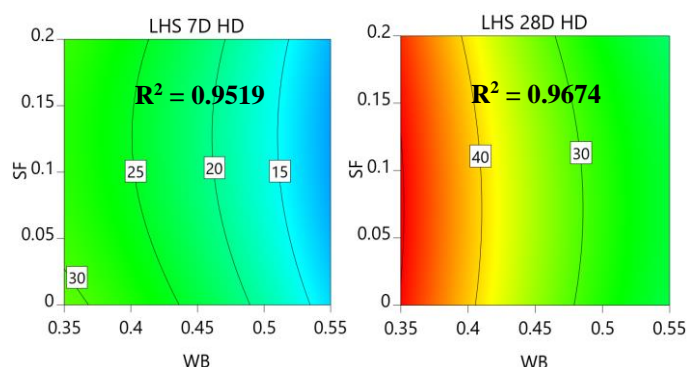


Figure 2 Contour plots of hardness models at 7 and 28 days with fly ash proportion at 0%

When the silica fume content is set at 0. It can be noticed from Fig.3 that 7days and 28 days results showed diverse contour patterns. 28 days contour plot displayed a collinear like characteristic, which is usually originated from the lack of sampling points or model selection. In reality, the cement-fly ash binary blending system usually

appears to be in radial shape, with increasing curvature along with increasing curing age. This is the synergic effect benefited from the continuous pozzolanic reaction in the mixes with medium ranges of WB and FA.

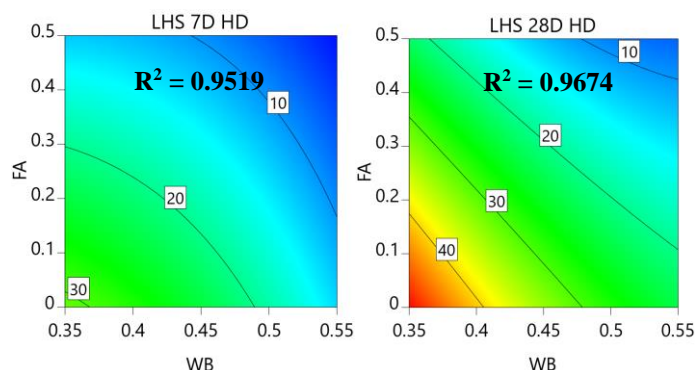


Figure 3 Contour plots of hardness models at 7 and 28 days with silica fume proportion at 0%

#### 4.3. Testing 7 days hardness test model

14 regular interval sampling points were used as the test points for the model. Among the 14 test mixes, 12 mix proportions resulted in acceptable predicted hardness value (percentage differences between  $\pm 13\%$ ). It means the model had grasped the general behavior of the ternary blending system, yet 2 deviating mixes (highlighted in Fig.4) showed the deficiency of the model in capturing the details in the boundary area at which drastic changes often take place. The combination of LHS and RSM enables an efficient exploration and approximation of concrete material system. Meanwhile, the experimental results suggested a poor coverage on in the boundary area. In view of the model underlain from engineering experience could possibly further improve this process.

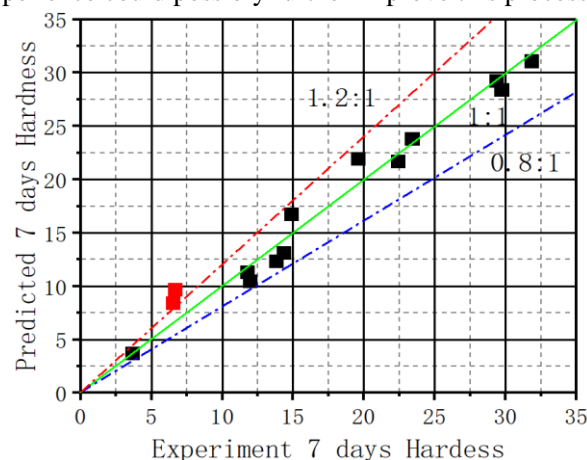


Figure 4 Correlation plot of predicted and experiment hardness value

#### References

- [1] Mckay, M. D., et al. "Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code." *Technometrics* 21.2(1979):239-245.