# Numerical Study on Vehicle Static and Dynamic Load Identification with Lane Detection from Bridge Acceleration and Inclination Data using Particle Filter Method

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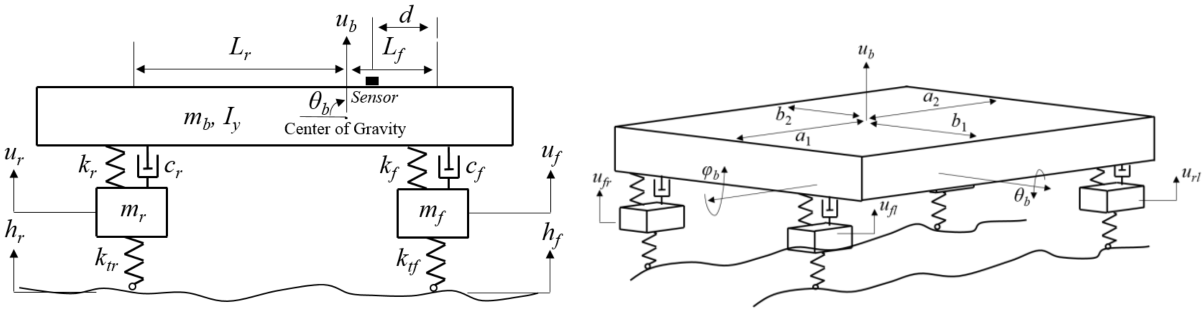
## 1. INTRODUCTION

The static and dynamic components of the vehicle passing load need to be evaluated as it may potentially cause problems including fatigue or even failure of the bridge in some extreme cases. Many bridge weigh-in-motion systems and moving force identification methods have been proposed to estimate vehicle load through strain gauges. However, the installation of strain gauges are usually costly and time-consuming, limiting its practical applications (Wang et al, 2017). Moreover, in the traditional influence line-based method, only static vehicle load is identified (Moses, 1979). In this paper, an identification of vehicle static and dynamic load using only accelerometers is investigated. A data assimilation technique known as particle filter is employed. A numerical example proves the feasibility of the proposed method.

## 2. VEHICLE AND BRIDGE MODEL

### 2.1 Vehicle Model

There are many different vehicle models in the field of vehicle dynamics, each of which has its own detailed features and scope of applications. The most frequently used models are the half-car model and the full-car model, which are shown in Fig. 1 (a) and (b), respectively.




(a) Half-car model (b) Full-car model

Fig. 1. Half-car and full-car vehicle model

### 2.2 Bridge Model

The bridge is modelled by a two-span continuous slab with a support in the middle. Each span has a length of 20.5 m and a width of 10 m. From the modal decomposition method, the equation of motion of the bridge is written as a series of single-degree-of-freedom (SDOF) equations, each of which represents one vibration mode, as expressed in Eq. (1).

$$M_n\ddot{q}_n(t)+C_n\dot{q}_n(t)+K_nq_n(t)=P_n(t) \tag{1}$$

where $M_n$, $C_n$, and $K_n$ is the modal mass, damping, and stiffness of the $n^{th}$ mode, $q_n$ is the $n^{th}$ modal coordinate, and $P_n$ is the modal load, which equals to the dynamic load multiplied by the mode shape at the location of the force.

The mode shapes of the bridge, shown with the corresponding natural frequency in Fig. 2, are extracted from a real bridge in Yokohama, Japan. The damping ratio of each mode is set as 0.02. The bridge mass per area is set as 1100 kg/m$^2$. With these information, the mass, damping, and stiffness can be calculated through basic structural dynamics (Chopra, 2007).

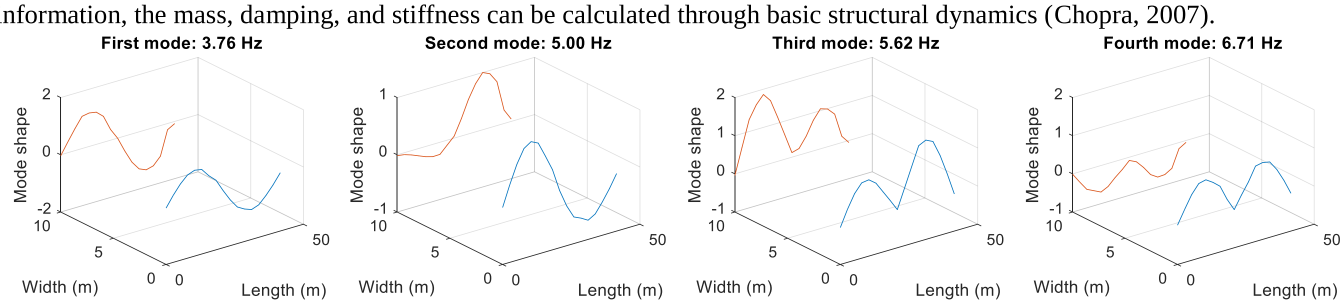



Fig. 2. Mode shapes of the bridge

## 3. PARTICLE FILTER AND ITS IMPLEMENTATION ON THE PROBLEM

Particle filter is a sequential data assimilation method to estimate system state using measured data (Gordon et al, 1993). The idea of particle filter is to use a large number of particles to represent the probability density function of dynamic state at each time step and estimate the optimal values of the state by sequentially introducing measured data. Two equations known as state equation and observation equation are used in particle filter step by step, as shown in Eq. (2).

$$\mathbf{x}_{k+1}=f_k(\mathbf{x}_k,\mathbf{w}_k),\mathbf{y}_k=h_k(\mathbf{x}_k,\mathbf{v}_k) \tag{2}$$

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in which $\mathbf{x}_k$ and $\mathbf{y}_k$ are the state vector and observation vector at time step $k$, and $\mathbf{w}_k$ and $\mathbf{v}_k$ are the system error and observation error following independent probability density function.

In this study, two loads, i.e., the front and rear axle loads $F_f$ and $F_r$, are identified together with the distance $d$ from the load path to the bridge lateral edge. These values are included in the state vector as well as the bridge modal coordinates $\mathbf{q}$, as shown in Eq. (3).

$$\mathbf{X}_k = \left(\mathbf{q}_k, \dot{\mathbf{q}}_k, \ddot{\mathbf{q}}_k, F_{f,k}, F_{r,k}, d_k\right)^{\mathrm{T}} \tag{3}$$

The evolution for each time step of the bridge response terms in the state vector is based on the equation of motion of the bridge. For the vehicle force terms, a random walk model is adopted, in which the mean value of prior PDF of the force at time step $k$+1 is taken as equal to the estimated value at time step $k$.

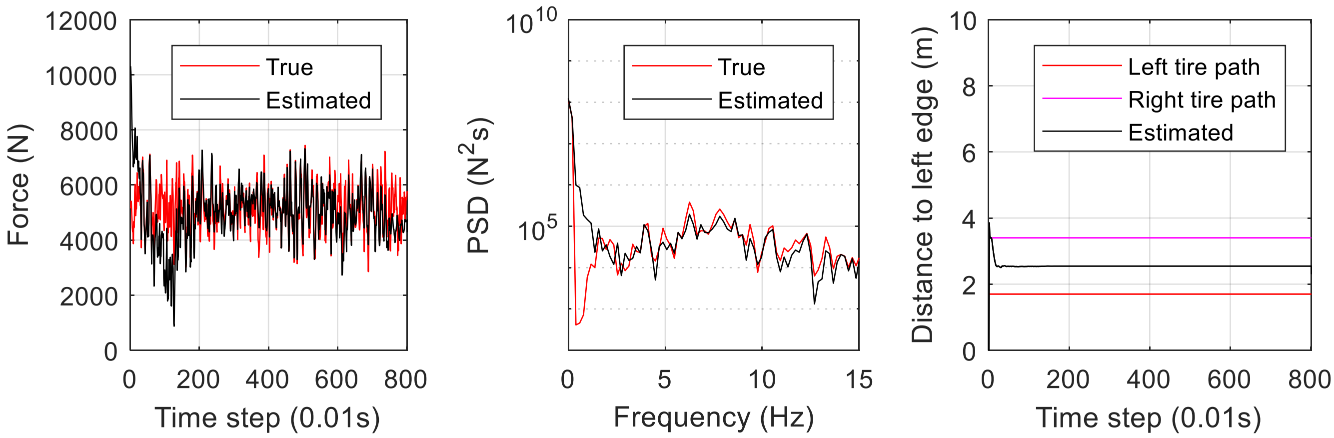
The observation vector is shown as Eq. (4), including two vectors containing bridge acceleration $\ddot{\mathbf{y}}$ and inclination $\mathbf{S}$ at each mid-span, respectively. From the mode shapes in Fig. 2, the mid-span has the largest responses in all these four dominant modes. Therefore, only four mid-span responses (left and right edge in both spans) are included in the observation vector. Note that in a real case, the inclination is obtained from the acceleration data projected in the bridge longitudinal direction (Nagayama and Zhang, 2017).

$$\mathbf{Y}_k = \left(\ddot{\mathbf{y}}, \mathbf{S}\right)^{\mathrm{T}} \tag{4}$$

## 4. NUMERICAL EXAMPLE

A vehicle represented by the full-car model shown in Fig. 1(b) is simulated to drive over the bridge. The distances from each tire path and the bridge left edge are 1.7 m and 3.4 m, respectively. The bridge acceleration and inclination shown in Eq. (4) are calculated as the measured responses after adding an artificial white noise with a standard deviation of 10 % of its RMS value.

The results of the front load identification is shown in Fig. 3(a) and (b) for time domain and frequency domain, respectively. In Fig. 3(a), the initial value of the front tire force is set twice as the true static axle load. After a few steps, both static and dynamic load are correctly estimated. In Fig. 3(c), the estimated path is between the true left and rear tire path. This is reasonable because the full-car model with four tires is adopted when calculating the bridge responses, while only two forces representing front and rear axle loads are used in the inverse identification.




(a) Front tire force (time domain) (b) Front tire force (frequency domain) (c) Lane detection

Fig. 3. Estimation results

## 5. CONCLUSIONS

A particle filter-based method is proposed to identify vehicle static and dynamic load from measured bridge responses. Compared to existing weigh-in-motion techniques, the method proposed in this paper used a limited number of accelerometers on both the left and right edges of the bridge. The passing lane of the vehicle is identified by including a parameter in the state vector. Both static and dynamic vehicle load are identified with a good accuracy.

## ACKNOWLEDGEMENT

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