A POINT PROCESS APPROACH OF BIVARIATE EXTREMES OF RAINFALL DATA

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1. INTRODUCTION

Recent years, natural disasters due to flood and typhoon have been posing a great threat to people's life and social property, which has promoted the research of extreme value analyses. Although the probability theory for these models is fairly promising, the statistical properties are not suitable. In this paper, we will introduce a Bivariate Point Process model (hereinafter, BPP) which is easier processed compared with the classical ones, which are Bivariate Block Maximum model (hereinafter, BBM) and Bivariate Generalized Pareto distribution model (hereinafter, BGP). And then we apply rainfall data which are observed in upstream and downstream to BPP.

2. NUMBER OF OCCURRENCES

We have 30 years of rainfall data at downstream site A and upstream site B. In Figure 1, the data show the daily maximum hourly precipitation at site A, and daily precipitation at B in the same day. We denote $K_a(U_a)$ to the number of occurrences over U_a and $K_b(U_b)$ over U_b. $K_{ab}(U_a,U_b)$ denoted as the number of joint occurrences where both observations are bigger than thresholds. $K_*(U_a,U_b)$ is the number of united occurrences where one of the observations are bigger than corresponding threshold. The probability of this situation is Poisson distribution.

$$p_1(K_a,K_b) = e^{-\{\lambda_{1,a}(U_a)+\lambda_{1,b}(U_b)-\lambda_{1,ab}(U_a,U_b)\}} \times \sum_{i=0}^{K_a\wedge K_b} \frac{(\lambda_{1,ab})^i}{i!} \frac{\{\lambda_{1,a}(U_a)-\lambda_{1,ab}(U_a,U_b)\}^{K_a-i}}{(K_a-i)!} \times \frac{\{\lambda_{1,b}(U_b)-\lambda_{1,ab}(U_a,U_b)\}^{K_b-i}}{(K_b-i)!} \tag{1}$$

$K_a \wedge K_b$ means the minimum one of them. $\lambda_{1,a}(U_a)$, $\lambda_{1,b}(U_b)$, $\lambda_{1,ab}(U_a,U_b)$ mean the rates of occurrences.

3. THE WEAKNESSES OF BBM AND BGP

When it comes to BBM, it should be noted that block maxima can hide the time structure, so we don't know whether the different components of the maxima occurred simultaneously (for example in the same day) or not. Taking the rainfall data of 1985 and 1986 as an example, in 1985 the annual maxima as shown in Figure 2 are observed on the same day, whereas

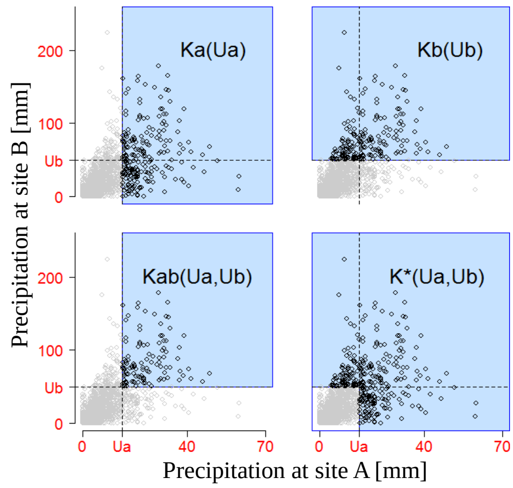

Figure 1. Four different kinds of occurrences

In 1986 the annual maxima are observed on different days. The BBM is dealt with componentwise maxima.

To avoid this problem exceedances over a high threshold can be considered (i.e. BGP). However we can't take account of the information of the block size in the likelihood function. This is a good fashion but disadvantage of BGP [1]. So we need the form of the rate of occurrences as follows.

$$\hat{\lambda}_{n,AB}(y_a,y_b) = \frac{\hat{\lambda}_{n,*}(U_a,U_b)\lambda_{ab}(y_a,y_b,\hat{\theta}_a,\hat{\theta}_b,\hat{\phi})}{\lambda_*(U_a,U_b,\hat{\theta}_a,\hat{\theta}_b,\hat{\phi})} \tag{2}$$

Where $\hat{\lambda}_{n,*}(U_a,U_b)=(n/B)\times K_*(U_a,U_b)$, n is the target time length, B is the time of the observation. But this equation contains three kinds of rates, which is not well feasible to be used in more advanced models.

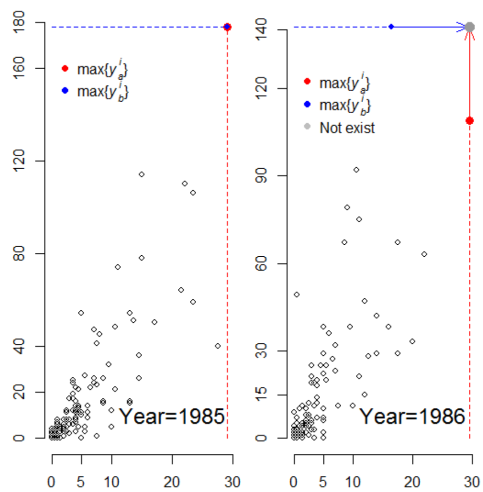

Figure 2. Two kinds of occurrences in BMM

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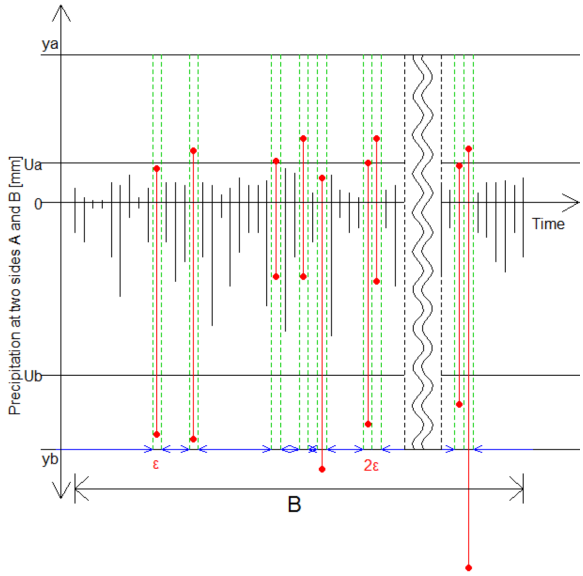

Figure 3. The occurrences in BPP

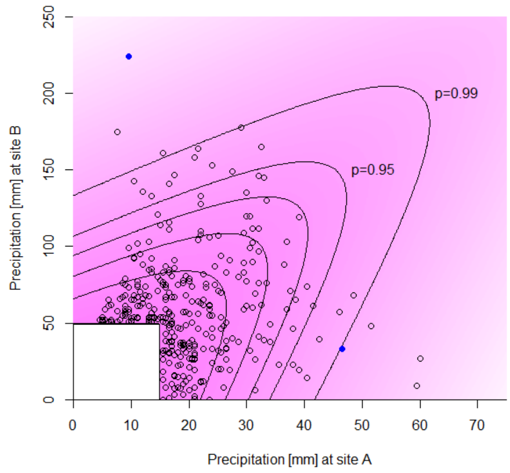

Figure 4. The density of BPP

4. MODEL REASONED BY POINT PROCESS

Figure 3 shows two time-series of precipitations at site A and site B along the time. Assuming a moment $\varepsilon \to 0$, the density function[2] is obtained from Poisson distribution equation (1):

$$\frac{\partial^2 p_\varepsilon(1,1)}{\partial y_a \partial y_b} \approx \varepsilon \frac{\partial^2 \lambda_{1,ab}}{\partial y_a \partial y_b} \tag{3}$$

In this figure, there are totally K_* excess data whose time length is $\varepsilon \cdot K_*$. The likelihood of K_* excess data is the product of equation (3). There are no excess data in the left time of $B - \varepsilon \cdot K_*$.The following likelihood obtained from equation (1):

$$P_{(B-k_{*\varepsilon})}(0,0) \approx e^{-B \cdot \lambda_{1,*}(U_a, U_b)} \tag{4}$$

Finally, the total likelihood is obtained by multiplying equation (4) and product of equation (3) as follows.

$$L(\theta) \propto e^{-B \cdot \lambda_{1,*}(u_a, u_b)} \times \prod^{K_*} \frac{\partial^2 \lambda_{1,ab}}{\partial y_a \partial y_b} \tag{5}$$

5. DEMONSTRATION BY PRECIPITATION DATA

By maximizing equation (5), we can obtain the estimates of parameters and then draw the contour of density and return period. In Figure 4, the contour of density labeled "p = 0.99" contains 99 percent of the whole data of which the number is K_* = 318. This fact leads to the excepted number of points outside this contour is around 4. But there are seven points outside this contour. After investigating these data, the two solid points as shown in the figure are found to be two consecutive days. It is because the precipitation observed at upstream site B in previous day was observed at downstream site A in latter day. This model can be improved by treating these two points as a simultaneous occurrence.

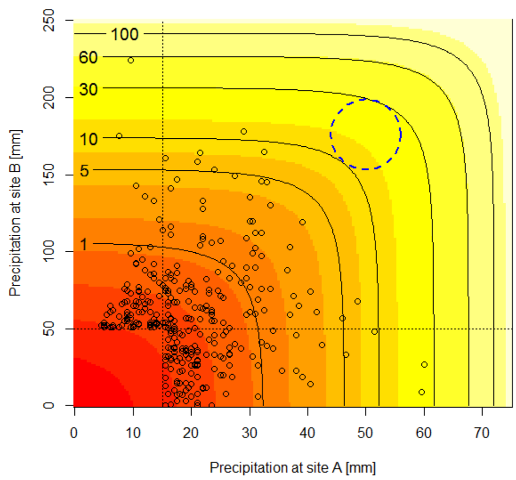

Figure 5. The return period of joint occurrences

The contour of the return period of joint occurrences is shown in Figure 5. Our observed data don't occur inside the dotted circle as shown in this plot, but it'll probably happen in the future of 30 years. For flood prevention design of river bank, we should pay more attention to the simultaneous occurrences of hourly precipitation around 60mm at site A and daily precipitation around 200mm at site B.

REFERENCE

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