# ESTIMATION OF MODAL CHARACTERISTICS FROM AMBIENT VIBRATIONS USING CONTINUOUS WAVELET TRANSFORM

*Saitama University Student Member ○Ejaz Ali*
*Saitama University Regular Member Yasunao Matsumoto*

## 1. INTRODUCTION

Traditionally, damping ratios have been identified in time or frequency domains using ambient vibrations. Recently, Continuous Wavelet Transform (CWT) is used for modal identification of structures using ambient vibrations in time-frequency domain. CWT is a linear representation of multi-component signal which makes it possible to decompose a multi-component signal into its components, Staszewski (1997). Previous studies of Kijewski et al. (2003) and Staszewski (1997) have highlighted that damping ratios can be estimated accurately through CWT. The aim of this paper is to extract the modal properties of different vibration modes of a bridge from its response to ambient forces considering different lengths of response using CWT.

## 2. CONTINUOUS WAVELET TRANSFORM

CWT is defined as Mallat (1999):

$$W(\tau, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \Psi^* \left( \frac{t-\tau}{s} \right) dt \tag{1}$$

Where, $x(t)$ is the signal to be analyzed using the analyzing wavelet $\Psi^* \left( \frac{t-\tau}{s} \right)$ and $s$ is the scaling parameter.

A minimum criterion was set for center frequency of Morlet wavelet in the study of Kijewski et al. (2003) as:

$$f_c = 2\alpha \frac{f_{1,2}}{2\pi\sqrt{2}\Delta f_{1,2}} \tag{2}$$

Where, a minimum value of 2 for $\alpha$ was suggested to identify two closely spaced frequencies $f_1$ and $f_2$ and $f_{1,2}$ is the average of those two frequencies. $f_c$ is the minimum center frequency required to separate two closely spaced frequencies $f_1$ and $f_2$. When the wavelet is placed at the start of signal, a part of it is outside the signal giving wavelet coefficients based on incomplete signal which leads to phenomena of edge effects. Such edge effects result in inaccurate estimation of modal properties if not mitigated properly. This edge effect was mitigated by Kijewski et al. (2003). Eq. (3) gives the temporal resolution at frequency $f_i$ while the accurate portion $t_j$ of wavelet transform at frequency $f_i$ is obtained by omitting $\beta$ times temporal resolution both from the start and end of signal. A minimum value of 4 for $\beta$ is recommended by Kijewski et al. (2003).

$$\Delta t_i = \frac{f_c}{f_i\sqrt{2}} \tag{3}$$

Keywords: Ambient Vibrations, Continuous Wavelet Transform, Damping Identification
Contact Address: Shimo Okubo 255, Sakura-ku Saitama city, Saitama, 338-8570. Email: ymatsu@mail.saitama-u.ac.jp

## 3. CASE STUDY

Ambient vibrations of a single span PC girder bridge having total of 34 girders were collected. A truck was run multiple times. A total of six sensors were installed in pairs beneath girders 4, 16 and 33. Fourier Transform of the vibrations showed three dominant frequencies of around 8, 11.5 and 17.5 Hz. Minimum center frequency based on Eq. (3) for $1^{st}$, $2^{nd}$ and $2^{nd}$, $3^{rd}$ frequencies came out to be 1.25 and 1.1 Hz respectively. Hence, a center frequency of 2 Hz was used in each analysis. To mitigate the edge effects, $\beta$ was kept 4 for each mode of vibration. Natural frequency of each mode was estimated by taking the average values of instantaneous frequencies along the ridge. A linear least squares curve was fitted along the natural log of modulus of wavelet coefficients of each ridge to estimate the slope. This slope is the product of damping ratio and natural frequency. Hence, dividing slope with the natural frequency gives damping ratio of that mode. Different time lengths “t” of the free vibration were considered to check the stability of the modal parameters identified using CWT. Eigensystem Realization Algorithm was adopted as a time domain technique for modal parameters identification to compare results. Size of Hankel matrix was 1000×500 which accommodated around 1.5 seconds of data. Stabilization diagrams were used to distinguish structural modes from artificial modes. The construction of stabilization diagrams is based on the comparison of the poles associated to a given model order with those obtained from a one-order lower model. The chosen stabilization criteria were 1% for frequency and 90% for MAC values. Damping ratios greater than 10% were also disregarded.

**Table 1 Comparison of Modal Parameters Identified Using CWT & ERA**

| Identification of Modal Parameters by CWT | | | | | | | | | | By ERA | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| t=3 sec | | t=2.5 sec | | t=2 sec | | t=1.75 sec | | t=1.5 sec | | t=1.5 sec | |
| N.F | D.R | N.F | D.R | N.F | D.R | N.F | D.R | N.F | D.R | N.F | D.R |
| 8.03 | 0.0235 | 8.02 | 0.0233 | 8.02 | 0.0233 | 8.02 | 0.0233 | 8.01 | 0.0238 | 8.01 | 0.0246 |
| 11.44 | 0.0240 | 11.43 | 0.0240 | 11.42 | 0.0239 | 11.41 | 0.0238 | 11.40 | 0.0238 | 11.40 | 0.0247 |
| 17.51 | 0.0228 | 17.51 | 0.0231 | 17.51 | 0.0231 | 17.52 | 0.0232 | 17.51 | 0.0232 | 17.50 | 0.0228 |

## 4. CONCLUSIONS

This paper presents the results of a study to determine natural frequencies and damping ratios of a single span PC girder bridge using Continuous Wavelet Transform. It is verified that a value of 4 for $\beta$ is adequate to completely neutralize edge effects. Edge effect is maximum for lowest mode as largest scale is used to capture lower frequency component. For the first mode around 8 Hz, time resolution was 0.177 sec from Eq. (3). After mitigating edge effects as described in section 2, only 0.1 sec of data was left for damping estimation and still gave accurate result. Results also show a very close proximity in damping ratios which are identified using different time lengths in CWT as can be seen in Table 1. Results show that modal parameters identified using CWT & ERA are in good agreement.

## 5. REFERENCES

Kijewski, T., Kareem, A.: Wavelet Transforms for System Identification in Civil Engineering, Computer Aided Civil and Infrastructure Engineering 18 (2003), pp. 339-355

Mallat, S.: A wavelet Tour of Signal Processing, Academic Press, San Diego, London, New York, 1999.

Staszewski, W. J.: Identification of Damping in MDOF Systems Using Time-Scale Decomposition, Journal of Sound and Vibration (1997) 203(2), pp.283-305.