Misfit Criteria for Quantitative Comparison of Phase-Shift-Modified Signals

Dept. of Civil Engineering, The University of Tokyo, Japan Regular Member, ○ T. Jothi Saravanan
Regular Member, Di Su
Regular Member, Tomonori Nagayama

1. INTRODUCTION:

Quantitative assessment of time signals is frequently essential in many problems. There are many time-frequency misfit and goodness-of-fit criteria in the past (Kristeková et al. 2006; Kristeková et al. 2009). Comparison of two measured signals considerably benefits in the analysis and interpretation of the process under investigation. A single-valued integral quantity is more appropriate if a set of signals is to be compared with the another set of signals. Finally, it is not clear whether they are able to accurately measure the difference. Therefore, there is necessity to use time-frequency envelope and phase misfit criteria to properly quantify and characterize a difference between two signals. In this paper, this modification is proposed to use Hilbert Transform (HT) technique. The numerically obtained signals (rail track profiles) from Augmented State Kalman Filter (ASKF) as explained in Saravanan et al. (2017), is employed for phase correction using the proposed misfit criteria through HT and compared using statistical metrics.

2. METHODOLOGY:

In Kalman Filter estimation results, it suffers from phase-shift comparing to the true signal. Hence, there is a need to do phase correction using misfit criteria through HT. HT returns a complex helical sequence, sometimes called the analytic signal, from a real data sequence. The discrete-time analytic signal comprises of real part, which is the original data, and an imaginary part, which contains HT. The imaginary part is a version of the original real sequence with a 90° phase shift. The Hilbert transformed series has the same amplitude and frequency content as the original sequence and includes phase information that depends on the phase of the original. HT is useful in calculating instantaneous attributes of a time series, especially the amplitude and frequency. The instantaneous amplitude is the amplitude of the complex HT and the instantaneous frequency is the time rate of change of the instantaneous phase angle. For a pure sinusoid, the instantaneous amplitude and frequency are constant. The instantaneous phase, however, is a saw-tooth, reflecting how the local phase angle varies linearly over a single cycle. For mixtures of sinusoids, the attributes are short term, or local, averages spanning no more than two or three points. HT is applied to true signal and analytic signal is obtained as shown below,

$$x_{true} = Hilbert\,Transform\,(x_{true\_real}) \tag{1}$$

Discrete-time analytic signal,

$$x_{true} = x_{real} + i * x_{img} \tag{2}$$

The phase of the true signal is obtained using the four-quadrant inverse tangent, which must be real,

$$\theta_{true} = tan^{-1}\left(\frac{x_{img}}{x_{real}}\right) \tag{3}$$

Similarly, HT is applied to the estimated signal and the phase of the estimated signal is modified with the phase of the true signal obtained in Eq. (3),

$$x_{estimated} = Hilbert\,Transform\,(x_{est\_real}) \tag{4}$$

$$x_{estimated} = x_{e\_real} + i * x_{e\_img} \tag{5}$$

$$x_{updated} = absolute\,(x_{estimated}) * e^{i\theta_{true}} \tag{6}$$

$$x_{updated\_real} = real\,(x_{updated}) \tag{7}$$

Phase correction,

$$x_{est\_real} \rightarrow x_{updated\_real} \tag{8}$$

Estimated signal is updated with the phase angle of the true signal, thus, x_{true_real} and $x_{updated\_real}$ can be compared.

3. QUANTIFICATION THROUGH VARIOUS STATISTICAL METRICS:

To validate the estimation algorithm, the following metrics have been used to quantify the signature variations.
(a) Root-Mean-Square Deviation (RMSD) - signifies the sample standard deviation of the differences between estimated values and true values. It is a normalized value and a non-dimensional metrics.

$$RMSD(\%) = \sqrt{\frac{\Sigma\left(x_i^0 - x_i^1\right)^2}{\Sigma\left(x_i^0\right)^2}} \text{ x } 100 \tag{9}$$

where x_i^0, x_i^1 are the i^{th} value of reference series and compared series respectively.
(b) Correlation Coefficient (CC) - identifies similarity and dependency amongst two waveforms. An exact resemblance indicates that CC is unity (or -1).

$$CC = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{s_X}\right)\left(\frac{Y_i - \bar{Y}}{s_Y}\right) \tag{10}$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad s_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} \tag{11}$$

Based on the sample of the paired data (X_i, Y_i), the CC is derived by the Eq. (10), where sample mean and sample standard deviation are specified in Eq. (11) respectively.

Keywords: misfit criteria, Hilbert transform, statistical metrics, phase corrections, rail track profile
Contact: Bridge & Structure Laboratory, Dept. of Civil Eng., The University of Tokyo, 113-8656, Japan, Tel: 03-58416009

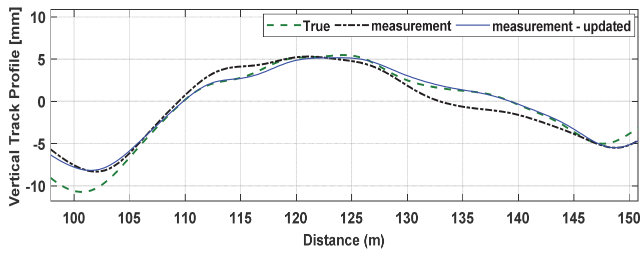
(c) Root-Mean-Square Error (RMSE) - serves to aggregate error into a single measure of predictive power. It is a dimensional error estimation value,

$$RMSE(mm) = \sqrt{\frac{\Sigma\left(x_i^0 - x_i^1\right)^2}{N}} \quad (12)$$

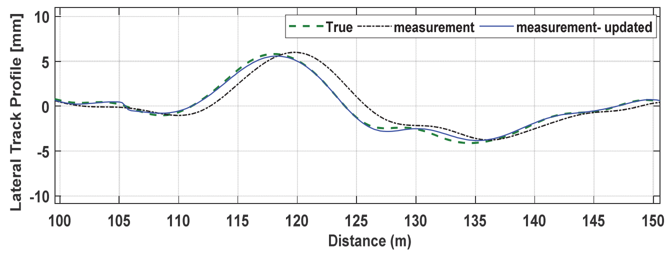
where x_i^0, x_i^1 are the i^{th} value of reference series and compared series respectively and N is the number of data samples.

4. NUMERICAL STUDY:

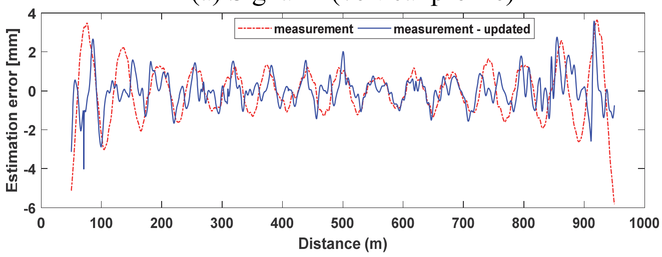
From numerical study, the two signals (vertical and lateral track profile) are obtained from ASKF method which need to be modified for phase shift error when compared to 'True' signal (respective original track profiles), as shown in Figs. 1(a-b) (zoomed section). By applying the proposed misfit criteria, the phase shift correction is carried out and represented as 'measurement – updated', as shown in the Figs. 1(a-b). The comparison of estimation error along the track length is obtained by subtracting the estimated signal from true signal. It is shown in the Figs. 1(c-d) for both the signals and it can be seen that updated waveform has lower error value. Over all the performance is improved along the track length. Now the statistical metrics are evaluated for both old and updated waveform obtained from ASKF method and it is tabulated in the Table 1. After misfit criteria, the single-valued metrics show good improvement comparing to the old values. CC value is improving drastically because of the modified phase shift waveform. Since RMSE and RMSD are the aggregate error, it is a single- valued index. Still, the real part of the analytic signal is also affected. By comparing the amplitude level for the estimated signal and updated estimated signal (with misfit criteria), the real is modified slightly (< 0.5 mm to 1 mm), which can be allowable for the railway track maintenance.



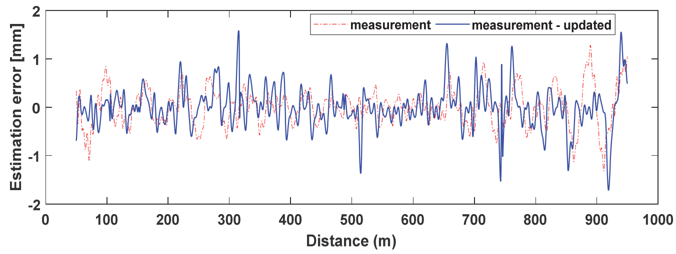
(a) Signal-1 (vertical profile)



(b) Signal-2 (lateral profile)



(c) Estimation error for Signal -1



(d) Estimation error for Signal -2

Figure 1. Phase Correction for quantitative comparison of two waveforms

Table 1. Comparison of statistical metrics for misfit criteria (for track length, 50 – 950 m)

Description	Statistical Metrics	Signal -1 (vertical profile)		Signal -2 (lateral profile)	
		Measurement	Measurement -updated	Measurement	Measurement - updated
Phase correction for quantitative comparison of two waveforms	RMSD (%)	33.38	21.1	16.1	10.4
	CC	0.94	0.99	0.96	0.99
	RMSE (mm)	1.32	0.83	0.47	0.38

4. CONCLUSION:

In order to obtain the quantitative comparison of two waveforms, proposed misfit criteria through HT can be used for phase-shift modified signals. The statistical metrics can be utilized for obtaining single-valued index and its value shows not only the performance of Kalman Filter estimation but also the performance of misfit criteria itself.

REFERENCES

Kristeková, M., Kristek, J., Moczo, P., and Day, S. M. (2006). Misfit criteria for quantitative comparison of seismograms. *Bulletin of the seismological Society of America*, 96 (5), 1836-1850.

Kristeková, M., Kristek, J., and Moczo, P. (2009). Time-frequency misfit and goodness-of-fit criteria for quantitative comparison of time signals. *Geophysical Journal International*, 178 (2), 813-825.

Saravanan, T.J., Boyu, Z., Su, D., and Nagayama, T. (2017). Numerical study for railway track geometry estimation using Augmented State Kalman Filter, *JSCE 19th International Summer Symposium*, ID#CS-059, Fukuoka, Japan.