IDENTIFICATION OF MODE SHAPE SCALING PARAMETERS IN THE SYSTEM IDENTIFICATION OF A SMALL-SCALE TIMBER FRAME

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1. INTRODUCTION

In theoretical dynamic analysis, it is commonly assumed in simple frame structures that masses are discretely distributed at each floor, and that columns and non-structural components (i.e. dampers, partitions) are massless. However, this is unsuitable for experimental analysis considering the high sensitivity of modal properties to mass distribution, as demonstrated by Assi et al. (2016). In order to identify structural stiffness, a reasonable approximation of the mass matrix is required as an input to modal analysis. This study describes a method of approximating the mass matrix in the context of a small-scale bi-level wooden frame. The frame was constructed and subjected to white noise ground motion using a shaking table, and modal parameters were extracted based on the response of each level. These parameters are then used as input for modal equations based on two assumed forms of the mass matrix. The results are then compared using the physical properties of the frame itself.

2. EXPERIMENTAL BACKGROUND

A bi-level timber frame of approximately 600 mm by 300 mm by 600 mm was constructed, as shown in Fig. 1. As the original purpose of this frame is to be used for the testing of cladding systems, it was outfitted with aluminum slotted rails in the direction of motion. An additional mass of approximately 6 kg was added to each level. The total mass of the frame, without the baseplate, is around 17.5 kg.

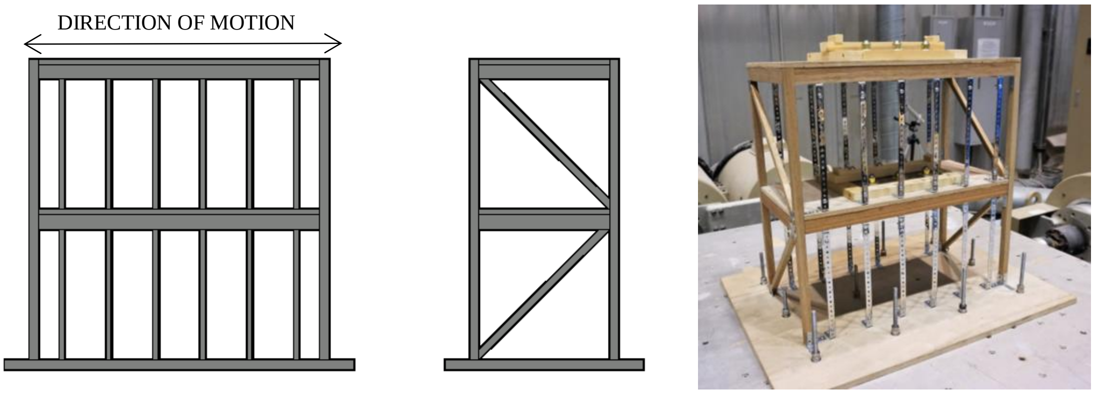

Fig. 1: Drawings of the frame design and a photo of the actual frame.

The above frame was subjected to 5 minutes of white noise ground motion, with a maximum amplitude of 0.05 g. The resulting displacement and acceleration histories were recorded at each level.

3. THEORETICAL ANALYSIS

Using the motion of the shaking table as the input and the resulting motion of the frame as the output, the frequency response function (FRF) was obtained. The natural frequencies of the two dominant modes were determined to be 15.1 rad/s and 42.6 rad/s respectively. The imaginary amplitudes of the corresponding peaks in the FRF at each level were used to determine the mode shapes. However, since it is assumed that there are only two modes in the structure, the mode shapes are not correctly scaled. The unscaled mode shapes were determined to be:

$$\hat{\phi}_1 = \begin{bmatrix} -23.6 \\ -42.1 \end{bmatrix}, \quad \hat{\phi}_2 = \begin{bmatrix} -10.7 \\ 6.44 \end{bmatrix} \tag{1}$$

where $\hat{\phi}_1$ is the unscaled mode shape of the first mode and $\hat{\phi}_2$ is that of the second mode. In order to determine the stiffness of the structure, it is necessary to obtain the properly scaled mode shapes (Yu & Song, 2017).

To ensure the orthogonality of modes, the modal mass matrix should be reduced to the identity matrix, hence producing the mass-normalized mode shapes ϕ_1 and ϕ_2, as shown in Eq. (2). Based on the design of the frame, the form of the stiffness matrix can be approximated as shown in Eq. (3), where k is the total combined lateral stiffness per level.

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$$\phi_i = \frac{\hat{\phi}_i}{\sqrt{\hat{\phi}_i{}^T M \hat{\phi}_i}} \quad for\ i = 1,2 \tag{2}$$

$$\mathbf{K} = \begin{bmatrix} k & -k \\ -k & 2k \end{bmatrix} \tag{3}$$

With the scaled mode shapes, the modal equations can then be combined and simplified as follows:

$$\frac{\phi_{11}^2 - 2\phi_{11}\phi_{21} + 2\phi_{21}^2}{\phi_{12}^2 - 2\phi_{12}\phi_{22} + 2\phi_{22}^2} = \frac{\omega_1^2}{\omega_2^2} \tag{4}$$

However, the scaling factor for the mode shapes is largely determined by the structure of the mass matrix. In this case, the stiffness of the two floor can be assumed to be equal, but the mass distribution evidently cannot be simplified as such. To provide a reasonable 2-by-2 approximate mass matrix, the mass discretization must be carefully considered.

3.1 Diagonal Mass Matrix

In the simplest case used in most theoretical analysis, the mass matrix is assumed to be a diagonal matrix of the form:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{5}$$

where m_1 and m_2 represent the lumped mass at each level. From Fig. 1, it can be assumed that m_1 is greater than m_2, which can be represented by:

$$m_1 = \alpha m_2 \tag{6}$$

where α is some factor greater than 1. Substituting Equations (1), (2), (5), and (6) into Eq. (4), however, resulted in α = -0.06. Not only does this suggest that the mass of the first level is less than that of the second, but it is also illogical in that the mass matrix is no long positive-definite. Hence, the possibility of a non-diagonal mass matrix must be considered.

3.2 Non-Diagonal Mass Matrix

If the mass matrix is assumed to be non-diagonal, the form can be written as:

$$\mathbf{M} = \begin{bmatrix} m_1 & \hat{m} \\ \hat{m} & m_2 \end{bmatrix} \tag{7}$$

where $\hat{m}$ is the off-diagonal mass term. This term does not have to be positive like the diagonal terms, but note that the mass matrix must still be symmetric and positive definite. Similarly, this term can be expressed in terms of m_2:

$$\hat{m} = \beta m_2 \tag{8}$$

where β is some real factor. Substituting Equations (2), (7), and (8) into Eq. (4), the following equality can be obtained:

$$\frac{\hat{\phi}_{12}^2\alpha + 2\hat{\phi}_{12}\hat{\phi}_{22}\beta + \hat{\phi}_{22}^2}{\hat{\phi}_{11}^2\alpha + 2\hat{\phi}_{11}\hat{\phi}_{21}\beta + \hat{\phi}_{21}^2} = \left(\frac{\hat{\phi}_{12}^2 - 2\hat{\phi}_{12}\hat{\phi}_{22} + 2\hat{\phi}_{22}^2}{\hat{\phi}_{11}^2 - 2\hat{\phi}_{11}\hat{\phi}_{21} + 2\hat{\phi}_{21}^2}\right)\frac{\omega_1^2}{\omega_2^2} \tag{9}$$

From the physical properties of the frame, half of the mass between levels is attributed to each level, which gives a value of α = 1.045. Substituting this and the known modal properties into Eq. (9), the value of β can be found as 0.645.

4. DISCUSSION AND CONCLUSION

Based on the method illustrated above, it is possible to avoid illogical results in the scaling of mass-normalized mode shapes by assuming a non-diagonal form of the mass matrix. With enough known physical properties of the experimental specimen, it is possible to determine factors α and β so that the mode shapes can be scaled accordingly. Using these mode shapes, it is possible to determine the stiffness of each level of the frame.

It is important to note that this study only aims to illustrate the method and does not recommend the values of the coefficients as obtained in this example. White noise response from tests with different mass distributions will be assess the effect on these coefficients. The final results will be presented at the JSCE Summer Symposium in August 2018.

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